Fundamentals of Electromagnetic Fields_ EPM 112

CHAPTER (7)

Steady State Magnetic Fields

Lecture 3

Biot-Savart law
Ampere's Circuital Law
Magnetic Field Density
Magnetic Flux

Part 3: Magnetic field density & magnetic flux

Magnetic flux density $\overline{B}$

$$\overline{B} = \mu_0 \mu_r \overline{H}$$

Magnetic flux ϕ_m:

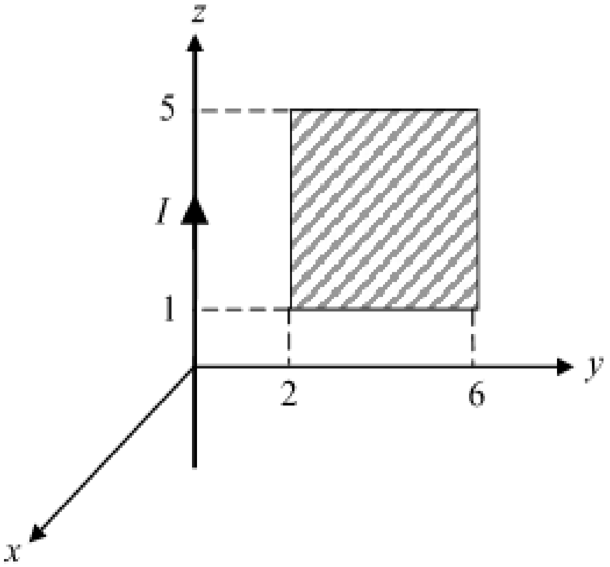
$$\phi_m = \iint \overline{B}.d\overline{S}$$

For a closed surface:

$$\phi_m = \oiint \overline{B}.d\overline{S} = 0$$

Example:

Find the total magnetic flux that crossing the area shown in figure.



Solution:

$$\overline{H} = \frac{I}{2\pi r_c}\hat{\phi}$$

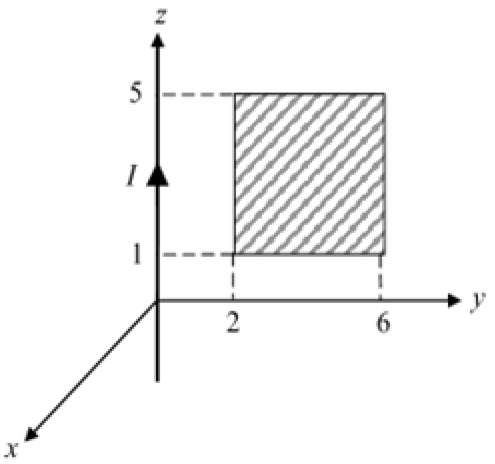
$$\overline{H} = \frac{I}{2\pi y}\hat{\phi}$$

$$\overline{B} = \mu_o\overline{H} = \frac{\mu_o I}{2\pi y}\hat{\phi}$$

$$\because\ d\overline{H} = \frac{Id\overline{l}\times\hat{a}_R}{4\pi R^2} = [\qquad]\hat{\phi}$$

$$\therefore\ \hat{\phi} = \hat{z}\times\hat{y} = -\hat{x}$$

$$\therefore\ \overline{B} = -\frac{\mu_o I}{2\pi y}\hat{x}$$



$$d\overline{S} = dydz(-\hat{x})$$

$$\phi_m = \int_1^5 \int_2^6 \frac{\mu_o I}{2\pi y}\, dydz$$

$$\phi_m = \frac{\mu_o I}{2\pi} (\ln y)_2^6 (z)_1^5$$

$$\phi_m = \frac{4\mu_o I}{2\pi} [\ln 6 - \ln 2] \qquad \text{Weber}$$

Example:

