Fundamentals of Electromagnetic Fields_ EPM 112

CHAPTER (7)

Steady State Magnetic Fields

Lecture 2

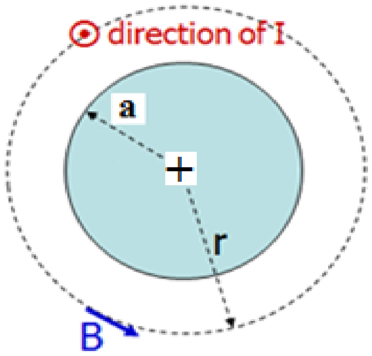
Biot-Savart law
Ampere's Circuital Law
Magnetic Field Density
Magnetic Flux

Part 2: Ampere's circuital law

Ampere's circuital law:

It states that the line integral of the tangential component of magnetic field intensity around a closed path is equal to the current enclosed by the path

$$\oint \overline{H}.d\overline{l} = I_{en}$$



Or the line integral of **H** about any closed path is exactly to the dc current enclosed by the path.

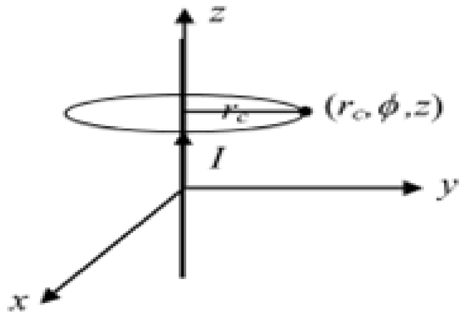
Conditions for application of Ampere's law:

$\overline{H}$ **must be constant on the loop.**

Example:

Find the magnetic field intensity at a point (r_c, ϕ, z) due to infinite wire of current *I*.

Solution:



1-Ampere's circuital law

$$\oint \overline{H}.d\overline{l} = I_{en}$$

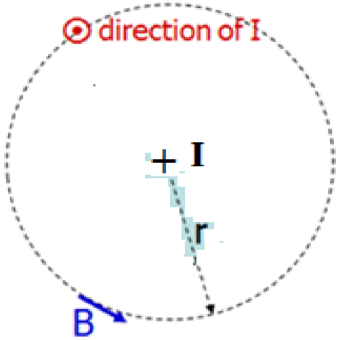
2-Choice of Amperian loop

3- $I_{en} = \mathrm{I}$

4- $\oint \overline{H}.d\overline{l} = H.2\pi r_c$

5- $H.2\pi r_c = I$

6- $\therefore \overline{H} = \dfrac{I}{2\pi r_c}\hat{\phi}$ **A/m**



Example:

Find $\overline{H}$ inside and outside a conductor (magnetic material) of infinite length carrying current I and of radius a.

Solution:

Region (I) $r_c < a$

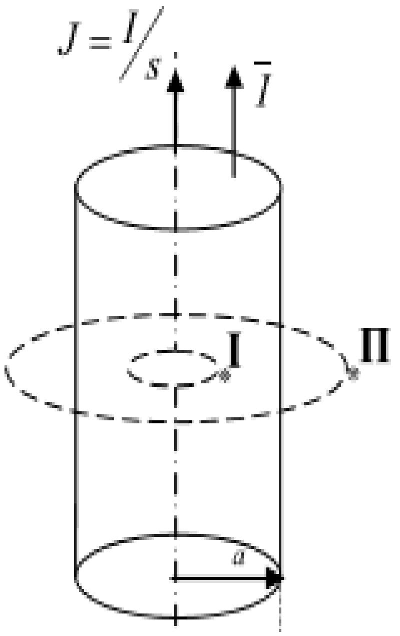
1- Ampere's circuital law

$$\oint \overline{H}.d\bar{l} = I_{en}$$

2- Choice of Amperian loop

3- $I_{en} = J.S = \frac{I}{\pi a^2}.\pi r_c^2$

4- $\oint \overline{H}.d\bar{l} = H.2\pi r_c$



5- $H.2\pi r_c = \frac{I}{\pi a^2}.\pi r_c^2$

6- $\overline{H} = \frac{I r_c}{2\pi a^2}\hat{\phi} \rightarrow (1)$

Region (II) $r_c > a$

1-Ampere's circuital law

$$\oint \overline{H}.d\overline{l} = I_{en}$$

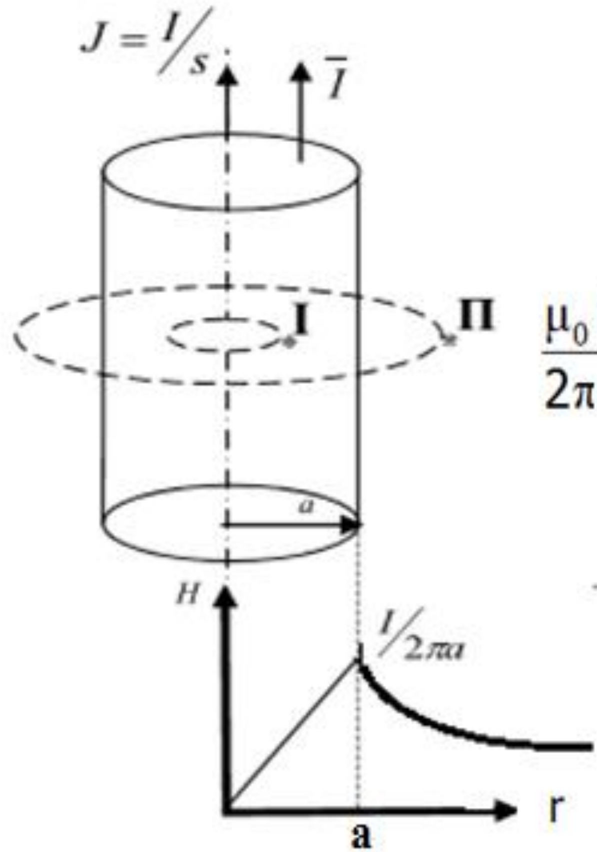
2-Choice of Amperian loop

3- $I_{en} = J.S = \mathrm{I}$

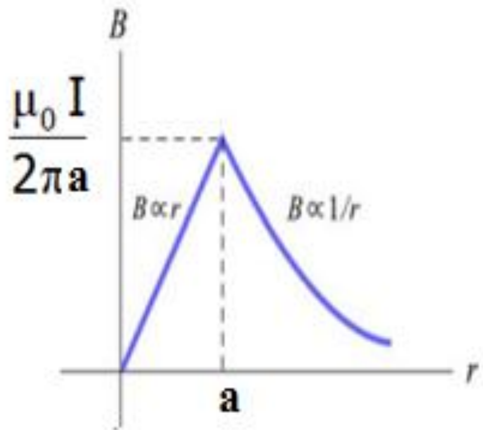
4- $\oint \overline{H}.d\overline{l} = H.2\pi r_c$

5- $H.2\pi r_C = I$

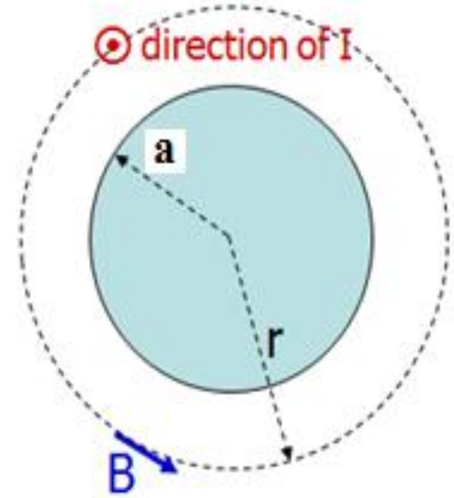
6- $\overline{H} = \frac{I}{2\pi r_c}\hat{\phi} \rightarrow (2)$



