Recommender system

Mohamed Elhaj-Abdou

Recommender systems

Problem Formulations

Content based approach

Collaborative Filtering

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Types of Recommender systems

Problem Formulations

Content based approach

Collaborative Filtering

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Problem Formulation

Lets say you are in Amazon or Netflix companies

We want to predict The series rating

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Problem Formulation

We want to predict
The series rating

Series name

House MD

The office

The modern Family

Lucifer

Users

Mohamed Ahmed Samar Sara

User rates Series using 1-5 stars

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Problem Formulation

We want to predict
The series rating

Series name

	Mohamed	Ahmed	Samar	Sara
House MD	5	5	?	0
The office	?	4	3	0
The modern Family	4	0	5	4
Lucifer	0	0	0	?

User rates Series using 1-5 stars

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Problem Formulation

Series name	Mohamed	Ahmed	Samar	Sara
House MD	5	5	?	0
The office	?	4	3	0
The modern Family	4	0	5	4
Lucifer	0	0	0	?

Notations

N_u → No. Users

4

N_m → No. Movies

4

r(i, j) → 1 if user j has rates movie I
0 other wise
y(i, j) → rating (1-5) given by user j to movie I

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Problem Formulation

Notations

N_u → Number of users

N_m → Number of Movies

r(i, j)

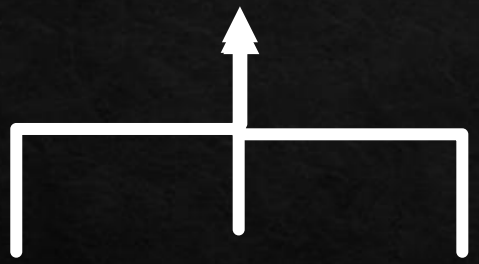
1 → if user j has rates movie i

0 → other wise

y(i, j) → rating (1-5) stars given by user j to movie i (defined only of r(i, j)=1) → this means that the user make a rates not ?

Problem Formulation

Given Dataset



r(i,j)

y(i,j)

No users

No Movies

Looks all the missing what question marks should be

As the question marks means that the user did not watched the movie

We want to predict the rating (?) for each user

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Problem Formulation

Depends on the predicted rating we can recommend the movie that have high rating to the use

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Example

Series name	Mohamed	Ahmed	Samar	Sara
House MD	5	5	?	0
The office	?	4	3	0
The modern Family	4	0	5	4
Lucifer	0	0	0	?

Mohamed→ likes mestry and family series types, because he likes (house and modern family

Mohamed→ donot likes fantasy he gives to Lucifer zero rating

Example

Series name	Mohamed	Ahmed	Samar	Sara
House MD	5	5	?	0
The office	4	4	3	0
The modern Family	4	0	5	4
Lucifer	0	0	0	?

Mohamed→ likes mestry and family series types, because he likes (house and modern family

Mohamed→ donot likes fantasy he gives to Lucifer zero rating

The office seems to be funny and family

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Example

Series name	Mohamed	Ahmed	Samar	Sara
House MD	5	2	?	0
The office	4	4	3	0
The modern Family	4	3	5	4
Lucifer	0	5	0	?

Ahmed→ likes the office and, and likes fantasy so he likes Lucifer

Ahmed → seems little bit likes modern fam but not much

House Md seems to not be the best choice for him

Recommender systems

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Content based R.S

Series name	Mohamed	Ahmed	Samar	Sara
House MD	5	?	?	0
The office	?	4	3	0
The modern Family	4	3	5	4
Lucifer	0	5	0	?

How we can predict these ? Marks to recommend the right movie

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Content based R.S

Suppose that you have for each movie a set of features for them→ lets say three

Funny Family Mystery

These features is a measurements for the degree of funny or family or mystery

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Content based R.S

	Mohamed	Ahmed	Samar	Sara	X1 Funny	X2 Family	X3 Mystery
House MD	5	?	?	0	0.3	0	0.9
The office	?	4	3	0	1	0.4	0
The modern Family	4	3	5	4	0.4	1	0
Lucifer	0	5	0	?	1	0	0.4

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Content based R.S

	Mohamed	Ahmed	Samar	Sara	X1 Funny	X2 Family	X3 Mystery
House MD	5	?	?	0	0.3	0	0.9
The office	?	4	3	0	1	0.4	0
The modern Family	4	3	5	4	0.4	1	0
Lucifer	0	5	0	?	1	0	0.4

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Content based R.S

By this each movie can be represented with a feature vector

Feature vectors 1

House MD→ X^1

$$X^1 = \begin{bmatrix} 0.3 \\ 0 \\ 0.9 \end{bmatrix}$$

Feature vectors 2

The office → X^2

$$X^2 = \begin{bmatrix} 1 \\ 0.4 \\ 0 \end{bmatrix}$$

Feature vectors 3

The modern Family → X^3

$$X^3 = \begin{bmatrix} 0.4 \\ 1 \\ 0 \end{bmatrix}$$

And So on .....