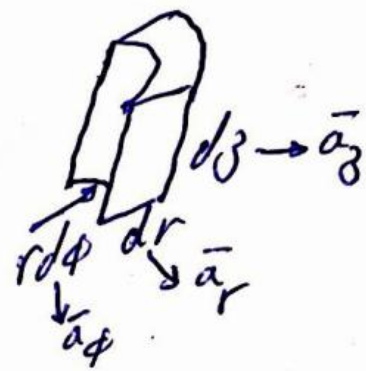
For $\bar{H} = 10\, r\, \bar{a}_\phi$ A/m (cylindrical coordinates), find the flux Φ that passes through a plane surface defined by $\phi = \pi/2$, $2 \leq r \leq 4$ and $0 \leq z \leq 2$.

solution

$$\bar{B} = \mu_0 \bar{H} = (4\pi * 10^{-7})\, 10 r\, \bar{a}_\phi \quad wb/m^2$$

$$= (4\pi r) * 10^{-6}\, \bar{a}_\phi$$

$$\Phi = \int_S \bar{B} \cdot \bar{ds} \quad \text{where } \bar{ds} = (dr\,dz)\,\bar{a}_\phi$$

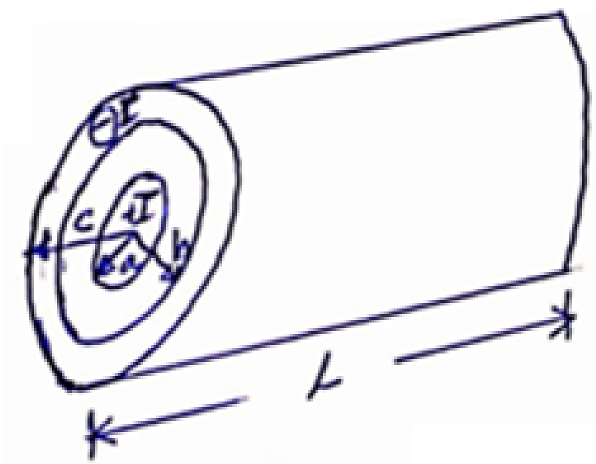


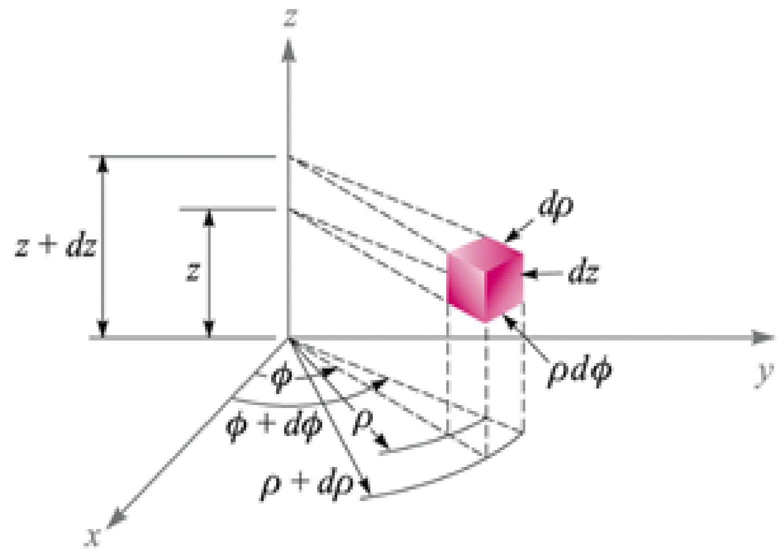
$$\therefore \Phi = \int_{z=0}^{2}\int_{r=2}^{4} (4\pi r) * 10^{-6}\, \bar{a}_\phi \cdot (dr\,dz)\,\bar{a}_\phi$$

$$= 4\pi * 10^{-6} * \left(\frac{r^2}{2}\right)_{r=2}^{4} \left(z\right)_0^2 = 15.1 * 10^{-6} \quad wb$$

Example:

A coaxial conductor with an inner conductor radius ‘a’ and outer radii ‘b’ and ‘c’, respectively, carries current I in the inner conductor. Find the magnetic flux per unit length crossing a plane φ=constant between the conductors $a \leq \rho < b$.





