Introduction

Computational Intelligence, Lecture 1, part 1

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Content

- Motivation
- Motivating example
  - fmincon
  - quadprog
  - SVD-based solution
  - CVX-based solution
- Homework

Motivation

Statement

A lot of modern methods, computations and work in general in Robotics are backed by numerical optimization tools.

What we want?

To go from a "I hope it works" to a solid understanding of the mathematics and use-cases of those tools.

Why we want it?

It should allow us to solve a much wider range of problems, and solve them much more effectively.

Motivating example

We have the following problem: find such $\mathbf{x}$ that minimizes $\mathbf{x}^\top \mathbf{M}\mathbf{x}$, while $\mathbf{C}\mathbf{x} = \mathbf{y}$. In other words:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{x}^\top \mathbf{M}\mathbf{x}, \\ \text{subject to} \quad & \mathbf{C}\mathbf{x} = \mathbf{y}. \end{aligned} \tag{1}$$

More concrete:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \\ \text{subject to} \quad & \begin{bmatrix} 1 & 7 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1. \end{aligned} \tag{2}$$

How do we solve it?

Motivating example

fmincon

One very popular way of doing it is by use of a general-purpose local optimization solver, such as `fmincon` provided by MATLAB. Here is one possible solution:

```
M = [1 0 1; 0 5 0; 1 0 3];
C = [1 7 2];
y = 1;

fnc = @(x) x'*M*x;
con = @(x) deal([], C*x-y);
x = fmincon(fnc, zeros(3, 1), [],[],[],[],[],[], con)
```

Average solution time is **4.8** ms (this depends on many factors, so treat it only as a relative information). Solution is $\mathbf{x} = \begin{bmatrix} 0.0442 & 0.1239 & 0.0442 \end{bmatrix}$.

Motivating example
quadprog

A more sophisticated, but still a very straightforward approach is to use a dedicated solver for this class of problems `quadprog` provided by MATLAB. Here is the solution:

```
M = [1 0 1; 0 5 0; 1 0 3];
C = [1 7 2];
y = 1;

x = quadprog(M,[],[],[],C,y)
```

Average solution time is **0.56** ms, an order of magnitude less than with `fmincon`.

Motivating example

SVD-based solution

We can use an algebraic solution, based on SVD decomposition (or its derivative methods - null space and pseudo-inverse), as follows:

```
M = [1 0 1; 0 5 0; 1 0 3];
C = [1 7 2];
y = 1;

tol = 10^(-5);
[P, N] = pinv_null(C, tol);
x = ( eye(3) - N*((N'*M*N) \ (N'*M)) ) * P*y
```

Where `pinv_null` is a function combining `pinv` and `null`, obtained from a single SVD decomposition.
Average solution time is **0.027** ms, $\sim$ 20 times faster than `quadprog` and $\sim$ 200 times faster than `fmincon`.

Finally, we can invoke one of the most powerful convex optimization tools with a user-friendly coding style - CVX:

```
M = [1 0 1; 0 5 0; 1 0 3];
C = [1 7 2];
y = 1;

cvx_begin
variables x(3);
minimize( x' * M * x );
subject to
    C*x == y;
cvx_end
```

However, we will see that the overhead for the call to the solver for this task is excessive. Average solution time is **282** ms, which is ~ 60 times slower than `fmincon`.

Homework

Solve problem (2) by the method you know and understand.

Lecture slides are available via Moodle.

You can help improve these slides at:
`https://github.com/SergeiSa/Linear-Control-Slides-Spring-2020`

Check Moodle for additional links, videos, textbook suggestions.