Optimal Sizing of Hyprid Solar/Wind/Hydroelectric Pumped Storage Energy System in Egypt

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***Abstract*—Providing access to clean, reliable, and affordable energy by adopting hybrid power systems is important for countries looking to achieve their sustainable development goals. This paper presents the results of application multiple optimization methods for sizing a hybrid system including photovoltaic (PV), wind turbines with a hydroelectric pumped storage system. A comprehensive study has been carried out between Simulated Annealing Algorithm (SA), Genetic Algorithm (GA), Particle Swarm Algorithm (PSO), and Firefly Algorithm (FA) to validate each one. The purpose of the optimization process is to minimize the cost of energy from this hybrid system while satisfying the operation constraints including high reliability of the hybrid power supply, small fluctuation in the energy injected to the grid, and high utilization of the photovoltaic and wind complementary properties. MATLAB software package has been used to evaluate each optimization algorithm for solving the considered optimization problem.**

***Index Terms*—Simulated Annealing, Genetic Algorithm, Particle Swarm, Firefly Algorithm.**

I. Introduction

A. Problem Formulation

The specific problem addressed is the optimal sizing of a hybrid solar/wind/hydroelectric pumped storage energy system for a practical case study in Egypt. By using hybrid power systems, countries can produce affordable, reliable and clean energy.Using this renewable hybrid power system will also reduce the usage of fossil fuels as an energy source which means reducing the negative effects fossil fuels can give in the process. To optimally size the hybrid power system in the most efficient manner possible,The goal is to minimize the cost of energy (COE) along with minimizing other quantities like fluctuation of power in the system, the probability of losing power from the proposed hybrid system, while also satisfying the design constraints such as providing the load with a reliable power supply, and selecting appropriate size of volume for the tank.

1) Hyprid System Modeling:

- **Solar Photovoltaic System Modeling:**
  It is the energy obtained from a PV module in terms of the solar radiation and the ambient temperature and can be expressed as:

$$E_{\text{PV}}(t) = \frac{n_{\text{PV}} P_{\text{PV}} \eta_{\text{PV}} \eta_{\text{INV}} \eta_{\text{Wire}} \cdot I_{\text{rad}}(t) \cdot (1 - \beta_T \cdot (T_C(t) - T_{C\text{nom}}))}{I_{\text{nom}}} \tag{1}$$

where:

n_{PV} = number of PV modules,
P_{PV} = installed capacity of the PV module,
η_{PV} = conversion efficiency of the PV module,
η_{INV} = inverter efficiency,
η_{Wire} = wire efficiency,
$I_{\text{rad}}(t)$ = ambient solar radiation intensity,
I_{nom} = intensity of solar radiation under standard conditions,
β_T = temperature coefficient of power of the selected PV modul
$T_C(t)$ = cell temperature,
$T_{C\text{nom}}$ = cell temperature under standard conditions of operation.

- **Wind Energy System Modeling:**
  Wind energy system modeling. Depending on the fundamentals of wind energy, the expected energy supplied by a wind turbine can be described as follows:

$$P_{\text{WT}} = \begin{cases} 0 & \text{if } u(t) < u_{\text{cut-in}} \\ n_{\text{wind}} \eta_{\text{wind}} P_{\text{R\_WT}} \frac{u^2(t) - u^2_{\text{cut-in}}}{u^2_{\text{rated}} - u^2_{\text{cut-in}}} & \text{if } u_{\text{cut-in}} \le u(t) < u_{\text{rated}} \\ n_{\text{wind}} \eta_{\text{wind}} P_{\text{R\_WT}} & \text{if } u_{\text{rated}} \le u(t) < u_{\text{cut-off}} \\ 0 & \text{if } u(t) \ge u_{\text{cut-off}} \end{cases} \tag{2}$$

where:

P_{WT} = actual power generated from the wind turbine,
$P_{\text{R\_WT}}$ = wind turbine rated power,
n_{wind} = wind turbine number,
η_{wind} = efficiency of the wind system,
$u(t)$ = wind speed,
$u_{\text{cut-in}}$ = cut-in wind speed at which the turbine starts operation,
u_{rated} = wind speed at rated power,
$u_{\text{cut-off}}$ = cut-off wind speed after which the wind turbine must be s

