Definitions:

- x : process model states
- ϕ : state-transition matrix
- H : measurement matrix
- z : measurement matrix
- $\hat{x}_0^-$: initial conditions of states
- P_o^- : initial error covariance matrix
- R : variance of measurement error matrix
- Q : variance of process model noise
- λ : forgetting factor $0 < \lambda < 2$
- λ^* : initial forgetting factor

Notations:

- $\hat{}$: estimate
- $\hat{x}_k^-$: a prior estimate of x_k
- $\hat{x}_k^+$: a posterior estimate of x_k

Discrete Kalman Filter:

1. Guess initial values of P_0^- and $\hat{x}_0^-$
2. Calculate the gain :
   - $K_k = P_k^- H^T \left(H P_k^- H^T + R \right)^{-1}$
3. Update estimate :
   - $\hat{x}_k^+ = \hat{x}_k^- + K_k \left(z_k - H \hat{x}_k^- \right)$
4. Update error :
   - $P_k^+ = (I - K_k H) P_k^-$
5. Project ahead :
   - $\hat{x}_{k+1}^- = \phi \hat{x}_k^+$
   - $P_{k+1}^- = \phi P_k^+ \phi^T + Q$
   - $P_{k+1}^- = \frac{P_{k+1}^- + P_{k+1}^{-T}}{2}$

Discrete Kalman Filter with Forgetting Factor λ:

1. Guess initial values of P_0^- and $\hat{x}_0^-$
2. Calculate the gain :
   - $K_k = P_k^- H^T \left(H P_k^- H^T + R\lambda \right)^{-1}$
3. Update estimate :
   - $\hat{x}_k^+ = \hat{x}_k^- + K_k \left(z_k - H \hat{x}_k^- \right)$
4. Update error :
   - $P_k^+ = (I - K_k H) \frac{P_k^-}{\lambda}$
5. Project ahead :
   - $\hat{x}_{k+1}^- = \phi \hat{x}_k^+$
   - $P_{k+1}^- = \phi P_k^+ \phi^T + Q$
   - $P_{k+1}^- = \frac{P_{k+1}^- + P_{k+1}^{-T}}{2}$

Discrete Kalman Filter with Varying Forgetting Factor λ:

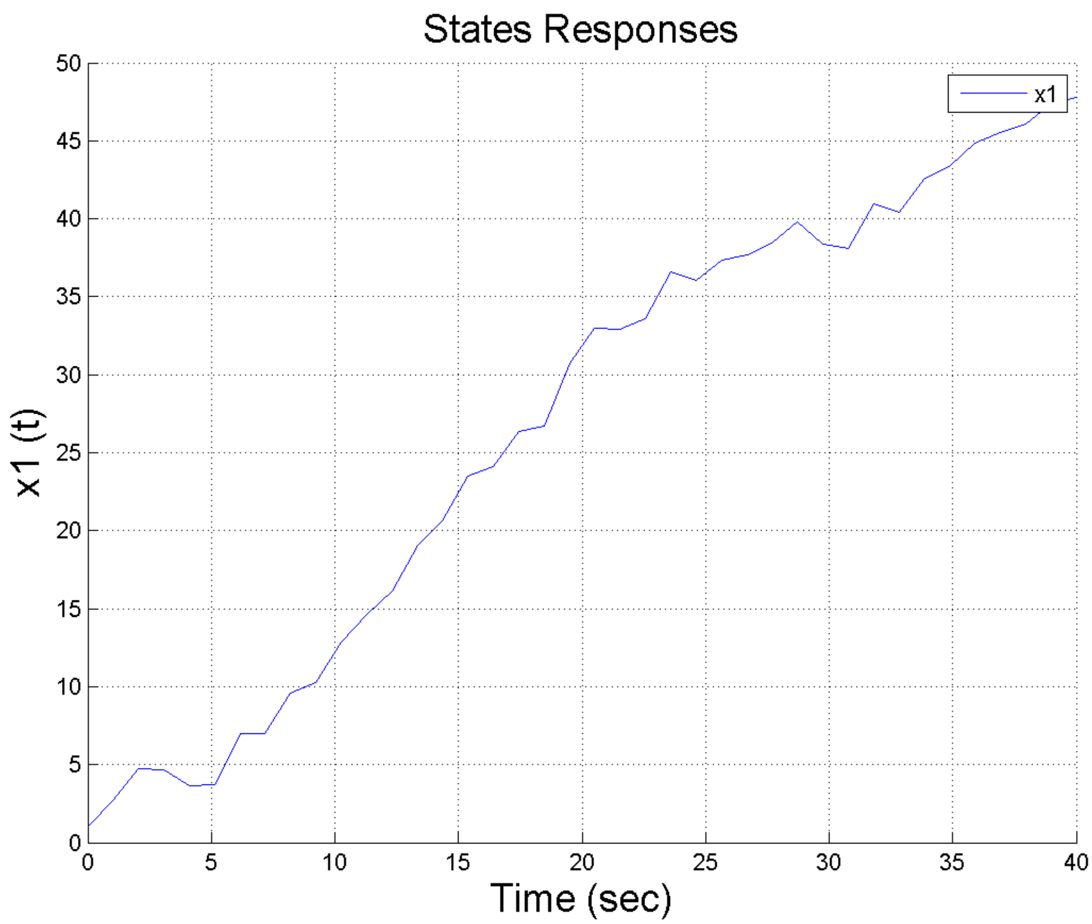
1. Guess initial values of P_0^- , $\hat{x}_0^-$ and λ^*
2. Calculate the gain :
   - $K_k = P_k^- H^T \left(H P_k^- H^T + R\lambda_k\right)^{-1}$
3. Update estimate :
   - $\hat{x}_k^+ = \hat{x}_k^- + K_k \left(z_k - H\hat{x}_k^-\right)$
4. Update error :
   - $P_k^+ = (I - K_k H) \frac{P_k^-}{\lambda_k}$
5. Project ahead :
   - $\hat{x}_{k+1}^- = \phi\hat{x}_k^+$
   - $P_{k+1}^- = \phi P_k^+ \phi^T + Q$
   - $P_{k+1}^- = \frac{P_{k+1}^- + P_{k+1}^{-T}}{2}$
6. Forgetting factor λ:
   - $\epsilon = \left(z_k - H\hat{x}_k^+\right)$
   - $\lambda_{k+1} = 1 - \frac{\left(1 - \hat{x}_k^{+T} K_k\right)\epsilon^2}{\sigma^2(\epsilon)\mu(\epsilon)}$
   - $\lambda_{k+1} = \begin{cases} > 0.95 & \lambda_{k+1} = 0.95 \\ < 0.3 & \lambda_{k+1} = 0.3 \\ else & \lambda_{k+1} = \lambda_{k+1} \end{cases}$