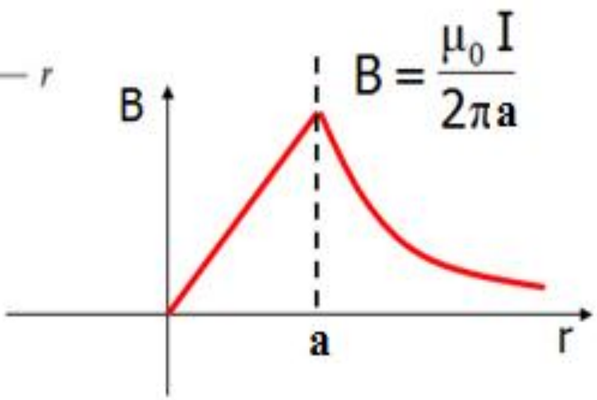
$J = I/s$
$\overline{I}$
I
II
a
H
$I/2\pi a$
r
a



B
$\frac{\mu_0 I}{2\pi a}$
$B \propto r$
$B \propto 1/r$
r
a

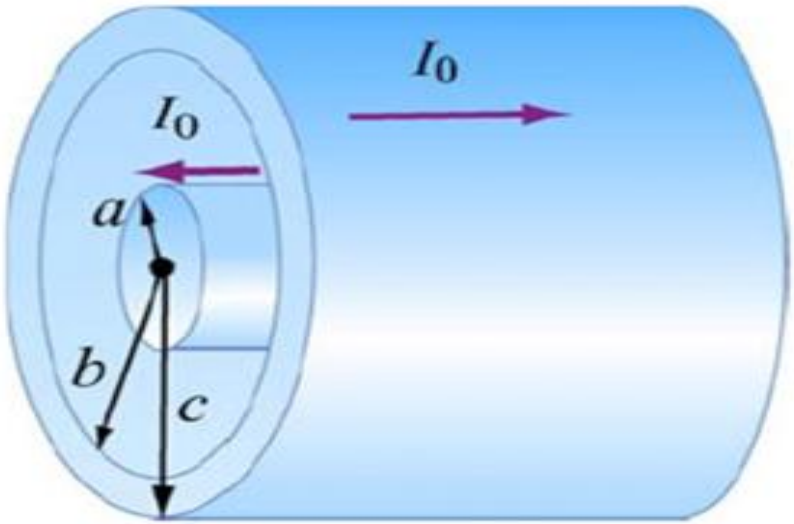


direction of I
a
r
B



B
$B = \frac{\mu_0 I}{2\pi a}$
a
r

Case of a coaxial Cable

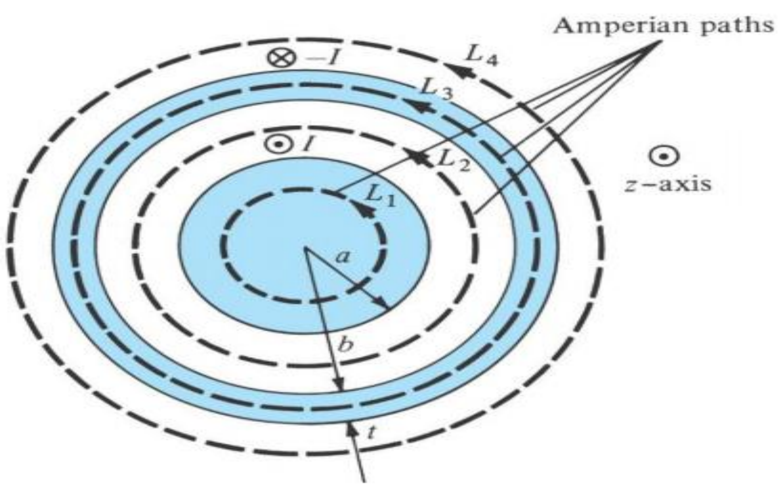


(a) $r < a$;

(b) $a < r < b$;

(c) $b < r < c$;

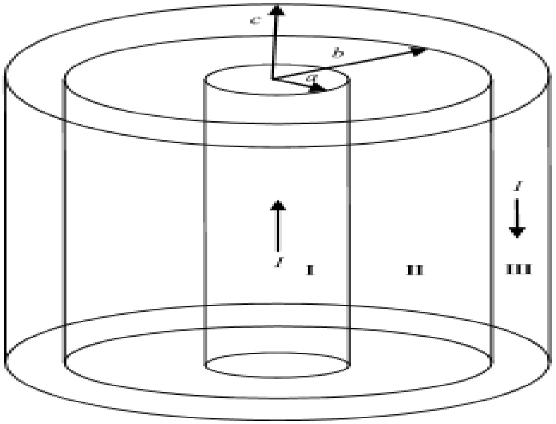
(d) $r > c$.