- **Hydraulic Pumped Storage System Modeling:**
  The upper reservoir is assumed to have a cube shape where h_2 is the height of the tank. The potential of energy storage depends on the volume of water stored V. During the process of charging and discharging, the head of water changes with time as a result of the additional head h_{add} beside the main head of the hydraulic system h_3.

$$h_{\text{add}} = V(t-1) \cdot \text{area} \tag{3}$$

When the system operates in the generating mode, the energy supplied by the turbine-generator set E_H is presented in the following equation:

$$E_H(t) = \min\left(\min\left(\frac{V(t-1)}{3600}, Q_T\right), \eta_T \eta_{WP} \rho g (h_{\text{add}} + h_3), |E_B|\right) \quad (4)$$

where Q_T is the discharge rate (m³/s), η_{WP} is the efficiency of the pipeline, η_T is the efficiency of the turbine-generator set, ρ is the water density (kg/m³), and E_B is the energy balance, which measures the generated energy with respect to the load demand E_D:

$$E_B = E_{PV} + E_{WT} - E_D \quad (5)$$

When the energy delivered from PV and wind turbine exceeds the load demand, the system operates in the pumping mode. In this condition, the energy consumed by the motor-pump set E_{pump} is described as follows:

$$E_{\text{pump}}(t) = \min\left(\min\left(V_{\max} - V(t-1), \frac{3600}{Q_P}\right), \eta_P \eta_{WP} \rho g (h_{\text{add}} + h_3), |E_B|\right) \quad (6)$$

where Q_P is the charging rate (m³/s), $V_{\max}$ is the maximum capacity of the upper reservoir, and η_P is the efficiency of the motor-pump unit.

2) Optimization Strategy Parameter Evaluation: Loss of power supply probability (LPSP) is a factor used as a system design, which measures the probability of insufficient operation of the hybrid system when it fails to meet the load requirements:

$$LPSP = \frac{\sum_{n=1}^{8760}[P_{load}(t_i) - (P_{PV}(t_i) + P_{WT}(t_i) + P_H(t_i))]}{\sum_{n=1}^{8760}[P_{load}(t_i)]} \quad (7)$$

Where $P_H(t_i)$ is the power supplied by the turbine-generator set.

Complementary characteristics of renewable sources. To make a full use of the PV and wind complementary characteristics, the relative fluctuation rate is adopted for the renewable generation with respect to the load power:

$$D_{load} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_{PV}(t_i) + P_{WT}(t_i) - P_{load}(t_i))^2}}{\overline{P}_{load}} \quad (8)$$

where Pload is the average load power. The smaller value of Dload means that the power generation is closer to the load demand and ensures better utilization of the sources.

Cost of energy (COE) a) Annual capital cost. The annual interest of capital cost for each individual component in the hybrid system is considered as follows:

$$C_{ann_c ap_i} = C_{cap_i} \times CRF(r, M_i) \quad (9)$$

where i refers to each component in the system including PV modules, wind turbine, turbine-generator set, motor-pump set, and converter. CRF is the capital recovery factor. r is the rate of interest. C_{cap} is the initial capital investment cost of installation of each subsystem. M_i is the lifetime of each subsystem:

$$CRF(r, m) = \frac{r(1+r)^M}{(1+r)^M - 1} \quad (10)$$

The operating and maintenance cost. The operating and maintenance cost of the hybrid renewable energy system is the major cost of the system as there is no fuel cost, and is defined as follows:

$$C_{o\&m} = C_{pv_{o\&m}} t_{wt} + C_{wt_{o\&m}} t_{wt} + C_{hydro_{o\&m}} t_{hydro} + C_{pump_{o\&m}} t_{pump} \quad (11)$$

where $C_{pv_{o\&m}}$, $C_{wt_{o\&m}}$, $C_{hydro_{o\&m}}$, and $C_{pump_{o\&m}}$ are the maintenance and operating cost of PV, wind turbine, turbine-generator unit, and motor-pump set, while t_{pv}, t_{wt}, t_{hydro}, and t_{pump} are the number of hours of operating during the study period for PV modules, wind system, turbine, and pump, respectively.

The replacement cost. The replacement cost for each subsystem, i, is calculated from the following equation:

$$C_{rep} = C_{cap_i} \frac{(M_{sys} - M_i)}{M_i} \quad (12)$$

where M_sys is the lifetime of the whole hybrid system.

