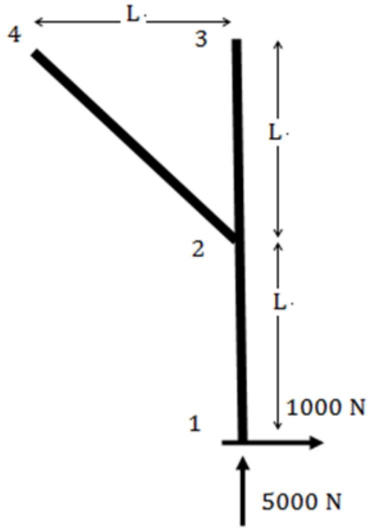
Under Carriage:

Problem Statement:

For the following under carriage,



- $E = 2.06e11$
- $\mu = 50\; kg\; per\; unit\; length$
- $L = 1\; m$
- $A = 0.01\; m^2$
- $I = 0.00001\; m^4$
- $C = M + 0.00005K$

Mass Matrix:

$$
\begin{aligned}
Cx &= cos(\theta) \\
Cy &= sin(\theta) \\
M &= \mu L \\
ML &= M\; L \\
ML2 &= ML\; L
\end{aligned}
$$

$$
m = \left[\begin{matrix}
[M\,Cx^2/3 + 13/35\,M\,Cy^2 & -4/105\,M\,Cx\,Cy & -11/210\,ML\,Cy \\
-4/105\,M\,Cx\,Cy & M\,Cy^2/3 + 13/35\,M\,Cx^2 & 11/210\,ML\,Cx - 11/210\,ML\,Cy \\
M\,Cx^2/6 + 9/70\,M\,Cy^2 & 4/105\,M\,Cx\,Cy & -13/420\,ML\,Cy \\
4/105\,M\,Cx\,Cy & M\,Cy^2/6 + 9/70\,M\,Cx^2 & 13/420\,ML\,Cx \\
13/420\,ML\,Cy & -13/420\,ML\,Cx & -1/140\,ML2
\end{matrix}\right.
$$

$$
\left.\begin{matrix}
M\,Cx^2/6 + 9/70\,M\,Cy^2 & 4/105\,M\,Cx\,Cy & 13/420\,ML\,Cy \\
4/105\,M\,Cx\,Cy & M\,Cy^2/6 + 9/70\,M\,Cx^2 & -13/420\,ML\,Cx \\
-13/420\,ML\,Cy & 13/420\,ML\,Cx & -1/140\,ML2 \\
M\,Cx^2/3 + 13/35\,M\,Cy^2 & -4/105\,M\,Cx\,Cy & 11/210\,ML\,Cy \\
-4/105\,M\,Cx\,Cy & M\,Cy^2/3 + 13/35\,M\,Cx^2 & -11/210\,ML\,Cx \\
11/210\,ML\,Cy & -11/210\,ML\,Cx & ML2/105
\end{matrix}\right]
$$

Stiffness Matrix:

$$
\begin{aligned}
cx &= cos(theta) \\
cy &= sin(\theta) \\
A &= E(i)\ A/L \\
B &= E\ I/L \\
C &= B/L \\
D &= C/L
\end{aligned}
$$

$$
k = \begin{bmatrix}
A\,cx^2 + 12\,D\,cy^2 & (A - 12\,D)\,cx\,cy & -6\,C\,cy \\
(A - 12\,D)\,cx\,cy & A\,cy^2 + 12\,D\,cx^2 & 6\,C\,cx \\
-6\,C\,cy & 6\,C\,cx & 4\,B \\
-A\,cx^2 - 12\,D\,cy^2 & (-A + 12\,D)\,cx\,cy & 6\,C\,cy \\
(-A + 12\,D)\,cx\,cy & -A\,cy^2 - 12\,D\,cx^2 - 6\,C\,cx & \\
-6\,C\,cy & 6\,C\,cx & 2\,B
\end{bmatrix}
$$

$$
\begin{matrix}
-A\,cx^2 - 12\,D\,cy^2 & (-A + 12\,D)\,cx\,cy & -6\,C\,cy \\
(-A + 12\,D)\,cx\,cy & -A\,cy^2 - 12\,D\,cx^2 6\,C\,cx & \\
6\,C\,cy & -6\,C\,cx & 2\,B \\
A\,cx^2 + 12\,C\,cy^2 & (A - 12\,D)\,cx\,cy & 6\,C\,cy \\
(A - 12\,D)\,cx\,cy & A\,cy^2 + 12\,D\,cx^2 & -6\,C\,cx \\
6\,C\,cy & -6\,C\,cx & 4\,B
\end{matrix}
\Bigg]
$$

Beam 12:

$$
m_1 = \begin{bmatrix}
18.571 & 0 & -2.619 & 6.4286 & 0 & 1.5476 \\
0 & 16.667 & 0 & 0 & 8.3333 & 0 \\
-2.619 & 0 & 0.47619 & -1.5476 & 0 & -0.35714 \\
6.4286 & 0 & -1.5476 & 18.571 & 0 & 2.619 \\
0 & 8.3333 & 0 & 0 & 16.667 & 0 \\
1.5476 & 0 & -0.35714 & 2.619 & 0 & 0.47619
\end{bmatrix}
$$

$$
k_1 = \begin{bmatrix}
2.472e+07 & 0 & -1.236e+07 & -2.472e+07 & 0 & -1.236e+07 \\
0 & 2.06e+09 & 0 & 0 & -2.06e+09 & 0 \\
-1.236e+07 & 0 & 8.24e+06 & 1.236e+07 & 0 & 4.12e+06 \\
-2.472e+07 & 0 & 1.236e+07 & 2.472e+07 & 0 & 1.236e+07 \\
0 & -2.06e+09 & 0 & 0 & 2.06e+09 & 0 \\
-1.236e+07 & 0 & 4.12e+06 & 1.236e+07 & 0 & 8.24e+06
\end{bmatrix}
$$

Beam 23:

$$m_2 \ = \begin{bmatrix} 18.571 & 0 & -2.619 & 6.4286 & 0 & 1.5476 \\ 0 & 16.667 & 0 & 0 & 8.3333 & 0 \\ -2.619 & 0 & 0.47619 & -1.5476 & 0 & -0.35714 \\ 6.4286 & 0 & -1.5476 & 18.571 & 0 & 2.619 \\ 0 & 8.3333 & 0 & 0 & 16.667 & 0 \\ 1.5476 & 0 & -0.35714 & 2.619 & 0 & 0.47619 \end{bmatrix}$$

$$k_2 = \begin{bmatrix} 2.472e+07 & 0 & -1.236e+07 & -2.472e+07 & 0 & -1.236e+07 \\ 0 & 2.06e+09 & 0 & 0 & -2.06e+09 & 0 \\ -1.236e+07 & 0 & 8.24e+06 & 1.236e+07 & 0 & 4.12e+06 \\ -2.472e+07 & 0 & 1.236e+07 & 2.472e+07 & 0 & 1.236e+07 \\ 0 & -2.06e+09 & 0 & 0 & 2.06e+09 & 0 \\ -1.236e+07 & 0 & 4.12e+06 & 1.236e+07 & 0 & 8.24e+06 \end{bmatrix}$$

Beam 24:

$$m_3 = \begin{bmatrix} 24.917 & 1.3469 & -3.7039 & 10.438 & -1.3469 & 2.1887 \\ 1.3469 & 24.917 & -3.7039 & -1.3469 & 10.438 & 2.1887 \\ -3.7039 & -3.7039 & 1.3469 & -2.1887 & -2.1887 & -1.0102 \\ 10.438 & -1.3469 & -2.1887 & 24.917 & 1.3469 & 3.7039 \\ -1.3469 & 10.438 & -2.1887 & 1.3469 & 24.917 & 3.7039 \\ 2.1887 & 2.1887 & -1.0102 & 3.7039 & 3.7039 & 1.3469 \end{bmatrix}$$

$$k_3 = \begin{bmatrix} 7.3269e+08 & -7.2395e+08 & -4.3699e+06 & -7.3269e+08 & 7.2395e+08 & -4.3699e+06 \\ -7.2395e+08 & 7.3269e+08 & -4.3699e+06 & 7.2395e+08 & -7.3269e+08 & -4.3699e+06 \\ -4.3699e+06 & -4.3699e+06 & 5.8266e+06 & 4.3699e+06 & 4.3699e+06 & 2.9133e+06 \\ -7.3269e+08 & 7.2395e+08 & 4.3699e+06 & 7.345e+08 & -7.2395e+08 & 4.3699e+06 \\ 7.2395e+08 & -7.3269e+08 & 4.3699e+06 & -7.2395e+08 & 7.3269e+08 & 4.3699e+06 \\ -4.3699e+06 & -4.3699e+06 & 2.9133e+06 & 4.3699e+06 & 4.3699e+06 & 5.8266e+06 \end{bmatrix}$$

Mass Matrix for Free Nodes $(1, 2, 3, 4, 5, 6)$**:**

$$M_{free} = \begin{bmatrix} 18.571 & 0 & -2.619 & 6.4286 & 0 & 1.5476 \\ 0 & 16.667 & 0 & 0 & 8.3333 & 0 \\ -2.619 & 0 & 0.47619 & -1.5476 & 0 & -0.35714 \\ 6.4286 & 0 & -1.5476 & 62.06 & 1.3469 & -3.7039 \\ 0 & 8.3333 & 0 & 1.3469 & 58.25 & -3.7039 \\ 1.5476 & 0 & -0.35714 & -3.7039 & -3.7039 & 2.2993 \end{bmatrix}$$