Example:

Find $\overline{H}$ in all region for coaxial cable of radii *a*, *b*, *c* carrying *I* in inner conductor and *I* in outer conductor

Solution:



Region (I) $r_c < a$

1- Ampere's circuital law

$$\oint \overline{H}.d\overline{l} = I_{en}$$

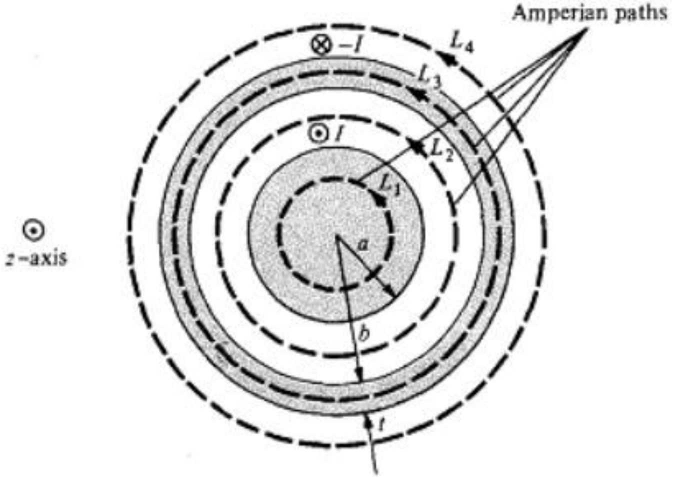
2- Choice of Amperian loop

3- $I_{en} = J.S = \frac{I}{\pi a^2}.\pi r_c^2$

4- $\oint \overline{H}.d\overline{l} = H.2\pi r_c$

5- $H.2\pi r_c = \frac{I}{\pi a^2}.\pi r_c^2$

6- $\overline{H} = \frac{I r_c}{2\pi a^2}\hat{\phi} \rightarrow (1)$



Region (II) $a < r_c < b$

1-Ampere's circuital law

$$\oint \overline{H}.d\overline{l} = I_{en}$$

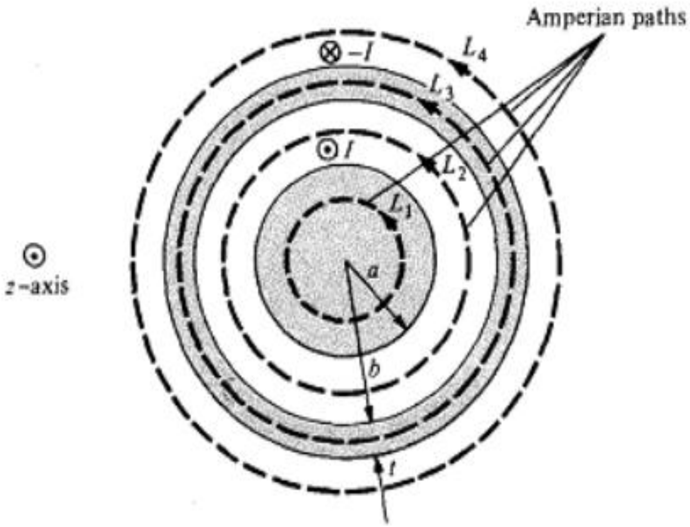
2-Choice of Amperian loop

