Simple Pendulum

A simple pendulum also exhibits periodic motion.

It consists of a particle-like bob of mass *m* suspended by a light string of length *L*.

The motion occurs in the vertical plane and is driven by gravitational force.

The motion is very close to that of the SHM oscillator if the angle is <10°

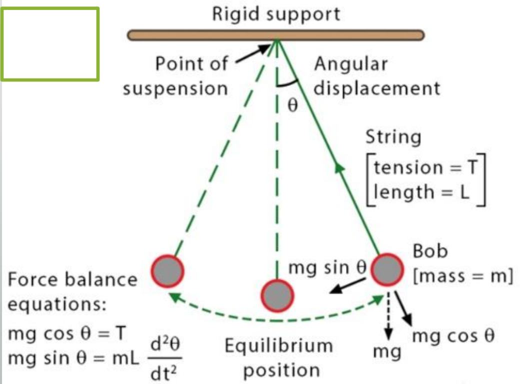
Simple Pendulum, 2

The forces acting on the bob are the tension and the weight.

1) $\vec{T}$ is the force exerted on the bob by the string.

2) $\mathbf{m}\vec{g}$ is the gravitational force.

The tangential component of the gravitational force is a restoring force.



Simple Pendulum, 3

Projection along x-axis, $F_x = ma_x$

$$x = L\theta, \qquad \dot{x} = L\dot{\theta}, \qquad \ddot{x} = L\ddot{\theta}$$

$$-mg\sin\theta = mL\ddot{\theta}$$

Projection along y-axis $mg\cos\theta = T$

The length L of the pendulum is constant, and for small values of θ.

$$-mg\theta = mL\ddot{\theta}$$

$$mL\ddot{\theta} + mg\theta = 0$$

$$divided\ by\ mL, we\ obtain:$$

$$\ddot{\theta} + \frac{g}{L}\theta = 0$$

This confirms the mathematical form of the motion is the same as for SHM.

Simple Pendulum, 4

- The function θ can be written as

$$\theta = \theta_m \cos(\omega t + \varphi)$$

- The angular frequency is:

$$\omega = \sqrt{\frac{g}{L}}$$

- The period is:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

- The frequency is:

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad Or\ f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Simple Pendulum, Summary

- The period and frequency of a simple pendulum depend only on the length of the string and the acceleration due to gravity.
- The period is independent of the mass.
- All simple pendula that are of equal length and are at the same location oscillate with thesame period.

Damped Oscillations

In many real systems, non-conservative forces are present.

1. This is no longer an ideal system (the type we have dealt with so far).
2. Friction and air resistance are common non-conservative forces.

In this case, the mechanical energy of the system diminishes in time, the motion is said to be ***damped***.

Damped Oscillation, Example

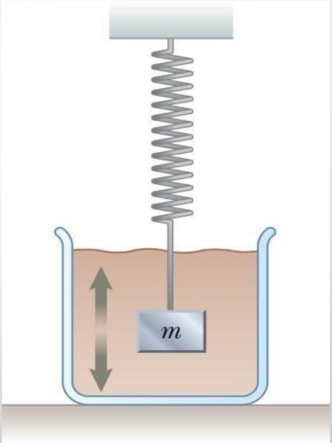
One example of damped motion occurs when an object is attached to a spring and submerged in a viscous liquid.

The retarding force can be expressed as

$$\vec{R} = -b\vec{v}$$

b is a constant

b is called the ***damping***



Damped Oscillations, Graph

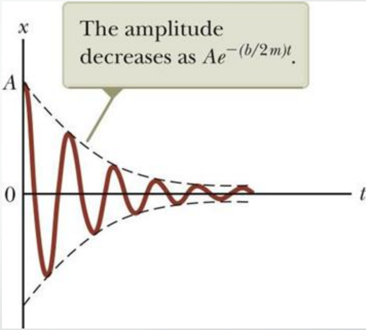
A graph for a damped oscillation.

The amplitude decreases with time.

The blue dashed lines represent the ***envelope*** of the motion.

Use the active figure to vary the mass and the damping constant and observe the effect on the damped motion.

The restoring force is – *kx*.



Damped Oscillations, Equations

From Newton's Second Law

$$\sum \vec{F} = m\vec{a}$$

$$\overrightarrow{F_s} + \vec{R} = m\vec{a}$$

$$-kx - bv = ma_x$$

$$or\ a_x = \ddot{x}\ and\ v = \dot{x}$$

$$so\ m\ddot{x} + b\dot{x} + kx = 0$$

$$divided\ by\ m, we\ obtain:$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

When the retarding force is small compared to the maximum restoring force we can determine the expression for *x*.

- This occurs when *b* is small.

The position can be described by

$$x(t) = Ae^{\frac{-bt}{2m}} \cos(\omega t + \varphi)$$

The angular frequency will be

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Damped Oscillations, Natural Frequency

- When the retarding force is small, the oscillatory character ofthe motion is preserved, but the amplitude decreases exponentially with time.
- The motion ultimately ceases.
- Another form for the angular frequency:

where ω_0 is the angular frequency in the absence of the retarding force and is called the natural frequency of the system.

Types of Damping

If the restoring force is such that $\frac{b}{2m} < \omega_0$, the system is said to be **underdamped.**

When b reaches a critical value b such that $\frac{b}{2m} = \omega_0$, the system will not oscillate. The system is said to be **critically damped**.

If the restoring force is such that $\frac{b}{2m} > \omega_0$, the system is said to be **overdamped**.