Stiffness for Free Nodes $(1, 2, 3, 4, 5, 6)$**:**

$$K_{free} = \begin{bmatrix} 2.472e+07 & 0 & -1.236e+07 & -2.472e+07 & 0 \\ 0 & 2.06e+09 & 0 & 0 & -2.06e+09 \\ -1.236e+07 & 0 & 8.24e+06 & 1.236e+07 & 0 \\ -2.472e+07 & 0 & 1.236e+07 & 7.8213e+08 & -7.2395e+08 \\ 0 & -2.06e+09 & 0-7.2395e+08 & 4.8527e+09 & \\ -1.236e+07 & 0 & 4.12e+06 & -4.3699e+06 & -4.3699e+06 \end{matrix}$$

$$\begin{matrix} -1.236e+07 \\ 0 \\ 4.12e+06 \\ -4.3699e+06 \\ -4.3699e+06 \\ 2.2307e+07 \end{bmatrix}$$

Reduced Mass Matrix for Free Nodes $(1, 2, 4, 5)$**:**

$$\begin{aligned} \begin{Bmatrix} F \\ M \end{Bmatrix} &= \begin{bmatrix} C & D \\ A & B \end{bmatrix} \begin{Bmatrix} \delta \\ \theta \end{Bmatrix} \\ A &= \begin{bmatrix} -2.619 & 0 & -1.5476 & 0 \\ 1.5476 & 0 & -3.7039 & -3.7039 \end{bmatrix} \\ B &= \begin{bmatrix} 0.47619 & -0.35714 \\ -0.35714 & 2.2993 \end{bmatrix} \\ C &= \begin{bmatrix} 18.571 & 0 & 6.4286 & 0 \\ 0 & 16.667 & 0 & 8.3333 \\ 6.4286 & 0 & 62.06 & 1.3469 \\ 0 & 8.3333 & 1.3469 & 58.25 \end{bmatrix} \\ D &= \begin{bmatrix} -2.619 & 1.5476 \\ 0 & 0 \\ -1.5476 & -3.7039 \\ 0 & -3.7039 \end{bmatrix} \end{aligned}$$

$$M_{reduced} = C - DB^{-1}A \begin{bmatrix} 4.0812 & 0 & -3.0811 & -0.75972 \\ 0 & 16.667 & 0 & 8.3333 \\ -3.0811 & 0 & 45.381 & -7.5229 \\ -0.75972 & 8.3333 & -7.5229 & 51.497 \end{bmatrix}$$

Reduced Stiffness Matrix for Free Nodes $(1, 2, 4, 5)$:

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{bmatrix} C & D \\ A & B \end{bmatrix} \begin{Bmatrix} \delta \\ \theta \end{Bmatrix}$$

$$A = \begin{bmatrix} -1.236e+07 & 0 & 1.236e+07 & 0 \\ -1.236e+07 & 0 & -4.3699e+06 & -4.3699e+06 \end{bmatrix}$$

$$B = \begin{bmatrix} 8.24e+06 & 4.12e+06 \\ 4.12e+06 & 2.2307e+07 \end{bmatrix}$$

$$C = \begin{bmatrix} 2.472e+07 & 0 & -2.472e+07 & 0 \\ 0 & 2.06e+09 & 0 & -2.06e+09 \\ -2.472e+07 & 0 & 7.8213e+08 & -7.2395e+08 \\ 0 & -2.06e+09 & -7.2395e+08 & 4.8527e+09 \end{bmatrix}$$

$$D = \begin{bmatrix} -1.236e+07 & -1.236e+07 \\ 0 & 0 \\ 1.236e+07 & -4.3699e+06 \\ 0 & -4.3699e+06 \end{bmatrix}$$

$$K_{reduced} = C - DB^{-1}A \begin{bmatrix} 4.2936e+06 & 0 & -9.4002e+06 & -1.3339e+06 \\ 0 & 2.06e+09 & 0 & -2.06e+09 \\ -9.4002e+06 & 0 & 7.5809e+08 & -7.2623e+08 \\ -1.3339e+06 & -2.06e+09 & -7.2623e+08 & 4.8517e+09 \end{bmatrix}$$

Reduced Damping Matrix for Free Nodes $(1, 2, 4, 5)$:

$$C_{reduced} = M_{reduced} + 0.00005K_{reduced} = \begin{bmatrix} 218.76 & 0 & -473.09 & -67.453 \\ 0 & 1.0302e+05 & 0 & -1.0299e+05 \\ -473.09 & 0 & 37950 & -36319 \\ -67.453 & -1.0299e+05 & -36319 & 2.4264e+05 \end{bmatrix}$$

Reduced Forces Vector for Free Nodes $(1, 2, 4, 5)$:

$$F = \begin{bmatrix} 1000 & 5000 & 0 & 0 \end{bmatrix}$$

Eigenvalue Values λ Eigenvectors:

$$K^{-1}_{reduced} M_{reduced} - \lambda I = 0$$
$$\lambda = \begin{bmatrix} 9.681e-07\,, & 8.3355e-08\,, & 3.1179e-08\,, & 3.9357e-09 \end{bmatrix}$$

$$\Phi = [\phi_1 \, , \ \phi_2 \, , \ \phi_3 \, , \ \phi_4] = \begin{bmatrix} -0.99991 & 0.51065 & -0.17022 & 0.12947 \\ -0.0034963 & 0.31836 & 0.76385 & -0.87794 \\ -0.012229 & 0.75015 & -0.36997 & 0.059584 \\ -0.0034526 & 0.27415 & 0.50068 & 0.45706 \end{bmatrix}$$

Natural Frequancy:

$$W_n = [1016.3 \, , \ 3463.7 \, , \ 5663.3 \, , \ 15940]$$

Orthonormal Eigenvectors:

$$\alpha = \sqrt{\frac{1}{\phi^T M_{reduced} \phi}} = [0.49956 \, , \ 0.18916 \, , \ 0.16251 \, , \ 0.24545]$$

$$X = \alpha \Phi = \begin{bmatrix} -0.49952 & 0.096594 & -0.027663 & 0.031779 \\ -0.0017466 & 0.06022 & 0.12413 & -0.21549 \\ -0.0061092 & 0.1419 & -0.060123 & 0.014625 \\ -0.0017248 & 0.051858 & 0.081364 & 0.11219 \end{bmatrix}$$

Transformation Axes (q):

$$Q = X^T F_{reduced} = \begin{bmatrix} -508.25 \\ 397.7 \\ 593 \\ -1045.7 \end{bmatrix}$$

$$\overline{M} = X^T M_{reduced} X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 00 & 1 & 0 & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{K} = X^T K_{reduced} X = \begin{bmatrix} 1.033e+06 & 0 & 0 & 0 \\ 0 & 1.1997e+07 & 0 & 0 \\ 0 & 0 & 3.2073e+07 & 0 \\ 0 & 0 & 0 & 2.5409e+08 \end{bmatrix}$$

$$\overline{C} = X^T C_{reduced} X = \begin{bmatrix} 52.648 & 0 & 0 & 0 \\ 0 & 600.84 & 0 & 0 \\ 0 & 0 & 1604.7 & 0 \\ 0 & 0 & 0 & 12705 \end{bmatrix}$$

Damping Coefficient ζ:

$$\zeta = \frac{\overline{C}}{2W_n} = [0.025901 \, , \ 0.086736 \, , \ 0.14167 \, , \ 0.39853]$$

Damping Frequancy W_d:

$$W_d = W_n\sqrt{1-\zeta^2} = [1016\ ,\ \ 3450.6\ ,\ \ 5606.2\ ,\ \ 14619]$$

Displacement in Transformation Axes (q):

$$q(t) = \frac{Q}{\overline{M}W_n^2}\left[1 - e^{-\zeta W_n t}\left(cos(W_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\, sin(W_d t)\right)\right]$$

Velocity in Transformation Axes (q):

$$\dot{q}(t) = \frac{Q}{\overline{M}W_n^2}\left[e^{-\zeta W_n t}\zeta W_n\left(cos(W_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\, sin(W_d t)\right)\right.$$

$$\left. - e^{-\zeta W_n t}\left(\frac{\zeta W_d}{\sqrt{1-\zeta^2}}\, cos(W_d t) - W_d sin(W_d t)\right)\right]$$

Acceleration in Transformation Axes (q):

$$\ddot{q}(t) = \frac{Q}{\overline{M}W_n^2}\left[-e^{-\zeta W_n t}\zeta^2 W_n^2\left(cos(W_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\, sin(W_d t)\right)\right.$$

$$+2\, e^{-\zeta W_n t}\zeta W_n\left(\frac{\zeta W_d}{\sqrt{1-\zeta^2}}\, cos(W_d t) - W_d\, sin(W_d t)\right)$$

$$\left. + e^{-\zeta W_n t}\left(W_d^2 cos(W_d t) + \frac{\zeta W_d^2}{\sqrt{1-\zeta^2}}\, sin(W_d t)\right)\right]$$