3- $I_{en} = J.S = \mathbf{I}$

4- $\oint \overline{H}.d\overline{l} = H.2\pi r_c$

5- $H.2\pi r_C = I$

6- $\overline{H} = \dfrac{I}{2\pi r_c}\hat{\phi} \rightarrow (2)$



Region (III) $b < r_c < c$

1-Ampere's circuital law

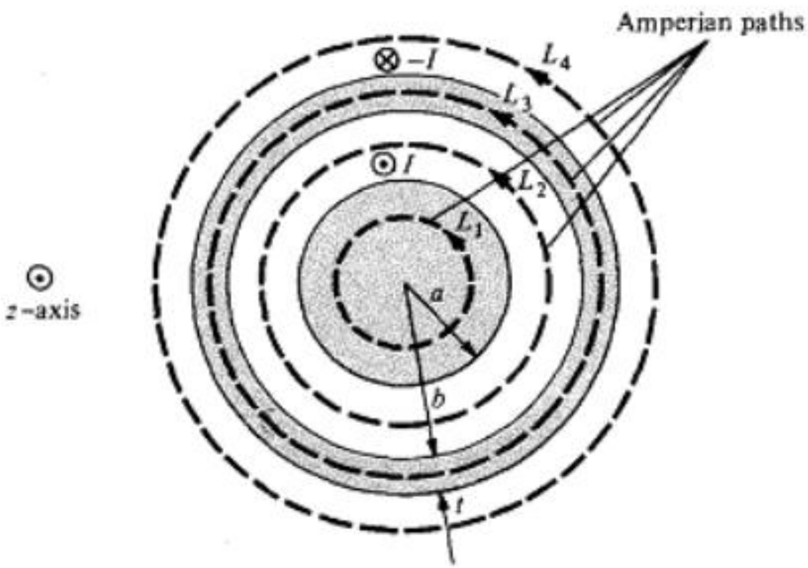
$$\oint \overline{H}.d\overline{l} = I_{en}$$

2-Choice of Amperian loop

3- $I_{en} = I - J.S$

$$= I - \left[\frac{I}{\pi(c^2 - b^2)} \; \pi(r_c^2 - b^2)\right]$$

$$= I\left(1 - \frac{r_c^2 - b^2}{c^2 - b^2}\right)$$



4- $\oint \overline{H}.d\overline{l} = H.2\pi r_c$

5- $H.2\pi r_C = I\left(1 - \frac{r_c^2 - b^2}{c^2 - b^2}\right)$

6- $\overline{H} = \frac{I}{2\pi r_c}\left(1 - \frac{r_c^2 - b^2}{c^2 - b^2}\right)\hat{\phi}$ **(3)**