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Content based R.S

For Each user j, learn a parameter $\theta^{(j)}$ predict user j as rating movie i with this following equation

$$\theta^{(j)T} x^{i}$$

Separate linear regression problem for each user

Parameter to be optimized

Feature i

Content based R.S

Lets take a concrete example

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	Mohamed $\theta^{(1)}$	Ahmed $\theta^{(2)}$	Samar $\theta^{(3)}$	Sara $\theta^{(4)}$	X1 Funny	X2 Family	X3 Mystery
House MD	5	?	?	0	0.3	0	0.9
The office	?	4	3	0	1	0.4	0
The modern Family	4	3	5	4	0.4	1	0
Lucifer	0	5	0	?	1	0	0.4

The parameters associated with each user

Now lets make a prediction for Mohamed about the office

	Mohamed $\theta^{(1)}$	Ahmed $\theta^{(2)}$	Samar $\theta^{(3)}$	Sara $\theta^{(4)}$	X1 Funny	X2 Family	X3 Mystery
House MD	5	?	?	0	0.3	0	0.9
The office	?	4	3	0	1	0.4	0
The modern Family	4	3	5	4	0.4	1	0
Lucifer	0	5	0	?	1	0	0.4

Now lets make a prediction for Mohamed about the office

$$X^2 = \begin{bmatrix} 1 \\ 0.4 \\ 0 \end{bmatrix} \qquad \theta^{(1)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

The office parameters

Some how the parameters are arrived with this values. Later we will talking about it how we can get it

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	Mohamed $\theta^{(1)}$	Ahmed $\theta^{(2)}$	Samar $\theta^{(3)}$	Sara $\theta^{(4)}$	X1 Funny	X2 Family	X3 Mystery
House MD	5	?	?	0	0.3	0	0.9
The office	2	4	3	0	1	0.4	0
The modern Family	4	3	5	4	0.4	1	0
Lucifer	0	5	0	?	1	0	0.4

Now lets make a prediction for Mohamed about the office

$$X^2 = \begin{bmatrix} 1 \\ 0.4 \\ 0 \end{bmatrix} \qquad \theta^{(1)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

$$\theta^{(1)^T} x^3 = 0.4 * 5 = 2$$

The office parameters

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Problem Formulation

r(i, j) 1→ if user j has rates movie i (0 → otherwise)

y(i, j)→ rating by user j to movie i (if defined)

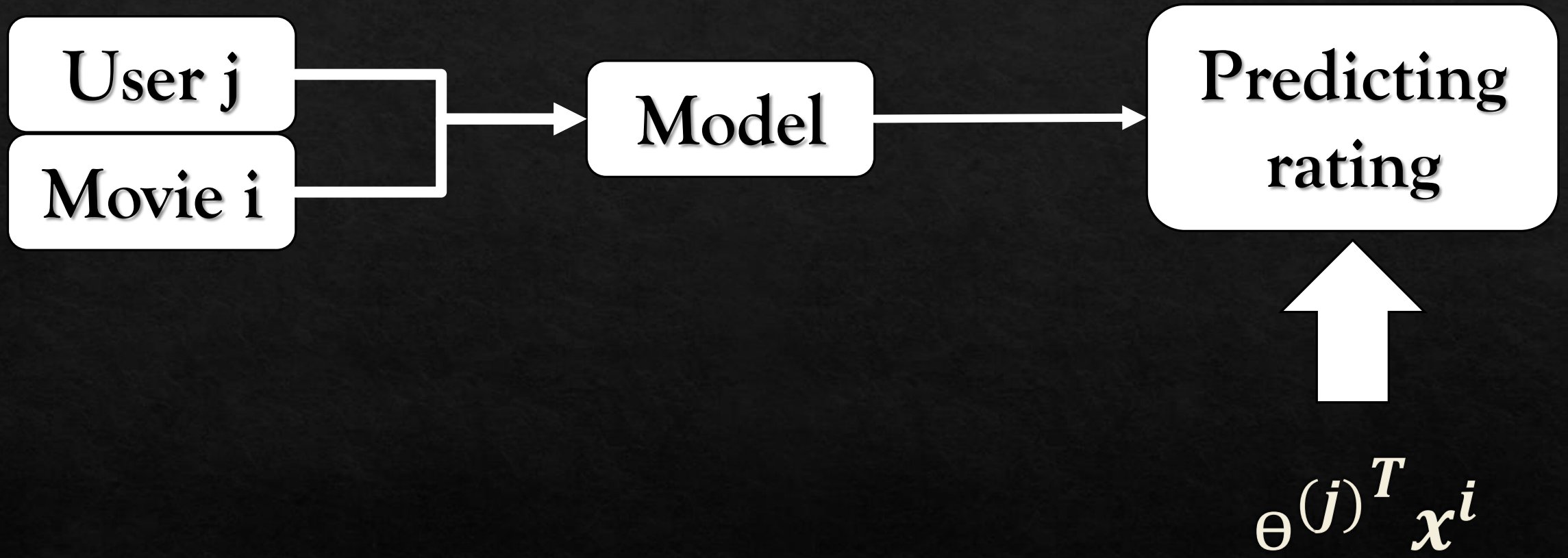
$\theta^{(j)}$ →parameter vector for user j