Solution:

we know before that $\bar{H}$ for the inner conductor is:

$$\boxed{\bar{H} = \frac{I}{2\pi r}\,\bar{a}_\phi} \qquad a \le r \le b$$

then $\bar{B}$ is given by $\bar{B} = \mu_0 \bar{H}$

$$\therefore \quad \bar{B} = \frac{\mu_0 I}{2\pi r}\,\bar{a}_\phi$$, the flux is given by

$$\Phi = \int_S \bar{B} \cdot \bar{ds} \qquad \text{where} \qquad \bar{ds} = (dr\,dz)\,\bar{a}_\phi$$

$$\therefore \quad \Phi = \int_{z=0}^{L}\int_{r=a}^{b} \frac{\mu_0 I}{2\pi r}\,dr\,dz = \frac{\mu_0 I}{2\pi}\,(\ln r)\Big|_{r=a}^{b}\,(z)\Big|_{z=0}^{L}$$

$$= \frac{\mu_0 I L}{2\pi}\ln(b/a) \qquad \text{wb}$$

Then, the flux per unit length is:

$$\boxed{\Phi/L = \frac{\mu_0 I}{2\pi}\ln(b/a) \qquad \text{wb/m}}$$

Chapter 8

Self Inductance & Magnetic Circuits

This chapter is divided into Two main parts as:

Part 1: Self Inductance

Part 2: Magnetic Circuits

Part I

Self Inductance

Conductor, turn and Winding

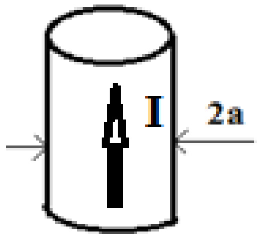


Fig. a

Conductor of radius a

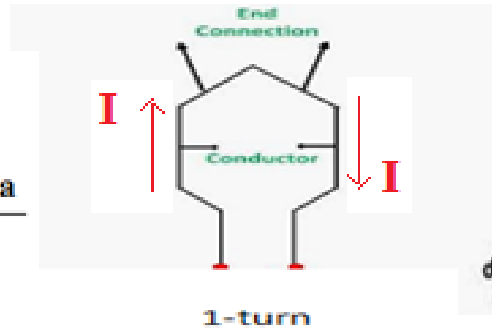


Fig. b

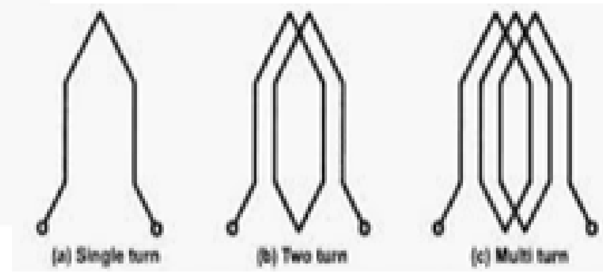


Fig. c

1-turn= 2conductors
Winding= group of turns

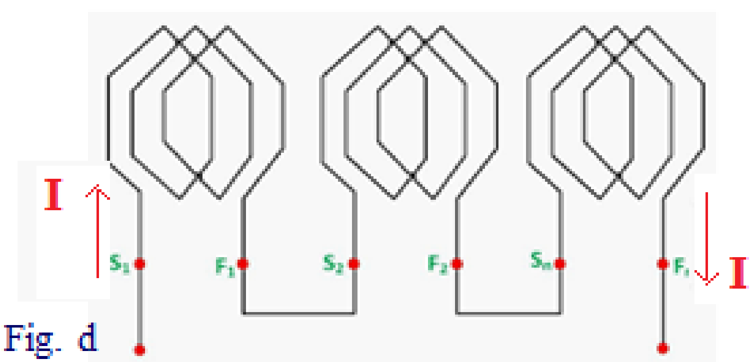
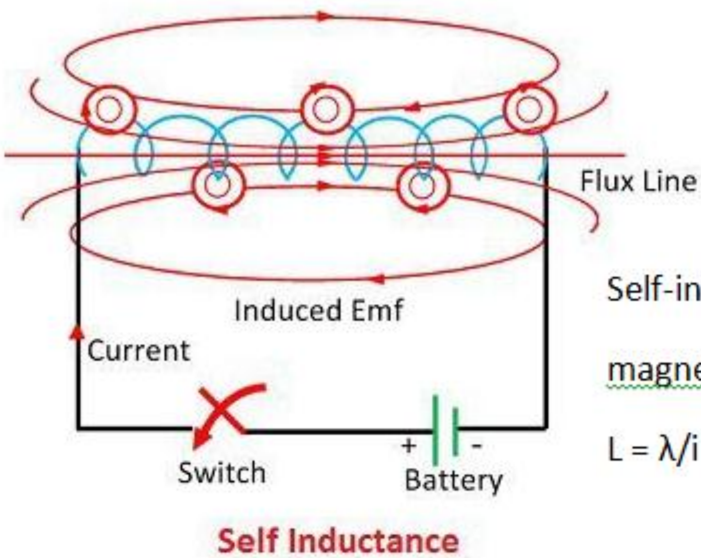