The annual cost of the hybrid renewable system is calculated depending on the annual investment cost $C_{ann-tot}$:

$$C_{ann-tot} = C_{ann-cap} + C_{ann-rep} + C_{ann-O\&M} \quad (13)$$

The net present value (NPV) of the system is calculated as follows:

$$NPV = \frac{C_{ann-tot}}{CRF} \quad (14)$$

The cost of energy (COE) from the hybrid system is in dollars per kilowatt hour and formulated in the following equation:

$$COE = \frac{C_{ann-tot}}{\sum_{h=1}^{8760} P_{load}} = \frac{NPC}{\sum_{h=1}^{8760} P_{load}} \times CRF \quad (15)$$

where P_{load} is the value of the hourly load demand.

Since the goal is to optimal sizing of the hybrid energy system, it is assumed that all components in the system is operating in their ideal state as such, all efficiencies are equal to 1. Most data obtained for the parameters in some equations are obtained from meteorological data in the area of study which is the Ataka region in Egypt. The study lifetime of this case is 25 years, the average wind speed is 6.72 m/s, the peak load power is 420 kW and the peak interest rate (r) is 0.06. Data for the rest of the parameters in the previous equations are obtained from the figures below.

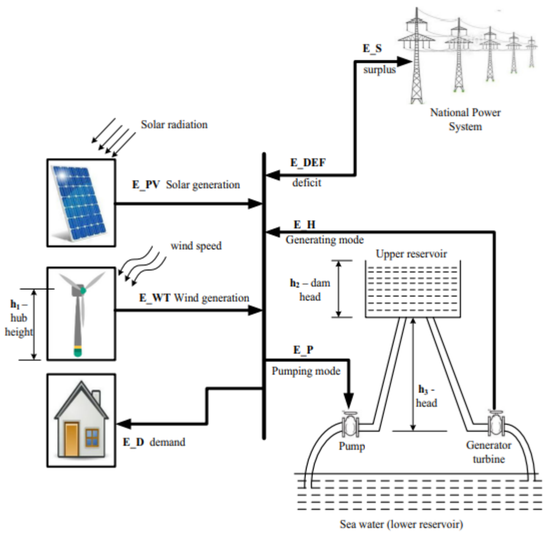

Fig. 1: Operational diagram of the hybrid energy system

Parameters of PV modules	
Model	PV-MLT260HC
Max. power	260 W
Length	1625 mm
Width	1019 mm
Thickness	46 mm
Operating temperature	47 °C
Temperature coefficient	0.45%
Initial cost	112 $
O&M cost	1%
Lifetime	25 years
Parameters of wind turbine	
Model	Fuhrländer FL 30
Rated power	30 kW
Rotor diameter	12.8 m
Swept area	13 m^2
Cut-in speed	2.5 m/s
Rated speed	12 m/s
Cut-off speed	25 m/s
Initial cost	58564.79 $
O&M cost	3%
Replacement cost	34553.226 $
Lifetime	20 years
Parameters of turbine-generator set	
Power	1 kW
Efficiency	86%
Initial cost	1000 $
Replacement cost	750 $
O&M cost	1%
Lifetime	10 years
Parameters of pump-motor set	
Rated power	45 kW
Initial cost/kW	10749 $
Replacement cost	10749 $
O&M cost	1%
Lifetime	10 years
Parameters of Reservoir	
Initial cost/m^3	170 $
Lifetime	35 years

Fig. 2: Technical and economic characteristics of the system components

B. Objective Functions

Minimizing the cost of energy (COE), D_{load}, and $LPSP$ are taken as objective functions for the optimization algorithm, and they can be expressed as:

$$minf = min(COE) \tag{16}$$

$$minf = min(D_{load}) \tag{17}$$

$$minf = min(LPSP) \tag{18}$$

Weighted Total Objective Function

$$minf = min(0.94 \times COE + 0.05 \times D_{load} + 0.01 \times LPSP) \tag{19}$$

C. Decision Variables

The decision variable used in the optimization problem are

1) Number of PV Panels (units)
2) Number of wind Panels (units)
3) Volume of reservoir (m^3)
4) Pump Flowrate (m^3/sec)
5) Turbine Flowrate (m^3/sec)

D. Constraints

The following are the constrains of the proposed method: The proposed system should provide the load with a reliable power supply and satisfy the constraint of reliability:

$$LPSP \leq \beta_L \tag{20}$$

According to the principle of operation of the hydraulic system, the state of charge (SOC) of the upper reservoir is constrained as follows:

$$V_{min} \leq V_{maxproposed} \leq V_{max} \tag{21}$$

Where V_{min} and V_{max} are the minimum and maximum allowable volume of water stored in the tank. V_{min} refers to the amount of water left in the upper reservoir for use in emergency conditions.

