# One Sample Tests of Hypothesis



Chapter 10

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## Learning Objectives

- LO1 Define a hypothesis.
- LO2 Explain the five-step hypothesis-testing procedure.
- LO3 Describe Type I and Type II errors.
- LO4 Define the term test statistic and explain how it is used.
- LO5 Distinguish between a one-tailed and two-tailed hypothesis
- LO6 Conduct a test of hypothesis about a population mean.
- LO7 Compute and interpret a p-value.
- LO8 Conduct a test of hypothesis about a population proportion.
- LO9 Compute the probability of a Type II error.

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## What is a Hypothesis?

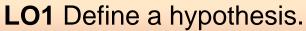
HYPOTHESIS A statement about the value of a population parameter developed for the purpose of testing.

population mean

Example: The mean monthly cell phone bill of this city is  $\mu = \$42$ 

population proportion

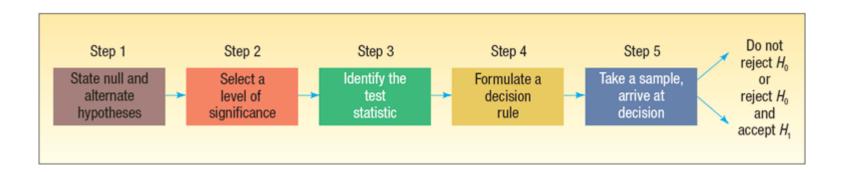
Example: The proportion of adults in this city with cell phones is p = .68



LO2 Explain the five-step hypothesis-testing procedure.

### Hypothesis Testing

HYPOTHESIS TESTING A procedure based on sample evidence and probability theory to determine whether the hypothesis is a reasonable statement.



## Null and Alternate Hypothesis

NULL HYPOTHESIS A statement about the value of a population parameter developed for the purpose of testing numerical evidence.

ALTERNATE HYPOTHESIS A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false.



# The Null Hypothesis, H<sub>0</sub>

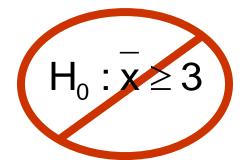
States the assumption (numerical) to be tested

Example: The average number of TV sets in U.S. Homes is at least three (  $H_0: \mu \ge 3$  )

 Is always about a population parameter, not about a sample statistic



$$H_0: \mu \geq 3$$





## The Null Hypothesis, H<sub>0</sub>

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- Refers to the status quo
- Always contains "=", "≤" or "≥" sign
- May or may not be rejected

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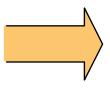
### The Alternate Hypothesis, H<sub>1</sub> or H<sub>A</sub>

- Is the opposite of the null hypothesis
  - e.g.: The average number of TV sets in U.S. homes is less than 3 ( $H_A$ :  $\mu$  < 3)
- Challenges the status quo
- Never contains the "=", "≤" or "≥" sign
- May or may not be accepted
- Is generally the hypothesis that is believed (or needs to be supported) by the researcher

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### **Hypothesis Testing Process**

Claim: the population mean age is 50. (Null Hypothesis:





**Population** 

 $H_0$ :  $\mu = 50$ )



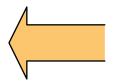
Now select a random sample

Is x=20 likely if  $\mu = 50$ ?

If not likely,

**REJECT** 

**Null Hypothesis** 



Suppose the sample mean age

is 20:  $\bar{x} = 20$ 





### Important Things to Remember about H<sub>0</sub> and H<sub>1</sub>

- H<sub>0</sub>: null hypothesis and H<sub>1</sub>: alternate hypothesis
- H<sub>0</sub> and H<sub>1</sub> are mutually exclusive and collectively exhaustive
- H<sub>0</sub> is always presumed to be true
- H<sub>1</sub> has the burden of proof
- A random sample (n) is used to "reject H<sub>0</sub>"
- If we conclude 'do not reject H<sub>0</sub>', this does not necessarily mean that the null hypothesis is true, it only suggests that there is not sufficient evidence to reject H<sub>0</sub>; rejecting the null hypothesis then, suggests that the alternative hypothesis may be true.
- Equality is always part of  $H_0$  (e.g. "=", "≥", "≤").
- "≠" "<" and ">" always part of H<sub>1</sub>

