Dynamics of Nonlinear Robotics Systems

Home Assignment 5

Submitted by: **Mohamed Moustafa Elsayed Ahmed**

Email: **o.ahmed@innopolis.university**

Program: **Robotics**

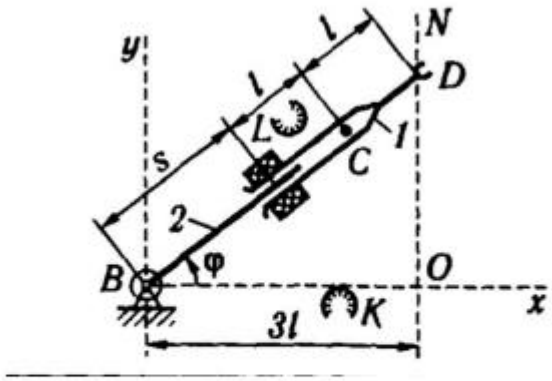
Date:

25/10/2020

Table of Contents

1- Robot Parameters and Scheme

Robot:



m1 = 2 kg (C – center mass)
m2 = 2 kg (B – center mass)
I1 = 1 kg*m^2
I2 = 2 kg*m^2
L = 0,2 m

I will change our references:

m1 = 2 kg (B - center mass)

m2 = 2 kg (C - center mass)

I1 = 2 kg*m^2

I2 = 1 kg*m^2

L = 0.2 m

2-forward kinematics

$$FK = Rz(q1) * Tx(q2) * Tx(2 * l)$$

FK =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 2\,l\cos(q_1) + q_2\cos(q_1) \\ \sin(q_1) & \cos(q_1) & 0 & 2\,l\sin(q_1) + q_2\sin(q_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3- joints & center of mass locations

Center of origin:

```
O0 = 3×1
     0
     0
     0
```

We consider Link 1 just revolute with no length so O1 will be:

```
O1 = 3×1
     0
     0
     0
```

Link 2 is prismatic joint with length 2*l so O1 will be:

O2 =

$$\begin{pmatrix} \cos(q_1) & (2\,l + q_2) \\ \sin(q_1) & (2\,l + q_2) \\ 0 & \end{pmatrix}$$

Center of mass for Link 1:

```
Oc1 = 3×1
      0
      0
      0
```

Center of mass for Link 2:

Oc2 =

$$\begin{pmatrix} \cos(q_1) & (l + q_2) \\ \sin(q_1) & (l + q_2) \\ 0 & \end{pmatrix}$$

4- get the Z

Joint 1 had rotation about z direction so:

```
Z0 = 3×1
     0
     0
     1
```

Joint 2 had translation in x direction so:

Z1 =

$$\begin{pmatrix} \cos(q_1) \\ \sin(q_1) \\ 0 \end{pmatrix}$$

5- Jv for joints

$$J_v^{(i)} = \begin{cases} z_{i-1}^0 & \text{for prismatic joint} \\ z_{i-1}^0 \times \left[o_c^0 - o_{i-1}^0 \right] & \text{for revolute joint} \end{cases}$$

$\boldsymbol{Jv1} = [\boldsymbol{ZO}\ \boldsymbol{X}\ (\boldsymbol{OC1} - \boldsymbol{Oo})\ , \boldsymbol{Zer}\ ]$

```
Jv1 = 3×2
     0     0
     0     0
     0     0
```

$\boldsymbol{Jv2} = [\boldsymbol{ZO}\ \boldsymbol{X}\ (\boldsymbol{OC2} - \boldsymbol{Oo})\ , \boldsymbol{Z1}\ ]$

Jv2 =

$$\begin{pmatrix} -\sin(q_1)\ (l + q_2) & \cos(q_1) \\ \cos(q_1)\ (l + q_2) & \sin(q_1) \\ 0 & 0 \end{pmatrix}$$

6- Jw for joints

$$J_{\omega}^{(i)} = \begin{cases} 0 & \text{for prismatic joint} \\ z_{i-1}^{0} & \text{for revolute joint} \end{cases}$$

$\boldsymbol{Jw1} = [\boldsymbol{ZO}, \boldsymbol{Zer}]$

```
Jw1 = 3×2
     0     0
     0     0
     1     0
```

$\boldsymbol{Jw2} = [\boldsymbol{ZO}, \boldsymbol{Zer}]$

```
Jw2 = 3×2
     0     0
     0     0
     1     0
```

R1 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R2 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