Region (IV) $r_c > c$

1-Ampere's circuital law

$$\oint \overline{H}.d\overline{l} = I_{en}$$

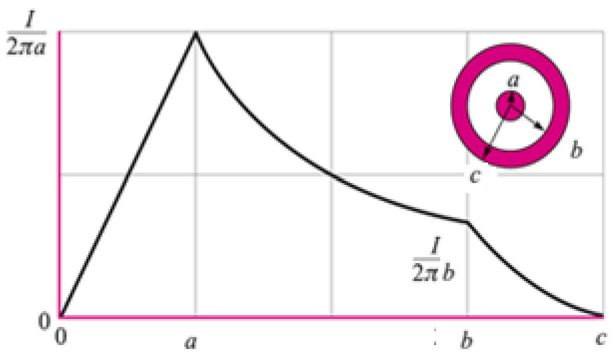
2-Choice of Amperian loop

3- $I_{en} = I - I = 0$

4- $\oint \overline{H}.d\overline{l} = H.2\pi r_c$

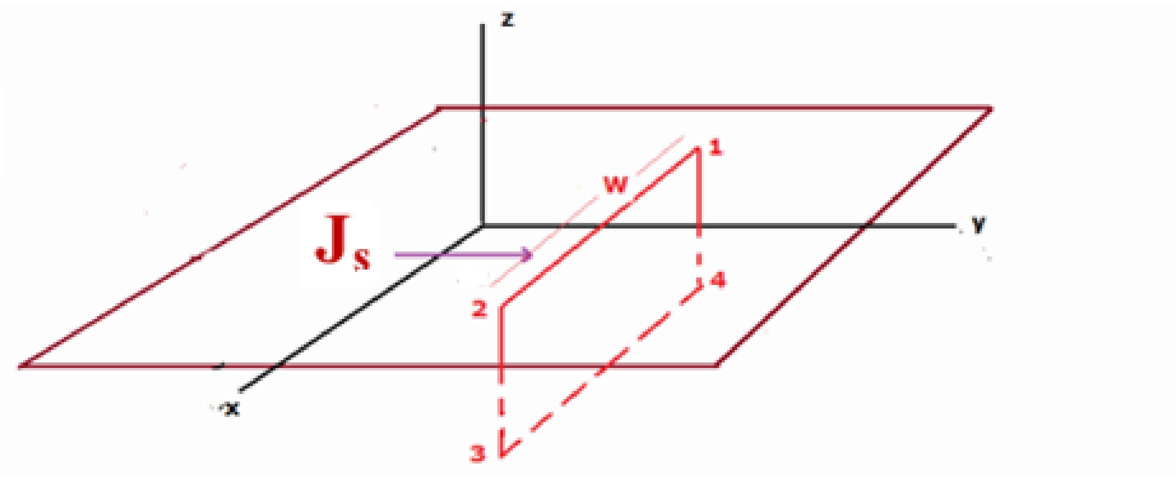
5- $H.2\pi r_c = 0$

6- $\overline{H} = 0$



The magnetic field intensity above and below a surface current distribution of infinite extended sheet with surface current density J_s

Derivation



1-Ampere's circuital law

$$\oint \overline{H}.d\overline{l} = I_{en}$$

2-Choice of Amperian loop (1-2-3-4) (x-z plane)

3- $I_{en} = J_{sy} * W$

4- $\oint \overline{H}.\overline{dl} = \int_1^2 \overline{H}.d\overline{l} + \int_2^3 \overline{H}.d\overline{l} + \int_3^4 \overline{H}.d\overline{l} + \int_4^1 \overline{H}.d\overline{l}$

above the surface :

$$d\overline{H} = \frac{Id\overline{l} \times \hat{a}_R}{4\pi R^2} = (\ )\hat{y} \times \hat{z} = \hat{x}$$

below :

$$d\overline{H} = (\quad)\hat{y} \times (-\hat{z}) = -(\ )\hat{x}$$

$$\oint \overline{H}.\overline{dl} = \int_1^2 H_x \hat{x}\,.dx\,\hat{x} + 0 + \int_3^4 H_x(-\hat{x}).dx(-\hat{x}) + 0$$