### Decisions and Errors in Hypothesis Testing

	Researcher			
Null Hypothesis	Does Not Reject  H <sub>0</sub>	Rejects <i>H</i> <sub>0</sub>		
$H_0$ is true	Correct decision	Type I error		
$\mathbf{H}_{0}$ is false	Type II error	Correct decision		



### Type of Errors in Hypothesis Testing

### Type I Error

- Defined as the probability of rejecting the null hypothesis when it is actually true.
- This is denoted by the Greek letter "α"
- Also known as the significance level of a test

### ■ Type II Error

- Defined as the probability of failing to reject the null hypothesis when it is actually false.
- This is denoted by the Greek letter "β"

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## Level of Significance, $\alpha$

- Defines unlikely values of sample statistic if null hypothesis is true
  - Defines rejection region of the sampling distribution
- Is designated by  $\alpha$ , (level of significance)
  - Typical values are .01, .05, or .10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test



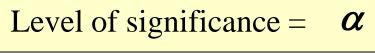
### Test Statistic and Critical Value

TEST STATISTIC A value, determined from sample information, used to determine whether to reject the null hypothesis.

Example: z, t, F,  $\chi^2$ 

CRITICAL VALUE The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.

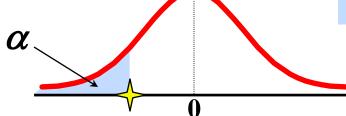
# Level of Significance and the Rejection Region



 $H_0: \mu \ge 3$ 

 $H_A$ :  $\mu < 3$ 

Lower tail test



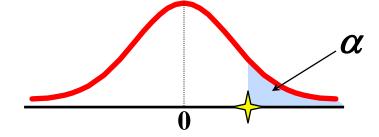
**♦** Represents critical value

Rejection region is shaded

$$H_0: \mu \le 3$$

$$H_{\Delta}$$
:  $\mu > 3$ 

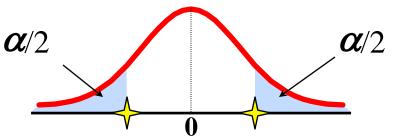
Upper tail test



$$H_0$$
:  $\mu = 3$ 

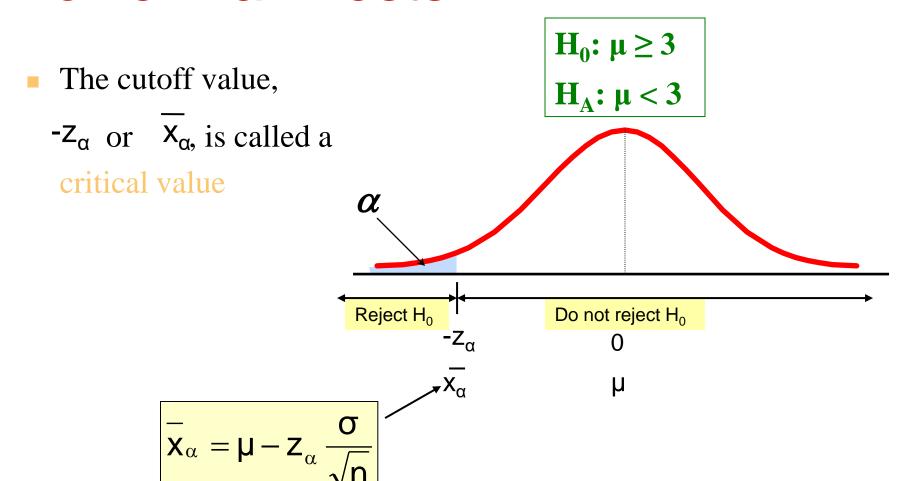
$$H_A$$
:  $\mu \neq 3$ 

Two tailed test



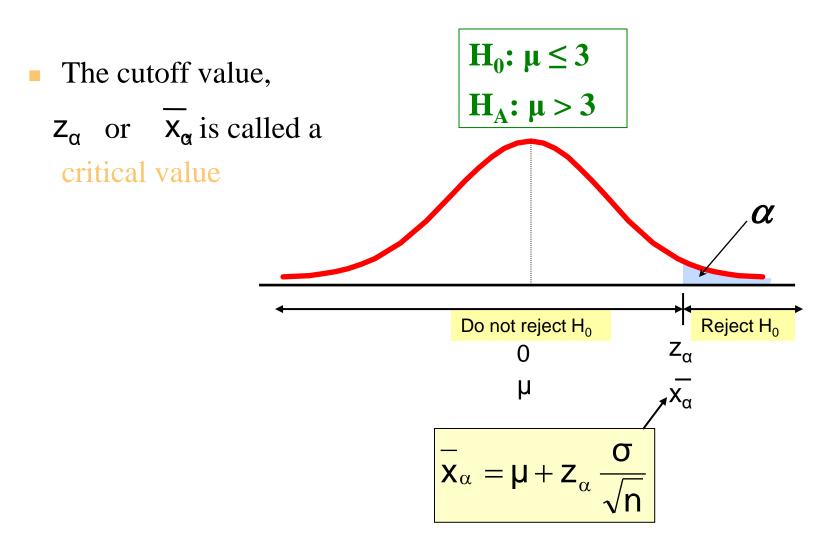
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### **Lower Tail Tests**