The hybrid system will ensure full utilization of renewable source characteristics if the relative fluctuation is smaller than the allowable reference (ε_L):

$$D_{load} \leq \varepsilon_L \tag{22}$$

II. Literature Review

Recent developments in the field of hybrid renewable energy systems, specifically the optimal sizing of solar/wind/HESS, have advanced considerably. [1] stands out for its simultaneous optimization of HESS sizing and EMS. The study introduces innovations such as an improved AMPC and employs a MOEA/D to address the cost-efficiency trade-off of supercapacitors and battery capacity degradation. Furthermore, the integration of the GWO algorithm and SVM in EMS underscores a multidisciplinary approach. [1] establishes performance metrics, including total energy loss, RMS of battery current, and Ah-throughput, providing benchmarks for evaluating future research. While significant, the study's applicability across diverse HESS configurations warrants further exploration.

Reference [2] presents a comprehensive study on the design of a grid-connected PV/wind/HESS for Egypt's Ataka region, focusing on ensuring low energy fluctuation and optimizing solar and wind resource utilization. The research involves a rigorous comparison of various optimization algorithms using MATLAB, with the WOA emerging as the most effective, followed by SSA, WCA, and GWO. WOA excels in achieving the lowest Cost of Energy (COE) and rapid convergence. The designed hybrid system showcases consistent COE and energy stability under diverse conditions, making it a promising and sustainable energy solution for the region. The study demonstrates an in-depth analysis by evaluating multiple optimization algorithms for sizing hybrid power systems, effectively addressing practical applications and introducing novel contributions regarding constraint parameter selection. Reference [2] rigorously models system components, set objectives and constraints, and adopts COE minimization as the primary objective function for optimization techniques, providing valuable insights into the evolving field of hybrid renewable energy systems.

Reference [3] investigates the design of a stand-alone hybrid energy system, integrating photovoltaics (PV), wind, and hydropower with a pumped-storage installation (HSPSI) in Xiaojin, Sichuan, China, aiming to optimize power supply reliability while minimizing investment cost. The analysis incorporates the curtailment rate (CR) of wind and PV power due to policy regulations, employing multi-objective particle swarm optimization (MOPSO) for trade-off assessment and comparing its performance with alternative optimization algorithms. Results demonstrate that the PV-wind-HSPSI hybrid system offers a reduced levelized cost of energy (LCOE) compared to PV-HSPSI and wind-HSPSI systems. The study scrutinizes the relationship between objectives and CR, highlighting CR's influence on investment cost. Conclusively, PSO proves superior to genetic algorithms (GA) and the simulated annealing method (SA) in LCOE optimization while addressing key decision variables, objective functions, and constraints pertaining to system reliability and cost efficiency.

III. Methodology

A. Optimization algorithms

To find the values of decision variables for optimal sizing of the hybrid energy system, simulated annealing (SA), Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Firefly Agorithm (FA) algorithms were used to perform a comprehensive study on the system.

1) Simulated Annealing Algorithm: Simulated annealing is a probabilistic technique for approximating the global optimum of a given function. It is a metaheuristic to approximate global optimization in a large search space for an optimization problem. For large numbers of local optima, SA can find the global optima.

2 different case studies were performed using the simulated annealing algorithm with different inputs for each. The first case study was performed using an input temperature of 100 degrees. The second case study was performed with an input temperature of 1000 degrees and 5 iterations per temperature change. The parameters for the equations remained constant for both cases

The above figures show the output from using simulated annealing on the first case study.