5- $H_x W + H_x W = J_{sy} W$

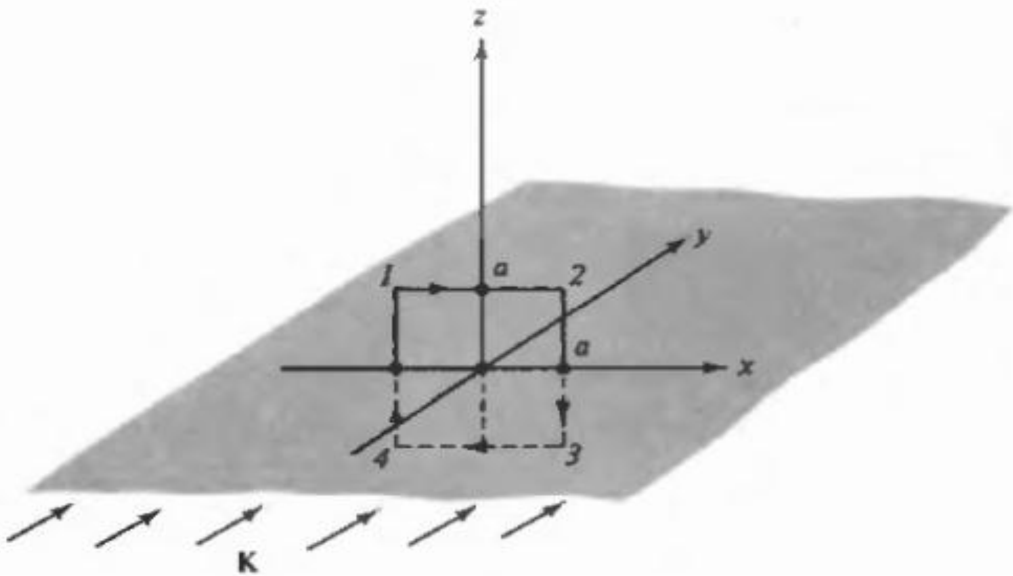
6- $H_x = \dfrac{J_{sy}}{2}$

Generally : $\overline{H} = \dfrac{1}{2}\overline{J_s} \times \hat{n}$

where $\overline{J_s} \equiv$ surface current denisty

$\hat{n} \equiv$ unit vector $\perp$ to the surface

EXAMPLE . An infinite current sheet lies in the $z = 0$ plane with $\mathbf{K} = K\mathbf{a}_y$, as shown in Fig. Find $\mathbf{H}$.



The Biot–Savart law and considerations of symmetry show that $\mathbf{H}$ has only an x component, and is not a function of x or y.

Applying Ampère's law to the square contour *12341*, and using the fact that $\mathbf{H}$ must be antisymmetric in z,

$$\oint \mathbf{H} \cdot d\mathbf{l} = (H)(2a) + 0 + (H)(2a) + 0 = (K)(2a) \qquad \text{or} \qquad H = \frac{K}{2}$$

Thus, for all $z > 0$, $\mathbf{H} = (K/2)\mathbf{a}_x$.

More generally, for an arbitrary orientation of the current sheet,

$$\boxed{\mathbf{H} = \tfrac{1}{2}\,\mathbf{K} \times \mathbf{a}_n}$$

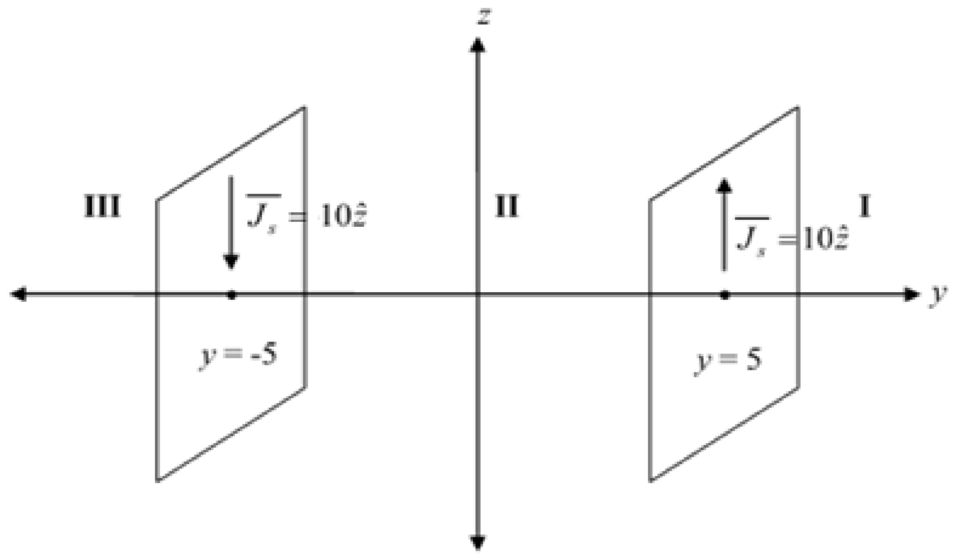
Observe that **H** is independent of the distance from the sheet.

Further, the directions of **H** above and below the sheet can be found

by applying the *right-hand rule* to a few of the current elements in the sheet.

Example:

Find $\overline{H}$ in all regions for the following current configuration shown in figure.



