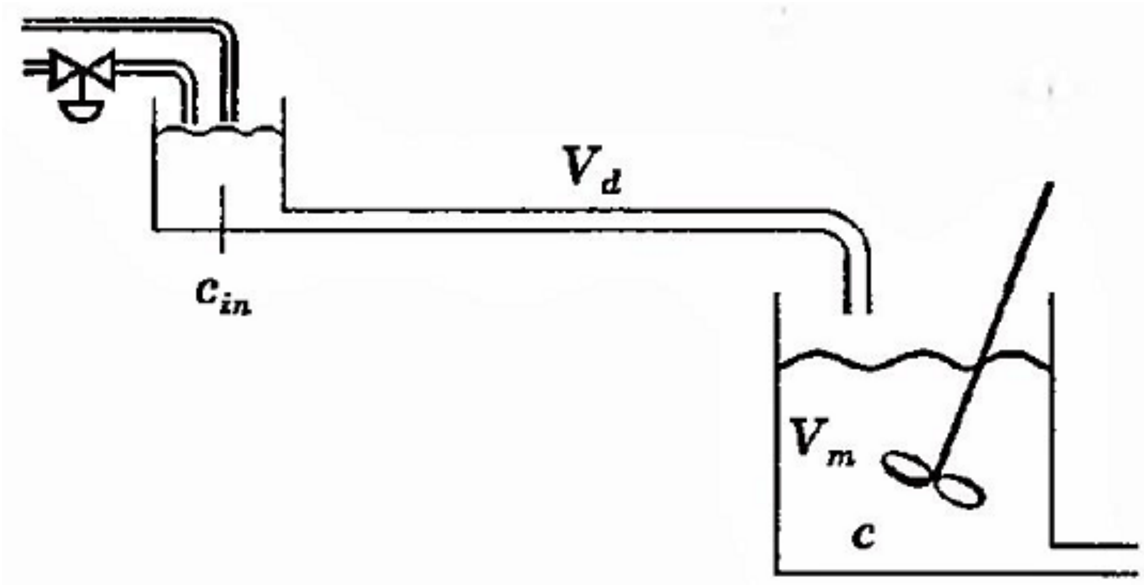
Concentration Control:

System Modeling:

- c_{in} : The concentration at the inlet of the pipe
- V_d : The pipe volume
- V_m : Tank volume
- q : Flow rate
- c : The concentration in the tank and at the outlet
- T_s : Sampling Time