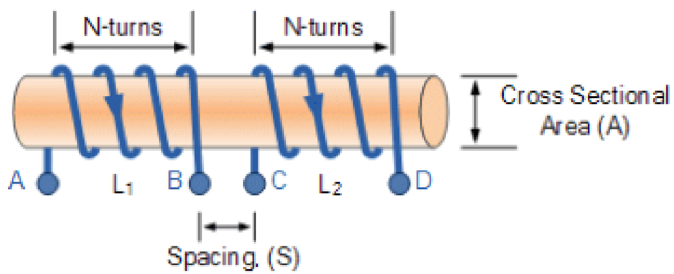
Fig. d

Winding



Self-inductance 'L' is defined as the ratio of the linking magnetic flux 'λ' to the current producing the flux or

$L = \lambda/i$



Mutual Inductance

Mutual Inductance is the interaction of one coils magnetic field on another coil as it induces a voltage in the adjacent coil

Inductance and Magnetic Circuits

INDUCTANCE

The *inductance L* of a conductor system may be defined as *the ratio of the linking magnetic flux to the current producing the flux*. For static (or, at most, low-frequency) current I and a coil containing N turns, as shown in Fig.

$$L = \frac{N\Phi}{I}$$

The units on L are *henries*, where $1\ \text{H} = 1\ \text{Wb/A}$. Inductance is also given by $L = \lambda/I$, where λ, the *flux linkage*, is $N\Phi$ for coils with N turns or simply Φ for other conductor arrangements.

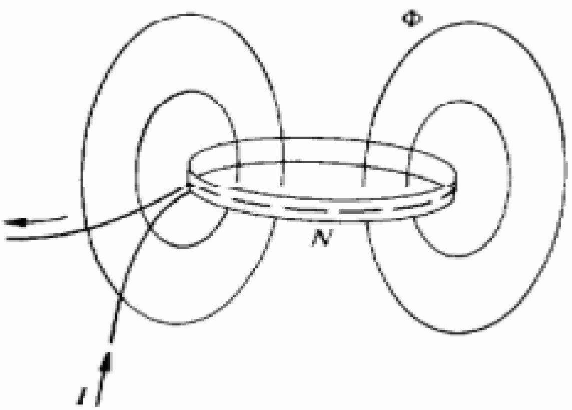


Fig.

Example

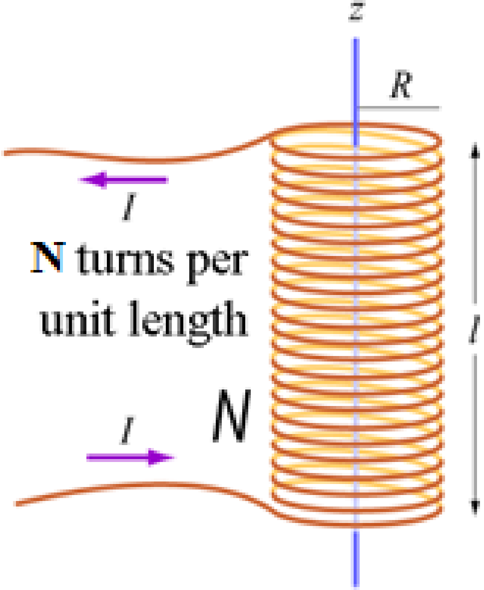
A long solenoid of length *L* and cross-sectional area *A* has *N* turns. Find its self-inductance. Assume that the field is uniform throughout the solenoid.