Displacement in Initial Axes (Z):

$$Z(t) = Xq(t)$$

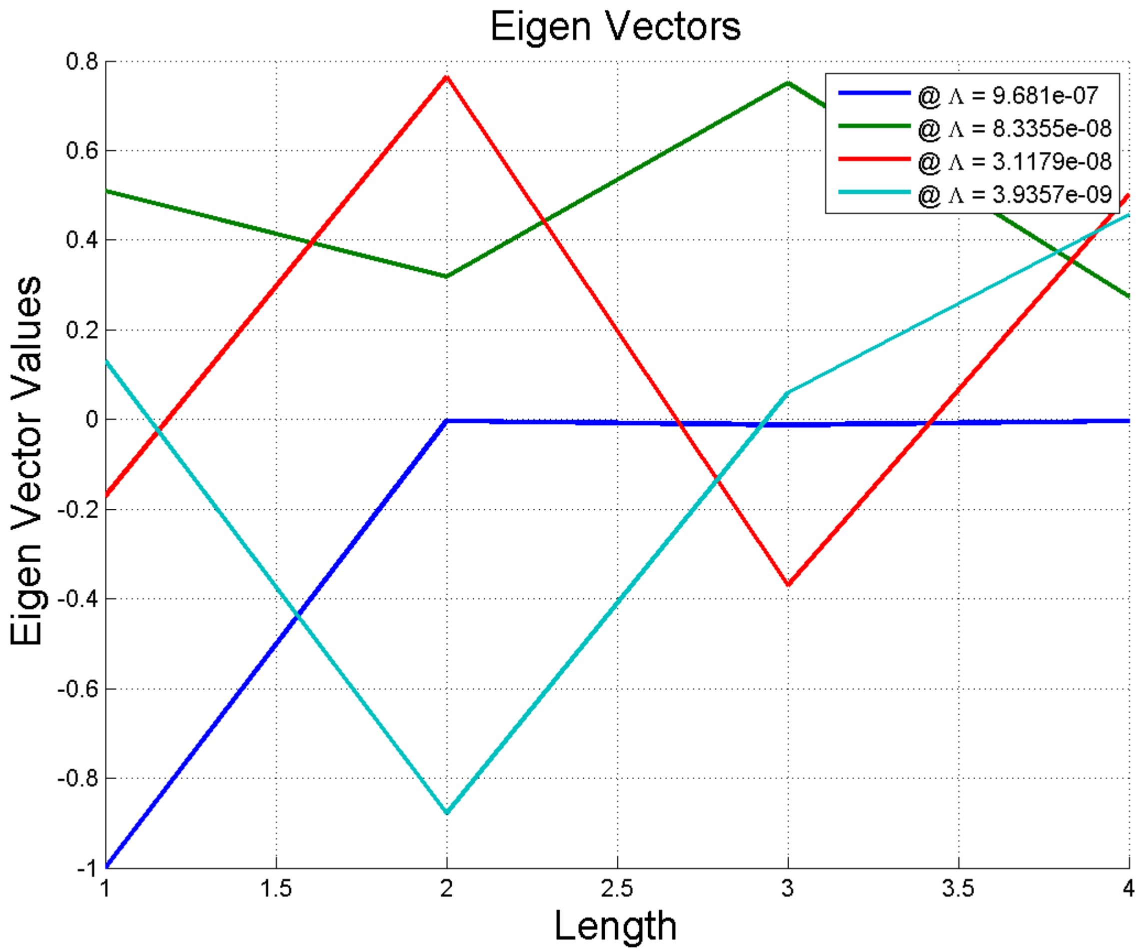
Velocity in Initial Axes (Z):

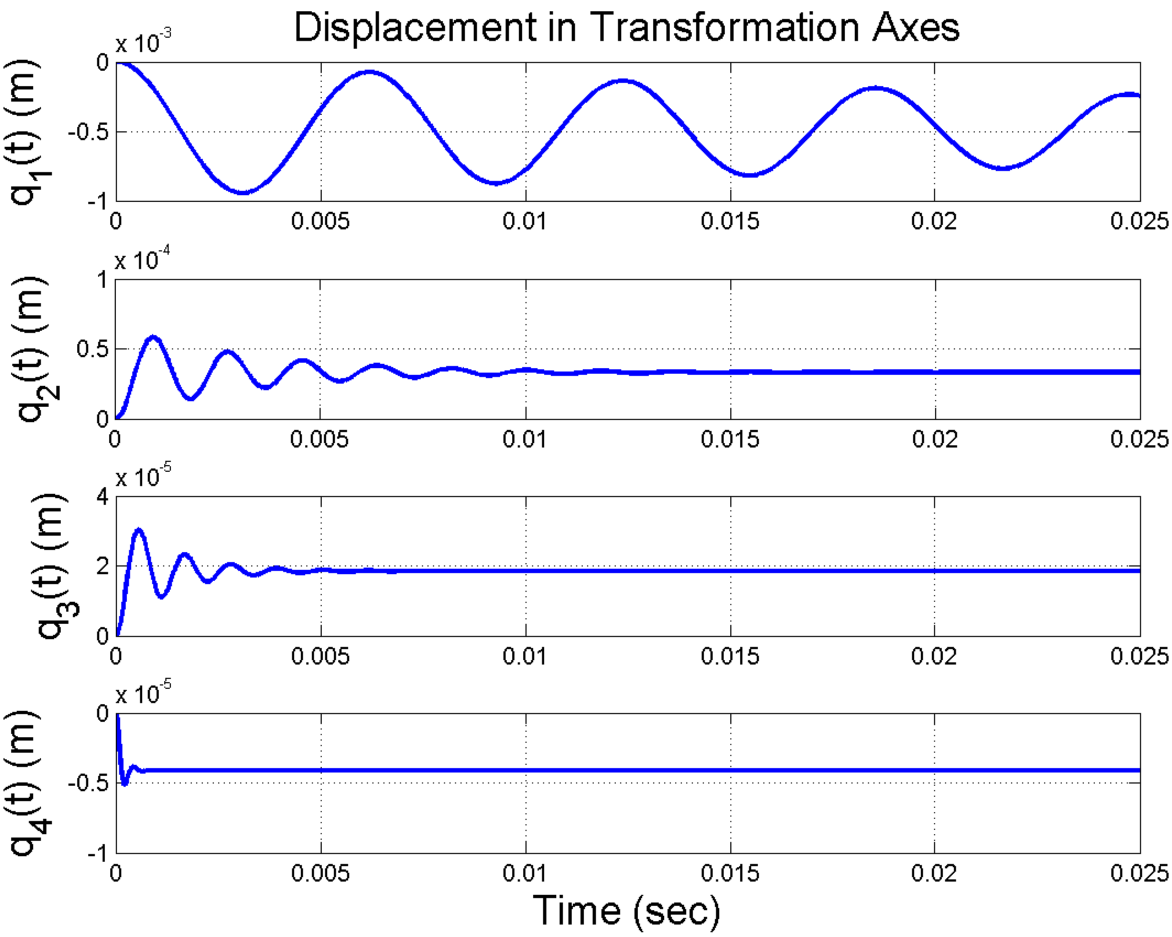
$$\dot{Z}(t) = X\dot{q}(t)$$

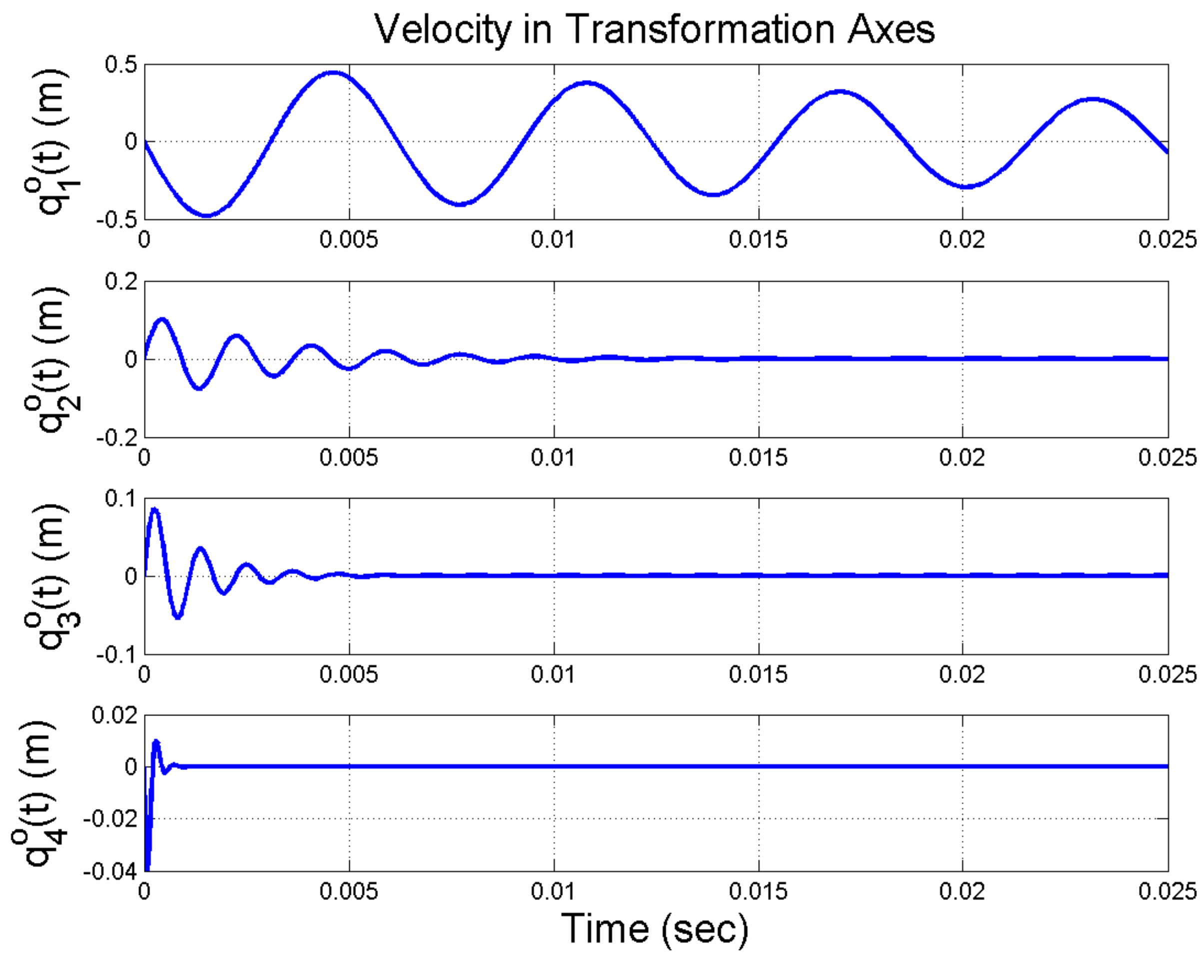
Acceleration in Initial Axes (Z):