7- Kinetic Energy Calculations

Kinetic Energy Equation:

$$\mathcal{K} = \tfrac{1}{2} m \left| v_c \right|^2 + \tfrac{1}{2} \omega^T \mathcal{I} \omega$$

the final form for the kinetic energy equation:

$$\mathcal{K} = \frac{1}{2} \dot{q}^T \left[\sum_{i=1}^{n} m_i J_{v_i}(q)^T J_{v_i}(q) + J_{\omega_i}(q)^T R_i(q) I R_i(q)^T J_{\omega_i}(q) \right] \dot{q}$$

The kinetic energy for our system is:

$$\mathcal{K} = \tfrac{1}{2} \left[m_1 \left| v_{c1} \right|^2 + \omega_1^T \mathcal{I}_1 \omega_1 \right] + \tfrac{1}{2} \left[m_2 \left| v_{c2} \right|^2 + \omega_2^T \mathcal{I}_2 \omega_2 \right]$$

Where:

$$v_{c1} = \begin{bmatrix} J_{v1}^{(1)} & J_{v1}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J_{v1}^{(1)} \dot{q}_1 + J_{v1}^{(2)} \dot{q}_2$$

$$v_{c2} = \begin{bmatrix} J_{v2}^{(1)} & J_{v2}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J_{v2}^{(1)} \dot{q}_1 + J_{v2}^{(2)} \dot{q}_2$$

$$\omega_1 = \begin{bmatrix} J_{\omega_1}^{(1)} & J_{\omega_1}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J_{\omega_1}^{(1)} \dot{q}_1 + J_{\omega_1}^{(2)} \dot{q}_2$$

$$\omega_2 = \begin{bmatrix} J_{\omega_2}^{(1)} & J_{\omega_2}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J_{\omega_2}^{(1)} \dot{q}_1 + J_{\omega_2}^{(2)} \dot{q}_2$$

Kinetic Energy For joint 1:

D1 =

$$\begin{pmatrix} I_1 & 0 \\ 0 & 0 \end{pmatrix}$$

Kinetic Energy For joint 2:

```
D2 =
```

$$\begin{pmatrix} I_2 + m_2 \cos(q_1)^2 \ (l + q_2)^2 + m_2 \sin(q_1)^2 \ (l + q_2)^2 & 0 \\ 0 & m_2 \cos(q_1)^2 + m_2 \sin(q_1)^2 \end{pmatrix}$$

Sum of Kinetic Energy For the 2 joints:

```
D = D1 + D2
```

$$\begin{pmatrix} m_2 l^2 + 2 m_2 l q_2 + m_2 {q_2}^2 + I_1 + I_2 & 0 \\ 0 & m_2 \end{pmatrix}$$

8- Potential Energy Calculations

The potential energy of the i th-link is:

$$\mathcal{P}_i = m_i g^T r_{c,i}$$

The total potential energy of the robot is then:

$$\mathcal{P} = \sum_{i=1}^{n} \mathcal{P}_i = \sum_{i=1}^{n} m_i g^T r_{c,i}$$

Potential Energy For joint 1:

$$P1 = 0$$

Potential Energy For joint 2:

$$P2 = m2 * g * (l + q2) * \sin(q1)$$

Sum of Potential Energy for both joints:

$$P = P1 + P2$$

G1:

$$G1 = \frac{\partial P}{\partial q1}$$

$$G1 = g\, m_2 \cos(q_1)\ (l + q_2)$$

G2:

$$G2 = \frac{\partial P}{\partial q2}$$

$$G2 = g\, m_2 \sin(q_1)$$

```
G= [G1 ; G2]
```

$$G = \begin{pmatrix} g\, m_2 \cos(q_1)\ (l + q_2) \\ g\, m_2 \sin(q_1) \end{pmatrix}$$

9- Coriolis Force Calculation

The inertia matrix is found from kinetic energy as:

We will need while getting carolios matrix elements

$$D(q) = \begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix}$$

Coriolis and centrifugal forces are associated with:

$$C(q, \dot{q}) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

To get value of elements of Coriolis matrix:

$$c_{kj} = \sum_{i=1}^{n} c_{ijk}(q)\dot{q}_i$$

$$c_{ijk}(q) = \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)$$

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1}$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1}$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2}$$

$$c_{122} = c_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1}$$

$$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2}$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2}$$

Our Corilios matrix for our RP Robot:

C =

$$\begin{pmatrix} \mathrm{dq}_2\, m_2\ (l + q_2) & \mathrm{dq}_1\, m_2\ (l + q_2) \\ -\mathrm{dq}_1\, m_2\ (l + q_2) & 0 \end{pmatrix}$$

10- Equation of motion

$$D(q)\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + C(q,\dot{q})\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + G(q) = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$$

tor =

$$\begin{pmatrix} \mathrm{ddq}_1\ \left(m_2\, l^2 + 2\, m_2\, l\, q_2 + m_2\, {q_2}^2 + I_1 + I_2\right) + g\, m_2 \cos(q_1)\ (l + q_2) + 2\, \mathrm{dq}_1\, \mathrm{dq}_2\, m_2\ (l + q_2) \\ -m_2\ (l + q_2)\, {\mathrm{dq}_1}^2 + \mathrm{ddq}_2\, m_2 + g\, m_2 \sin(q_1) \end{pmatrix}$$

D(q1, q2) =

$$\begin{pmatrix} 2\,{q_2}^2 + \frac{4\,q_2}{5} + \frac{77}{25} & 0 \\ 0 & 2 \end{pmatrix}$$

C(q1, q2, dq1, dq2) =

$$\begin{pmatrix} 4\,\mathrm{dq}_1\,\mathrm{dq}_2\ \left(q_2 + \frac{1}{5}\right) \\ -2\,{\mathrm{dq}_1}^2\ \left(q_2 + \frac{1}{5}\right) \end{pmatrix}$$

G(q1, q2) =

$$\begin{pmatrix} \frac{981\cos(q_1)\ \left(q_2 + \frac{1}{5}\right)}{50} \\ \frac{981\sin(q_1)}{50} \end{pmatrix}$$

11- Initial Positions, Velocities, Torques & Forces

Initial Position:

q1=0

q2=0

Initial Velocity:

q1_velocity=0

q2_velocity=0

Torques:

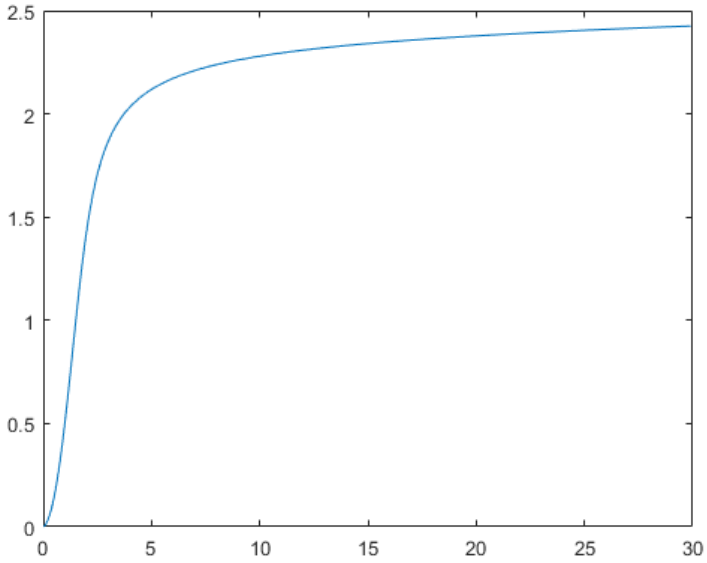
Torque applied to q1 = 300 N.m

Forces:

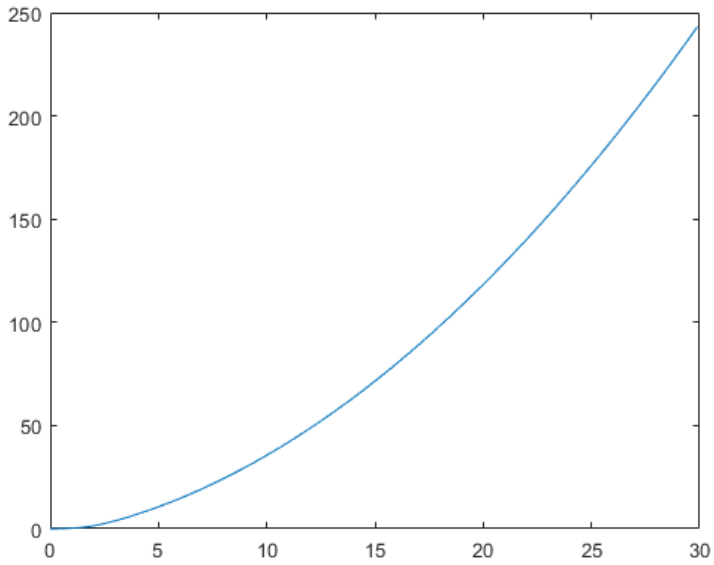
Force Applied to q2 = 100 N

12- Position Plots

Joint 1 Position vs time

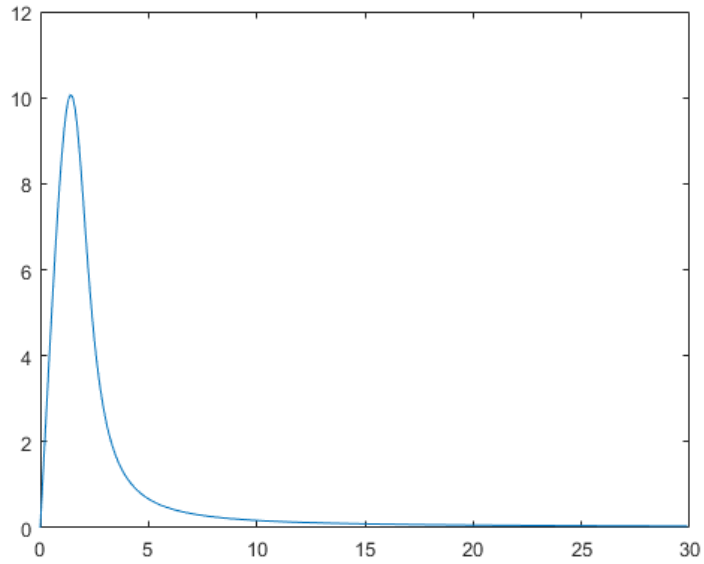


Joint 2 Position vs time

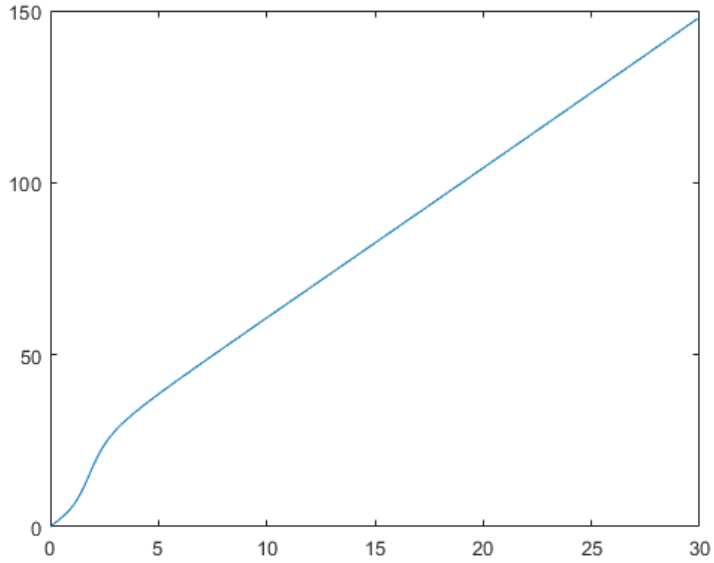


13- Velocity Plots

Joint 2 velocity vs time

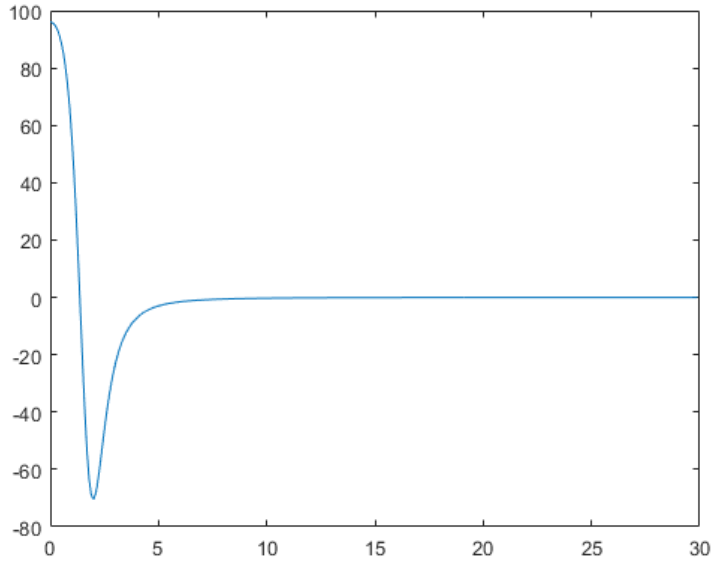


Joint 2 Velocity vs time