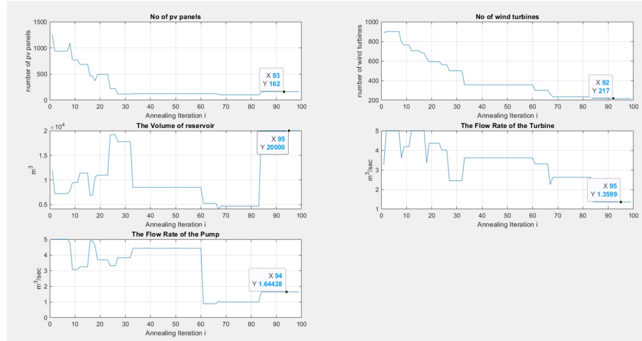

Fig. 3: Output using simulated annealing in first case study

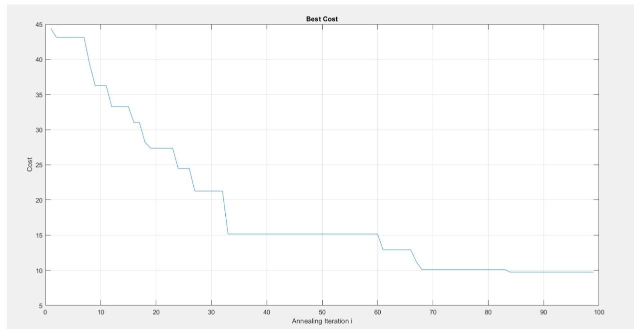

Fig. 4: Cost output from simulated annealing in first case study

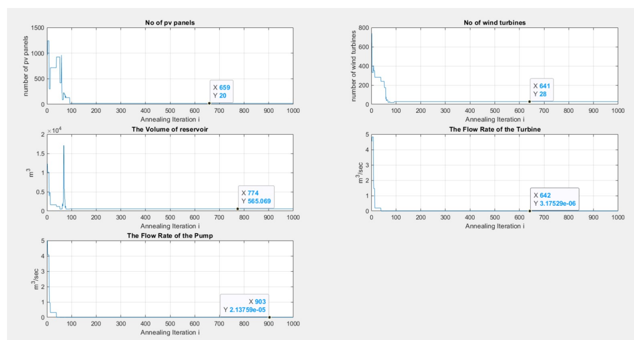

Fig. 5: Output using simulated annealing in second case study

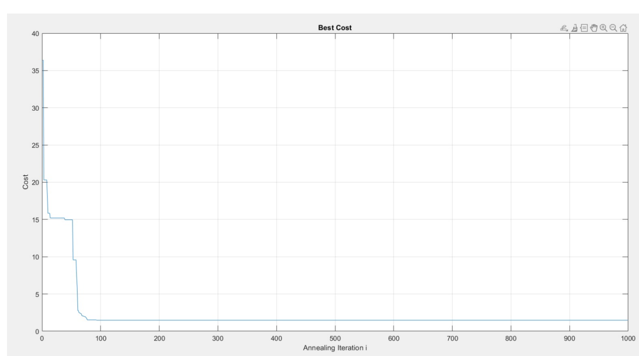

Fig. 6: Cost output from simulated annealing in second case study

The figures shown are the output from using simulated annealing on the first case study. In the first case study, 3 shows the final output that the system should have 162 pv panels, 217 wind turbines, the reservoir volume should be 20,000 m^2, the flow rate of the turbine should be 1.359 m^3/s, and the flow rate of the pump should be 1.64m^3/s and figure 3.3 shows that final cost output is about 10 according to figure 4.

The figures show the output from using simulated annealing on the second case study. In the second case study, figure 5 shows the final output that the system should have 20 pv panels, 28 wind turbines, the reservoir volume should be 565 m^2, the flow rate of the turbine should be 3.172e-06 m^3/s, and the flow rate of the pump should be 2.136e-05m^3/s and figure 6 shows that final cost output is about 2.

B. Genetic Algorithm

Genetic algorithm (GA) is a population based stochastic optimization algorithm inspired by the process of natural selection and genetics. Its basic idea is to mimic the process of natural evolution to evolve potential solutions to a problem.

1) Case Study 1: The effect of changing the value of alpha on the results

```
w_COE=0.9;
w_D_Load=0.02;
w_LSPS=0.08;



% Genetic Algorithm with Elitism, CrossOver and Mutation
% GA Parameters
populationSize = 50;
numGenes = 5;
numGenerations = 100;
elitePercentage = 0.2;
crossoverPercentage = 0.6;
mutationPercentage = 0.2;
alpha = 0.3;
```