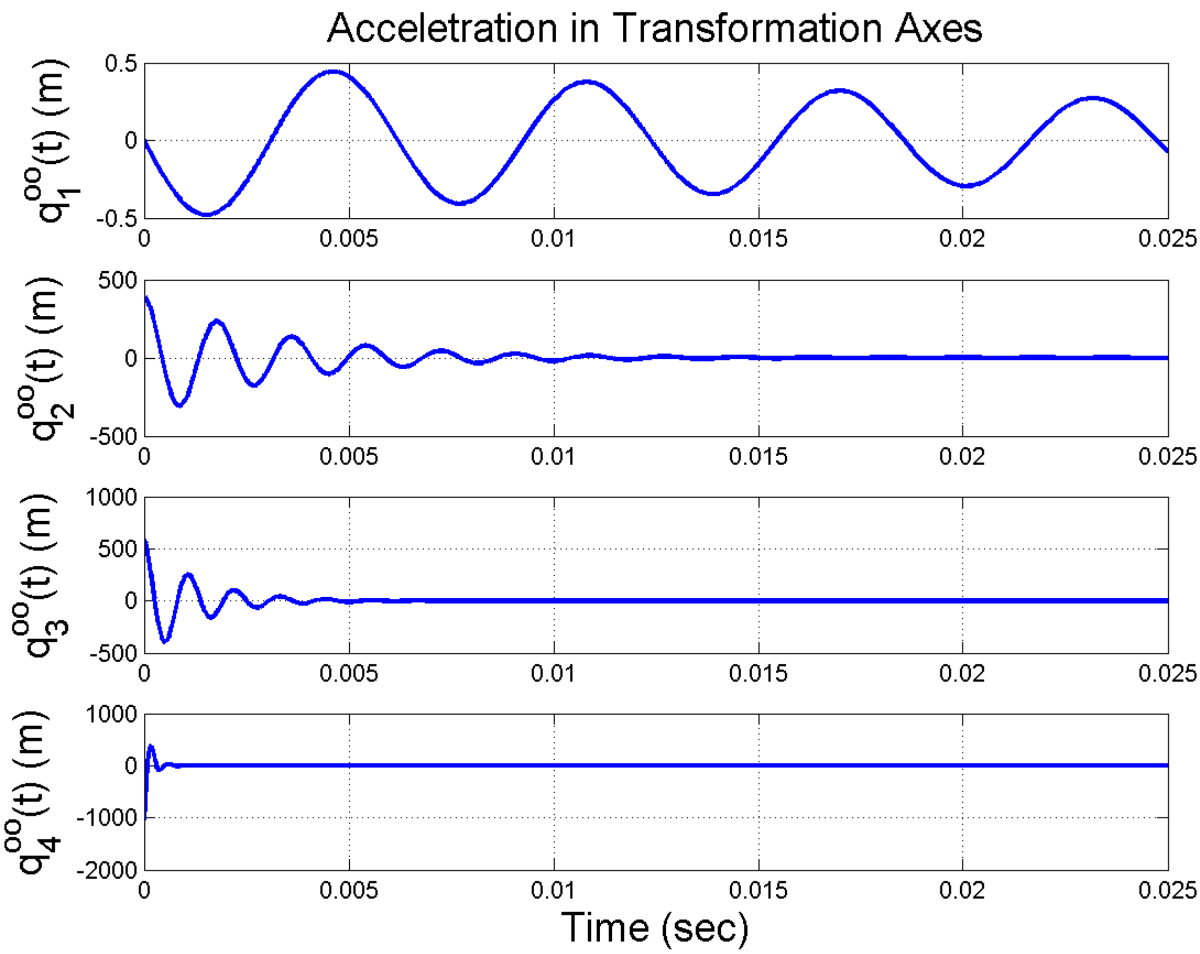
$$\ddot{Z}(t) = X\ddot{q}(t)$$

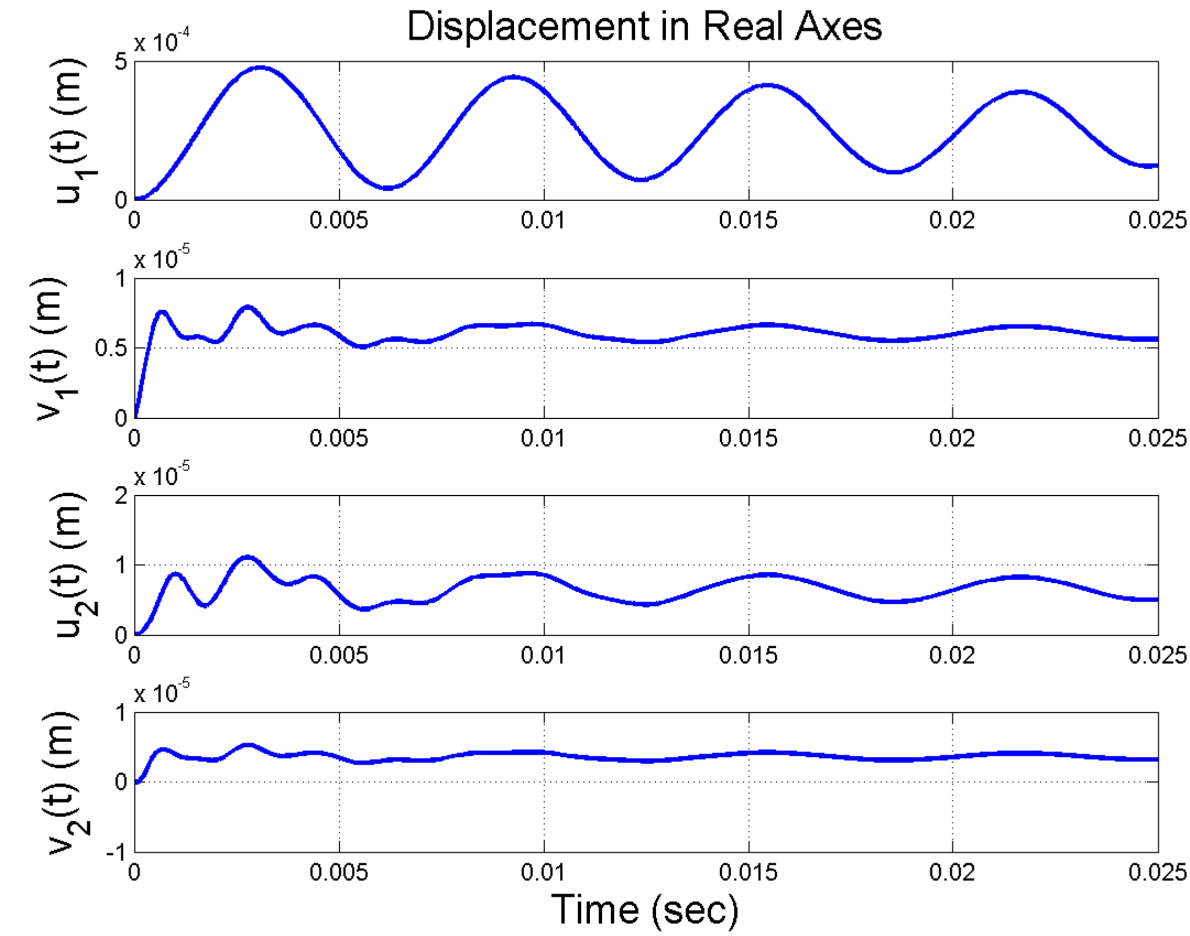
Results:

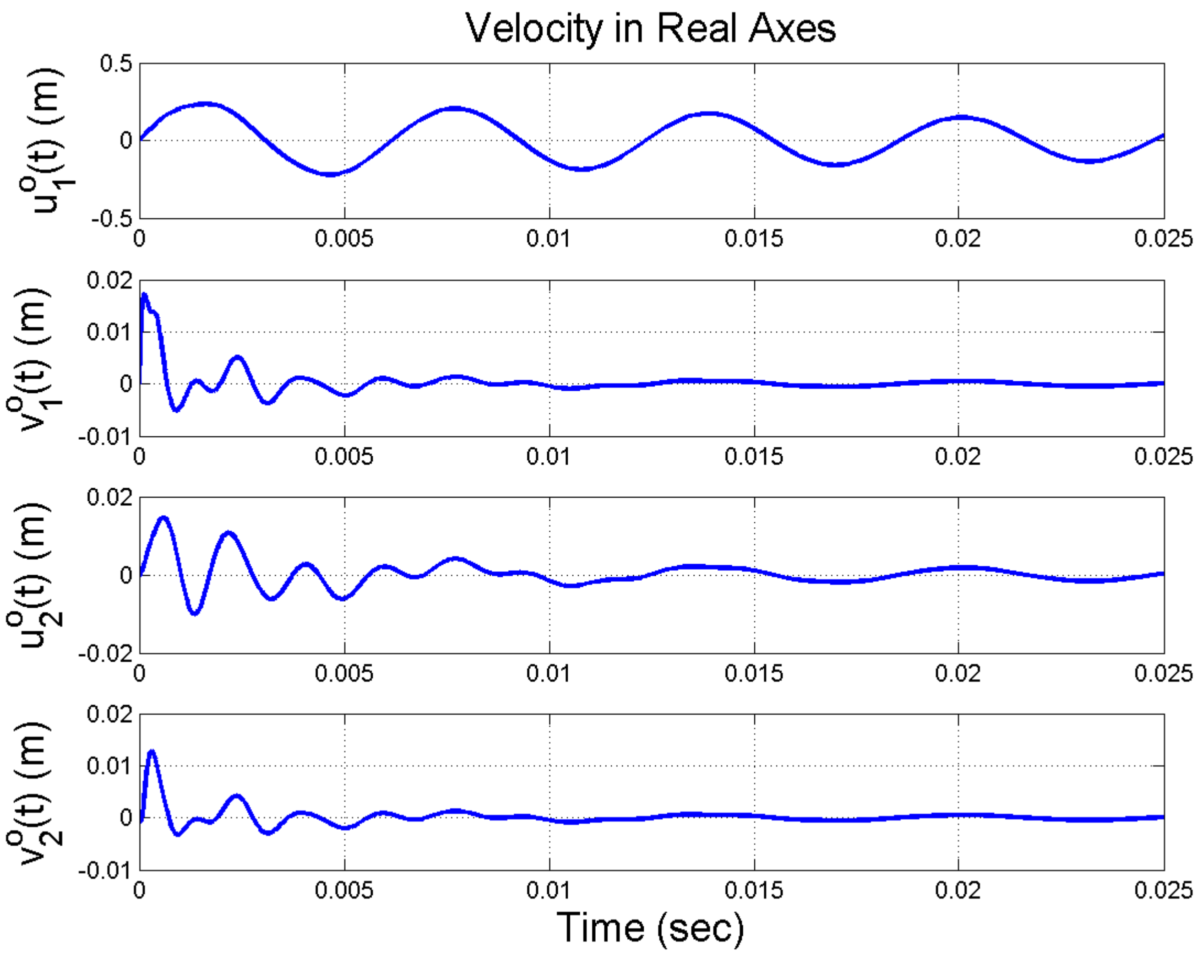

Eigen Vectors
@ Λ = 9.681e-07
@ Λ = 8.3355e-08
@ Λ = 3.1179e-08
@ Λ = 3.9357e-09
Eigen Vector Values
Length
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
1
1.5
2
2.5
3
3.5
4

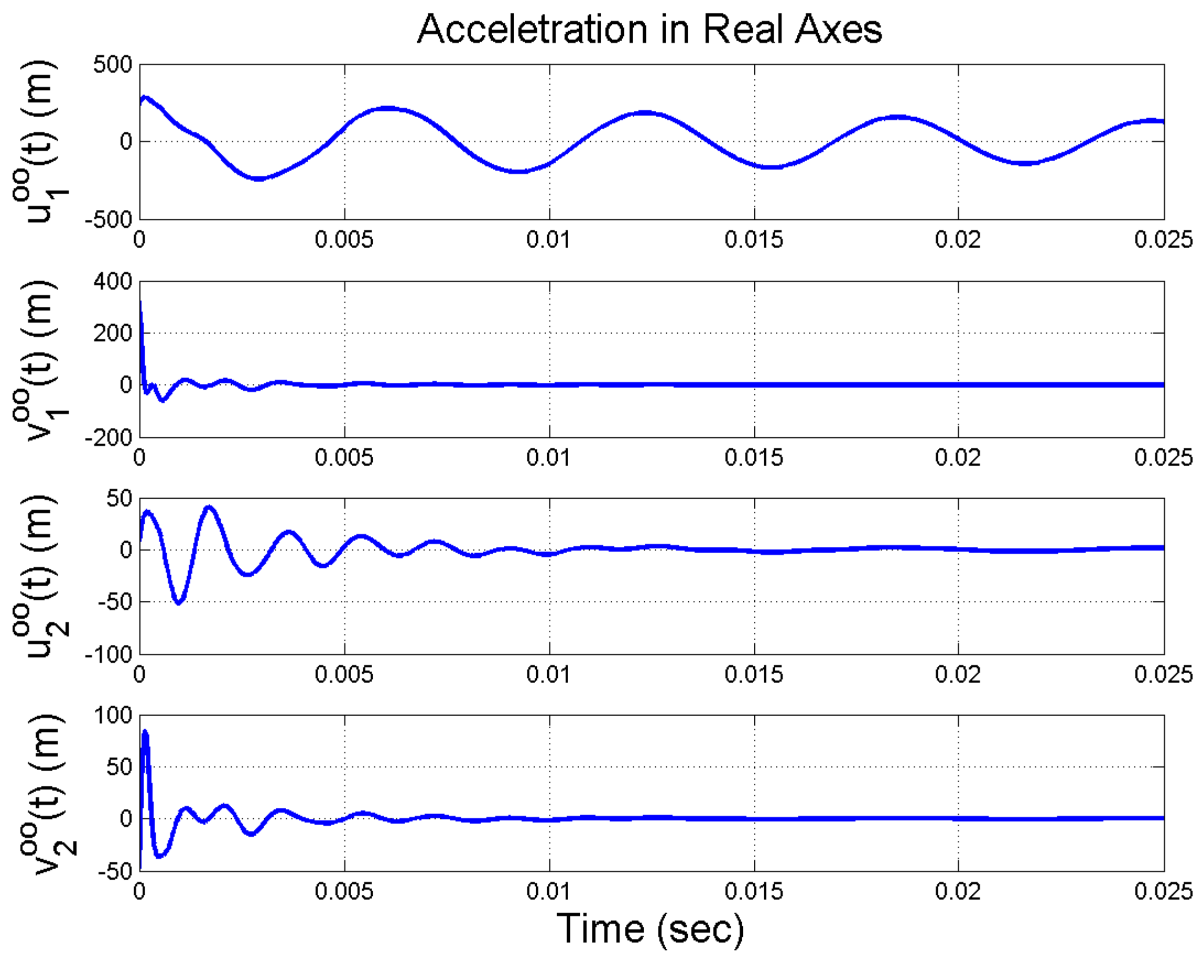

Displacement in Transformation Axes
x 10^-3
q1(t) (m)
x 10^-4
q2(t) (m)
x 10^-5
q3(t) (m)
x 10^-5
q4(t) (m)
0
0.005
0.01
0.015
0.02
0.025
Time (sec)

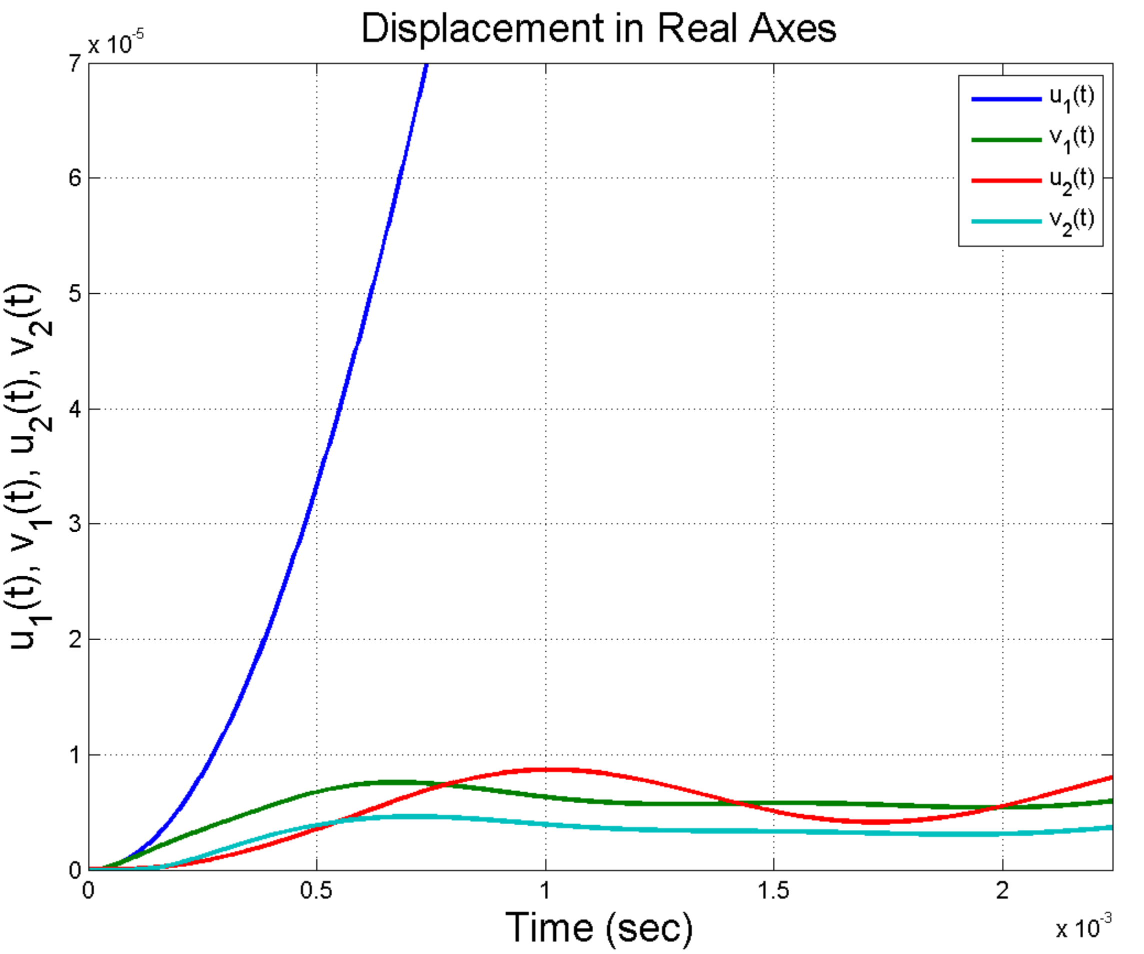

Velocity in Transformation Axes
$q_1^o(t)$ (m)
$q_2^o(t)$ (m)
$q_3^o(t)$ (m)
$q_4^o(t)$ (m)
Time (sec)