$x^{(i)}$ →feature vector for movie i

N_u→ Number of users

N_m→ Number of Movies

$m^{(j)}$→ Number of Movies rated by user j


User j
Movie i
Model
Predicting rating
$\theta^{(j)^T} x^i$

to learn $\theta^{(j)}$ we want to minimize the parameter theta we need to recap linear regression looks like

$$\boldsymbol{Min}\ \theta^{(j)} \rightarrow \frac{1}{2m^{(j)}} \sum_{i:r(i,j)=1}^{m} \left(\theta^{(j)^T} x^i - y_{true}\right)^2$$

Summing over all the movies that user j has rated

Prediction

True
y(i, j)

In content based Recommender system we need to make some modifications

$$\boldsymbol{Min}\ \theta^{(j)} \rightarrow \frac{1}{2m^{(j)}} \sum_{i:r(i,j)=1}^{m} \left(\theta^{(j)^T} x^i - \boldsymbol{y_{true}}\right)^2$$

We want to learn the parameters for N_u users

$$\boldsymbol{Min}\ \theta^{(j)} \rightarrow \frac{1}{2m^{(j)}} \sum_{j=1}^{N_u} \sum_{i:r(i,j)=1}^{m} \left(\theta^{(j)^T} x^i - \boldsymbol{y_{true}}\right)^2$$

In content based Recommender system we need to make some modifications

$$\boldsymbol{Min}\ \theta^{(j)} \rightarrow \frac{1}{2m^{(j)}} \sum_{j=1}^{N_u} \sum_{i:r(i,j)=1}^{m} \left(\theta^{(j)^T} x^i - \boldsymbol{y_{true}}\right)^2$$

Making prediction for all the users parameters ($\theta^{(j)}$) for all the movies

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And then use the gradient decent

Content base approach need x_features that represents how much the movie is action or family or funny

The problem is that its hard and rare to find a movies with this kind of data

collaborative filtering based approach is found

Recommender systems

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Content based approach

Collaborative Filtering

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Collaborative filtering

In Collaborative filtering we have no idea about the features

In Collaborative filtering use inferencing method to find the features

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	Mohamed θ^1	Ahmed θ^2	Samar θ^3	Sara θ^4	X1 Funny	X2 Family	X3 Mystery
House MD	5	?	?	0	?	?	?
The office	?	5	3	0	?	?	?
The modern Family	4	3	5	4	?	?	?
Lucifer	0	0	0	?	?	?	?

$$\theta^1 = \begin{bmatrix} 0 \\ 0 \\ \mathbf{5} \end{bmatrix} \quad \theta^2 = \begin{bmatrix} 0 \\ 0 \\ \mathbf{5} \end{bmatrix} \quad \theta^3 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix} \quad \theta^4 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix}$$

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	Mohamed θ^1	Ahmed θ^2	Samar θ^3	Sara θ^4	X1 Funny	X2 Family	X3 Mystery
House MD	5	5	?	0	?	?	?
The office	?	5	3	0	?	?	?
The modern Family	4	?	5	4	?	?	?
Lucifer	0	0	0	?	?	?	?

