Summary Quiz 1

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2.1 Equivalence Relations

Define $R = \{(x, y): x, y \in X, x \sim y\} \subseteq X \times X$
R: set of all pairs that are equivalent

$\sim$ is an equivalence relation if it satisfies:
- Reflexive: $x \sim x \; \forall x \in X$
- Symmetric: $x \sim y \leftrightarrow y \sim x$
- Transitive: If $x \sim y$ and $y \sim z$, then $x \sim z$

2.2 Equivalence Classes

$[x] = \{y \in X: y \sim x\}$

3.1 Well-Defined Operations

An operation $\cdot$ is well defined if:
$\left.\begin{matrix} x \sim y \\ w \sim z \end{matrix}\right\} \rightarrow (x \cdot w) \sim (y \cdot z)$

+ Theorems

- Let X be a set with an equivalence relation,
  then $[x] \cap [y] \neq \emptyset \rightarrow [x] = [y]$

- Equivalence classes are either disjoint or equal

- Let X be a set with an equivalence relation, then the equivalence classes form a partition of X

- Let R_j ($j \in J$, for some index set J) form a partition of X. Say that $x \sim y$ means $x, y \in R_j$ for some j, then $\sim$ is an equivalence relation on X

3.2 Number Theory

For every set $S \subseteq \mathbb{N}$, there's a minimum element $d \in S$ $(\forall x \in S,\; d \leq x)$
Proposition:
Let $a, b \in \mathbb{Z}, b > 0$, then $\exists!\, q, r \in \mathbb{Z}$ such that $a = bq + r,\; 0 \leq r < b$