Fig. 7: Parameters after changing the value of alpha

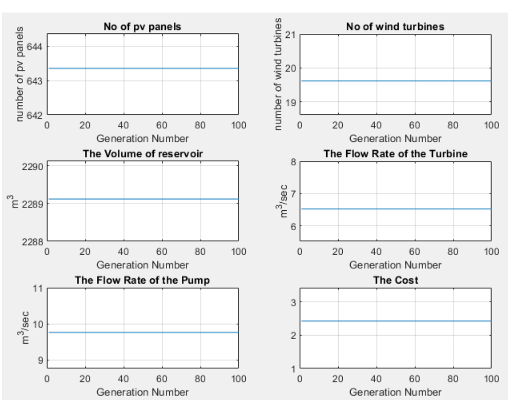

Fig. 8: Simulation Results after changing alpha value

2) Case Study 2: The effect of changing the size of elites and mutations on the results by increasing mutation size to increase exploration rate and reduced number of elites to reduce exploitation.

```
w_COE=0.9;
w_D_Load=0.02;
w_LSPS=0.08;


% Genetic Algorithm with Elitism, CrossOver and Mutation
% GA Parameters
populationSize = 50;
numGenes = 5;
numGenerations = 100;
elitePercentage = 0.1;
crossoverPercentage = 0.6;
mutationPercentage = 0.3;
alpha = 0.7;
```

Fig. 9: Parameters after changing elite and mutation

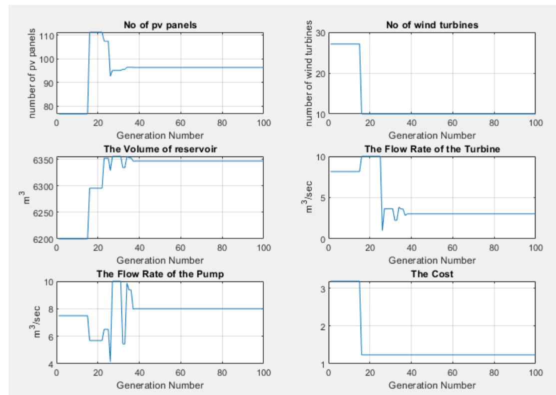

Fig. 10: Simulation Results after changing elite and mutation

```
% Genetic Algorithm with Elitism
% GA Parameters
populationSize = 50;
numGenes = 5;
numGenerations = 100;
elitePercentage = 0.2;
crossoverPercentage = 0.6;
mutationPercentage = 0.2;
alpha = 0.7;
```

```
clc ;
clear;
%Cost function weights and cost function evaluation
w_COE=0.9;
w_D_Load=0.02;
w_LSPS=0.08;
```

Fig. 11: GA and SA Algorithms Comparison Parameters.

3) Optimality: The following parameters were used to compare the GA and the SA performance

The results obtained by these parameters conducted by GA are used to be compared with the SA and are shown below in the following figure

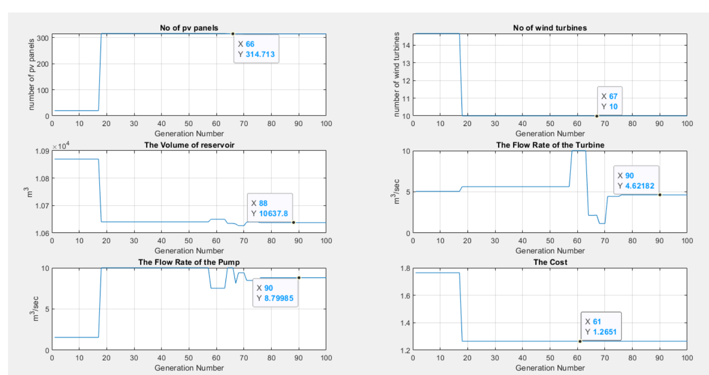

Fig. 12: GA obtained results used in SA Comparison

After performing 10 simulations using same parameters the best results were chosen for both GA and SA to be compared. By conducting 100 iterations to compare GA and SA, the fitness value of the SA is 27 as shown in the figure below while in the GA it reached value less than that so after 100 iterations GA showed less fitness value being more efficient.

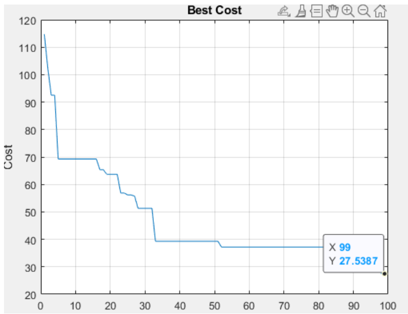

Fig. 13: SA Fitness Value

4) Repeatability: After performing 10 simulations using same parameters the best results were chosen for both GA and SA to be compared. Repeatability is used to compare which technique would converge faster to the answer. GA showed faster conversion than SA

Most of the times when running the genetic algorithm, the first elite value ends up being the most optimal choice because none of the children after the elite value are able to obtain the fitness lower than the first elite value and so throughout simulation process the elite value never changed due to the fact that not of its children in next generations could surpass this but it could change sometimes in the previous simulations.