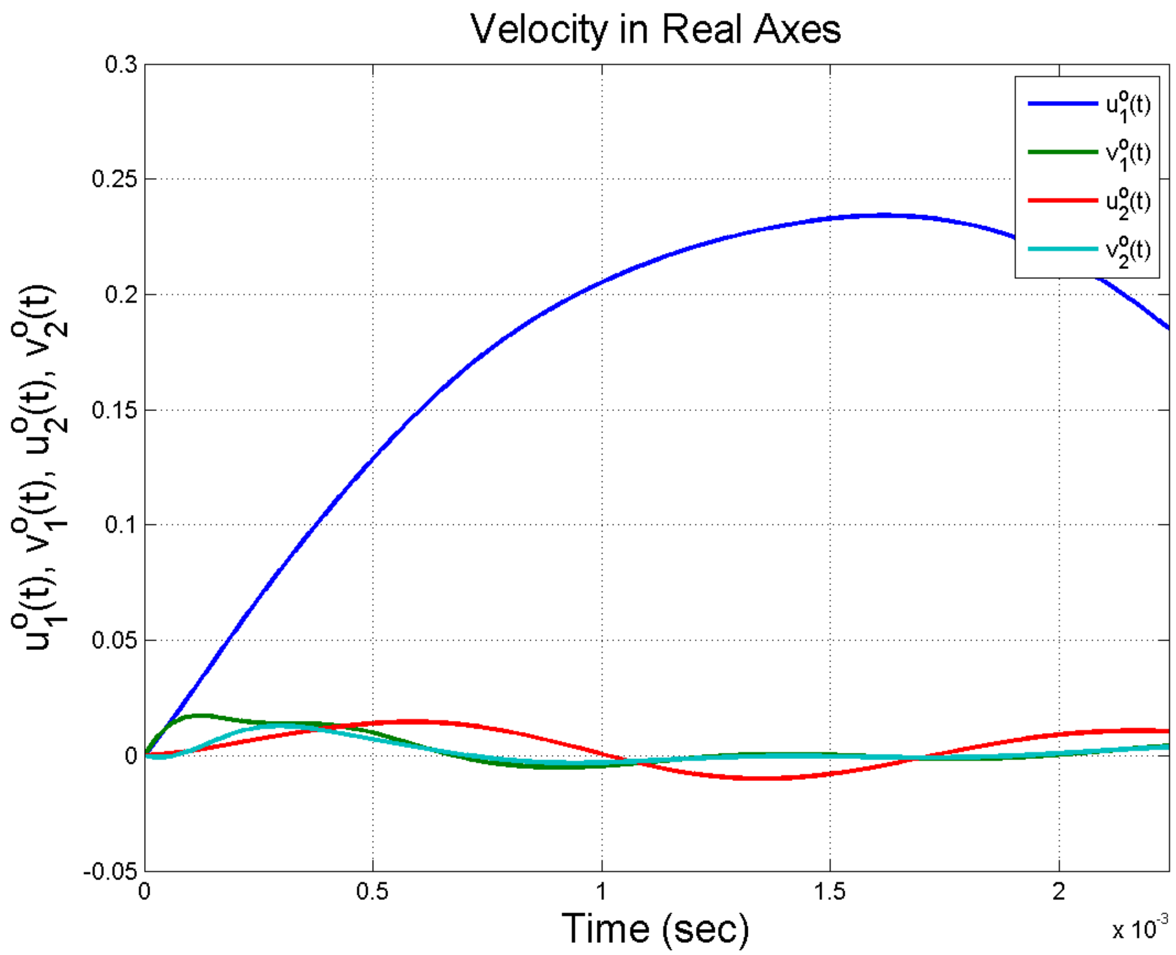

Acceletration in Transformation Axes
$q_1^{oo}(t)$ (m)
$q_2^{oo}(t)$ (m)
$q_3^{oo}(t)$ (m)
$q_4^{oo}(t)$ (m)
Time (sec)

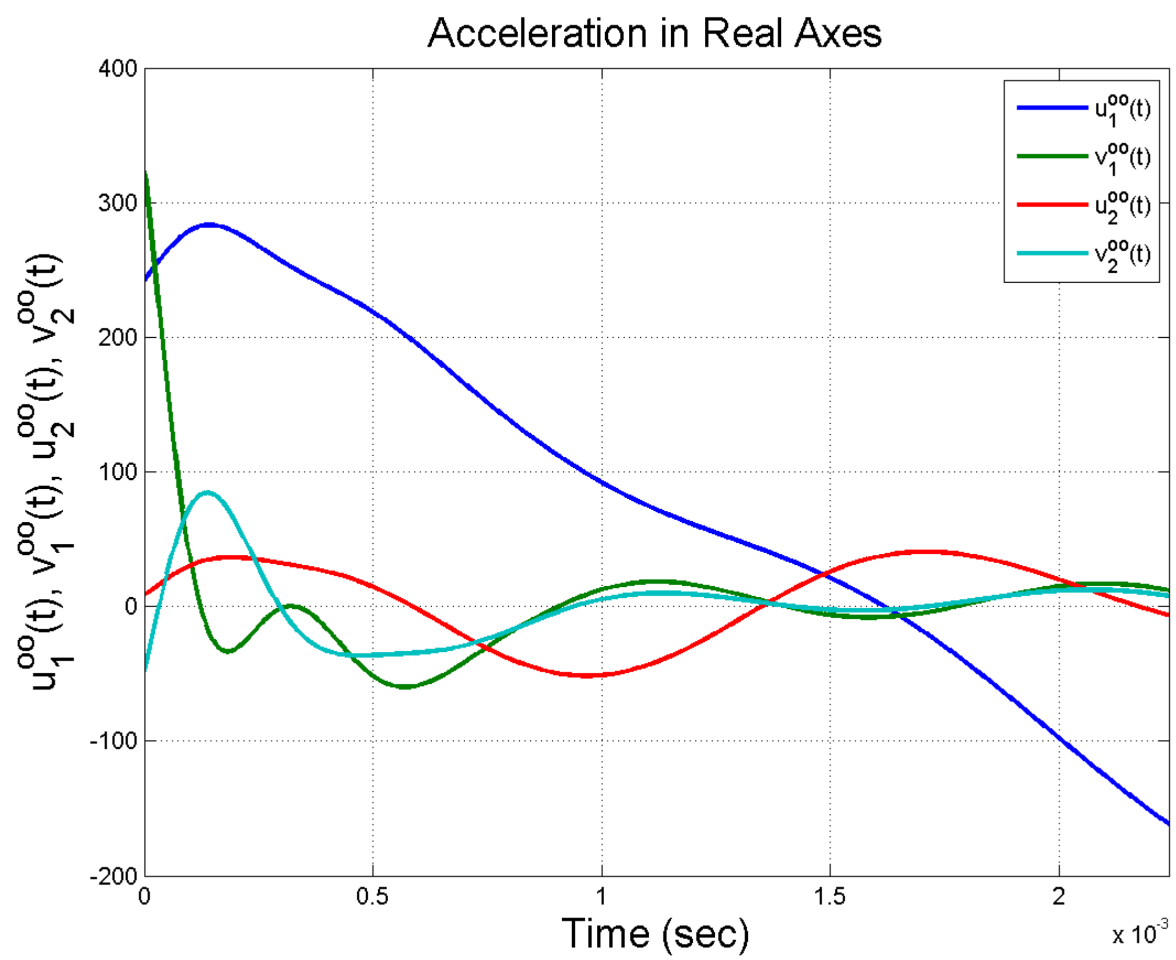

Displacement in Real Axes
x 10^-4
u1(t) (m)
v1(t) (m)
u2(t) (m)
v2(t) (m)
Time (sec)


Velocity in Real Axes
u°1(t) (m)
v°1(t) (m)
u°2(t) (m)
v°2(t) (m)
Time (sec)


Acceletration in Real Axes
u1oo(t) (m)
v1oo(t) (m)
u2oo(t) (m)
v2oo(t) (m)
Time (sec)


Displacement in Real Axes
x 10-5
u1(t)
v1(t)
u2(t)
v2(t)
u1(t), v1(t), u2(t), v2(t)
Time (sec)
x 10-3


