ICS 2020 Problem Sheet #5:

Problem 5.1:

a)

The largest number that can be represented is 4444 in base 5 which is equal to:

4*5^3+4*5^2+4*5+4 = 624 in base 10

And the smallest is -624 in base 10

b)

1 ÷ 5 = 0 with remainder of 1

So 1 in base 5 is equal to 0001 after adding (5-1) to the negative value of each digit we get 4443 we then add one to get -1 in b-complement which is equal to 4444.

8 ÷ 5 = 1 with remainder of 3
1 ÷ 5 = 0 with remainder of 1

So 8 in base 5 is equal to 0013 after adding (5-1) to the negative value of each digit we get 4431 we then add one to get -1 in b-complement which is equal to 4432.

c)

4444+4432 = 4431

To convert back to decimal:
-(4444 - (4431 - 1))=-0014= -(0 * 5^3 + 0 * 5^2 + 1 * 5 + 4) = -9 (base 10)

Problem 5.2:

let's first represent 273.15 in base 2:

let's start with 273:

273 ÷ 2 = 136 remainder =1;
136 ÷ 2 = 68 remainder =0;
68 ÷ 2 = 34 remainder = 0;
34 ÷ 2 = 17 remainder =0;
17 ÷ 2 = 8 remainder =1;
8 ÷ 2 = 4 remainder = 0;
4 ÷ 2 = 2 remainder = 0;
2 ÷ 2 = 1 remainder =0;
1 ÷ 2 = 0 remainder =1;

so 273 (in base 10) = 100010001 (in base 2) let’s now represent 0.15 in base 2 :

$$0.15 \times 2 = 0 + 0.3;$$
$$0.3 \times 2 = 0 + 0.6;$$
$$0.6 \times 2 = 1 + 0.2;$$
$$0.2 \times 2 = 0 + 0.4;$$
$$0.4 \times 2 = 0 + 0.8;$$
$$0.8 \times 2 = 1 + 0.6;$$
$$0.6 \times 2 = 1 + 0.2;$$
$$0.2 \times 2 = 0 + 0.4;$$
$$0.4 \times 2 = 0 + 0.8;$$
$$0.8 \times 2 = 1 + 0.6;$$
$$0.6 \times 2 = 1 + 0.2;$$
$$0.2 \times 2 = 0 + 0.4;$$
$$0.4 \times 2 = 0 + 0.8;$$
$$0.8 \times 2 = 1 + 0.6;$$
$$0.6 \times 2 = 1 + 0.2;$$
$$0.2 \times 2 = 0 + 0.4;$$
$$0.4 \times 2 = 0 + 0.8;$$
$$0.8 \times 2 = 1 + 0.6;$$
$$0.6 \times 2 = 1 + 0.2;$$
$$0.2 \times 2 = 0 + 0.4;$$
$$0.4 \times 2 = 0 + 0.8;$$
$$0.8 \times 2 = 1 + 0.6;$$
$$0.6 \times 2 = 1 + 0.2;$$
$$0.2 \times 2 = 0 + 0.4;$$

we didn’t get a fractional part that was equal to zero but we have exceeded the mantissa limit so :

0.15 (in base 10) = 0.0010 0110 0110 0110 0110 0110
(in base 2)

So:

273.15 (b=10) =
100010001.00100110011001100110 0110 (b=2)

273.15 (b=10) =
100010001.00100110011001100110 0110 x 2^0 (b=2)

273.15 (b=10) =
1.00010001001001100110011001100110 0110 x 2^8 (b=2)

Mantissa is 1.0001000100100110011001100110

We now need to adjust the exponent :

The new exponent is equal to 8 +127 = 135 (in base 10)
we now need to convert it to base 2:

135 ÷ 2 = 67 remainder =**1;**
67 ÷ 2 = 33 remainder = **1;**
33 ÷ 2 = 16 remainder = **1;**
16 ÷ 2 = 8 remainder = **0;**
8 ÷ 2 = 4 remainder = **0;**
4 ÷ 2 = 2 remainder = **0;**
2 ÷ 2 = 1 remainder = **0;**
1 ÷ 2 = 0 remainder = **1;**

135 (in base 10) = 10000111 (in base 2)

The last step is gonna be to normalize the mantissa by removing the first bit and adjust its length to 23 which gives us:

00010001001001100110011

The sign is negative so S gets the value 1 (first bit)

So:

-273.15 converted to 32 bit single precision IEEE 754 binary floating point is

1 - 1000 0111 - 000 1000 1001 0011 0011 0011

b)

-100010001.00100110011001100110011001 (base 2) =

=-(2^7 + 2^3+2^0+2^(-3)+2^(-4)+2^(-5)+2^(-8)...+2^(-23)=

=-273.149993896484375 (base 10)

Problem 5.3 :