C. Particle Swarm Optimization

Particle swarm optimization (PSO) is a powerful meta-heuristic optimization algorithm inspired by swarm behavior observed in nature, such as fish and bird schooling.

Figure 14 shows the PSO Algorithm with population size of 20 and the inertia, cognitive, and social factors as 0.792, 1.4944, and 1.4944, respectively.

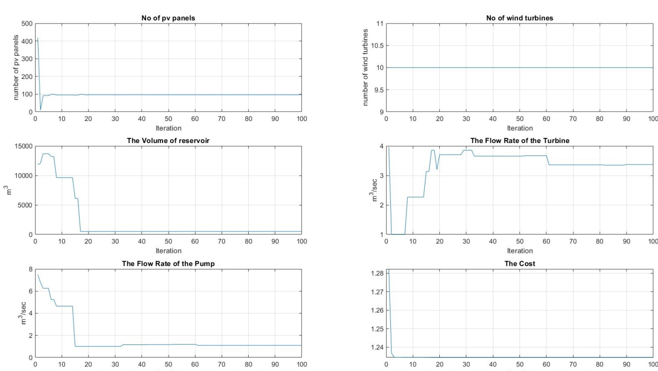

Fig. 14: PSO Algorithm Response

Three case studies were performed to gauge the effect of changing the population size and the PSO algorithm parameters on the PSO algorithm.

1) Case Study 1: Population Size: Reducing the population size from 20 to 3 seems to have slowed down the algorithm's convergence. The initial random values also led to a less favorable initial result. These factors, combined with the small population size, contributed to the slower convergence, as illustrated in Figure 15.

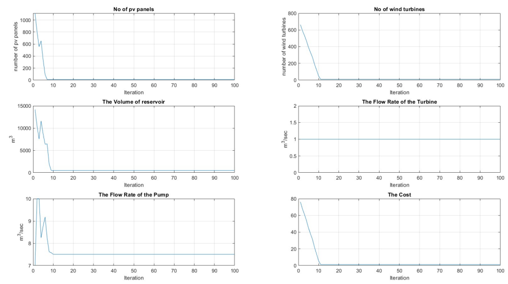

Fig. 15: PSO First Case Study

2) Case Study 2: Inertia Factor: Halving the inertia factor from 0.8 to 0.4 weakens the algorithm, leading to less exploration and convergence to local minima. This effect becomes more apparent when the population size is kept low (3) to induce a less favorable starting best value, as shown in Figure 17.

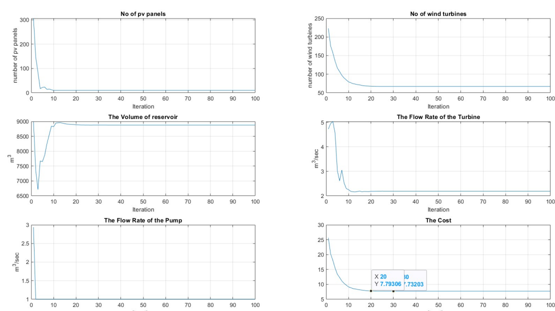

Fig. 16: PSO Second Case Study

3) Case Study 3: Soical & Cognitive Factors: Giving dominance to the Cognitive factor by reducing the value of the Social factor to 0.1, while keeping the Cognitive factor at 1.4944, leads to extremely slow convergence and easily results in getting stuck in local minima, as shown in Figure 17.

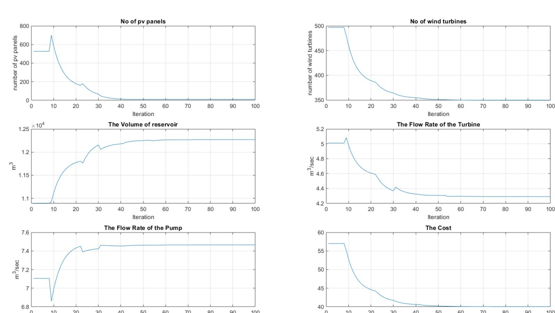

Fig. 17: PSO Third Case Study

D. *Firefly Algorithm*

Firefly algorithm is a metaheuristic algorithm inspired by the flashing behavior of fireflies and the phenomenon of bioluminescent communication.