Solution:

Region (I) : $y > 5$

$$\overline{H_1} = \frac{\overline{J_s}}{2} \times \hat{n} = \frac{10\hat{z}}{2} \times \hat{y} = \frac{-10}{2}\hat{x}$$

$$\overline{H_2} = \frac{-10}{2}\hat{z} \times \hat{y} = \frac{10}{2}\hat{x}$$

$$\overline{H_t} = \overline{H_1} + \overline{H_2} = 0$$

Region (II) : $-5 < y < 5$

$$\overline{H_1} = \frac{1}{2}\overline{J_{s1}} \times \hat{n} = \frac{1}{2} 10\,\hat{z} \times (-\hat{y}) = 5\hat{x}$$

$$\overline{H_2} = \frac{1}{2}\overline{J_{s2}} \times \hat{n} = \frac{1}{2}(-10\hat{z}) \times (\hat{y}) = 5\hat{x}$$

$$\overline{H_t} = 10\,\hat{x}$$

Region (III) : $y < -5$

$$\overline{H_1} = \frac{1}{2}(10\hat{z}) \times (-\hat{y}) = 5\hat{x}$$

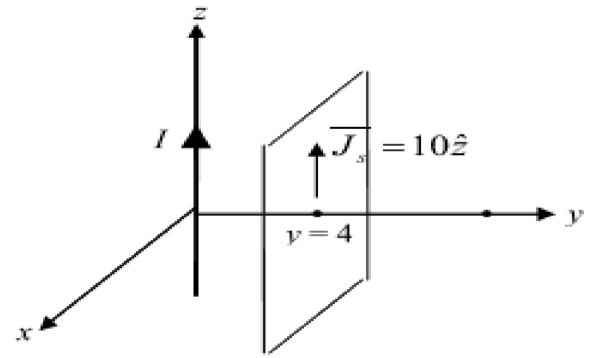
$$\overline{H_2} = \frac{1}{2}(-10\hat{z}) \times (-\hat{y}) = -5\hat{x}$$

$$\overline{H_t} = \text{zero}$$

Example:

Find $\overline{H}$ in all regions for the following current configuration shown in figure.

Solution:



Region (I): $y > 4$

$$\overline{H_s} = \frac{1}{2}\overline{J_S} \times \hat{n} = \frac{1}{2}(10\hat{z}) \times \hat{y} = -5\hat{x}$$

$$\oint \overline{H}.\overline{dl} = I_{en}$$

$$H.2\pi r_c = I$$

$$\overline{H} = \frac{I}{2\pi . r_c}\hat{\phi}$$

$$d\overline{H} = \frac{Id\overline{l} \times \hat{a}_R}{4\pi R^2} = \quad | \quad | \; \hat{\phi}$$

$$\hat{\phi} = \hat{z} * \hat{y} = -\hat{x}$$

$$\overline{H}_I = \frac{I}{2\pi . r_c}(-\hat{x})$$

$$\therefore \overline{H_t} = \overline{H_s} + \overline{H_l} = -5\hat{x} - \frac{I}{2\pi . r_c}\hat{x} \quad A/m$$

Region (II): $0 < y < 4$

$$\overline{H_s} = \frac{1}{2}\overline{J_S} \times \hat{n} = \frac{1}{2}(10\hat{z}) \times (-\hat{y}) = 5\hat{x}$$

$$\overline{H_l} = \frac{I}{2\pi . r_c}\hat{\phi}$$

$$\hat{\phi} = \hat{z} * \hat{y} = -\hat{x}$$

$$\overline{H_l} = \frac{I}{2\pi . r_c}(-\hat{x})$$

$$\overline{H_t} = \overline{H_s} + \overline{H_l} = 5\hat{x} - \frac{I}{2\pi . r_c}\hat{x} \quad A/m$$

Region (III): $y < 0$

$$\overline{H_s} = \frac{1}{2}\overline{J_S} \times \hat{n} = \frac{1}{2}(10\hat{z}) \times \left(-\hat{y}\right) = 5\hat{x}$$

$$\overline{H_l} = \frac{I}{2\pi . r_c}\hat{\phi}$$

$$\hat{\phi} = \hat{z} * -\hat{y} = \hat{x}$$

$$\overline{H_l} = \frac{I}{2\pi . r_c}(\hat{x})$$

$$\overline{H_t} = \overline{H_s} + \overline{H_l} = 5\hat{x} + \frac{I}{2\pi . r_c}\hat{x} \quad A/m$$

Thanks