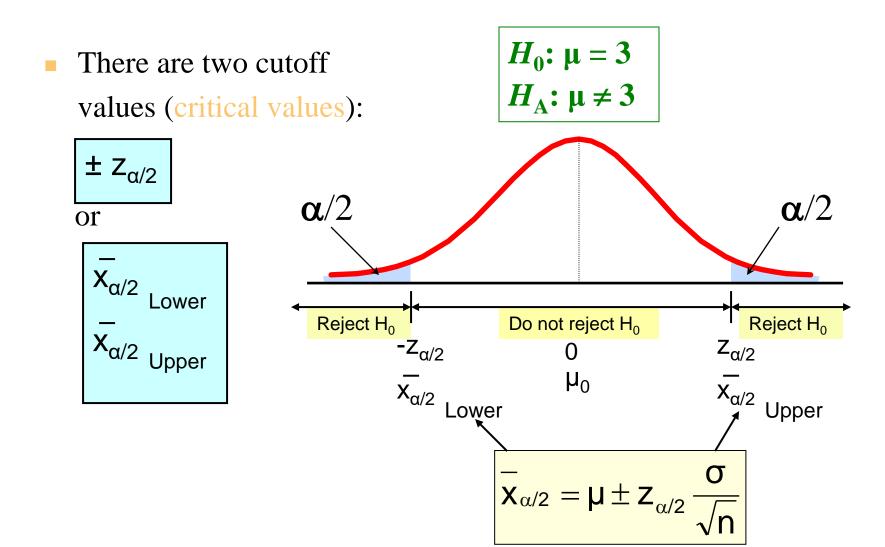
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## **Upper Tail Tests**



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### **Two Tailed Tests**



# Hypothesis Setups for Testing a Mean (μ)

H<sub>0</sub>: 
$$\mu$$
 = value  
H<sub>1</sub>:  $\mu \neq$  value  
Reject H<sub>0</sub> if:  
 $|Z| > Z_{\alpha/2}$   
 $|t| > t_{\alpha/2, n-1}$ 

H<sub>0</sub>: 
$$\mu \ge$$
 value  
H<sub>1</sub>:  $\mu <$  value  
Reject H<sub>0</sub> if:  
 $Z < -Z_{\alpha}$   
 $t < -t_{\alpha, n-1}$ 

H<sub>0</sub>: 
$$\mu \le \text{value}$$
  
H<sub>1</sub>:  $\mu > \text{value}$   
Reject H<sub>0</sub> if:  
 $Z > Z_{\alpha}$   
 $t > t_{\alpha, n-1}$ 

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$



### How to Set Up a Claim as Hypothesis

- In actual practice, the status quo is set up as H<sub>0</sub>
- If the claim is "boastful" the claim is set up as H<sub>1</sub>. Remember, H<sub>1</sub> has the burden of proof
- In problem solving, look for key words and convert them into symbols. Some key words include: "improved, better than, as effective as, different from, has changed, etc."

Keywords	Inequal ity Symbol	Part of:
Larger (or more) than	>	H <sub>1</sub>
Smaller (or less)	<	H <sub>1</sub>
No more than	<b>\leq</b>	H <sub>o</sub>
At least	2	H <sub>o</sub>
Has increased	>	H <sub>1</sub>
Is there difference?	<b>≠</b>	H <sub>1</sub>
Has not changed	=	H <sub>o</sub>
Has "improved", "is better than". "is more effective"	See left text	H <sub>1</sub>

Jamestown Steel Company manufactures and assembles desks and other office equipment. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of 200 and a standard deviation of 16. Recently, new production methods have been introduced and new employees hired. The VP of manufacturing would like to investigate whether there has been a *change* in the weekly production of the Model A325 desk. Is the mean number of desks produced different from 200 at the 0.01 significance level?