Figure 18 shows the FA Algorithm with population size of 20 and the Randomization (alpha), absorption (gamma), and Attraction (beta) factors as 1, 1, and 1.5, respectively.

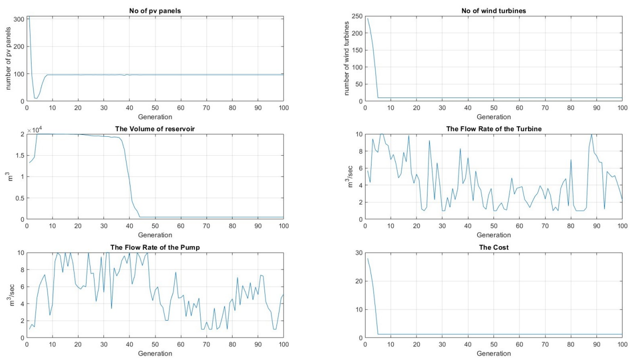

Fig. 18: Firefly Algorithm Results

We have performed two case studies to study the effect of changing the absorption and attraction coefficients on the algorithm

1) Case Study 1: Absorption coefficient: The algorithm continues to function close to normal when lowering the absorption coefficient. However, increasing it significantly to a large value, such as 30, leads to no convergence, and the particles move randomly, sometimes increasing the cost and sometimes lowering it, as shown in Figure 19.

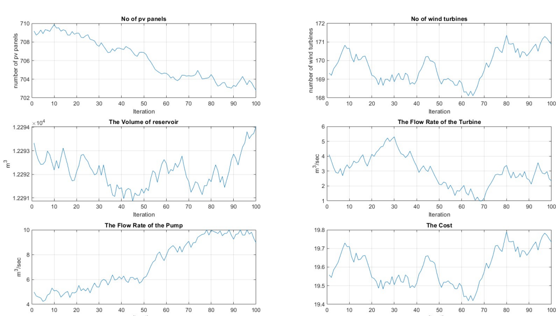

Fig. 19: FA First Case Study

2) Case Study 2: Attraction coefficient: In contrast to the absorption coefficient, greatly increasing the attraction coefficient does not cause the code to stop working. However, decreasing the attraction coefficient to a low number, such as 0.1, leads to very slow convergence, as shown in Figure 20.

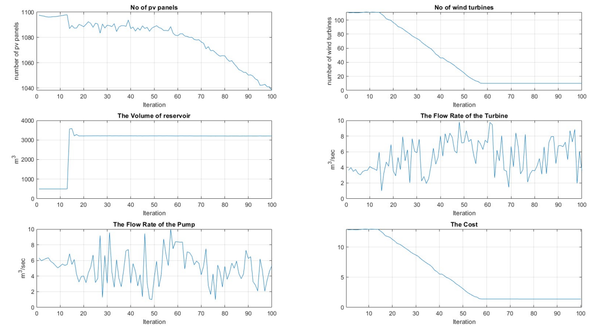

Fig. 20: FA Second Case Study

IV. Results and Discussion

The following table shows a comparison among the three implemented algorithms (SA - GA - PSO - FA)

Optimization Method	nPV	nWind	V_max	Q_T	Q_P
PSO	96.34487	10	500	2.95016	1
GA	31.012922	10	4819.6016	10	1
SA	455.7437	96.274	1675.83939	2.08160	1.70835
FA	96.34	10	500	1	6.081

TABLE I: Optimal Solution of the 5 parameters

Parameter	SA	GA	PSO	FA
Optimal Fitness Value	11.1585	1.23599	1.23444	1.234
Mean Fitness	23.47665	6.43196	1.23444	1.255
Standard Deviation	0.7563448	0.22151	$8.3377\text{x}10^{-14}$	0.001059
Computational Time	0.018879105	0.1042	0.096632	1.01457

TABLE II: Implemented Optimization methods KPI Comparison

In 20 runs, PSO algorithm reached the optimal solution with fitness value of 1.234. Also , the PSO showed the lowest standard deviation indicating its repeatability as well. Furthermore, the optimal fitness value reached by GA is very close to the optimal value produced by the PSO. SA was the fastest algorithm in terms of computational time. However, for population-based algorithms only, PSO was the fastest, FA was the slowest algorithm and second best algorithm in terms of results and standard deviation after PSO.

V. Conclusion and Future Recommendations

References

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