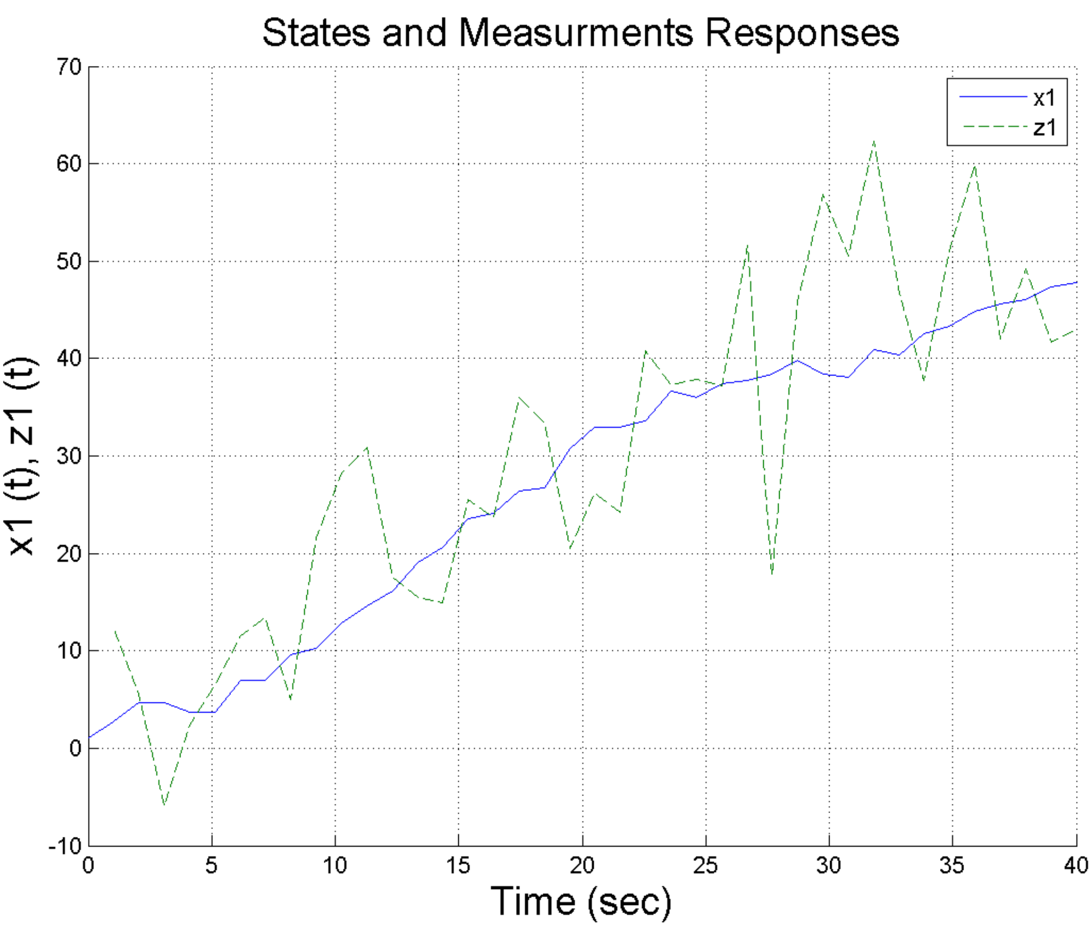
Example:

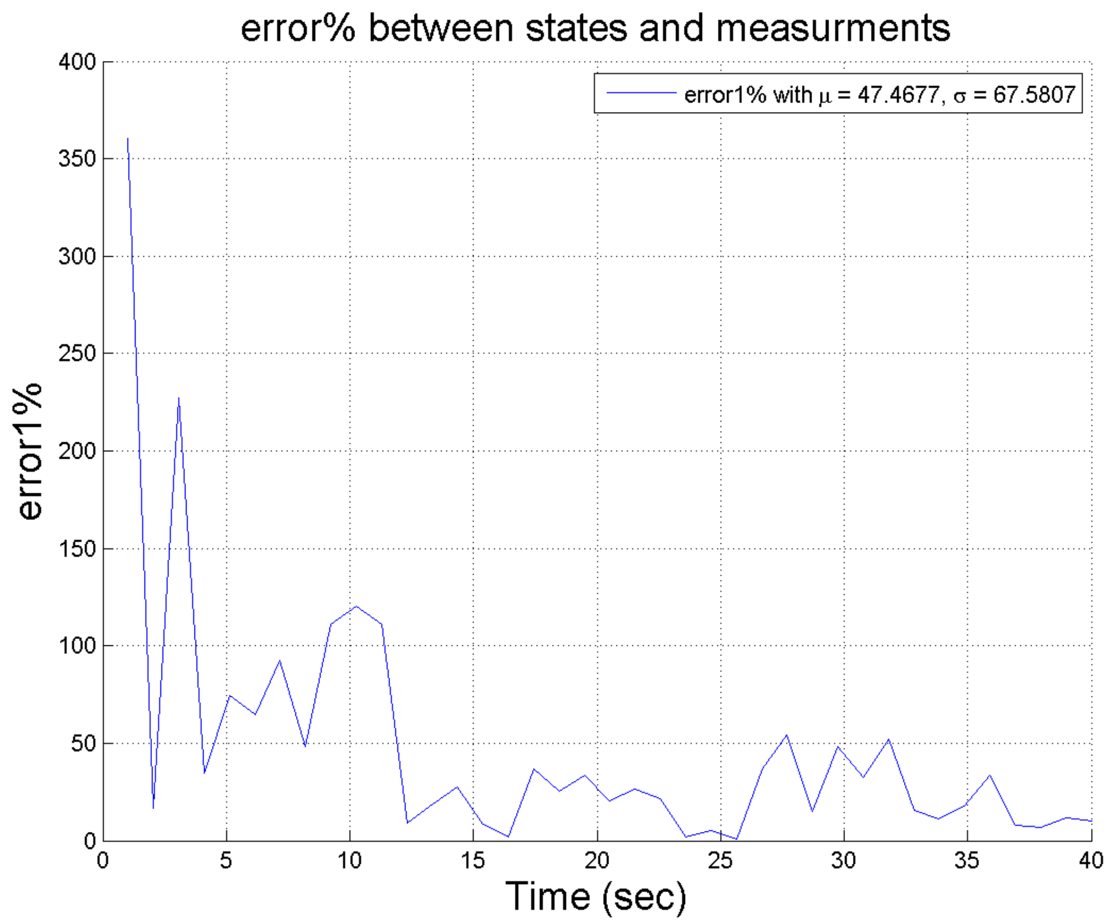
- $\phi = 1$
- $H = 1$
- $R = 100$
- $Q = 1$
- $\hat{x}_0^- = 1$
- $P_0^- = 0$
- $\Delta t = 1.0256$
- $x(1) = 1$
- $x(k+1) = \phi x(k) + \Delta t + normrnd(0, \sqrt{Q})$
- $z(k) = Hx(k) + normrnd(0, \sqrt{R})$
- $\lambda = 0.9$
- $\lambda^* = 0.9$

