

1st Semester

Instructor : Dr. Abbas Rammal

Duration : 90 minutes

Final Exam 2022-2023

Course of Mathematics

Linear Algebra

Exercise I :

Let S be a system defined by :

$$(S) : \begin{cases} x + y + z = 6 \\ 2y + 5z = -4 \\ 2x + 5y - z = 27 \end{cases}$$

1. Find the A and b such that $AX = b$.
2. Calculate the determinant of A . Deduce that A is invertible.
3. Calculate the adjoint matrix of A . Deduce A^{-1} .
4. Deduce the solution of the linear system $AX = b$.
5. Find LU decomposition of A .
6. Use L and U to refined the solution of the system $AX = b$.

Exercise II :

Let

$$U = \{(a, b, c, d) \in \mathbb{R}^4; a = c \text{ and } b = d\}$$

$$V = \{(a, b, c, d) \in \mathbb{R}^4; a - d = 0\}$$

1. Show that U and V are two subspaces of \mathbb{R}^4 over \mathbb{R} .
2. Compute a systems of generators of U , V and $U + V$.
3. Show that the system of generators of $U + V$ is a system of generators of \mathbb{R}^4 .
4. Deduce that $\mathbb{R}^4 = U \oplus V$

Exercise III :

Consider the A matrix defined by :

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$$

1. Determine the eigenvalues of A .
2. For each eigenvalue, determine a basic for the associated eigenspace.
- ~~3. Find a orthonormal basic for the associated eigenspace.~~
4. Find an invertible matrix $P \in M_3(\mathbb{R})$ and a diagonal matrix D such that $P^{-1}AP = D$. Deduce the inverse of P .
5. Compute large powers of A that is A^{99} .