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Mathematical logic.....Mathematical logic..? , It's not a popular definition , So what is the combine between Mathematics and Logic ?

Mathematical logic is a sub-field of mathematics exploring the applications of formal logic to mathematics. It has close connections to metamathematics, the foundations of mathematics, and theoretical computer science. The unifying themes of mathematical logic include the study of the expressive power of formal systems and the deductive power of formal proof systems and the main major of it is proof system , That shows us the **objective** of this topic.

Maybe you were wondering why we were doing this? So, let's get **introduction** about it.

Why is Mathematical Logic? It's a key issue since it teaches relationship. Logical thinking is the heart of math, according to set theory, it is a very basic theory used by many aspects of math and other sciences, especially physics and computer science. , Any conceptual framework that separates the form of statements from its content in order to establish abstract principles of logical thought for consistency and validity includes proofs of various methods, and this is so relevant not just in math but also in your real life, it's all about "proving your thesis" by different means. , For eg, while it might be true that all wealthy people have money, it is not true that all those that have money are wealthy.

In this research we will show you some subjects of mathematical logic like :

1 – Propositional Logic (Page 3,4,5) show us statements that only be true or false.

.2 – Logic Operators connectives (Page 6,7,8) understand us the logic symbols.

3 – Logical Equivalence (Page 8,9) shows us how to equality of two statements.

And we will Show You Some Solved Problems on this topics (Page 10)

Propositional Logic

A proposal is simply a statement. Propositional logic studies how declarations can interact with one another. It is important to remember that the content of the .statements does not really concern propositional logic

We agreed that we will use only proof that is true or false so that we will use two very important letters at this point .**T** that indicate true and **F** that indicate .false

For Example :

Cairo is the capital of Egypt" thats True

"10=2+8" thats True

My Uncle ate a red bannana" thats False"

Asia is the smallest Continent" thats False"

" What is The Time ?"

let's stop here because this statement Not expected the answer to be true or false but it will be another one , So that this kind of statement we can not say that it's Propositional Logic.

So

What is The Time?" Not propositions"

X+1=5" Not proposition"

Propositional Logic and math logic depend on variables and logic operations that get the validation by truth table.

Variables is like P , Q , T , Y and some alphabites like that and logic operations we will talk about it next slids, So now we will talk about Variables and truth table.

So…how to create this truth table to prove any problem?

The shape of truth table like that ⟶

Truth table contains rows and column , Number of Variables so if we had 2 variables we will draw 2 columns , if we had 3 variables we will draw 3 columns and so on.

Example : P ∧ Q * Do not give a mind to ∧ we will talk about it later*

P	Q

So . two variables then our truth table will be like that Regardless of columns

About columns either depends on Variables too but we must use this low 2^v

(v : number of variables)

From previous Example : P ∧ Q * row of variables is constant and does not count *

P	Q

So . we have now two variables , for columns we will draw two according to number of variables , for rows we will draw four according to the low 2^v so $2^2 = 4$.

Rest of the fields are blank?

Write content in the rest of the cells is filled with (T or F) but not filled randomly, but it fills the tiredness of the probability theory, because it cannot be believed that all probabilities are true or all wrong. , Therefore, the possibility could be right and the other wrong.

If we got one variable P , it truth table will be like that

P
T
F

One true and the other is false (probability theory)

If we got two variable P and Q , According to probability theory when P is true might Q is true too or False (two probability) and when P is False might Q is False too or True and so on in Q.

So It truth table will be like that

P	Q
T	T
T	F
F	F
F	T

And if you got three variables the same thing (probability theory) what is true in one maybe true in all of them or two is true and one is false or only one true and the other is false and so on

For Example : if we had P , Q , R

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

As we all see here

Using probability theory is very important in distributed the values in true or false in a different ways.

So that there's no one can prove or solve any problem without use probability theory.

Logic Operators connectives

Maybe you ask yourself what next ? How does the question come to prove it ?

But before talking about Logic Operators we must talk about Logic Symbols that create the operators to prove the statement

The symbols (Signs operators) :

Symbols	Read	Mean	Example
~	Not	opposite of the sign T will be F and So on	~ P
∧	And	If both true it will be true else = F	P ∧ Q
∨	Or	If both true it will be true and also If one of them are T and other F But it will be false if both are F	P ∨ Q
⟶	Implies	When the first variable is T And the other one is F in the Same order and the other blanks Will be T	P ⟶ Q
↔	If and only if	When both are T it will be T Also when both are F it will be T Other blanks will be F	P ↔ Q

Let's get some Example on every operation and know it mechanism

Example 1 : ~P

It's something like

P	$\sim P$
T	F
F	T

P : I will go to school

~ P : I will not go to school

Example 2 : P ∧ Q

It's something like

P : I will eat pizza , Q : I will drink Soda

P ∧ Q : I will eat pizza and drink Soda

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

Example 3 : P ∨ Q

It's something like

P : I Study Math , Q : I will watch TV

P ∨Q : I will Study Math or watch TV

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Example 4 : $P \longrightarrow Q$

It's something like :

If today is Friday, then 2+3=5" (always true)"

If today is Friday, then "2+3=6" (False only when today is Friday!!)