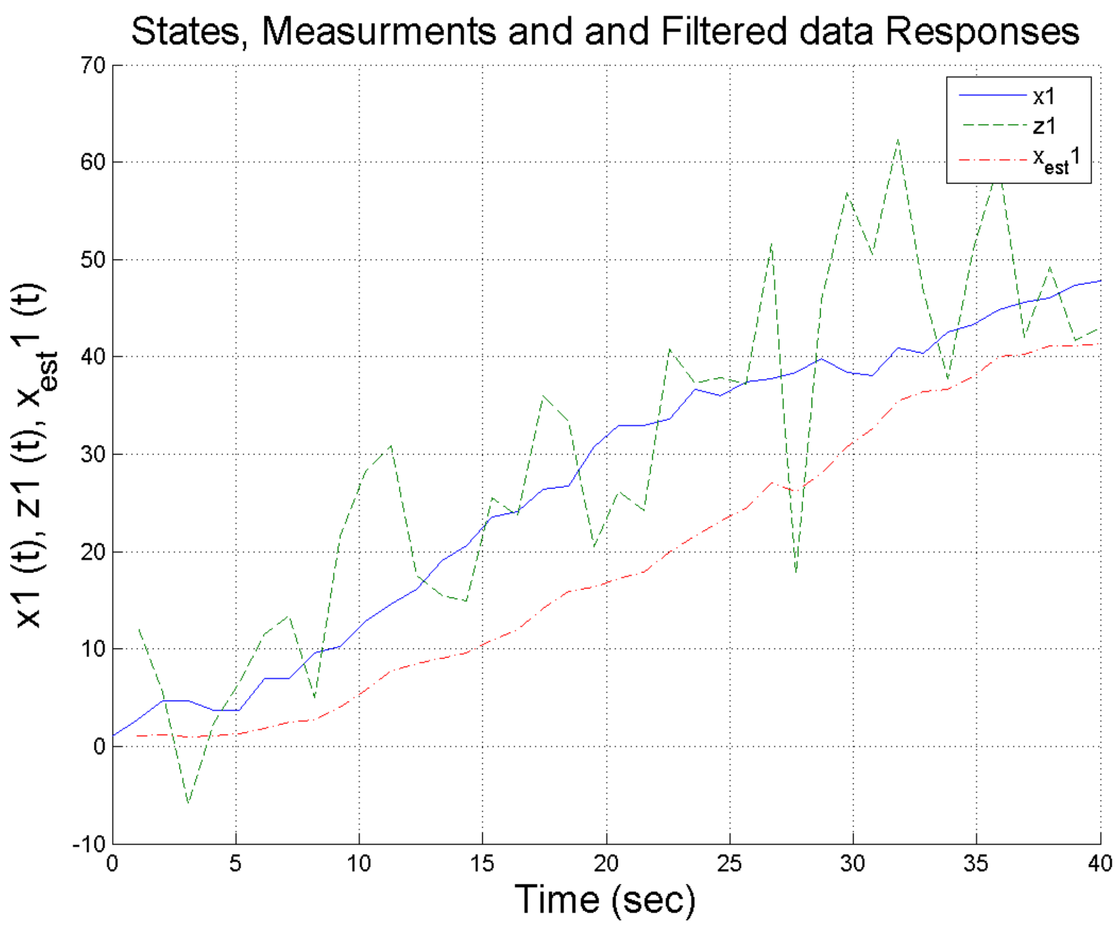
Results :

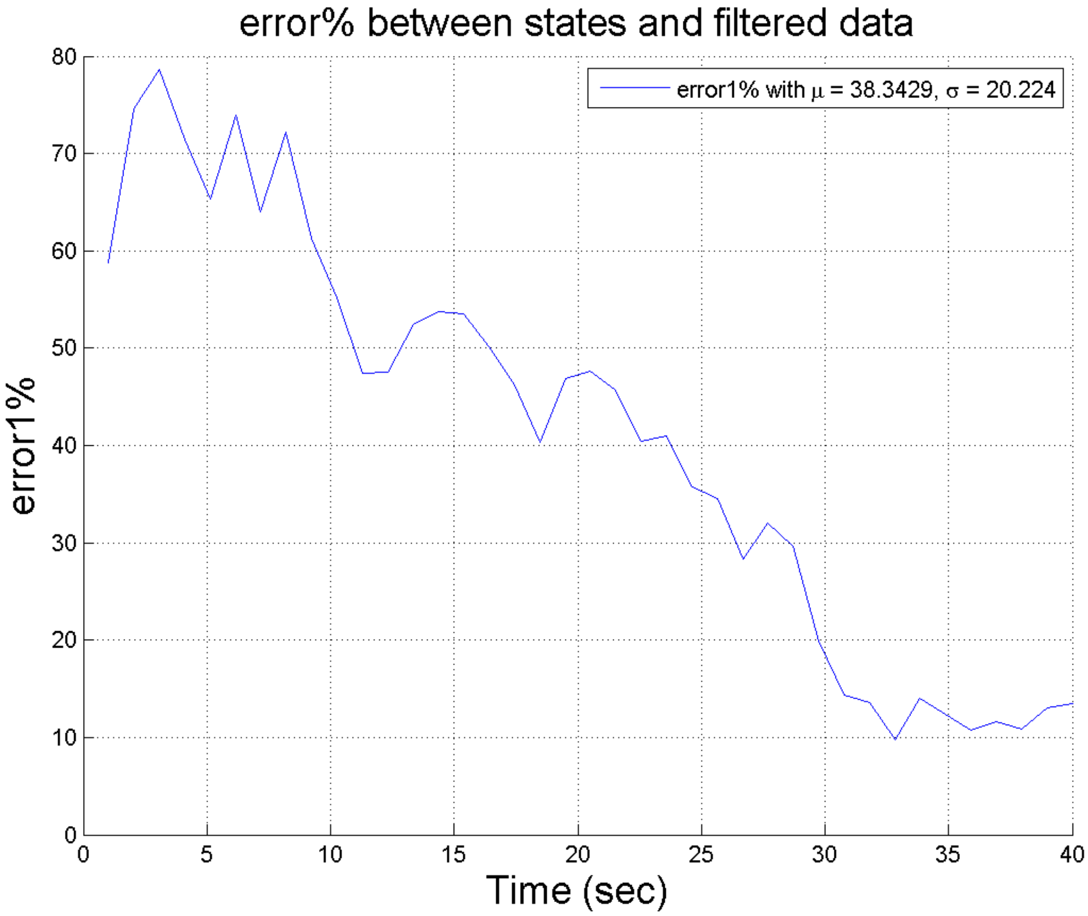
Discrete Kalman Filter:



