Problem 2.1:

a-) 1) to wich big O set do t_1 belong?

$t_1 \leqslant t_1$ for $m > m_0$ $m_0 = 1$

$5m^2 + 16 \leqslant 5m^2 + 16$

$5m^2 + 16 \leqslant 5m^2 + 16m^2$

$t_1 \leqslant 21m^2$

$t_1 \in O(m^2)$

2-) to wich big O set do t_2 belong?

$t_2 \leqslant t_2$ $m > m_0$ $m_0 = 0$

$6m^3 + m^2 + 18 \leqslant 6m^3 + m^2 + 18$

$6m^3 + m^2 + 18 \leqslant 6m^3 + m^3 + 18m^3$

$t_2 \leqslant 25m^3$

$t_2 \in O(m^3)$

b-) we know that $O(m^2) \subset O(m^3)$ so the entire program belongs to $O(m^3)$

c-) $f_1 \in O(g_1) \Leftrightarrow f_1 \leqslant k_1 g_1$ for some $m \geqslant m_0$

$f_2 \in O(g_2) \Leftrightarrow f_2 \leqslant k_2 g_2$ for some $m > m_0'$

$\Leftrightarrow (f_1 + f_2) \leqslant k_1 g_1 + k_2 g_2$

$\Leftrightarrow (f_1 + f_2) \leqslant k_1 \times \max(g_1, g_2) + k_2 \times \max(g_1, g_2)$

$\Leftrightarrow (f_1 + f_2) \leqslant (k_1 + k_2) \max(g_1, g_2)$

$(f_1 + f_2) \in O(\max(g_1, g_2))$

Problem 2.2:

for $m = 1$

$$\sum_{k=1}^{1} (2k-1)^2 = (2-1)^2 = 1 \quad \text{and} \quad \frac{2\times 1 (2\times 1 - 1)(2\times 1+1)}{6} = 1$$

the proposition is true for $m = 1$

Now we assume that $\sum_{k=1}^{m} (2K-1)^2 = \frac{2m(2m-1)(2m+1)}{6} = \frac{4m^3 - m}{3}$

let's prove that it is true for $(m+1)$

$$\sum_{k=1}^{m} (2k-1)^2 = \frac{4m^3 - m}{3} \iff \sum_{k=1}^{m} (2K-1)^2 + (2(m+1)-1)^2 = \frac{4m^3 - m}{3} + (2(m+1)-1)^2$$

$$\iff \sum_{k=1}^{m+1} (2K-1)^2 = \frac{4m^3 - m}{3} + \frac{3(2m+1)^2}{3}$$

$$\iff \sum_{k=1}^{m+1} (2K-1)^2 = \frac{4m^3 - m + 3(4m^2 + 4m + 1)}{3}$$

$$\iff \sum_{k=1}^{m+1} (2K-1)^2 = \frac{4m^3 + 12m^2 + 11m + 3}{3}$$

$$\iff \sum_{k=1}^{m+1} (2K-1)^2 = \frac{4m^3 + 12m^2 + 12m - m + 4 - 4 + 3}{3}$$

$$\iff \sum_{k=1}^{m+1} (2K-1)^2 = \frac{4m^3 + 12m^2 + 12m + 4 - m - 1}{3}$$

$$\iff \sum_{k=1}^{m+1} (2K-1)^2 = \frac{4(m^3 + 3m^2 + 3m + 1) - (m+1)}{3}$$

$$\iff \sum_{k=1}^{m+1} (2K-1)^2 = \frac{4(m+1)^3 - (m+1)}{3}$$

$$\Leftrightarrow \sum_{k=1}^{m+1} (2k-1)^2 = \frac{4(m+1)^3 - (m+1)}{3}$$

$$\Leftrightarrow \sum_{k=1}^{m+1} (2k-1)^2 = \frac{2(m+1)(2(m+1)-1)(2(m+1)+1)}{6}$$

So the proposition is true for $m+1$

we can then conclude that:

$$1^2 + 3^2 + 5^2 + \dots (2m-1)^2 = \sum_{k=1}^{m} (2k-1)^2 = \frac{2m(2m-1)(2m+1)}{6}$$

Problem 2.3:

a) the list comprehension (of) that returns all positive factors of 210 is:

[x | x <- [1... 210], 210 'mod' x == 0]

(~~we can easly replace another number~~)

we can easly replace 210 by another number to get all its positive factor

b) [(a, b, c) | a <- [1.. 100], b <- [1..100], c <- [1,.100], a*a + b*b == c]

this include the duplicates. I Didn't Know how to remove them.