

ICS 2020 Problem Sheet #8:

Problem 8.1:

a) $Y = (\neg A \uparrow \neg B) \uparrow (C \uparrow (A \uparrow B))$

b)

$$(\neg A \uparrow \neg B) = \neg (\neg A \wedge \neg B)$$

$$(C \uparrow (A \uparrow B)) = \neg (C \wedge \neg (A \wedge B))$$

$$Y = (\neg A \uparrow \neg B) \uparrow (C \uparrow (A \uparrow B)) = \neg (\neg (\neg A \wedge \neg B) \wedge \neg (C \wedge \neg (A \wedge B)))$$

$$= (\neg A \wedge \neg B) \vee (C \wedge \neg (A \wedge B))$$

$$= (\neg A \wedge \neg B) \vee (C \wedge (\neg A \vee \neg B))$$

$$Y = (\neg A \wedge \neg B) \vee (C \wedge (\neg A \vee \neg B))$$

Problem 8.2:

$$S = A \vee B \vee C_{in}; \quad C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \vee B))$$

Making the truth table:

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a)

$$DNF(S) = (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in})$$

$$DNF(C_{out}) = (A \wedge B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in})$$

b)

$$CNF(S) = (\neg A \vee \neg B \vee \neg C_{in}) \wedge (\neg A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee C_{in}) \wedge (A \vee B \vee \neg C_{in})$$

$$CNF(C_{out}) = (\neg A \vee \neg B \vee \neg C_{in}) \wedge (A \vee \neg B \vee C_{in}) \wedge (\neg A \vee B \vee C_{in}) \wedge (A \vee B \vee \neg C_{in})$$

$$\begin{aligned} \text{c) } X \vee \cdot Y &= ((X \vee Y) \wedge \neg(X \wedge Y)) \\ &= (\neg(\neg X \wedge \neg Y) \wedge \neg(X \wedge Y)) \\ &= (\neg X \uparrow \neg Y) \wedge (X \uparrow Y) \\ &= \neg((\neg X \uparrow \neg Y) \uparrow (X \uparrow Y)) \end{aligned}$$

$$S = A \vee \cdot B \vee \cdot C_{in}$$

$$A \vee \cdot B = \neg((\neg A \uparrow \neg B) \uparrow (A \uparrow B))$$

$$S = \neg(((\neg A \uparrow \neg B) \uparrow (A \uparrow B)) \uparrow \neg C_{in}) \uparrow (\neg((\neg A \uparrow \neg B) \uparrow (A \uparrow B)) \uparrow C_{in})$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \vee \cdot B))$$

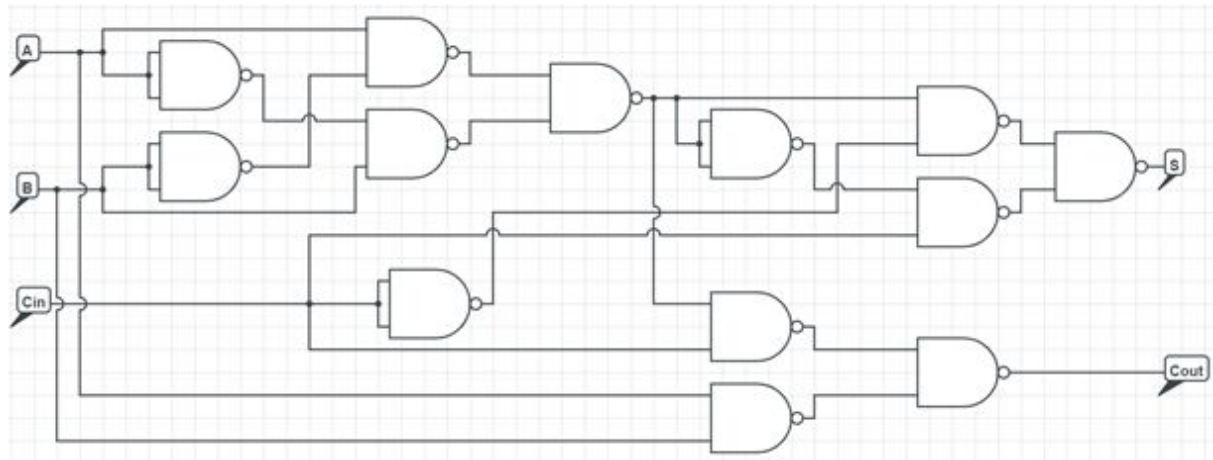
$$C_{out} = (A \wedge B) \vee (C_{in} \wedge \neg((\neg A \uparrow \neg B) \uparrow (A \uparrow B)))$$

$$C_{out} = \neg(\neg(A \wedge B) \wedge \neg(C_{in} \wedge \neg((\neg A \uparrow \neg B) \uparrow (A \uparrow B))))$$

$$C_{out} = \neg((A \uparrow B) \wedge (C_{in} \uparrow \neg((\neg A \uparrow \neg B) \uparrow (A \uparrow B))))$$

$$C_{out} = (A \uparrow B) \uparrow \neg(C_{in} \uparrow \neg((\neg A \uparrow \neg B) \uparrow (A \uparrow B)))$$

d)



Problem 8.3:

a)

fizz :: Int -> String

fizz n | n `mod` 15 == 0 = "FizzBuzz"

| n `mod` 3 == 0 = "Fizz"

| n `mod` 5 == 0 = "Buzz"

| otherwise = show n