P	Q	$P \longrightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

OR

P : I Study Math , Q : I will watch TV

P ∨Q : I will Study Math then I will watch TV

Example 5 : P ↔ Q

P	Q	P ↔ Q
T	T	T
T	F	F
F	F	T
F	T	F

It's something like

Let p and q be propositions. the biconditional statement p ↔ q is the

propositions "p if and only if ". It is true when p and q have the same truth value and is false otherwise.

OR

P : it's stop raining , Q : I'll buy a new car

P ↔ Q :

only if it's stop raining I'll buy a new car (OR) only if it's not stop raining I will not buy a new car

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Logical Equivalence

After you got to know Logic Operations and some Examples , we need to take another idea in this topic it's called Logic Equivalence , it's a way to prove the equality between two statement after prove it validation.

(P ∨ Q) Equal $\sim (P) \wedge \sim (Q)$

This is mean that we need to prove that the output from (P ∨ Q) is equal the output from $\sim (P) \wedge \sim (Q)$

The equal sign is (⇔) and also (≡) but in popular we use this (⇔)

So that if we got A ⇔ B

A : Ahmed had a online work ⇔ B : Ahmed had a laptop

Example 1 : $\sim(P \vee Q) \Leftrightarrow \sim(P) \wedge \sim(Q)$

P : I will eat pizza

Q : I will drink soda

P ∨ Q : I will eat pizza or I will drink soda.

$\sim(P \vee Q)$: it's not the case that i will eat pizza or i will drink soda

$\sim(P) \wedge \sim(Q)$: it's not the case that i will eat pizza and it is not the case that i will drink soda

P	Q	~P	$\sim Q$	P ∨ Q	$\sim(P \vee Q)$	$\sim(P) \wedge \sim(Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

That's mean that the right side equal the left one

So : $\sim(P \vee Q) \Leftrightarrow \sim(P) \wedge \sim(Q)$

Example 2 : (P ∨ T) ⇔ T

*T here mean true , So it'll be true all time *

So that (P ∨ T) Equal T

(P ∨ T) ⇔ T is right

P	T	(P ∨ T)
T	T	T
T	T	T
F	T	T
F	T	T

Solved Problems :

S.P. 1 : $P \rightarrow (Q \wedge R) \Leftrightarrow (P \rightarrow Q) \wedge (P \rightarrow R)$

P	Q	R	$Q \wedge R$	$P \rightarrow (Q \wedge R)$	$P \rightarrow Q$	$P \rightarrow R$	$(P \rightarrow Q) \wedge (P \rightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

So that $P \rightarrow (Q \wedge R)$ Equal $(P \rightarrow Q) \wedge (P \rightarrow R)$

S.P. 2 : $\sim((P \wedge Q) \rightarrow (Q \vee P))$

P	Q	$(P \wedge Q)$	$(Q \vee P)$	$\sim((P \wedge Q) \rightarrow (Q \vee P))$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	F
F	F	F	F	F

Conclusion :

You may not understand the importance of this department in the real life, only numbers and symbols written on a paper, and you are not sure if this will help you in your career But, if you go deeper, You will find it out, You will find that a very powerful field like computer programming is heavily depending on it; Because the computer doesn't understand from us except the language of true or false (0 , 1) , and not only the computer, but also any system that is connected to digital field , It also very important in teaching and mathematical proofs and educational system that's used in most of the colleges, its very important for lots of sciences such as Philosophy and psychology, Another field that mathematical logic is important for is the digital system as it used for developing SAT and SMT(satisfiability modulo theories) and used for formal verification, and automated reasoning for various problems.

It also important in making digital instruments such as watches, calculators, and used in modern cars digital systems

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Reference :

1 - Discrete Mathematics and Structures By Prof / Eman Fathy and Prof / Omar Saad

2 - A friendly introduction to mathematical logic Book By Prof / Christopher C.Leary and Prof / Lars Kristianceu

3 - Discrete Mathematics By Prof / Oscar Levin

4 - Discrete Mathematics and Applications By Prof / Kenneth H.Rosen