Step 1: State the null hypothesis and the alternate hypothesis.

 $H_0$ :  $\mu = 200$ 

 $H_1$ :  $\mu \neq 200$ 

(note: keyword in the problem "has changed")

Step 2: Select the level of significance.

 $\alpha$  = 0.01 as stated in the problem

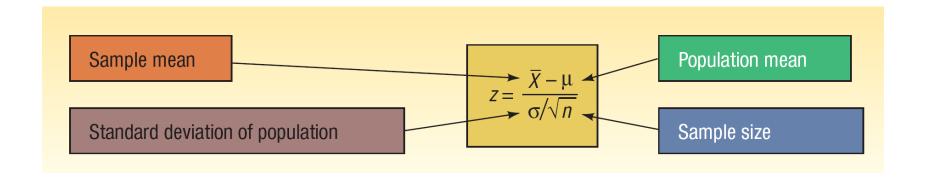
Step 3: Select the test statistic.

Use Z-distribution since σ is known



### **Step 3: Select the test statistic.**

Use Z-distribution since  $\sigma$  is known





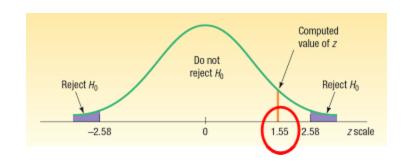
Step 4: Formulate the decision rule. Reject  $H_0$  if  $|Z| > Z_{\alpha/2}$  Sample: The mean number of produced desks last year (50 weeks because they took 2 weeks off) was 203.5.



### Step 4: Formulate the decision rule. Reject $H_0$ if $|Z| > Z_{\alpha/2}$

$$\begin{aligned} |Z| > Z_{\alpha/2} \\ \left| \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \right| > Z_{\alpha/2} \\ \left| \frac{203.5 - 200}{16 / \sqrt{50}} \right| > Z_{.01/2} \\ 1.55 \text{ is not} > 2.58 \end{aligned}$$

Sample: The mean number of produced desks last year (50 weeks because they took 2 weeks off) was 203.5.



#### Step 5: Make a decision and interpret the result.

Because 1.55 does not fall in the rejection region,  $H_0$  is not rejected. We conclude that the population mean is not different from 200. So we would report to the vice president of manufacturing that the sample evidence does not show that the production rate at the plant has changed from 200 per week.



Heinz, a manufacturer of ketchup, uses a particular machine to dispense 16 ounces of its ketchup into containers. From many years of experience with the particular dispensing machine, Heinz knows the amount of product in each container follows a normal distribution with a mean of 16 ounces and a standard deviation of 0.15 ounce. A sample of 50 containers filled last hour revealed the mean amount per container was 16.017 ounces. Does this evidence suggest that the mean amount dispensed is different

from 16 ounces? Use the .05 significance level.

- (a) State the null hypothesis and the alternate hypothesis.
- (b) What is the probability of a Type I error?
- (c) Give the formula for the test statistic.
- (d) State the decision rule.
- (e) Determine the value of the test statistic.
- (f) What is your decision regarding the null hypothesis?
- (g) Interpret, in a single sentence, the result of the statistical test.



Suppose in the previous problem the vice president wants to know whether there has been an increase in the number of units assembled. To put it another way, can we conclude, because of the improved production methods, that the mean number of desks assembled in the last 50 weeks was more than 200?

Recall:  $\sigma=16$ , mean=200,  $\alpha=.01$ 



Suppose in the previous problem the vice president wants to know whether there has been an *increase* in the number of units assembled. To put it another way, can we conclude, because of the improved production methods, that the mean number of desks assembled in the last 50 weeks was *more than 200*?

