Summary Quiz 3

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11.1 Subgroups

For a group G with operation $\cdot$, if $H \subseteq G$, and it's a group with the same operation $\cdot$, then H is a subgroup
<u>Notation:</u> If H is a subgroup of G: $H \leq G$, $H < G$ (if $H \neq G$)

11.2 Subgroup Test

Suppose H is a subset of G, then if:

- $H \neq \emptyset$
- $x, y \in H \to x \cdot y \in H$
- $x \in H \to x^{-1} \in H$

Then H is a subgroup

+ Theorems

- If $H \leq G$, then $\epsilon_G \in H$ and $\epsilon_H = \epsilon_G$
- If $H_1 \leq G$ and $H_2 \leq G$, then $H_1 \cap H_2 \leq G$
- If $K \leq H_1$ and $K \leq H_2$, then $K \leq H_1 \cap H_2$
- For $H_1 \leq G$ and $H_2 \leq G$:
  If $H_1 \cup H_2 \leq G$, then $H_1 \leq H_2$ or $H_2 \leq H_1$

11.3 Product Set

If $S \subseteq G$, then $\langle S \rangle$ is the set of all possible products of elements in S and their inverses

+ Theorems

- $S \subseteq G \to \langle S \rangle \leq G$
- If $H_1 \leq K$ and $H_2 \leq K$, then $\langle H_1 \cup H_2 \rangle \leq K$

11.3 Lattices

It's a diagram of subgroups, where each line connecting H and K (with K vertically higher than H in the diagram) means $H \leq K$

Note: If $H \leq K$, and we have some subgroup F such that $H \leq F \leq K$, then $F = H$ or $F = K$

<u>Lattice Example:</u>