Velocity in Real Axes
$u_1^o(t)$, $v_1^o(t)$, $u_2^o(t)$, $v_2^o(t)$
Time (sec)
x 10^{-3}
$u_1^o(t)$
$v_1^o(t)$
$u_2^o(t)$
$v_2^o(t)$
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
0.5
1
1.5
2


Acceleration in Real Axes
$u_1^{oo}(t)$, $v_1^{oo}(t)$, $u_2^{oo}(t)$, $v_2^{oo}(t)$
Time (sec)
x 10^{-3}
$u_1^{oo}(t)$
$v_1^{oo}(t)$
$u_2^{oo}(t)$
$v_2^{oo}(t)$
400
300
200
100
0
-100
-200
0
0.5
1
1.5
2

MATLAB Code of Solution:

```
% This code is used to solve under carriage problem
%% Coded by
% Mohamed Mohamed El-Sayed Atyya
% mohamed.atyya94@eng-st.cu.edu.eg
% 22 - 5 - 2016
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
close all; clear all; clc;
format shortG
%% DATA
    %% Beam-12
     theta(1)=90; % degree
     Length(1)=1; % m
     Area(1)=0.01; % m^2
     Inertia(1)=0.00001; % m^4
     mu(1)=50; % kg/unit length
     E(1)=2.06e11; % N/m^2
    %% Beam-23
     theta(2)=90; % degree
     Length(2)=1; % m
     Area(2)=0.01; % m^2
     Inertia(2)=0.00001; % m^4
     mu(2)=50; % kg/unit length
     E(2)=2.06e11; % N/m^2
    %% Beam-24
     theta(3)=90+45; % degree
     Length(3)=1*sqrt(2); % m
     Area(3)=0.01; % m^2
     Inertia(3)=0.00001; % m^4
     mu(3)=50; % kg/unit length
     E(3)=2.06e11; % N/m^2
    %% Damping parameter
    % C = alpha * M + beta * K
    alpha=1;
    beta=0.00005;
    %% Forces
    % Force=[f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12]
    Force=[1000, 5000, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];
    %% Response Time
    Time=linspace(0,0.025,10000);
    %% Plotting Control
    % Plot = 0 --> No Plotting data    Plot = 1 --> Plotting data
    Plot=1;
    %% Save figures control
    % Save = 0 --> No Saving figures    Save = 1 --> Saving figures
    % The figures will saved in running folder directory
    Save=1;
%% Mass Matrix
```

```
    %% global mass matrix of each beam
    for i=1:length(Area)
      Cx=cosd(theta(i));
      Cy=sind(theta(i));
      M=mu(i)*Length(i);
      ML=M*Length(i);
      ML2=ML*Length(i);
      m{i}=[M*Cx^2/3+13/35*M*Cy^2,      -4/105*M*Cx*Cy,            -11/210*ML*Cy,
M*Cx^2/6+9/70*M*Cy^2,       4/105*M*Cx*Cy,             13/420*ML*Cy ; ...
           -4/105*M*Cx*Cy,              M*Cy^2/3+13/35*M*Cx^2,      11/210*ML*Cx,
4/105*M*Cx*Cy,              M*Cy^2/6+9/70*M*Cx^2,       -13/420*ML*Cx ; ...
           -11/210*ML*Cy,               11/210*ML*Cx,               ML2/105,          -
13/420*ML*Cy,               13/420*ML*Cx,               -1/140*ML2 ; ...
           M*Cx^2/6+9/70*M*Cy^2,        4/105*M*Cx*Cy,              -13/420*ML*Cy,
M*Cx^2/3+13/35*M*Cy^2,      -4/105*M*Cx*Cy,             11/210*ML*Cy ; ...
           4/105*M*Cx*Cy,               M*Cy^2/6+9/70*M*Cx^2,       13/420*ML*Cx,     -
4/105*M*Cx*Cy,              M*Cy^2/3+13/35*M*Cx^2,      -11/210*ML*Cx ; ...
           13/420*ML*Cy,                -13/420*ML*Cx,              -1/140*ML2,
11/210*ML*Cy,               -11/210*ML*Cx,              ML2/105 ];
    end
    %% global stiffness matrix of under carriage
    clear M;
    M=zeros(12,12);
    M(1:6,1:3)=m{1}(1:6,1:3);
    M(1:3,4:6)=m{1}(1:3,4:6);
    M(7:9,4:9)=m{2}(4:6,1:6);
    M(4:6,7:9)=m{2}(1:3,4:6);
    M(10:12,4:6)=m{3}(4:6,1:3);
    M(10:12,10:12)=m{3}(4:6,4:6);
    M(4:6,10:12)=m{3}(1:3,4:6);
    M(4:6,4:6)=m{1}(4:6,4:6)+m{2}(1:3,1:3)+m{3}(1:3,1:3);
    %% mass matrix of free nodes (1,2,3,4,5,6)
    M_free=M(1:6,1:6);
    %% reduced mass matrix nodes (1,2,4,5)
    AM=[M_free(3,1:2),M_free(3,4:5);M_free(6,1:2),M_free(6,4:5)];
    BM=[M_free(3,3),M_free(3,6);M_free(6,3),M_free(6,6)];
    CM=[M_free(1:2,1:2),M_free(1:2,4:5);M_free(4:5,1:2),M_free(4:5,4:5)];
    DM=[M_free(1:2,3),M_free(1:2,6);M_free(4:5,3),M_free(4:5,6)];
    M_reduced=CM-DM*inv(BM)*AM;
%% Stiffness Matrix
    %% global stiffness matrix of each beam
    for i=1:length(Area)
      cx=cosd(theta(i));
      cy=sind(theta(i));
      A=E(i)*Area(i)/Length(i);
      B=E(i)*Inertia(i)/Length(i);
      C=B/Length(i);
      D=C/Length(i);
```

```
        k{i}=[A*cx^2+12*D*cy^2,    (A-12*D)*cx*cy,            -6*C*cy,        -A*cx^2-12*D*cy^2,        (-
A+12*D)*cx*cy,          -6*C*cy ; ...
              (A-12*D)*cx*cy,        A*cy^2+12*D*cx^2,      6*C*cx,        (-A+12*D)*cx*cy,            -
A*cy^2-12*D*cx^2         6*C*cx ; ...
              -6*C*cy,               6*C*cx,                4*B,           6*C*cy,                     -
6*C*cx,                 2*B ; ...
              -A*cx^2-12*D*cy^2,   (-A+12*D)*cx*cy,        6*C*cy,        A*cx^2+12*C*cy^2,          (A-
12*D)*cx*cy,            6*C*cy ; ...
              (-A+12*D)*cx*cy,      -A*cy^2-12*D*cx^2      -6*C*cx,       (A-12*D)*cx*cy,
A*cy^2+12*D*cx^2,        -6*C*cx ; ...
              -6*C*cy,               6*C*cx,                2*B,           6*C*cy,                     -
6*C*cx,                 4*B];
    end
    %% global stiffness matrix of under carriage
    K=zeros(12,12);
    K(1:6,1:3)=k{1}(1:6,1:3);
    K(1:3,4:6)=k{1}(1:3,4:6);
    K(7:9,4:9)=k{2}(4:6,1:6);
    K(4:6,7:9)=k{2}(1:3,4:6);
    K(10:12,4:6)=k{3}(4:6,1:3);
    K(10:12,10:12)=k{3}(4:6,4:6);
    K(4:6,10:12)=k{3}(1:3,4:6);
    K(4:6,4:6)=k{1}(4:6,4:6)+k{2}(1:3,1:3)+k{3}(1:3,1:3);
    %% siffness matrix of free nodes (1,2,3,4,5,6)
    K_free=K(1:6,1:6);
    %% reduced siffness matrix nodes (1,2,4,5)
    AK=[K_free(3,1:2),K_free(3,4:5);K_free(6,1:2),K_free(6,4:5)];
    BK=[K_free(3,3),K_free(3,6);K_free(6,3),K_free(6,6)];
    CK=[K_free(1:2,1:2),K_free(1:2,4:5);K_free(4:5,1:2),K_free(4:5,4:5)];
    DK=[K_free(1:2,3),K_free(1:2,6);K_free(4:5,3),K_free(4:5,6)];
    K_reduced=CK-DK*inv(BK)*AK;
%% Damping Matrix
    %% global damping matrix of under carriage
    C=alpha*M+beta*K;
    %% damping matrix of free nodes (1,2,3,4,5,6)
    C_free=alpha*M_free+beta*K_free;
    %% reduced damping matrix
    C_reduced=alpha*M_reduced+beta*K_reduced;
%% Forces
    %% forces vector of free nodes (1,2,3,4,5,6)
    Force_free=Force(1:6);
    %% reduced forces vector
    Force_reduced=[Force(1:2),Force(4:5)]';
%% EigenValues (lamda) EigenVectors
[Eigen_Vectors, Lamda]=eig(inv(K_reduced)*M_reduced);
%% Natural Frequancy
Wn=diag(sqrt(inv(Lamda)));
%% Orthonormal Eigen Vectors
```

```
for i=1:length(Lamda(1,:))
    Alpha(i)=sqrt(1./(Eigen_Vectors(:,i)'*M_reduced*Eigen_Vectors(:,i)));
    Orthonormal_Eigen_Vectors(:,i)=Alpha(i)*Eigen_Vectors(:,i);
end
%% Transformation Axes (q)
Q=Orthonormal_Eigen_Vectors'*Force_reduced;