Recall:  $\sigma$ =16, mean=200,  $\alpha$ =.01



## Step 1: State the null hypothesis and the alternate hypothesis.

 $H_0: \mu \le 200$ 

 $H_1$ :  $\mu > 200$ 

(note: keyword in the problem "an increase")

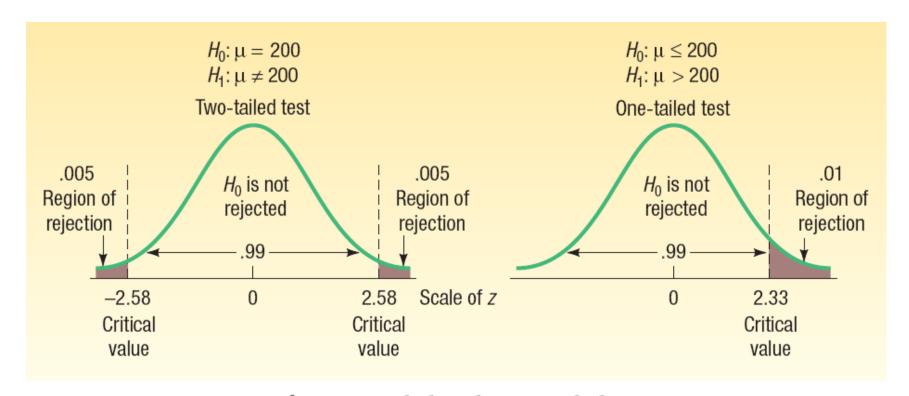
### Step 2: Select the level of significance.

 $\alpha = 0.01$  as stated in the problem

#### Step 3: Select the test statistic.

Use Z-distribution since σ is known

### One-Tailed Test versus Two-Tailed Test

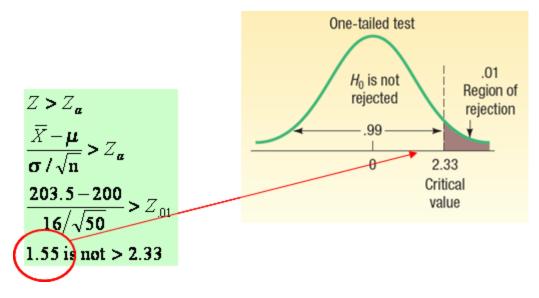


Rejection Regions for Two-Tailed and One-Tailed Tests,  $\alpha = .01$ 



#### **Step 4: Formulate the decision rule.**

Reject  $H_0$  if  $Z > Z_\alpha$ 



#### Step 5: Make a decision and interpret the result.

Because 1.55 does not fall in the rejection region, H<sub>0</sub> is not rejected. We conclude that the average number of desks assembled in the last 50 weeks is not more than 200



# Testing for the Population Mean: Population Standard Deviation Unknown

- When the population standard deviation (o) is unknown, the sample standard deviation (s) is used in its place
- The *t*-distribution is used as test statistic, which is computed using the formula:

**TESTING A MEAN, σ UNKNOWN** 

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

[10-2]

with n-1 degrees of freedom, where:

 $\overline{X}$  is the sample mean.

 $\mu$  is the hypothesized population mean.

s is the sample standard deviation.

*n* is the number of observations in the sample.

# Testing for the Population Mean: Population Standard Deviation Unknown - Example



The McFarland Insurance Company Claims Department reports the mean cost to process a claim is \$60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of 26 claims processed last month. The sample information is reported below.

At the .01 significance level is it reasonable a claim is now less than \$60?

¢45	\$40	\$60	\$40	¢42	¢61
\$45	\$49	\$62	\$40	\$43	\$61
48	53	67	63	78	64
48	54	51	56	63	69
58	51	58	59	56	57
38	76				
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# Testing for the Population Mean: Population Standard Deviation Unknown - Example



The McFarland Insurance Company Claims Department reports the **mean** cost to process a claim is \$60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of 26 claims processed last month. The sample information is reported below.

At the .01 significance level is it reasonable a claim is now less than \$60?

\$45	\$49	\$62	\$40	\$43	\$61
48	53	67	63	78	64
48	54	51	56	63	69
58	51	58	59	56	57
38	76				



# Step 1: State the null hypothesis and the alternate hypothesis.

 $H_0$ :  $\mu$  ≥ \$60

 $H_1$ :  $\mu$  < \$60

(note: keyword in the problem "now less than")

#### Step 2: Select the level of significance.

 $\alpha = 0.01$  as stated in the problem

### **Step 3: Select the test statistic.**

Use *t*-distribution since σ is unknown

# t-Distribution Table (portion)

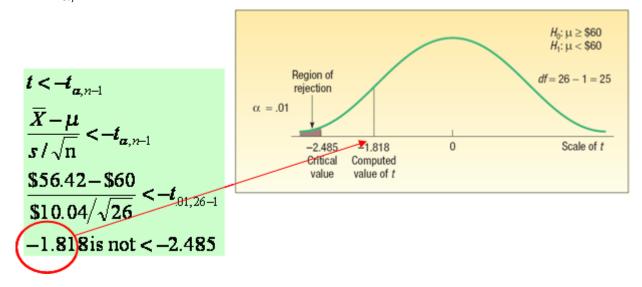
**TABLE 10–1** A Portion of the t Distribution Table

