Summary Mini Test 5

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18.1 Graphs

A graph G is made of a non-empty set V of vertices (nodes) together with a set E of edges

Each edge in S is an unordered pair $\{u, v\} \subseteq V$ with $u \neq v$
We write $G = (V, E)$

Note:
- Loops aren't allowed so $\{u, u\} = \{u\}$ is not a pair
- Parallel edges $\{\{u, v\}, \ \{u, v\}\} = \{\{u, v\}\}$ aren't allowed

Graphs without loops and parallel edges are simple, so we're only going to be working with simple graphs

+ Terminology

- **Adjacent:** u is adjacent to v if $\{u, v\}$ is an edge
- **Incident:** An edge e is incident to u if one of the two endpoints of e is u
- **Degree:** The degree of a vertex $v \in V$ is the number of edges incident to v

+ Theorems

- Handshaking Lemma: $\sum_{v \in V} \deg(v) = 2|E|$
- G has an even number of vertices with an odd degree

18.2 Paths

A path is a sequence of distinct vertices $v_0, \dots, v_l$ such that $\{v_i, v_{i+1}\} \in E$ for $0 \leq i < l$

It can also be described as $l - 1$ edges $\{v_0, v_1\}, \dots, \{v_{l-1}, v_l\}$
The vertices v_0 and v_l are the endpoints of the path and l it its length

If $\exists$ a path with endpoints $v, w \in V$, then v and w are connected
If all vertex-pairs are connected, then the graph is connected

20.1 Cycles

A cycle is a sequence of vertices $v_0, v_1, \dots, v_{l-1}, v_0$ such that:
- $v_0, v_1, \dots, v_{l-1}$ is a path
- $\{v_0, v_1\}, \{v_1, v_2\}, \dots, \{v_{l-1}, v_0\}$ are distinct edges

The length of this cycle is l

Cycles of length 0, 1 or 2 are not allowed by this definition

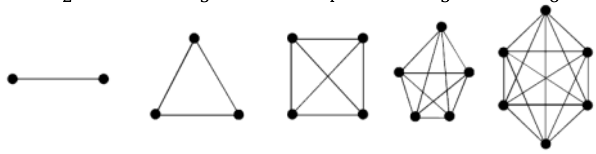
20.2 Walks

- A walk is a path where we allow repeated vertices
- A closed walk is a cycle where we allow repeated vertices

20.2 Families of Graphs

20.2.1 Complete Graphs K_n for $n \geq 1$

K_2 K_3 K_4 K_5 K_6

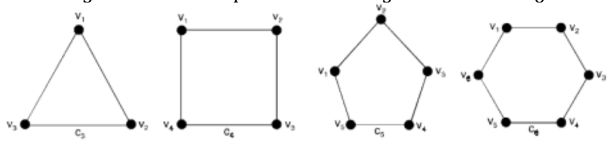


Every pair of vertices is connected by a unique edge
Each vertex is connected to $n - 1$ other vertices

Number of edges: $|E| = \dfrac{n(n-1)}{2} = O(n^2)$

20.2.2 Cycles C_n for $n \geq 3$

C_3 C_4 C_5 C_6



The whole graph is a single cycle with n vertices, the graph makes a closed chain

20.2.3 $(m \times n)$-grids for $n \geq m \geq 1$

$m = 2$, $n = 3$ $m = 1$, $n = 1$ $m = 3$, $n = 4$

20.3 Subgraphs

H is a subgraph of G, denoted $H \subseteq G$, is a graph $H = (V', E')$, where $V' \subseteq V$ and $E' \subseteq E$

20.4 Connected Components

A connected component of G is a subgraph consisting of:
- All vertices that are connected to a given vertex
- Together with all edges incident to them

20.4 Forests, Trees and Leaves

- **Forest:** A forest is a graph that has no cycle
- **Tree:** A tree is a connected forest
- **Leaf:** A leaf in a forest is a vertex of degree 1

+ Theorems

- For $G = (V, E)$ and $n = |V|, \ m = |E|$. If G is a forest, then $n > m$ and G has $n - m$ connected components
- For $G = (V, E)$, a tree, then $n = |V| = |E| + 1 = m + 1$

20.5 Spanning Trees

A spanning tree of a connected graph G is a subgraph of G that includes all vertices of G that is a tree