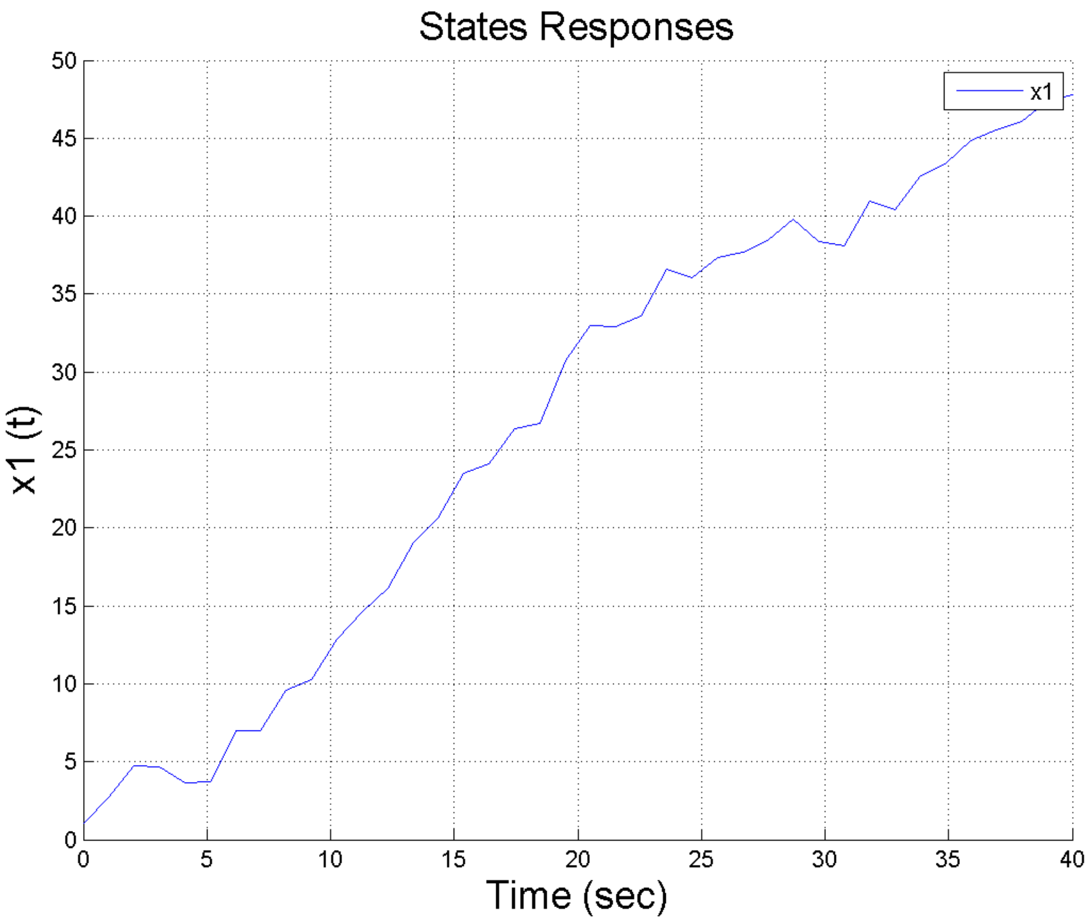


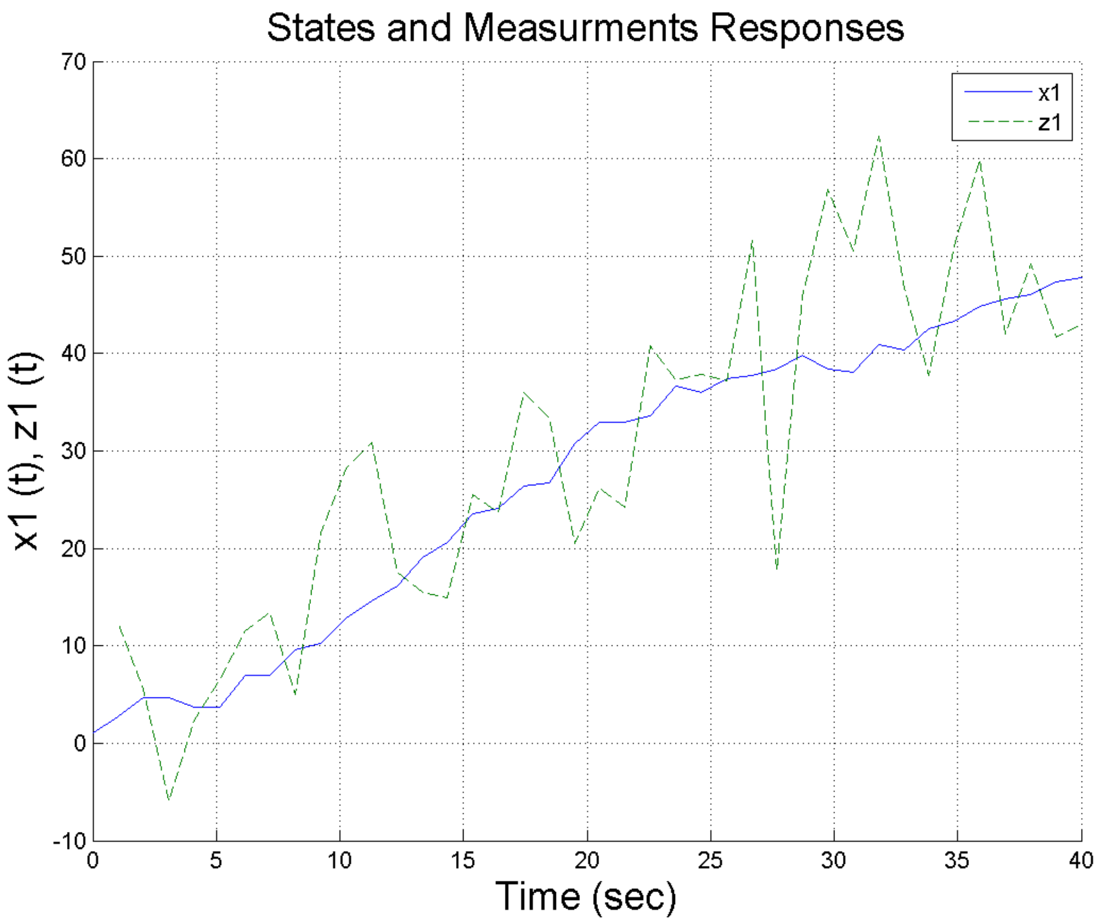

error% between states and measurments
error1% with μ = 47.4677, σ = 67.5807
error1%
Time (sec)
400
350
300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40

