

Second Order System Adjustment:

$$G(s) = \frac{y}{u} = \frac{s+5}{(s-1)(s+3)} = \frac{s+5}{s^2+2s-3}$$

$$G_m(s) = \frac{y_m}{u_c} = \frac{s+2}{(s+1)(s+2)} = \frac{s+2}{s^2+3s+2}$$

Let the control law : $u(t) = t_o u_c(t) + t_1 u_c(t-1) - s_o y(t) - s_1 y(t-1)$

$$u = (t_o + t_1 s) u_c - (s_o + s_1 s) y$$

$$y \frac{s^2+2s-3}{s+5} = (t_o + t_1 s) u_c - (s_o + s_1 s) y \Rightarrow \frac{y}{u_c} = \frac{t_o + t_1 s}{\frac{s^2+2s-3}{s+5} + s_o + s_1 s} = \frac{t_1 s^2 + (t_o + 5t_1)s + 5t_o}{(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o}$$

$$e = y - y_m = \frac{t_1 s^2 + (t_o + 5t_1)s + 5t_o}{(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o} u_c - y_m$$

$$\frac{\partial e}{\partial t_o} = \frac{s+5}{(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o} u_c$$

$$\frac{\partial e}{\partial t_1} = \frac{s^2+5s}{(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o} u_c$$

$$\frac{\partial e}{\partial s_o} = \frac{-[t_1 s^2 + (t_o + 5t_1)s + 5t_o](s+5)}{[(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o]^2} u_c = \frac{-(s+5)}{(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o} y$$

$$\frac{\partial e}{\partial s_1} = \frac{-[t_1 s^2 + (t_o + 5t_1)s + 5t_o](s^2+5s)}{[(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o]^2} u_c = \frac{-(s^2+5s)}{(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o} y$$

At steady state : $\frac{(s+5)}{(1+s_1)s^2 + (2+s_o+5s_1)s - 3+5s_o} = G_m(s)$

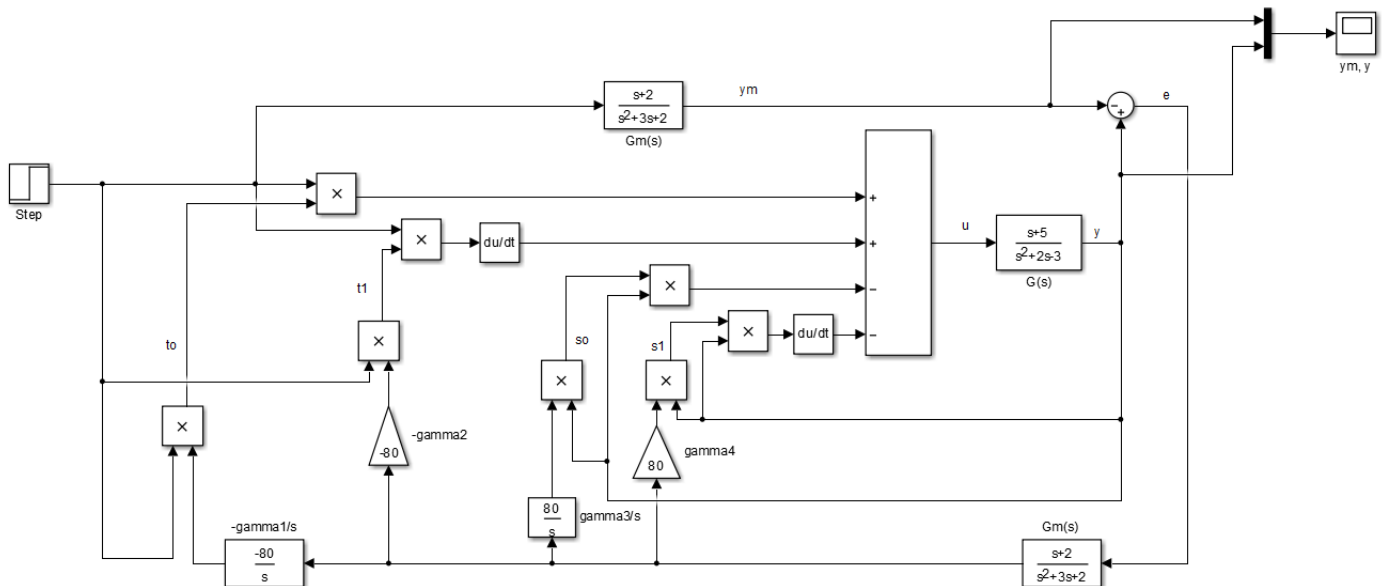
$$\frac{d\theta}{dt} = -\gamma \frac{\partial e}{\partial \theta} e$$