Solution:

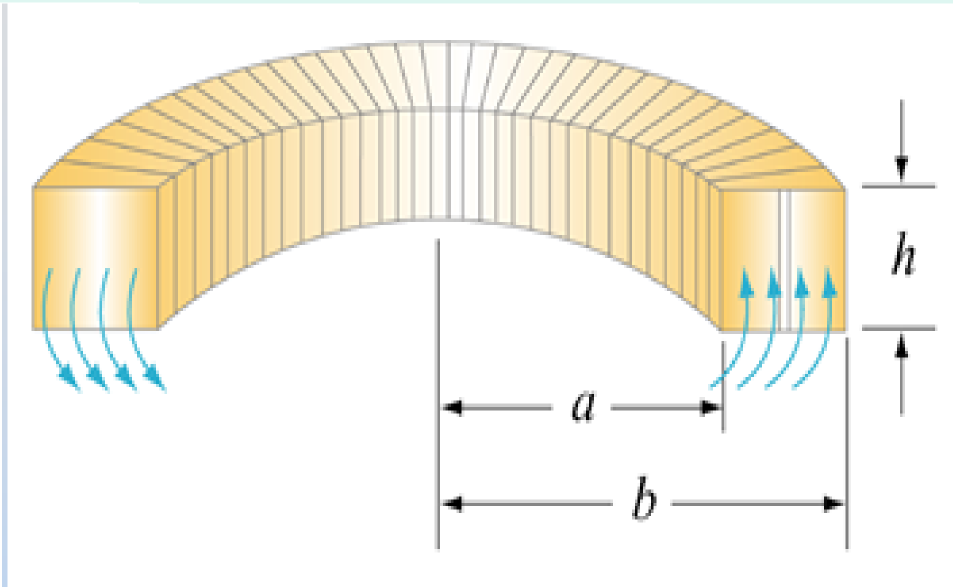
$$\Phi = BA = \mu_0 \frac{N}{l} IA$$

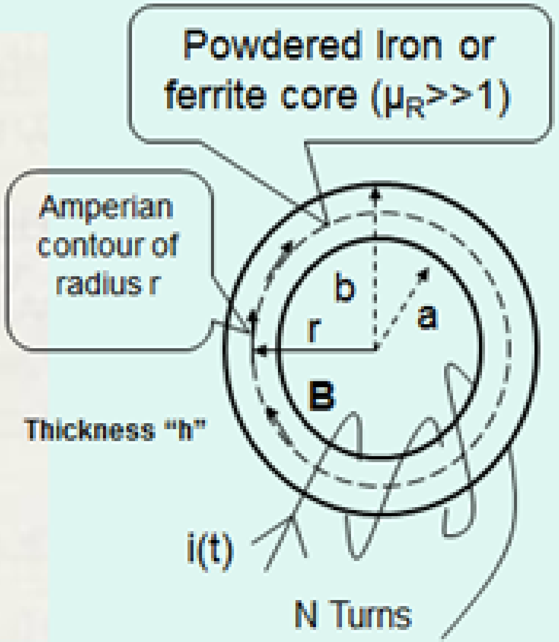
$$N\Phi = LI = N\mu_0 \frac{N}{l} IA$$

$$L = \mu_0 \frac{N^2}{l} A$$



Inductance of a toroidal inductor





From symmetry, we see that B and H will be directed in the i ϕ direction, pointing along the Amperian contour, thus

$$H_{\phi} \cdot 2 \cdot \pi \cdot r = N \cdot i$$

Thus $H_\phi = \dfrac{N \cdot i}{(2\pi \cdot r)}$ Inside the toroidal core, for a < r < b