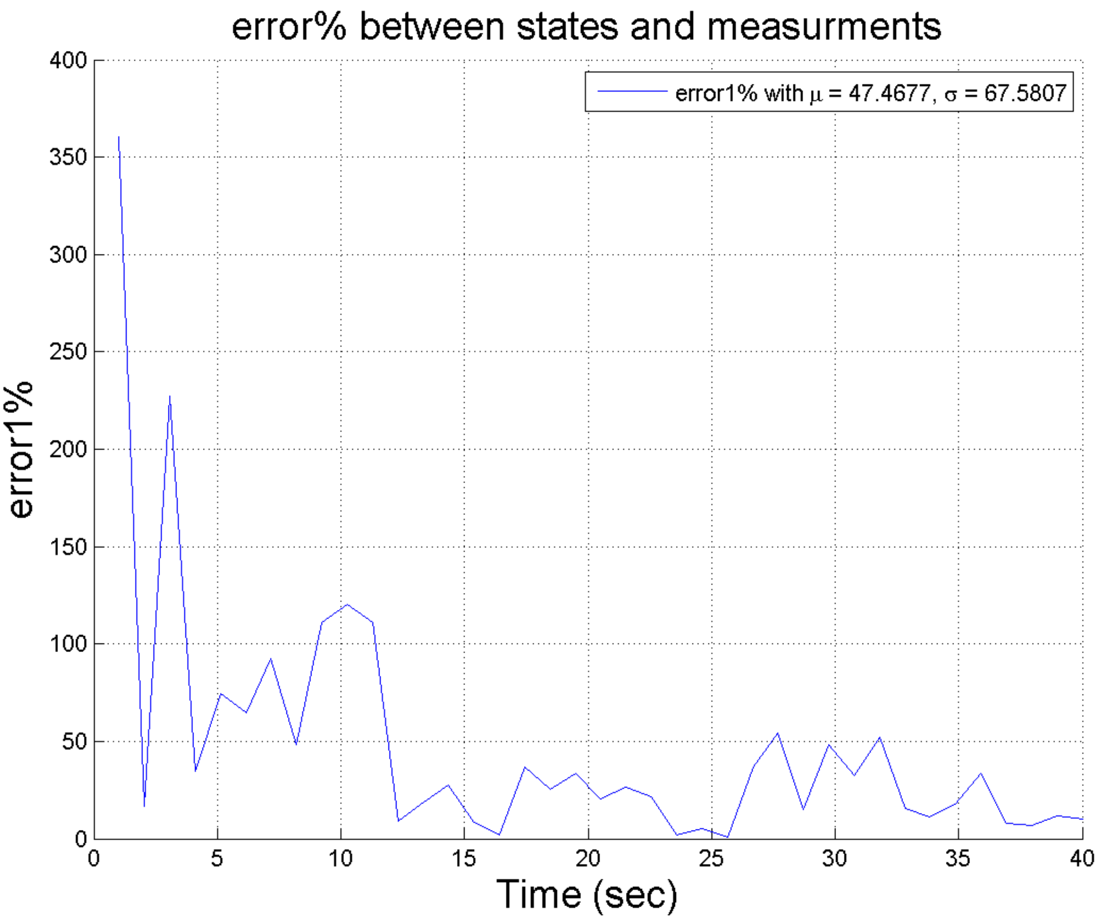

States, Measurments and and Filtered data Responses
x1
z1
$x_{est}1$
$x1\ (t),\ z1\ (t),\ x_{est}1\ (t)$
Time (sec)
70
60
50
40
30
20
10
0
-10
0
5
10
15
20
25
30
35
40

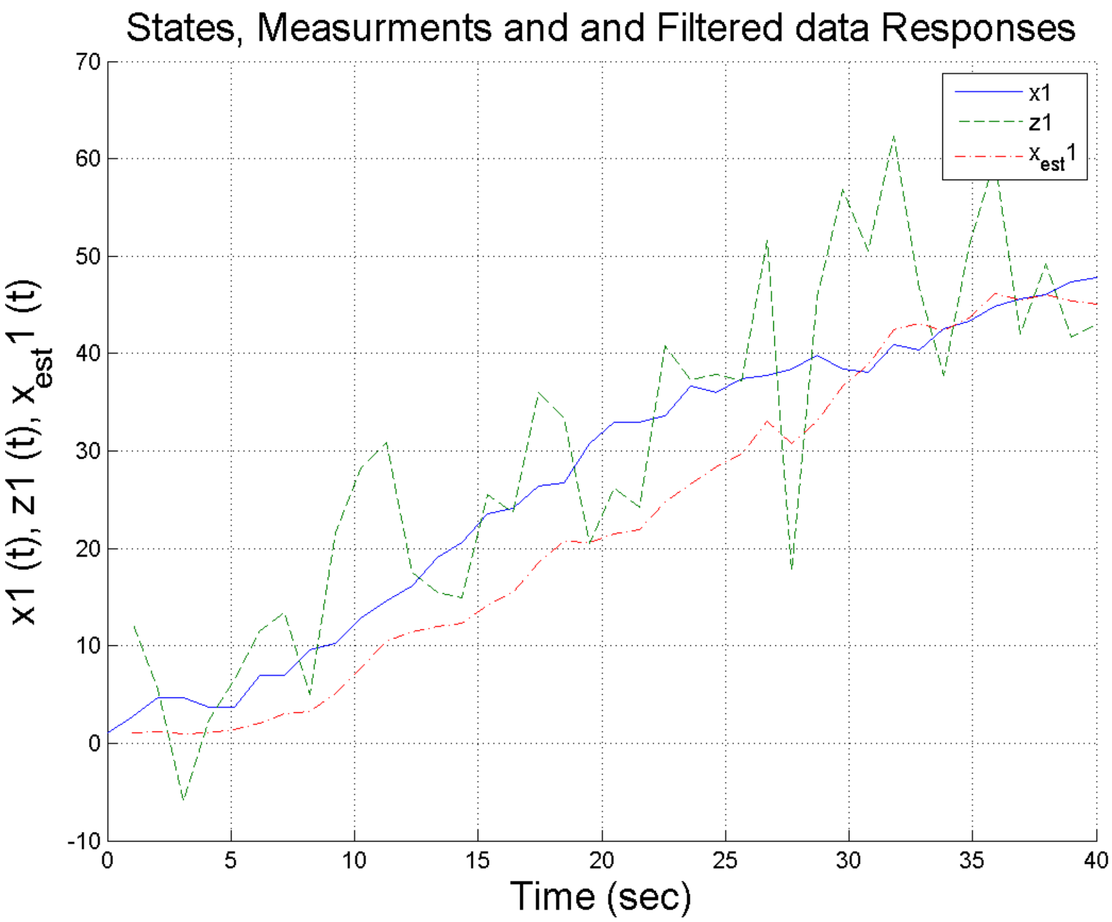


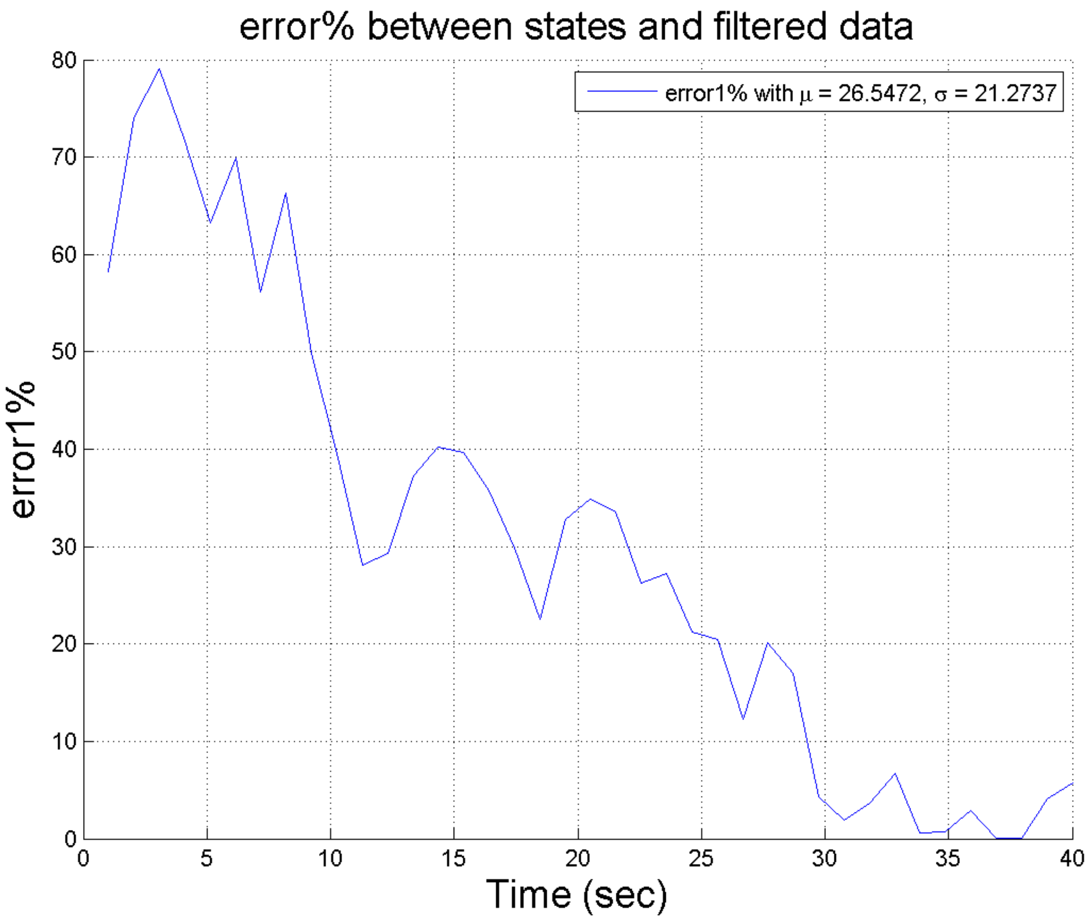
Discrete Kalman Filter with Forgetting Factor λ:



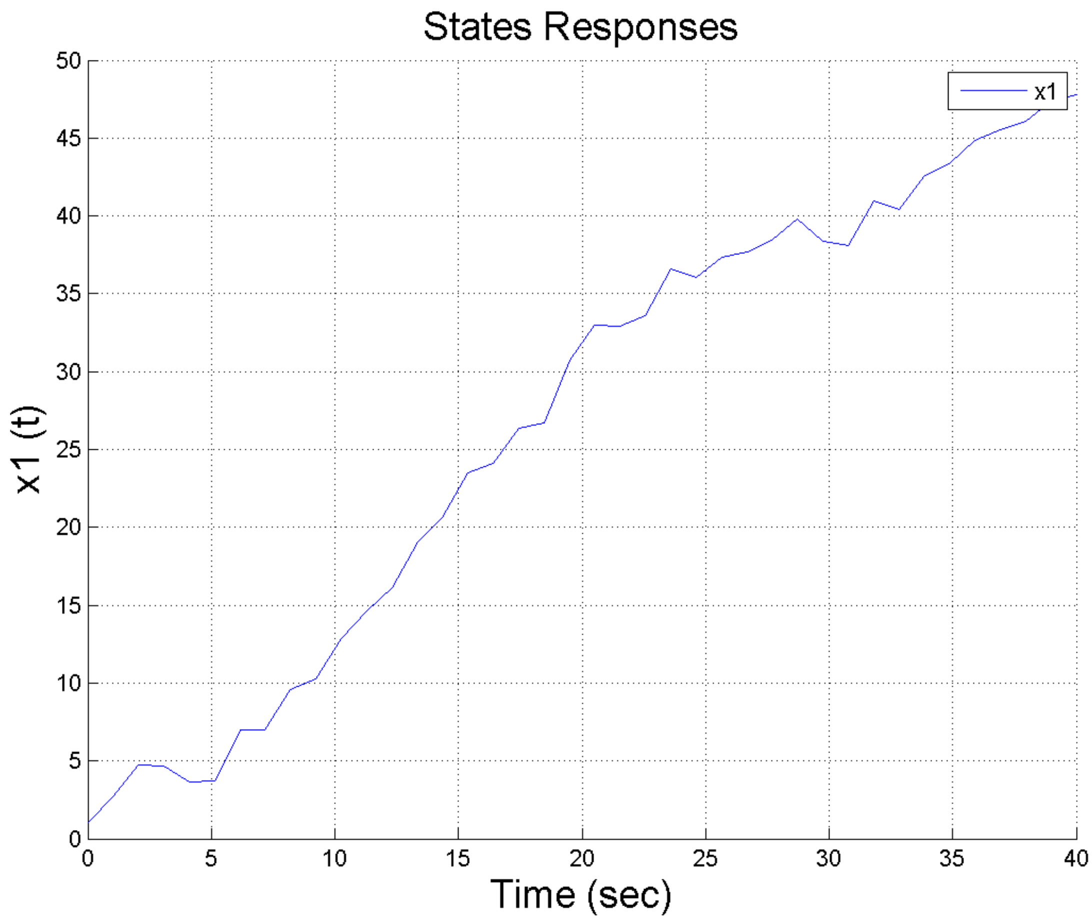

States and Measurments Responses
x1
z1
x1 (t), z1 (t)
Time (sec)
70
60
50
40
30
20
10
0
-10
0
5
10
15
20
25
30
35
40

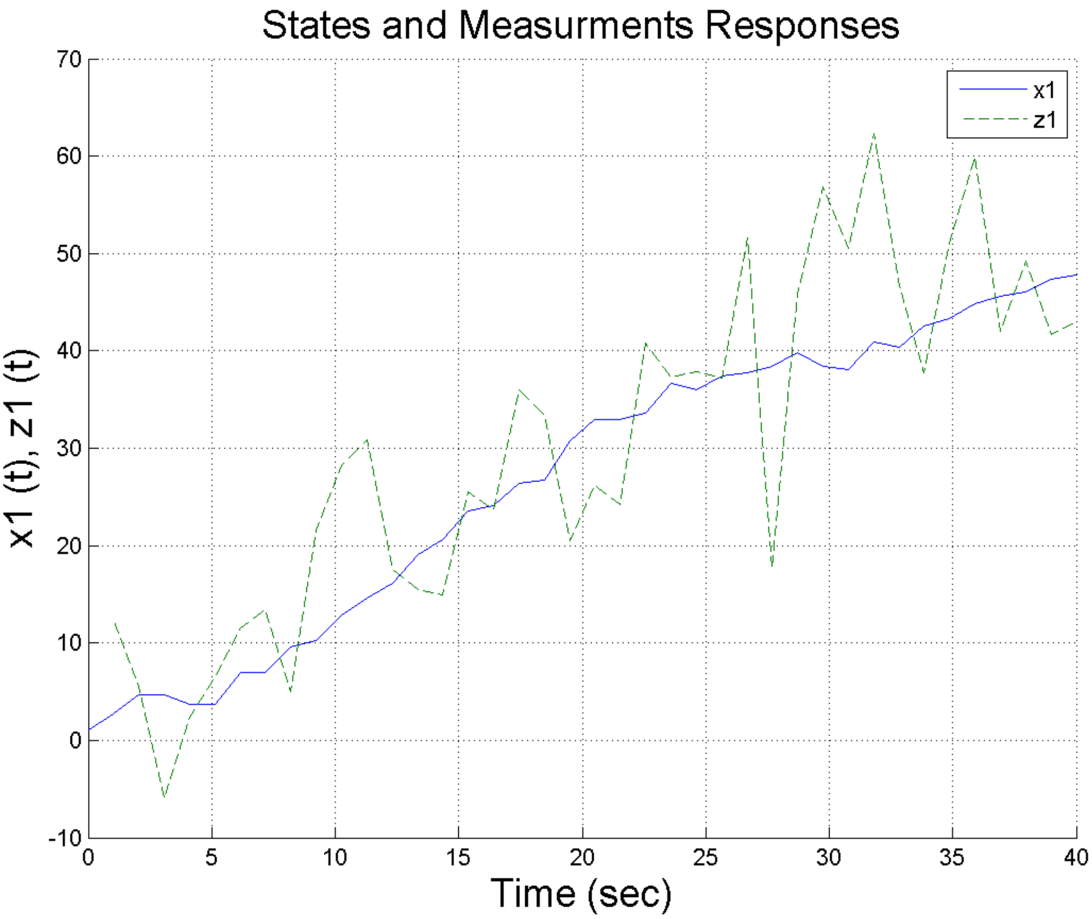

error% between states and measurments
error1% with μ = 47.4677, σ = 67.5807
error1%
Time (sec)
400
350
300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40