$$\frac{dt_o}{dt} = -\gamma_1 [G_m(s) u_c] e$$

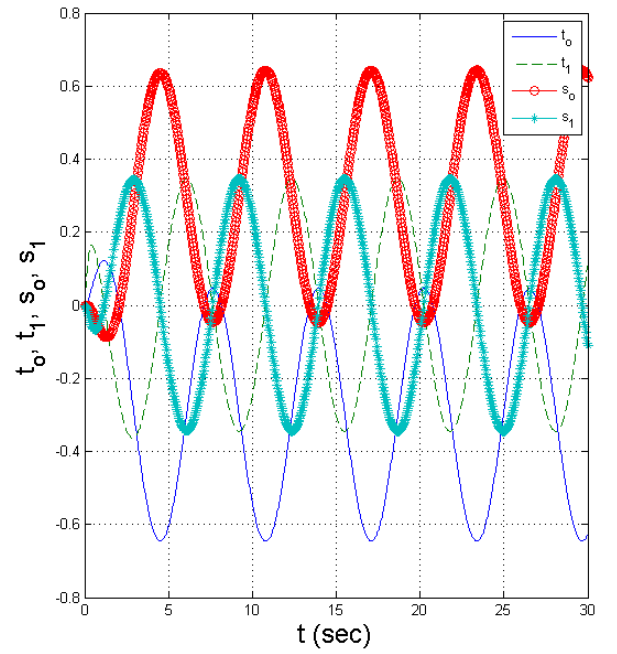
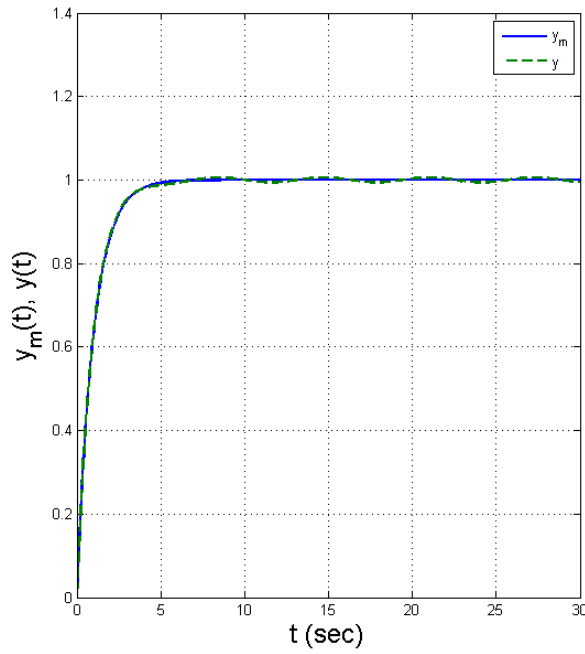
$$\frac{dt_1}{dt} = -s \gamma_2 [G_m(s) u_c] e$$