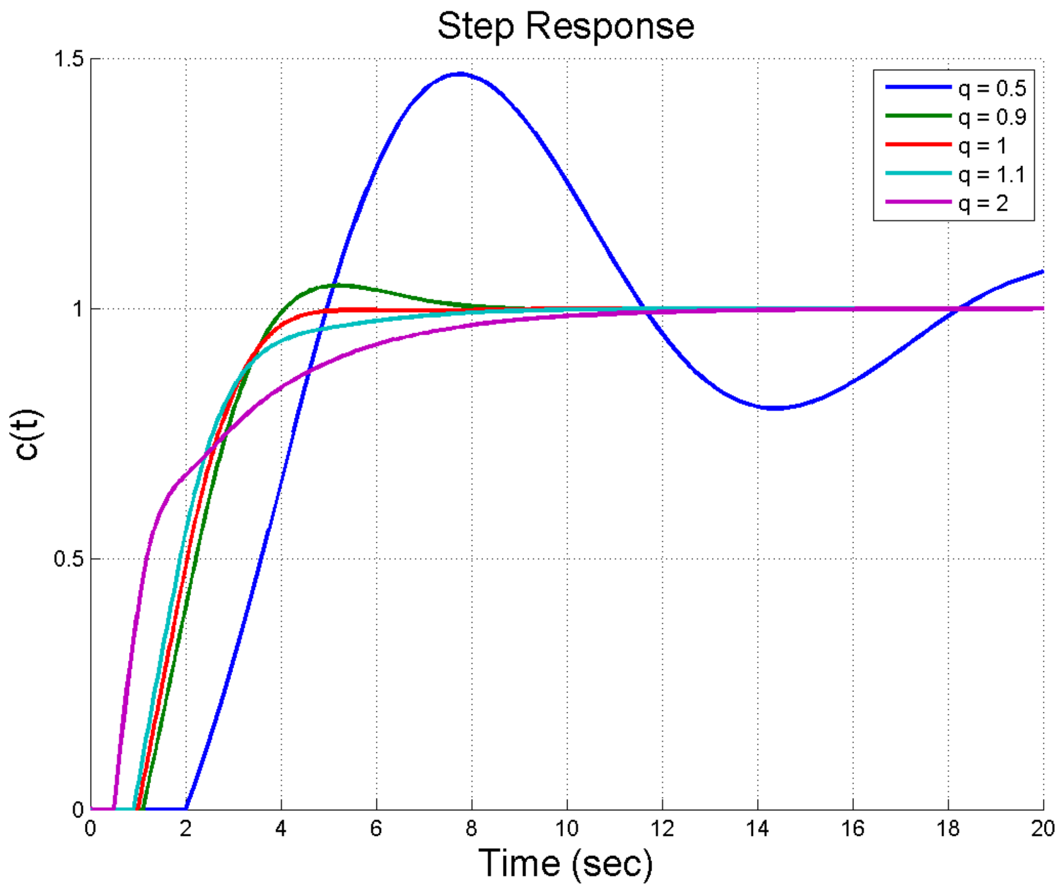


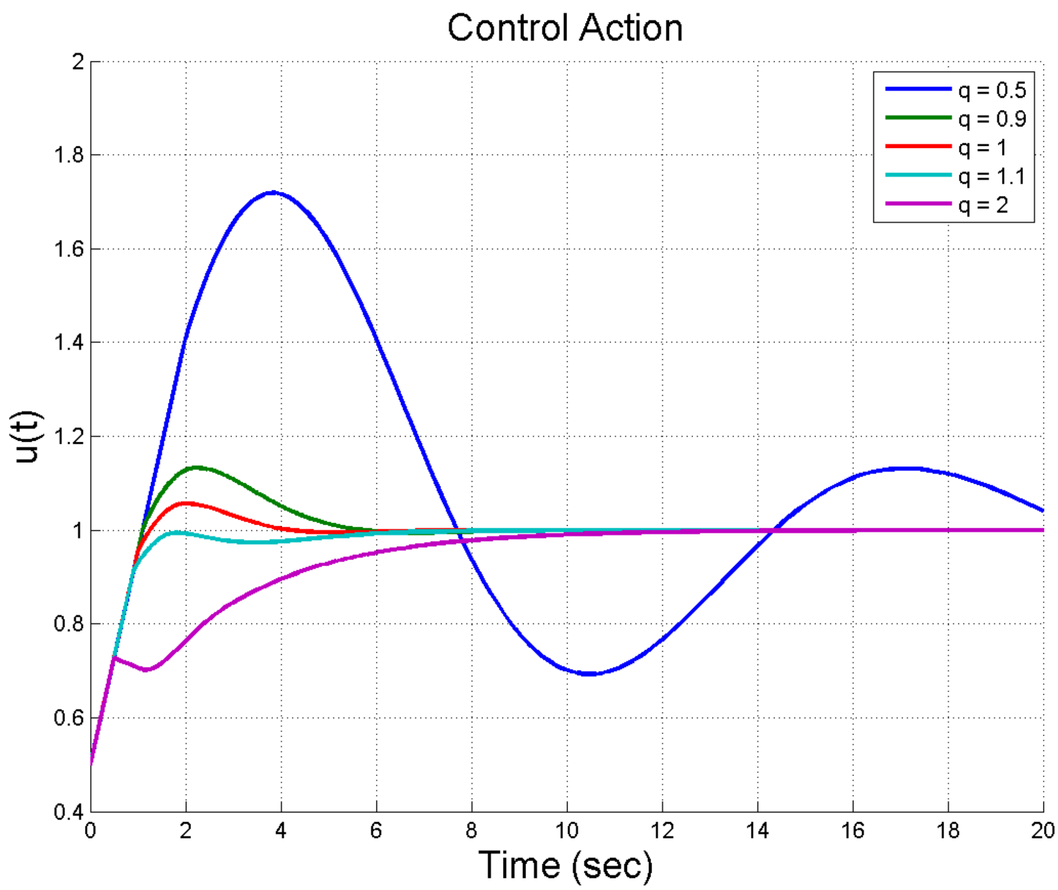
$$
\begin{aligned}
V_m \frac{dc}{dt} &= q(t)[c_{in}(t-\tau) - c(t)] \\
\tau &= \frac{V_d}{q(t)} \\
V_m cs &= q[c_{in}e^{-\tau s} - c] \\
G(s) &= \frac{c}{c_{in}} = \frac{qe^{-\tau s}}{V_m s + q} = \frac{e^{-\tau s}}{Ts+1} \; ; \quad T = \frac{V_m}{q} \\
G(z) &= Z\left\{\frac{1-e^{-T_s s}}{s}\frac{e^{-\tau s}}{Ts+1}\right\} = z^{-d}(1-z^{-1})Z\left\{\frac{1/T}{s(s+1/T)}\right\} \\
&= z^{-d}(1-z^{-1})\frac{(1-e^{-T_s/T})z^{-1}}{(1-z^{-1})(1-e^{-T_s/T}z^{-1})} = z^{-d}\,\frac{(1-e^{-T_s/T})z^{-1}}{1-e^{-T_s/T}z^{-1}} \\
\textbf{Where : } \tau &= dT_s \\
\textbf{Let : } a &= e^{-T_s/T} = e^{-T_s q/V_m} = e^{-T_s V_d/(\tau V_m)} = e^{-V_d/(V_m d)} \\
G(z) &= z^{-d}\,\frac{(1-a)z^{-1}}{1-az^{-1}}
\end{aligned}
$$

Closed Loop System Performance Without Gain Scheduling:

Let : $q = 1\,,\quad T = 1\,,\quad \tau = 1\,,\quad \Rightarrow\; G(s) = \frac{e^{-s}}{s+1}$

And use PI controller $G_c(s) = 0.5\left(1 + \frac{1}{1.1s}\right)$





Closed Loop System Performance With Gain Scheduling:

Use Ziegler-Nichols of PI controller of the open loop system,

$$K_c = \frac{0.9\tau}{T} = \frac{0.9V_d}{V_m}\,, \quad T_i = \frac{L}{0.3} = \frac{V_d}{0.3q}$$