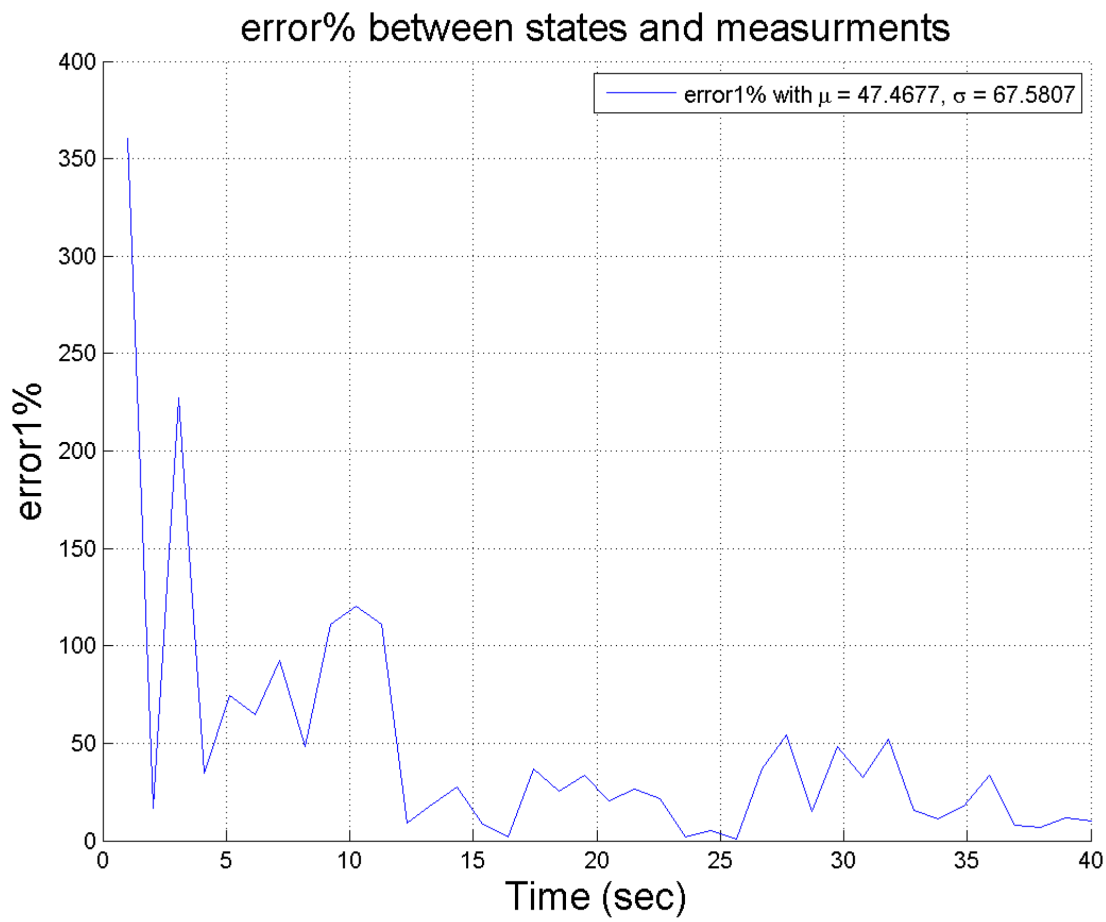

States, Measurments and and Filtered data Responses
x1
z1
x_est1
x1 (t), z1 (t), x_est1 (t)
Time (sec)
70
60
50
40
30
20
10
0
-10
0
5
10
15
20
25
30
35
40

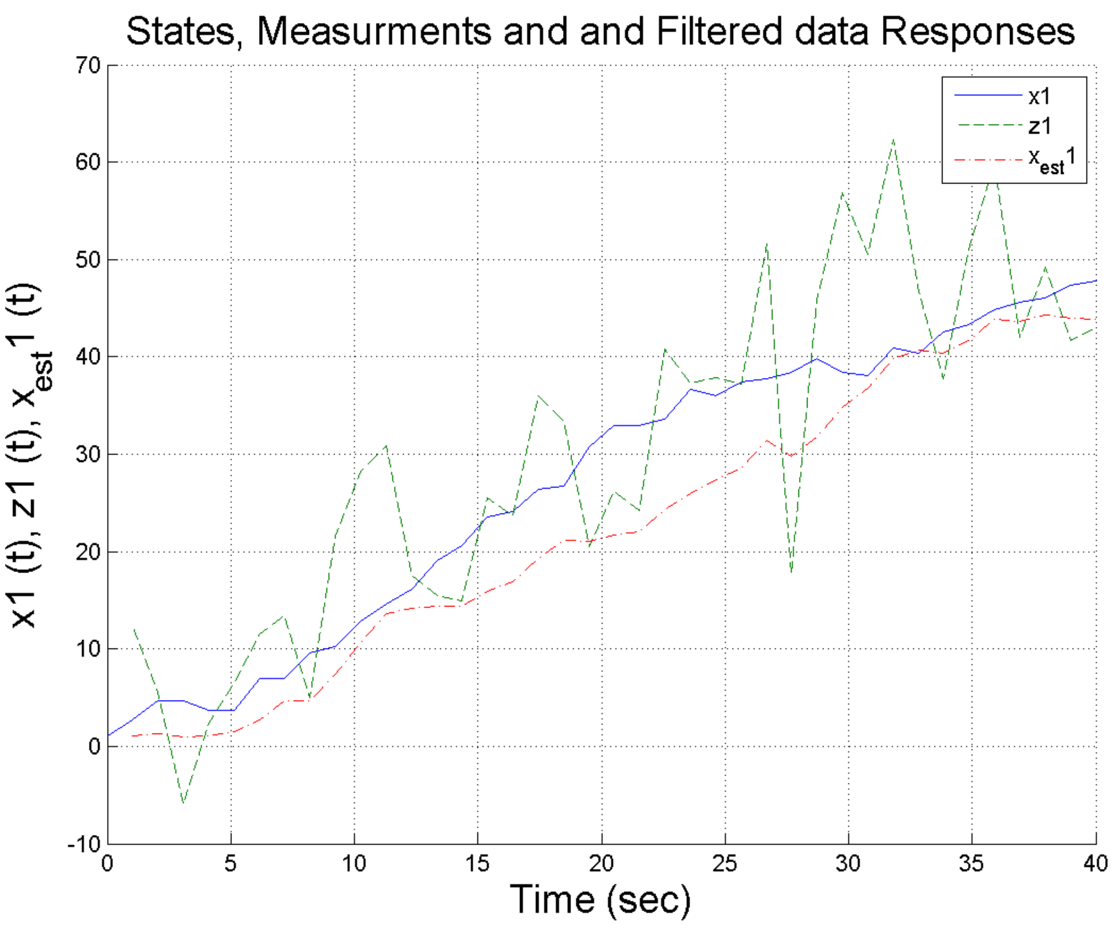

error% between states and filtered data
error1% with μ = 26.5472, σ = 21.2737
error1%
Time (sec)
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40