M_bar=Orthonormal_Eigen_Vectors'*M_reduced*Orthonormal_Eigen_Vectors;
K_bar=Orthonormal_Eigen_Vectors'*K_reduced*Orthonormal_Eigen_Vectors;
C_bar=Orthonormal_Eigen_Vectors'*C_reduced*Orthonormal_Eigen_Vectors;
%% Damping Coefficient (Zeta)
Zeta=diag(C_bar)./Wn/2;
%% Damping Frequancy (Wd)
Wd=Wn.*sqrt(1-Zeta.^2);
%% Displacement in Transformation Axes (q)
for i=1:length(Zeta)
    q(i,:)=Q(i)./(M_bar(i,i)*Wn(i)^2)*(1-exp(-Zeta(i)*Wn(i)*Time).*(cos(Wd(i)*Time)+Zeta(i)./sqrt(1-
Zeta(i)^2).*sin(Wd(i)*Time)));
end
%% Velocity in Transformation Axes (q_dot)
for i=1:length(Zeta)
    q_dot(i,:)=Q(i)./(M_bar(i,i)*Wn(i)^2)*(Zeta(i)*Wn(i)*exp(-Zeta(i)*Wn(i)*Time).*(cos(Wd(i)*Time)+Zeta(i)./sqrt(1-
Zeta(i)^2).*sin(Wd(i)*Time)) ...
                                                +exp(-Zeta(i)*Wn(i)*Time).*(Wd(i)*sin(Wd(i)*Time)-
Wd(i)*Zeta(i)./sqrt(1-Zeta(i)^2).*cos(Wd(i)*Time)));
end
%% Acceleration in Transformation Axes (q_dot2)
for i=1:length(Zeta)
    q_dot2(i,:)=Q(i)./(M_bar(i,i)*Wn(i)^2)*(-exp(-
Zeta(i)*Wn(i)*Time)*Zeta(i)^2*Wn(i)^2.*(cos(Wd(i)*Time)+Zeta(i)/sqrt(1-Zeta(i)^2)*sin(Wd(i)*Time)) ...
                                                +2*exp(-Zeta(i)*Wn(i)*Time)*Zeta(i)*Wn(i).*(Zeta(i)*Wd(i)/sqrt(1-
Zeta(i)^2)*cos(Wd(i)*Time)-Wd(i)*sin(Wd(i)*Time)) ...
                                                +exp(-
Zeta(i)*Wn(i)*Time).*(Wd(i)^2*cos(Wd(i)*Time)+Zeta(i)*Wd(i)^2/sqrt(1-Zeta(i)^2)*sin(Wd(i)*Time)));
end
%% Displacement in Initial Axes (Z)
Z=Orthonormal_Eigen_Vectors*q;
%% Velocity in Initial Axes (Z_dot)
Z_dot=Orthonormal_Eigen_Vectors*q_dot;
%% Acceleration in Initial Axes (Z_dot2)
Z_dot2=Orthonormal_Eigen_Vectors*q_dot2;
%% Plotting
if Plot ==1
        %% EigenVectors
        set(0,'defaultfigureposition',[0 50 1700 630]);
        figure('Name','Eigen Vectors','NumberTitle','off');
        set(gcf,'color','w')
        hold all;
        for i=1:length(diag(Lamda))
```

```
    plot(1:length(diag(Lamda)),Eigen_Vectors(:,i),'linewidth',2)
    LegenD{i}=['@ \Lambda = ' num2str(Lamda(i,i))];
end
grid on;
title('Eigen Vectors','fontsize',18)
ylabel('Eigen Vector Values','fontsize',18)
xlabel('Length','fontsize',18)
legend(LegenD)
%% Displacement in Transformation Axes
figure('Name','Displacement in Transformation Axes','NumberTitle','off');
set(gcf,'color','w')
subplot(4,1,1)
plot(Time,q(1,:),'linewidth',2)
title('Displacement in Transformation Axes','fontsize',18)
grid on;
ylabel('q_1(t) (m)','fontsize',18)
subplot(4,1,2)
plot(Time,q(2,:),'linewidth',2)
grid on;
ylabel('q_2(t) (m)','fontsize',18)
subplot(4,1,3)
plot(Time,q(3,:),'linewidth',2)
grid on;
ylabel('q_3(t) (m)','fontsize',18)
subplot(4,1,4)
plot(Time,q(4,:),'linewidth',2)
grid on;
ylabel('q_4(t) (m)','fontsize',18)
xlabel('Time (sec)','fontsize',18)
%% Velocity in Transformation Axes
set(0,'defaultfigureposition',[0 50 1700 630]);
figure('Name','Velocity in Transformation Axes','NumberTitle','off');
set(gcf,'color','w')
subplot(4,1,1)
plot(Time,q_dot(1,:),'linewidth',2)
title('Velocity in Transformation Axes','fontsize',18)
grid on;
ylabel('q_1^o(t) (m)','fontsize',18)
subplot(4,1,2)
plot(Time,q_dot(2,:),'linewidth',2)
grid on;
ylabel('q_2^o(t) (m)','fontsize',18)
subplot(4,1,3)
plot(Time,q_dot(3,:),'linewidth',2)
grid on;
ylabel('q_3^o(t) (m)','fontsize',18)
subplot(4,1,4)
plot(Time,q_dot(4,:),'linewidth',2)
```

```
grid on;
ylabel('q_4^o(t) (m)','fontsize',18)
xlabel('Time (sec)','fontsize',18)
%% Acceletration in Transformation Axes
set(0,'defaultfigureposition',[0 50 1700 630]);
figure('Name','Acceletration in Transformation Axes','NumberTitle','off');
set(gcf,'color','w')
subplot(4,1,1)
plot(Time,q_dot(1,:),'linewidth',2)
title('Acceletration in Transformation Axes','fontsize',18)
grid on;
ylabel('q_1^o^o(t) (m)','fontsize',18)
subplot(4,1,2)
plot(Time,q_dot2(2,:),'linewidth',2)
grid on;
ylabel('q_2^o^o(t) (m)','fontsize',18)
subplot(4,1,3)
plot(Time,q_dot2(3,:),'linewidth',2)
grid on;
ylabel('q_3^o^o(t) (m)','fontsize',18)
subplot(4,1,4)
plot(Time,q_dot2(4,:),'linewidth',2)
grid on;
ylabel('q_4^o^o(t) (m)','fontsize',18)
xlabel('Time (sec)','fontsize',18)
%% Displacement in Real Axes
figure('Name','Displacement in Real Axes','NumberTitle','off');
set(gcf,'color','w')
subplot(4,1,1)
plot(Time,Z(1,:),'linewidth',2)
title('Displacement in Real Axes','fontsize',18)
grid on;
ylabel('u_1(t) (m)','fontsize',18)
subplot(4,1,2)
plot(Time,Z(2,:),'linewidth',2)
grid on;
ylabel('v_1(t) (m)','fontsize',18)
subplot(4,1,3)
plot(Time,Z(3,:),'linewidth',2)
grid on;
ylabel('u_2(t) (m)','fontsize',18)
subplot(4,1,4)
plot(Time,Z(4,:),'linewidth',2)
grid on;
ylabel('v_2(t) (m)','fontsize',18)
xlabel('Time (sec)','fontsize',18)
%% Velocity in Real Axes
figure('Name','Velocity in Real Axes','NumberTitle','off');
```

```
        set(gcf,'color','w')
        subplot(4,1,1)
        plot(Time(1:length(Z_dot(1,:))),Z_dot(1,:),'linewidth',2)
        title('Velocity in Real Axes','fontsize',18)
        grid on;
        ylabel('u_1^o(t) (m)','fontsize',18)
        subplot(4,1,2)
        plot(Time(1:length(Z_dot(2,:))),Z_dot(2,:),'linewidth',2)
        grid on;
        ylabel('v_1^o(t) (m)','fontsize',18)
        subplot(4,1,3)
        plot(Time(1:length(Z_dot(3,:))),Z_dot(3,:),'linewidth',2)
        grid on;
        ylabel('u_2^o(t) (m)','fontsize',18)
        subplot(4,1,4)
        plot(Time(1:length(Z_dot(4,:))),Z_dot(4,:),'linewidth',2)
        grid on;
        ylabel('v_2^o(t) (m)','fontsize',18)
        xlabel('Time (sec)','fontsize',18)
        %% Acceletration in Real Axes
        figure('Name','Acceletration in Real Axes','NumberTitle','off');
        set(gcf,'color','w')
        subplot(4,1,1)
        plot(Time(1:length(Z_dot2(1,:))),Z_dot2(1,:),'linewidth',2)
        title('Acceletration in Real Axes','fontsize',18)
        grid on;
        ylabel('u_1^o^o(t) (m)','fontsize',18)
        subplot(4,1,2)
        plot(Time(1:length(Z_dot2(2,:))),Z_dot2(2,:),'linewidth',2)
        grid on;
        ylabel('v_1^o^o(t) (m)','fontsize',18)
        subplot(4,1,3)
        plot(Time(1:length(Z_dot2(3,:))),Z_dot2(3,:),'linewidth',2)
        grid on;
        ylabel('u_2^o^o(t) (m)','fontsize',18)
        subplot(4,1,4)
        plot(Time(1:length(Z_dot2(4,:))),Z_dot2(4,:),'linewidth',2)
        grid on;
        ylabel('v_2^o^o(t) (m)','fontsize',18)
        xlabel('Time (sec)','fontsize',18)
end
%% Save Figures
if Save ==1 && Plot ==1
    for S=1:7
        figure(S);
        saveas(gcf, [num2str(S) '.png']);
    end
end
```