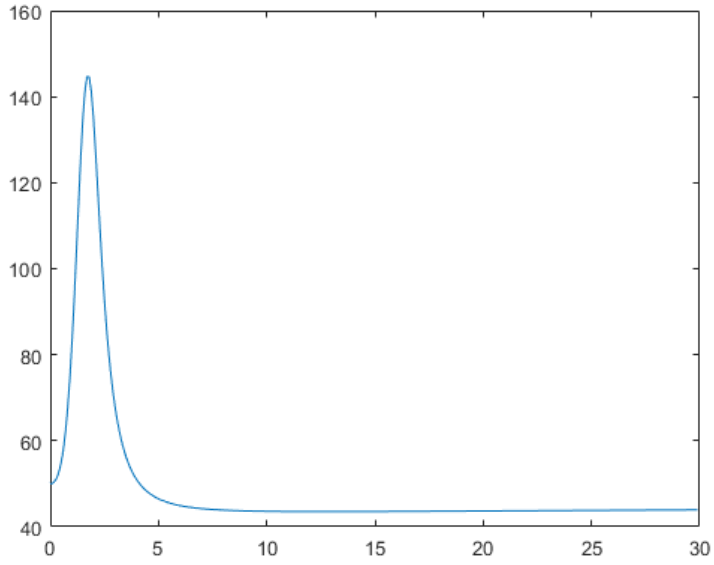


14- Acceleration Plots

Joint 1 Acceleration vs time

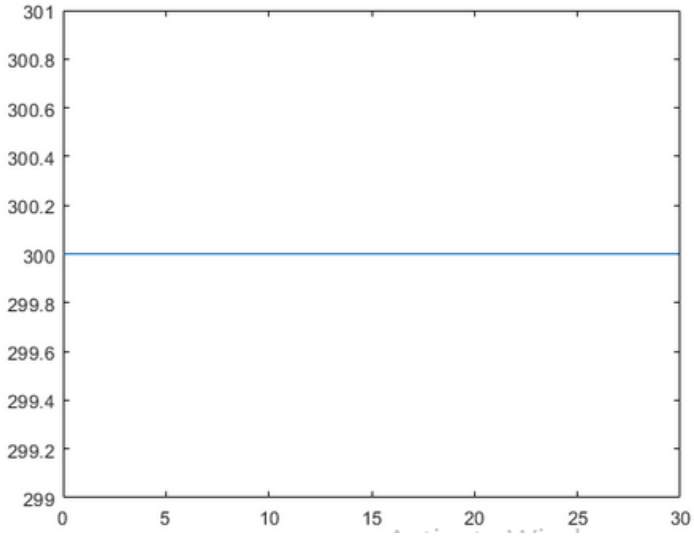


Joint 2 Acceleration vs time

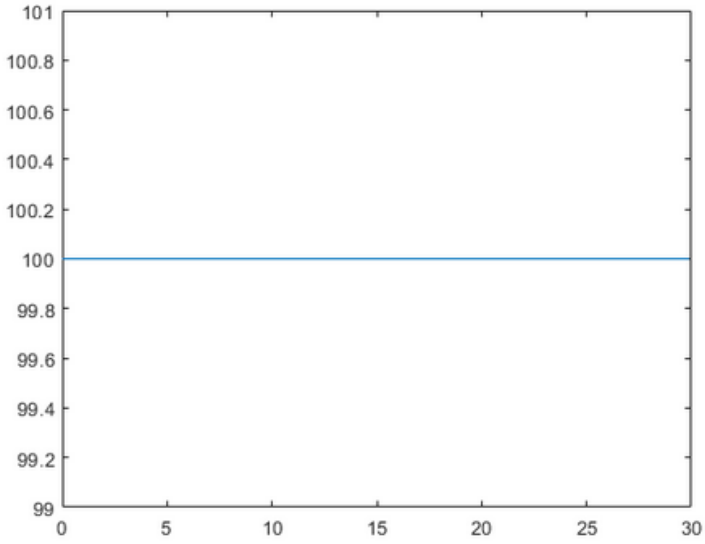


15- Torques & Forces Plots

Joint 1 Torque vs time



Joint 2 Force vs time



16- Link to github

https://github.com/Mohamed-Moustafa/Donrs_hw5.git