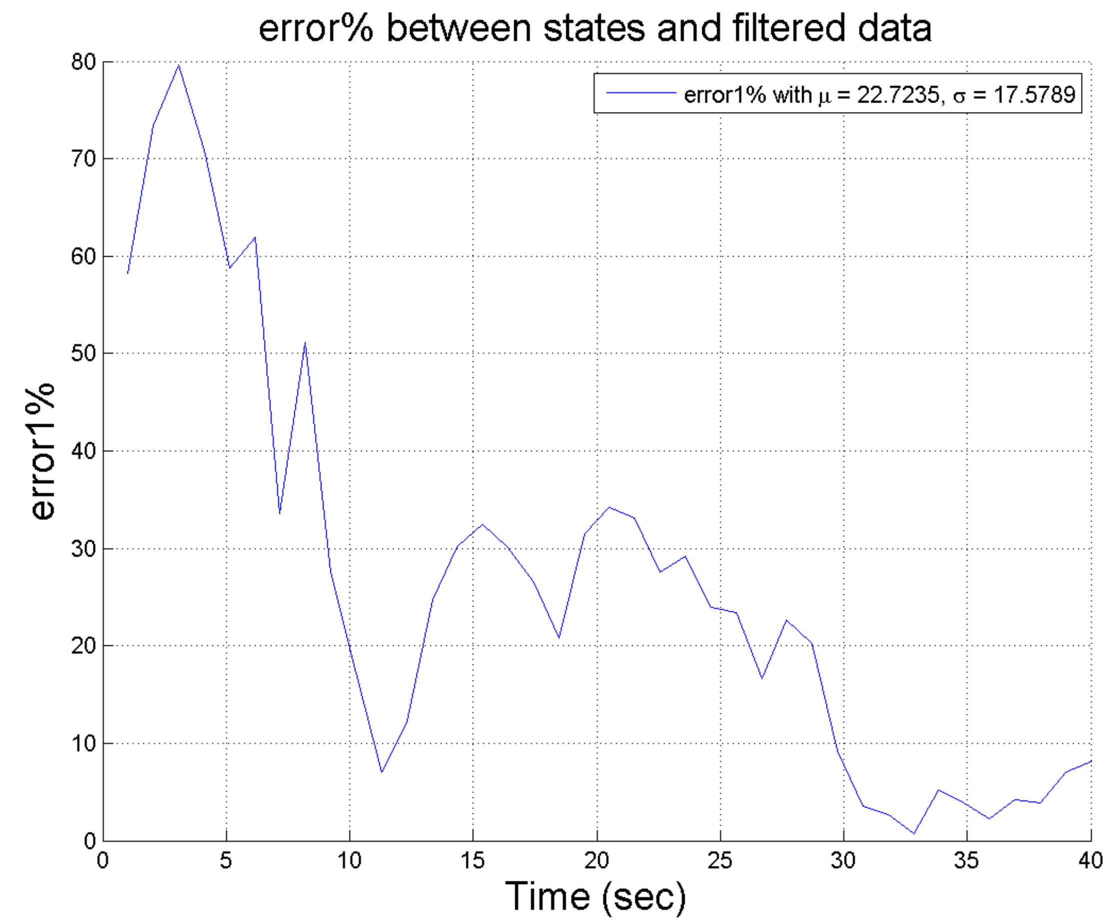
Discrete Kalman Filter with Varying Forgetting Factor λ:

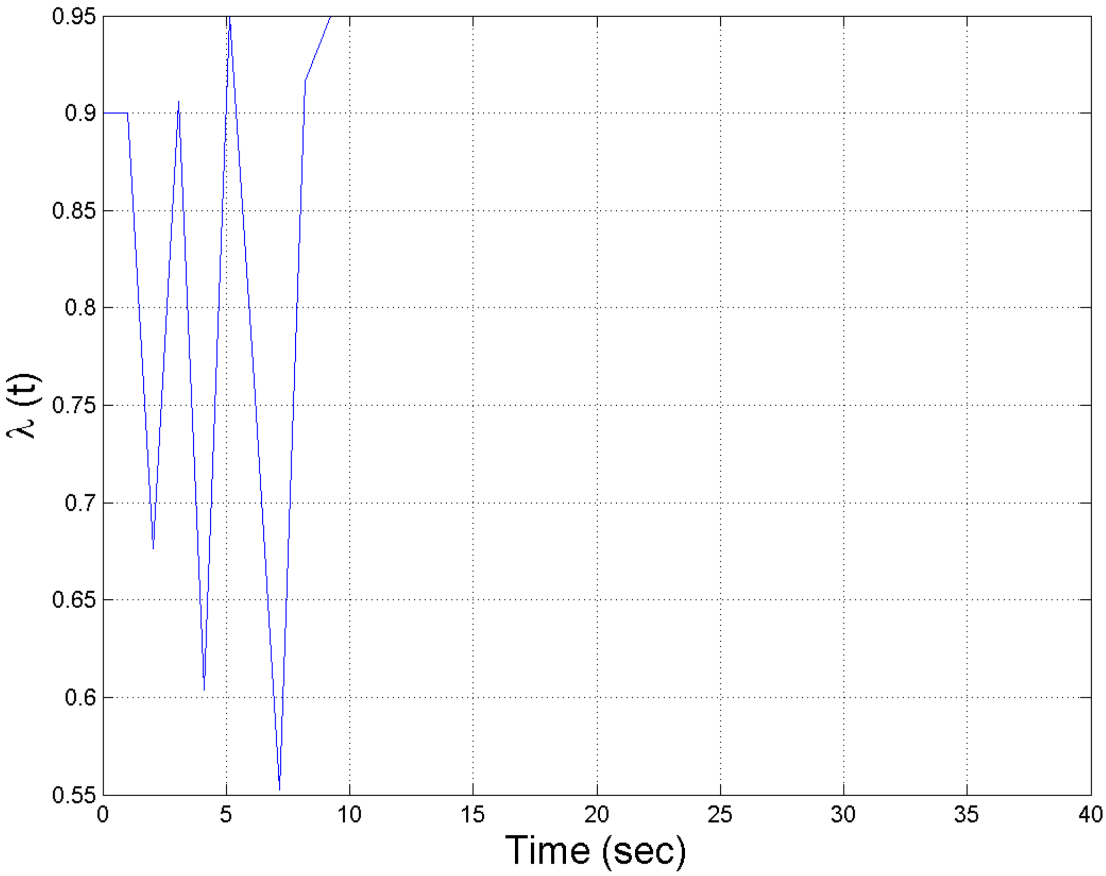





error% between states and measurments
error1% with μ = 47.4677, σ = 67.5807
error1%
Time (sec)


States, Measurments and and Filtered data Responses
x1
z1
x_est1
x1 (t), z1 (t), x_est1 (t)
Time (sec)





Comments:

From results we see that the $error\%$ have the data,

	DKF	DKF with λ	DKF with varying λ
μ	38.3429	26.5472	22.7235
σ	20.224	21.2737	17.5789

It's seems that DKF with varying λ has the min. μ and σ