$$\theta^1 = \begin{bmatrix} 0 \\ 0 \\ \mathbf{5} \end{bmatrix}$$

Mohamed likes mestry

$$\theta^2 = \begin{bmatrix} 5 \\ 0 \\ \mathbf{0} \end{bmatrix}$$

Ahmed likes Funny

$$\theta^3 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix}$$

Samar likes family

$$\theta^4 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix}$$

Sara likes family

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	Mohamed θ^1	Ahmed θ^2	Samar θ^3	Sara θ^4	X1 Funny	X2 Family	X3 Mystery
House MD	5	5	?	0	?	?	?
The office	?	5	3	0	?	?	?
The modern Family	4	?	5	5	?	?	?
Lucifer	0	0	0	?	?	?	?

$$\theta^1 = \begin{bmatrix} 0 \\ 0 \\ \mathbf{5} \end{bmatrix}$$

Mohamed likes mestry

$$\theta^2 = \begin{bmatrix} 0 \\ 0 \\ \mathbf{5} \end{bmatrix}$$

Ahmed likes mestry

$$\theta^3 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix}$$

Samar likes family

$$\theta^4 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix}$$

Sara likes family

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	Mohamed θ^1	Ahmed θ^2	Samar θ^3	Sara θ^4	X1 Funny	X2 Family	X3 Mystery
House MD	5	5	?	0	?	?	?
The office	?	5	3	0	?	?	?
The modern Family	4	?	5	5	?	?	?
Lucifer	0	0	0	?	?	?	?

$$\theta^1 = \begin{bmatrix} 0 \\ 0 \\ \mathbf{5} \end{bmatrix}$$

Mohamed likes mestry

$$\theta^2 = \begin{bmatrix} 0 \\ 0 \\ \mathbf{5} \end{bmatrix}$$

Ahmed likes mestry

$$\theta^3 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix}$$

Samar likes family

$$\theta^4 = \begin{bmatrix} 0 \\ 5 \\ \mathbf{0} \end{bmatrix}$$

Sara likes family

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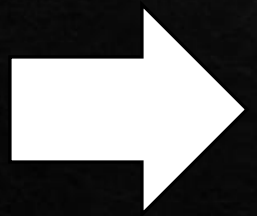
	Mohamed θ^1	Ahmed θ^2	Samar θ^3	Sara θ^4	X1 Funny	X2 Family	X3 Mystery
House MD	5	5	?	0	0	0	1
The office	?	5	3	0	?	?	?
The modern Family	4	?	5	5	?	?	?
Lucifer	0	0	0	?	?	?	?

Mohamed likes mestry

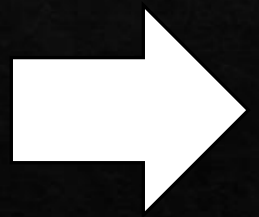
Ahmed likes mestry

Samar likes family

Sara likes family



As mohamed and Ahmed gives 5 starts to house MD



We can deduce that house MD is mestry series

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	Mohamed θ^1	Ahmed θ^2	Samar θ^3	Sara θ^4
House MD	5	5	?	0
The office	?	5	3	0
The modern Family	4	?	5	5
Lucifer	0	0	0	?

X1 Funny	X2 Family	X3 Mystery
0	0	1
?	?	?
0	1	0
?	?	?

Mohamed likes mestry

Ahmed likes mestry

Samar likes family

Sara likes family

As Samar and sara rates 5 stars on modern family

We can deduce that modern family is family series

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Given the features → Estimate the parameters

Given the parameters → Estimate the Features

Given $x^{(1)}, \ldots, x^{(n_m)}$, estimate $\theta^{(1)}, \ldots, \theta^{(n_u)}$:

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2$$

Given $\theta^{(1)}, \ldots, \theta^{(n_u)}$, estimate $x^{(1)}, \ldots, x^{(n_m)}$:

$$\min_{x^{(1)},\ldots,x^{(n_m)}} \frac{1}{2} \sum_{i=1}^{n_m} \sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^{n} (x_k^{(i)})^2$$

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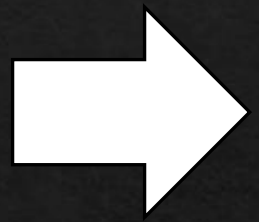
What if we want the both



We can go back and forth to solve the problem simultaneously

Given $x^{(1)}, \ldots, x^{(n_m)}$, estimate $\theta^{(1)}, \ldots, \theta^{(n_u)}$:

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2$$

Given $\theta^{(1)}, \ldots, \theta^{(n_u)}$, estimate $x^{(1)}, \ldots, x^{(n_m)}$:

$$\min_{x^{(1)},\ldots,x^{(n_m)}} \frac{1}{2} \sum_{i=1}^{n_m} \sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^{n} (x_k^{(i)})^2$$

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Content based

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