Energy of the SHM Oscillator

Mechanical energy is associated with a system in which a particle undergoes simple harmonic motion.

- For example, assume a spring-mass system is moving on a frictionless surface.

Because the surface is frictionless, the system is isolated.

- This tells us the total energy is constant.

The elastic potential energy can be found by

$$PE = \frac{1}{2}kx^2$$

$$x(t) = A\cos(\omega t + \varphi)$$

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$$

The kinetic energy can be found by

$$KE = \frac{1}{2} m v^2$$

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi)$$

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \varphi)$$

- Assume a massless spring, so the mass is the mass of the block.

Mechanical energy

Summation of the kinetic energy and elastic potential energy

$$ME = KE + PE$$

$$ME = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \varphi) + \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$$

$$Or\ \ \omega^2 = \frac{k}{m}$$

$$So\ ME = \frac{1}{2}m\frac{k}{m}A^2\sin^2(\omega t + \varphi) + \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$$

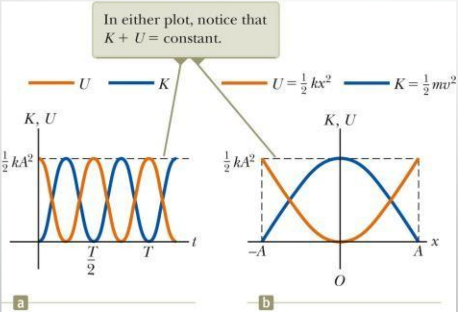
$$ME = \frac{1}{2}kA^2\sin^2(\omega t + \varphi) + \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$$

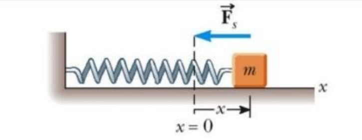
$$ME = \tfrac{1}{2}kA^2[\sin^2(\omega t + \varphi) + \cos^2(\omega t + \varphi)]$$

$$With\ \sin^2(\omega t + \varphi) + \cos^2(\omega t + \varphi) = 1,$$

$$Then\ \boldsymbol{ME = \frac{1}{2}kA^2}$$

- The mechanical energy of the system of a block and a spring is equal to the sum of the potential energy stored in the spring plus the kinetic energy of the block and is proportional to the square of the amplitude.
- The mechanical energy of the system is conserved in simple harmonic motion.

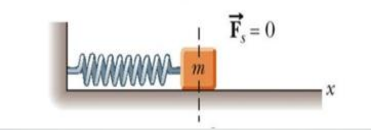




X=A

- KE= 0 because the block is at rest
- $PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$

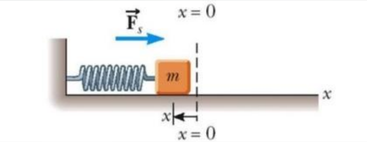
Maximum stretched when $\cos^2(\omega t + \varphi)$=1 at t=0 and x=A



X=0

- PE =0
- $KE = \frac{1}{2}kA^2\sin^2(\omega t + \varphi)$

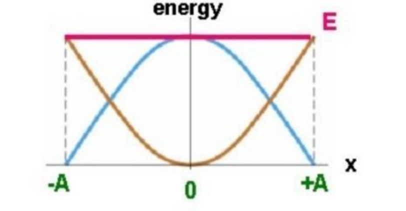
Maximum velocity when $\sin^2(\omega t + \varphi)$=1 at $\frac{T}{4}$ and x=0



X=-A

- KE= 0 because the block is at rest
- $PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$ Maximum compressed when $\cos^2(\omega t + \varphi)$=1 at $\frac{T}{2}$ and x=-A

X	KE	PE
A	$\frac{1}{2}kA^2$	0
0	0	$\frac{1}{2}kA^2$
-A	$\frac{1}{2}kA^2$	0



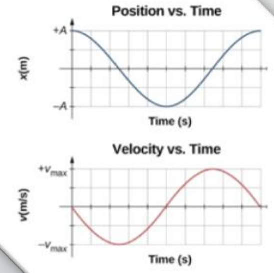


fig (a), $x = +A$

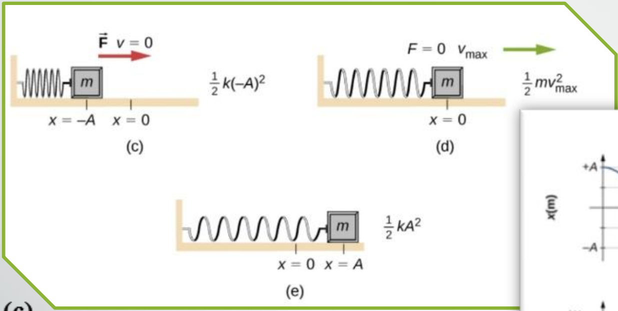
all the energy is stored as potential energy in the spring $\frac{1}{2}kA^2$.

The kinetic energy is equal to zero because the velocity of the block is zero.

fig (b)

As the block moves toward $x = -A$, the block crosses the equilibrium position ($x = 0$). At this point, the spring is neither stretched nor compressed, so the potential energy stored in the spring is zero. At $x = 0$, the mechanical energy is all kinetic energy where

$$KE = \frac{1}{2}m(-v_{max})^2$$



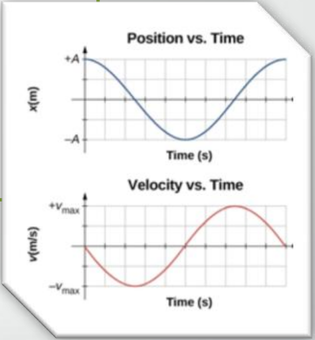


fig (c)

The block continues to move until it reaches $x = -A$ where the block stops and starts moving toward $x = +A$. At the position $x = -A$, the total energy is stored as potential energy in the compressed $PE = \frac{1}{2}k(-A)^2$ and the kinetic energy is equal to zero

fig (d)

As the block passes the equilibrium position ($x = 0$) , the kinetic energy is $KE = \frac{1}{2}m(v_{max})^2$ and the potential energy stored in the spring is zero.

fig (e)

Velocity at a Given Position

The equation for the energy associated with simple harmonic motion can be solved to find the velocity at any position:

$$ME = KE + PE$$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$mv^2 = kA^2 - kx^2$$

$$v^2 = \frac{k}{m}A^2 - \frac{k}{m}x^2 = \frac{k}{m}\left(A^2 - x^2\right) \quad \text{then} \qquad v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$