$$\frac{ds_o}{dt} = \gamma_3 [G_m(s) y] e$$

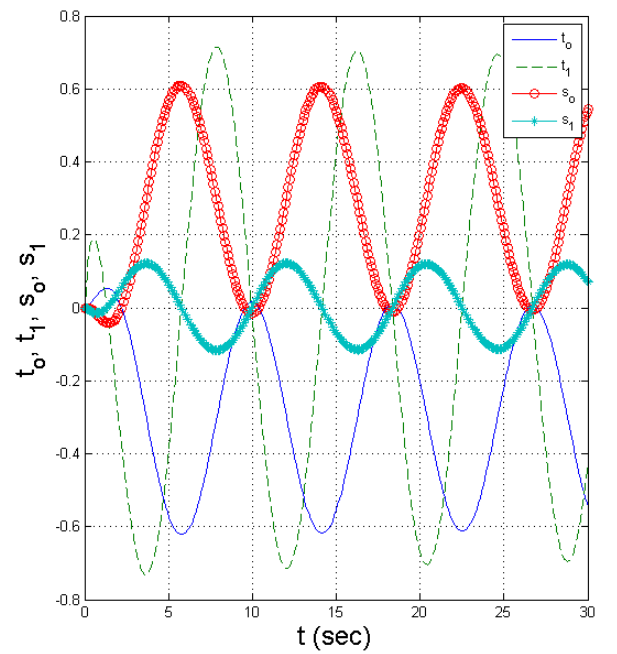
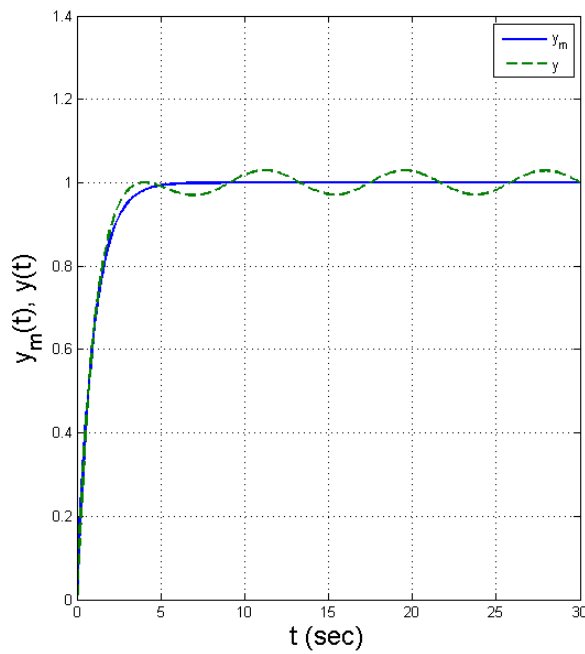
$$\frac{ds_1}{dt} = s \gamma_4 [G_m(s) y] e$$



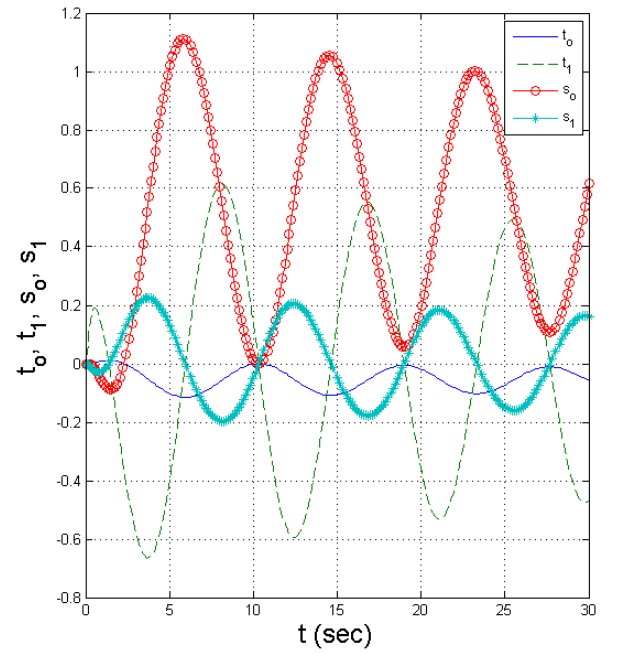
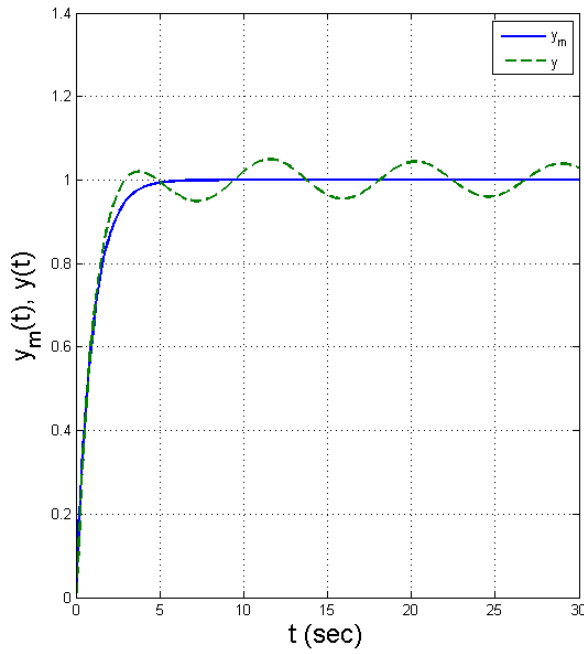
At $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 80$



