Analysis Model: A Particle in Simple Harmonic Motion

- Model the block as a particle.
- The representation will be **particle in simple harmonic motion model.**
- Choose x as the axis along which the oscillation occurs.
- Acceleration

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

- We let

$$\omega^2 = \frac{k}{m} \longrightarrow a = -\omega^2 x$$

A Particle in Simple Harmonic Motion, 2

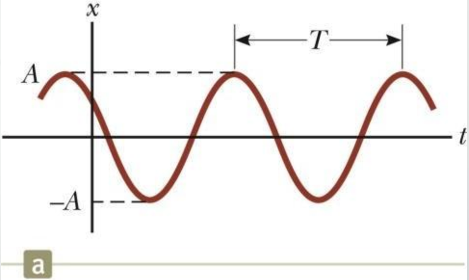
- A function that satisfies the equation is needed.
- Need a function $x(t)$ whose second derivative is the same as the original function with a negative sign and multiplied by ω^2.
- The sine and cosine functions meet these requirements.
- Simple harmonic motion is described by a single bounded trigonometric function like sine and cosine function having single frequency

Simple Harmonic Motion – Graphical Representation

- A solution is

$$x(t) = A\cos(\omega t + \phi)$$

- A, ω, φ are all constants
- A cosine curve can be used to give physical significance to these constants.



Simple Harmonic Motion

- A is the amplitude of the motion.
- This is the maximum position of the particle in either the positive or negative x direction.
- ω is called the angular frequency in rad/s

$$\omega = \sqrt{\frac{k}{m}} > 0$$

- φ is the phase constant or the initial phase angle.

Simple Harmonic Motion, cont.

- A and φ are determined uniquely by the position and velocity of the particle at t = 0.

- If the particle is at x = A at t = 0, then $\varphi = 0$.

- The **phase** of the motion is the quantity $(\omega t + \varphi)$.

- x (t) is periodic and its value is the same each time ωt increases by 2π radians.

Period

- The ***period***, T, of the motion is the time interval required for the particle to go through one full cycle of its motion.
- The values of x and v for the particle at time t equal the values of x and v at $t + T$.

$$T = \frac{2\pi}{\omega}$$

Frequency

- The inverse of the period is called the ***frequency*** expressed in Hertz (Hz)

- The frequency represents the number of oscillations that the particle undergoes per unit time interval.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Summary Equations – Period and Frequency

- The frequency and period equations can be rewritten to solve for ω.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- The period and frequency can also be expressed as:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- The frequency and the period depend only on the mass of the particle and the force constant of the spring
- They do not depend on the parameters of motion.
- The frequency is larger for a stiffer spring (large values of *k*) and decreases with increasing mass of the particle.

Motion Equations for Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \varphi)$$

$$V(t) = \frac{dx}{dt} = A\cos(\omega t + \varphi) = -\omega A\sin(\omega t + \varphi)$$

$$a(t) = \frac{d^2x}{dt^2} = \frac{dV(t)}{dt} = \frac{d}{dt}\big(A\cos(\omega t + \varphi)\big) = -\omega^2 A\cos(\omega t + \varphi)$$

- Simple harmonic motion is one-dimensional and so directions can be denoted by + or - sign.
- Remember, simple harmonic motion is **not** uniformly accelerated motion.

Maximum Values of v and a

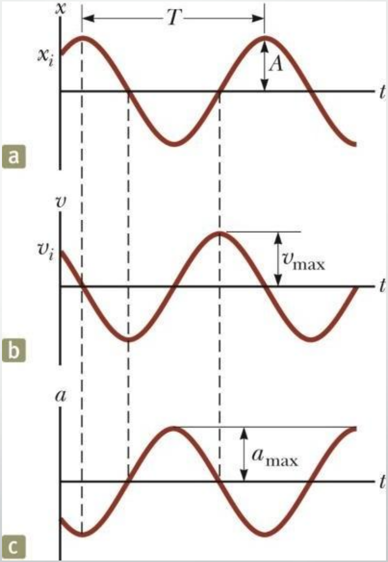
- Because the sine and cosine functions oscillate between +1 and -1, we can easily find the maximum values of velocity and acceleration for an object in SHM.

$$V_{max} = \omega A = \sqrt{\frac{k}{m}} A$$

$$a_{max} = \omega^2 A = \frac{k}{m} A$$

Graphs

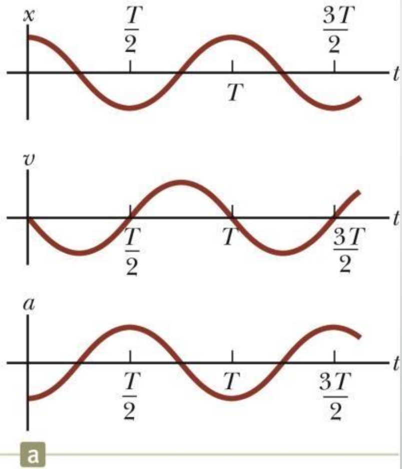
- The graphs show:
  - (a) displacement as a function of time
  - (b) velocity as a function of time
  - (c) acceleration as a function of time
- The velocity is 90º out of phase with the displacement and the acceleration is 180º out of phase with the displacement.



SHM Example 1

- Initial conditions at $t = 0$ are
  - $x(0) = A$
  - $v(0) = 0$

- This means $\varphi = 0$
- The acceleration reaches extremes of $\pm\,\omega^2 A$ at $\pm A$.
- The velocity reaches extremes of $\pm\,\omega A$ at $x = 0$.



SHM Example 1

$\underline{\boldsymbol{at\ t = 0\quad x(0) = A, V(0) = 0}}$

$$x(t) = x_{max}\cos(\omega t + \varphi)$$

$$\boldsymbol{A = x_{max}\cos(\varphi) > 0}$$

$$V(t) = -\omega x_{max}\sin(\omega t + \varphi)$$

$$\boldsymbol{0 = -\omega x_{max}\sin(\varphi)}$$

$$\sin(\varphi) = 0\ , \varphi = 0\ or\ \varphi = \pi$$

$$but\quad \cos(\varphi) > 0\ \ then\ \boldsymbol{\varphi = 0}$$

$$A = x_{max}\cos(0) = x_{max}$$

$$x(t) = A\cos(\omega t), V(t) = -\omega A\sin(\omega t)$$
$$a(t) = -\omega^2 A\cos(\omega t)$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos\left(\omega t + \frac{\pi}{2}\right) = \cos(\omega t)\cos\frac{\pi}{2} - \sin(\omega t)\sin\frac{\pi}{2}$$

$$\cos\frac{\pi}{2} = 0 \text{ and } \sin\frac{\pi}{2} = 1$$

So $\cos\left(\omega t + \frac{\pi}{2}\right) = \cos(\omega t) \times 0 - \sin(\omega t) \times 1 = -\sin(\omega t)$

$$\cos(\omega t + \pi) = \cos(\omega t)\cos\pi - \sin(\omega t)\sin\pi$$

$$\cos\pi = -1 \text{ and } \sin\pi = 0$$

So $\cos(\omega t + \pi) = \cos(\omega t) \times (-1) - \sin(\omega t) \times 0 = -\cos(\omega t)$

$$x(t) = A\cos(\omega t),\ V(t) = \omega A \cos\left(\omega t + \frac{\pi}{2}\right)$$

$$a(t) = \omega^2 A \cos(\omega t + \pi)$$