The magnetic flux that circulates around the toroidal core and cuts each turn of the coil must be

$$\phi = \int_0^h \int_a^b \mu \cdot H_\phi \, dr \, dz = \int_0^h \int_a^b \mu \cdot \frac{N \cdot i}{(2\pi \cdot r)} \, dr \, dz$$

$$\phi = h \cdot \mu \cdot N \cdot i \cdot \frac{\ln\left(\frac{b}{a}\right)}{2\pi}$$

By definition of inductance, the inductance of the toroid becomes:

$$L = \frac{N \cdot \phi}{i} = \mu \cdot \left(\frac{N^2 \cdot h}{2 \cdot \pi}\right) \cdot \ln\left(\frac{b}{a}\right)$$

Numerical example

Consider a toroid with a ferrite core $\mu_R := 250$

With $a := 5\cdot cm$ $b := 10\cdot cm$ $h := 1\cdot cm$

And $N := 50$ turns $\mu_0 := 4\cdot\pi\cdot 10^{-7}\cdot\frac{H}{m}$

$$L := \mu_0\cdot\mu_R\cdot\left(\frac{N^2\cdot h}{2\cdot\pi}\right)\cdot\ln\left(\frac{b}{a}\right)$$

$$L = 8.664\times 10^{-4}\,H$$

Example

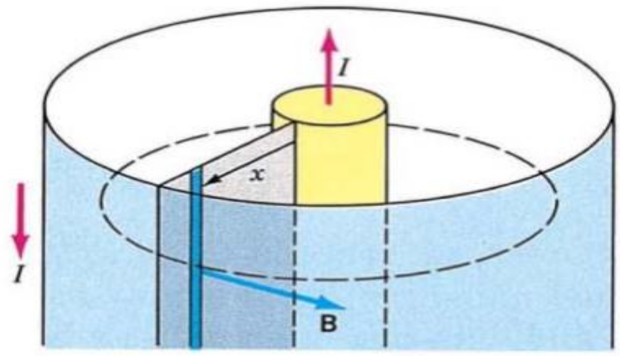
A coaxial cable consists of an inner wire of radius *a* that carries a current I upward, and an outer cylindrical conductor of radius *b* that carries the same current downward. Find the self-inductance of a coaxial cable of length *L*. Ignore the magnetic flux within the inner wire.

Solution:

$$B = \frac{\mu_0 I}{2\pi x}, \quad d\Phi = B dA = \frac{\mu_0 I}{2\pi x}\ell dx$$

$$\Phi = \int_a^b \frac{\mu_0 I}{2\pi x}\ell dx = \frac{\mu_0 I \ell}{2\pi}\ln\frac{b}{a} = LI$$

$$L = \frac{\mu_0 \ell}{2\pi}\ln\frac{b}{a}$$



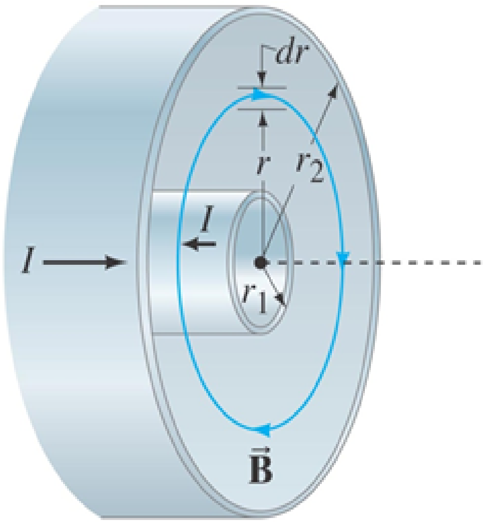
Hint1: The direction of the magnetic field.

Hint2: What happens when considers the inner flux?

Self-Inductance

Example: Coaxial cable inductance.