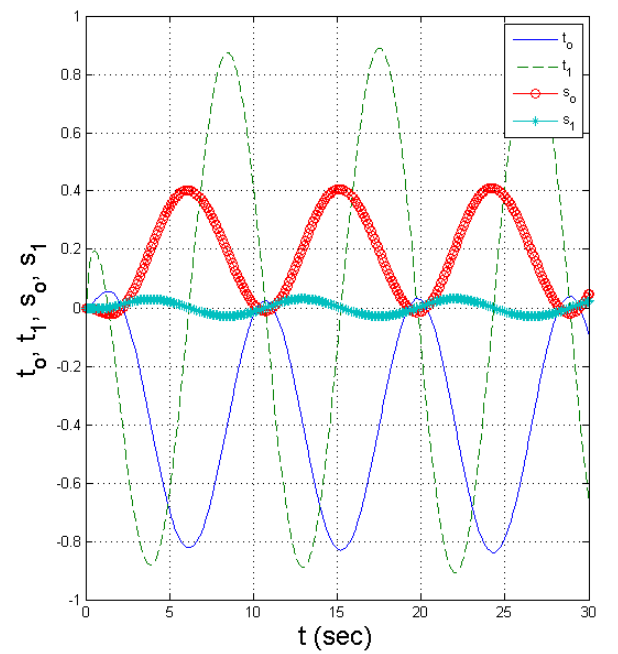
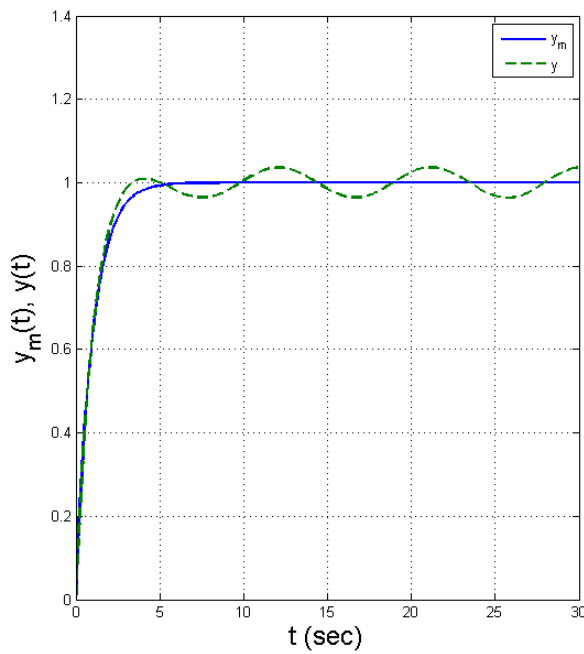
At $\gamma_1 = 10, \gamma_2 = 30, \gamma_3 = 10, \gamma_4 = 5$



At $\gamma_1 = 1$, $\gamma_2 = 15$, $\gamma_3 = 10$, $\gamma_4 = 5$



At $\gamma_1 = 10$, $\gamma_2 = 30$, $\gamma_3 = 5$, $\gamma_4 = 1$



Conclusion:

γ_1 , γ_2 , γ_3 , and γ_4 must be 80 or higher to get min. error.