		0	onfidence Int	ervals		
	80%	90%	95%	98%	99%	99.9%
		Level o	f Significance	for One-Taile	d Test, α	
df	0.100	0.050	0.025	0.010	0.005	0.0005
	Level of Significance for Two-Tailed Test, $lpha$					
	0.20	0.10	0.05	0.02	0.01	0.001
:	:	:	:	:	:	:
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646

## Testing for a Population Mean with a Known Population Standard Deviation- Example

#### **Step 4: Formulate the decision rule.**

Reject 
$$H_0$$
 if  $t < -t_{\alpha,n-1}$ 



#### Step 5: Make a decision and interpret the result.

Because -1.818 does not fall in the rejection region,  $H_0$  is not rejected at the .01 significance level. We have not demonstrated that the cost-cutting measures reduced the mean cost per claim to less than \$60. The difference of \$3.58 (\$56.42 - \$60) between the sample mean and the population mean could be due to sampling error.

## Testing for a Population Mean with an Unknown Population Standard Deviation- Example

The current rate for producing 5 amp fuses at Neary Electric Co. is 250 per hour. A new machine has been purchased and installed that, according to the supplier, will increase the production rate. A sample of 10 randomly selected hours from last month revealed the mean hourly production on the new machine was 256 units, with a sample standard deviation of 6 per hour.

At the .05 significance level can Neary conclude that the new machine is faster?

## Testing for a Population Mean with an Unknown Population Standard Deviation- Example

The current rate for producing 5 amp fuses at Neary Electric Co. is 250 per hour. A new machine has been purchased and installed that, according to the supplier, will increase the production rate. A sample of 10 randomly selected hours from last month revealed the mean hourly production on the new machine was 256 units, with a sample standard deviation of 6 per hour.

At the .05 significance level can Neary conclude that the new machine is faster?



#### Testing for a Population Mean with an Unknown Population Standard Deviation- Example

Step 1: State the null and the alternate hypothesis.

 $H_0$ :  $\mu \le 250$  $H_1$ :  $\mu > 250$ 

Step 2: Select the level of significance. It is .05.

Step 3: Find a test statistic.

Use the *t* distribution because the population standard deviation is not known and the sample size is less than 30.



## Testing for a Population Mean with an Unknown Population Standard Deviation- Example

Step 4: State the decision rule. There are 10 - 1 = 9 degrees of freedom. The null hypothesis is rejected if t > 1.833.

$$t = \frac{\overline{X} - \mu}{s / \sqrt{n}} = \frac{256 - 250}{6 / \sqrt{10}} = 3.162$$

Step 5: Make a decision and interpret the results.

The null hypothesis is rejected. The mean number produced is more than 250 per hour.

## p-Value in Hypothesis Testing

- p-VALUE is the probability of observing a sample value as extreme as, or more extreme than, the value observed, given that the null hypothesis is true.
- In testing a hypothesis, we can also compare the p-value to the significance level ( $\alpha$ ).
- Decision rule using the p-value:

Reject  $H_0$  if p-value < significance level



## p-Value in Hypothesis Testing - Example

Recall the previous problem where the hypothesis and decision rules were set up as:

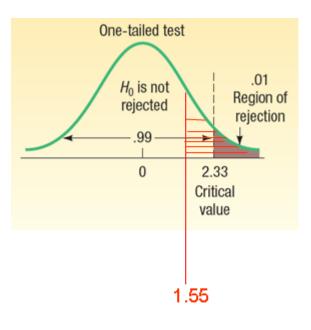
 $H_0$ :  $\mu \le 200$ 

 $H_1$ :  $\mu > 200$ 

Reject  $H_0$  if  $Z > Z_{\alpha}$ where Z = 1.55 and  $Z_{\alpha} = 2.33$ 

Reject  $H_0$  if p-value  $< \alpha$ 0.0606 is not < 0.01

Conclude: Fail to reject H<sub>0</sub>



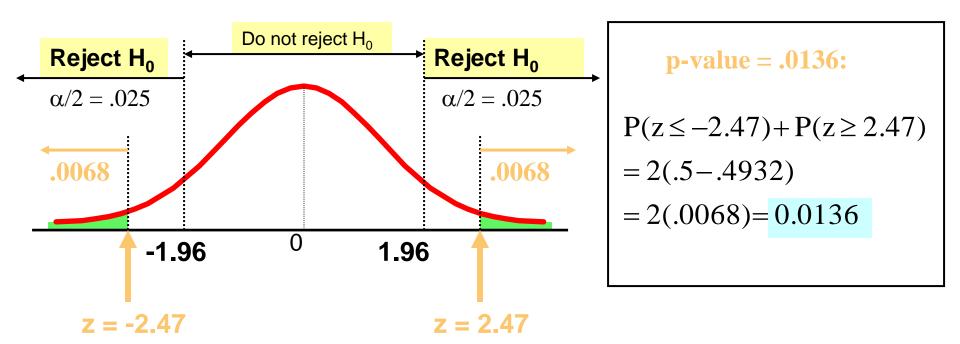
P(Z>1.55)=.5000 -.4394 P-value = .0606

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## p - Value Example 2

#### Calculate the p-value and compare to $\alpha$

(For a two sided test the p-value is always two sided)



Reject  $H_0$  since p-value = .0136 <  $\alpha$  = .05



## What does it mean when p-value is less than

- (a) .10, we have some evidence that  $H_0$  is not true.
- (b) .05, we have strong evidence that  $H_0$  is not true.
- (c) .01, we have very strong evidence that  $H_0$  is not true.
- (d) .001, we have extremely strong evidence that  $H_0$  is not true.

LO8 Conduct a test of hypothesis about a population proportion.

## **Tests Concerning Proportion**

- A Proportion is the fraction or percentage that indicates the part of the population or sample having a particular trait of interest.
- The sample proportion is denoted by p and is found by x/n
- The test statistic is computed as follows:

**TEST OF HYPOTHESIS, ONE PROPORTION** 

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}}$$
 [10–3]

#### where:

 $\pi$  is the population proportion.

p is the sample proportion.

n is the sample size.



# Assumptions in Testing a Population Proportion using the z-Distribution

- A random sample is chosen from the population.
- It is assumed that the binomial assumptions discussed in Chapter 6 are met:
  - (1) the sample data collected are the result of counts;
  - (2) the outcome of an experiment is classified into one of two mutually exclusive categories—a "success" or a "failure";
  - (3) the probability of a success is the same for each trial; and
  - (4) the trials are independent
- The test is appropriate when both  $n\pi$  and  $n(1-\pi)$  are at least 5.
- When the above conditions are met, the normal distribution can be used as an approximation to the binomial distribution

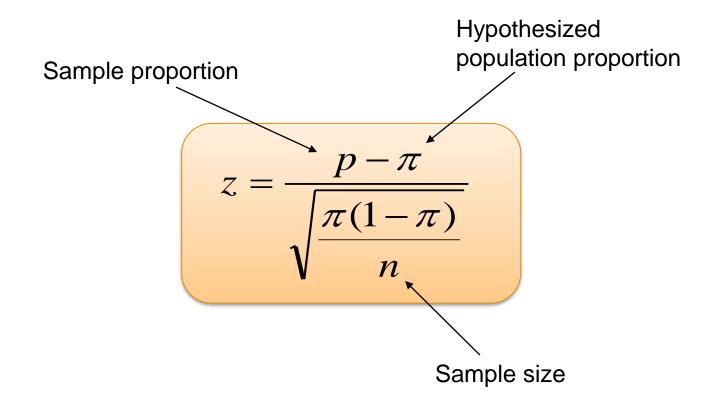
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# Hypothesis Setups for Testing a Proportion $(\pi)$

H<sub>0</sub>:  $\pi$  = value H<sub>1</sub>:  $\pi \neq$  value Reject H<sub>0</sub> if:  $|Z| > Z_{\alpha J2}$  H<sub>0</sub>:  $\pi \ge$  value H<sub>1</sub>:  $\pi$  < value Reject H<sub>0</sub> if:  $Z < -Z_{\alpha}$  H<sub>0</sub>:  $\pi \le \text{value}$ H<sub>1</sub>:  $\pi > \text{value}$ Reject H<sub>0</sub> if:  $Z > Z_{\alpha}$ 

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}}$$

# Test Statistic for Testing a Single Population Proportion





Suppose prior elections in a certain 3 state indicated it is necessary for a candidate for governor to receive at least 80 percent of the vote in the northern section of the state to be elected. The incumbent governor is interested in assessing his chances of returning to office and plans to conduct a survey of 2,000 registered voters in the northern section of the state. Using the hypothesis-testing procedure, assess the governor's chances of reelection.