Determine the inductance per unit length of a coaxial cable whose inner conductor has a radius r_1 and the outer conductor has a radius r_2. Assume the conductors are thin hollow tubes so there is no magnetic field within the inner conductor, and the magnetic field inside both thin conductors can be ignored. The conductors carry equal currents I in opposite directions.



Solution

The solution is typical as the previous example.

INTERNAL INDUCTANCE

Magnetic flux occurs within a conductor cross section as well as external to the conductor. This internal flux gives rise to an *internal inductance*, which is often small compared to the external inductance and frequently ignored. In Fig. (*a*) a conductor of circular cross section is shown, with a current I assumed to be uniformly distributed over the area. (This assumption is valid only at low frequencies, since *skin effect* at higher frequencies forces the current to be concentrated at the outer surface.) Within the conductor of radius a, Ampère's law gives

$$\mathbf{H} = \frac{Ir}{2\pi a^2}\mathbf{a}_\phi \qquad \text{and} \qquad \mathbf{B} = \frac{\mu_0 Ir}{2\pi a^2}\mathbf{a}_\phi$$

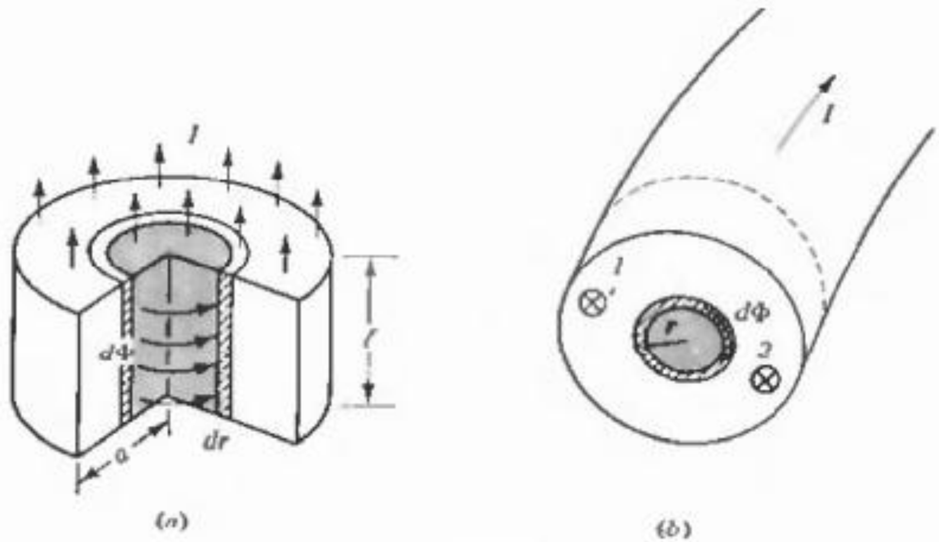


Fig.

The straight piece of conductor shown in Fig.(a) must be imagined as a short section of an infinite torus, as suggested in Fig. (b). The current filaments become circles of infinite radius. The lines of flux $d\Phi$ through the strip $\ell\, dr$ encircle only those filaments whose distance from the conductor axis is smaller than r. Thus, an open surface bounded by one of those filaments is cut once (or an odd number of times) by the lines of $d\Phi$; It follows that $d\Phi$ links only with the fraction $\pi r^2/\pi a^2$ of the total current, so that the total flux linkage is given by the weighted "sum"

$$\lambda = \int \left(\frac{\pi r^2}{\pi a^2}\right) d\Phi = \int_0^a \left(\frac{\pi r^2}{\pi a^2}\right) \frac{\mu_0 I r}{2\pi a^2} \ell\, dr = \frac{\mu_0 I \ell}{8\pi}$$

and

$$\frac{L}{\ell} = \frac{\lambda/I}{\ell} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7}\ \mathrm{H/m}$$

This result is independent of the conductor radius. The total inductance is the sum of the external and internal inductances. If the external inductance is of the order of $\frac{1}{2} \times 10^{-7}$ H/m, the internal inductance should not be ignored.

Thanks