

- **heapify:**
  - Ensures the Max-Heap property for a subtree rooted at index  $i$ .
  - Compares the root with its left and right children and swaps if the root is not the largest.
  - Recursively fixes the affected subtree if needed.
- **heap\_sort:**
  - Builds a Max-Heap from the array.
  - Repeatedly swaps the root (largest element) with the last element, reduces the heap size, and rebuilds the heap.
  - Continues until the entire array is sorted.

#### **Execution Steps:**

1. Build a Max-Heap from the array.
2. Extract the largest element (root) and place it at the end of the array.
3. Reduce the heap size and rebuild the Max-Heap for the remaining elements.
4. Repeat until the array is sorted.

#### **Example:**

Input: [4, 1, 3, 9, 7]

- After building Max-Heap: [9, 7, 3, 1, 4]
- After sorting: [1, 3, 4, 7, 9]

#### **Time Complexity:**

- Building the heap:  $O(n)$ .
- Sorting using the heap:  $O(n \log n)$ .
- Total:  $O(n \log n)$ .