## Test Statistic for Testing a Single Population Proportion - Example

Step 1: State the null hypothesis and the alternate hypothesis.

 $H_0$ : π ≥ .80

 $H_1$ :  $\pi$  < .80

(note: keyword in the problem "at least")

Step 2: Select the level of significance.

 $\alpha = 0.01$  as stated in the problem

**Step 3: Select the test statistic.** 

Use Z-distribution since the assumptions are met and  $n\pi$  and  $n(1-\pi) \ge 5$ 



# Testing for a Population Proportion - Example

**Step 4: Formulate the decision rule.** 

Reject  $H_0$  if  $Z < -Z_{\alpha}$ 

The sample revealed that 1550 are planning to vote for.

P = 0.775



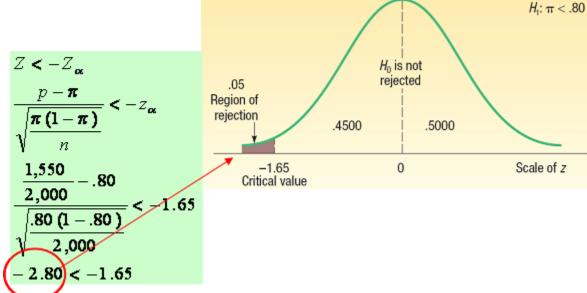
 $H_0: \pi \ge .80$ 

# Testing for a Population Proportion - Example

**Step 4: Formulate the decision rule.** 

Reject  $H_0$  if  $Z < -Z_\alpha$ 

The sample revealed that 1550 are planning to vote for. P = 0.775



#### Step 5: Make a decision and interpret the result.

The computed value of z (-2.80) is in the rejection region, so the null hypothesis is rejected at the .05 level. The difference of 2.5 percentage points between the sample percent (77.5 percent) and the hypothesized population percent (80) is statistically significant. The evidence at this point does not support the claim that the incumbent governor will return to the governor's mansion for another four years.

## Type II Error

Recall Type I Error, the level of significance, denoted by the Greek letter "α", is defined as the probability of rejecting the null hypothesis when it is actually true.

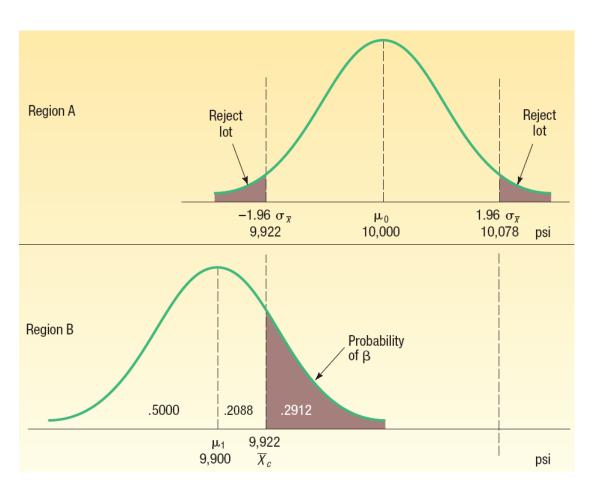
■ Type II Error, denoted by the Greek letter "β",is defined as the probability of "accepting" the null hypothesis when it is actually false.

## Type II Error - Example

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is 10,000 psi and that the standard deviation,  $\sigma$ , is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: "Take a sample of 100 steel bars. At the .05 significance level if the sample mean strength falls between 9,922 psi and 10,078 psi, accept the lot. Otherwise the lot is to be rejected."

### Type I and Type II Errors Illustrated

Suppose the unknown population mean of an incoming lot is 9900psi. What is the probability that the quality control inspector will fail to reject the shipment (Type II error)



$$z = \frac{\overline{X}_c - \mu_1}{\sigma/\sqrt{n}} = \frac{9,922 - 9,900}{400/\sqrt{100}} = \frac{22}{40} = 0.55$$

 $\rightarrow$  area = 0.2088

Probability of type II error = 0.5-0.2088