Lebanese University
Faculty of Information
Section 1 Unesco

الجامعة اللبنانية
كلية الإعلام
الفرع الأول

1ˢᵗ Semester.
Instructor : Dr. Abbas Rammal
Duration : 90 minutes

Final Exam 2021-2022
Course of Mathematics
Linear Algebra

Exercise I :

Let S be a system defined by :

$$(S) : \begin{cases} x + y + z = 1 \\ 4x + 3y - z = 6 \\ 3x + 5y + 3z = 4 \end{cases}$$

1. Find the A and b such that $AX = b$.
2. Calculate the determinant of A. Deduce that A is invertible.
3. Calculate the adjoint matrix of A. Deduce A^{-1}
4. Deduce the solution of the linear system $AX = b$.
5. Find LU decomposition of A.
6. Use L and U to refined the solution of the system $AX = b$.

Exercise II :

Let

$$U = \{(x, y, z) \in \mathbb{R}^3 ; x = 0\}$$
$$V = \{(x, y, z) \in \mathbb{R}^4 ; y = 0\}$$

1. Show that U and V are two subspaces of $\mathbb{R}^4$ over $\mathbb{R}$.
2. Compute a systems of generators of U, V and $U + V$.
3. Show that $\mathbb{R}^4 = U + V$, But $\mathbb{R}^4$ is not the direct sum of U and V.

Exercise III :

Consider the A matrix defined by :

$$A = \begin{pmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{pmatrix}$$

1. Determine the eigenvalues of A.
2. For each eigenvalue, determine a basic for the associated eigenspace.

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3. Find a orthonormal basic for the associated eigenspace.
4. Find an invertible matrix $P \in M_3(\mathbb{R})$ and a diagonal matrix D such that $P^{-1}AP = D$. Deduce the inverse of P.
5. Compute large powers of A that is A^{99}.

Good work...