Simulating the Chaotic behavior of Chua circuit and how to control it

MATH 202 project

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1.. Abstract

"Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?!", Philip Merilees concocted this as a title for a talk Edward Lorenz was to present in 1972, however, that title displays that even a small change of air pressure made by the butterfly flap can cause something great as a tornado on the long term, and that simple is chaos. Chua circuit is one of the simplest electrical samples of a chaotic system which displays a new kind of attractors, double scroll attractor, that will be demonstrated in upcoming simulation, graph… etc. Chua circuit is used in varies application, after using a certain control system needed for a certain purpose in applications.

2.. Introduction

Edward Lorenz was the first one to define chaos (When the present determines the future, but the approximate present does not approximately determine the future),also there is no excepted definition to chaos till now. Robert L Devaney presents the most used definition dynamic systems which posses certain properties can be defined as chaotic system: it could respond sensitively in various initial conditions, it should be act as mixed system according to the concepts of topology and it must possess periodic orbits with noticeable density(fractals).

To see how simple and beautiful chaos is create the following system, draw 3 equal distance vertices of equilateral triangle and label each vertex with 2 numbers from 0 to 6 as in figure (1), choose a start point randomly from inside the 3 vertices, (triangle), and roll a dice, and mark the point midway between your start point and the vertex with the same number that appeared on the dice, use a ruler of course, roll the dice again and repeat the past procedures like a 10,000 times and you will get Sierpenski triangle figure (2).

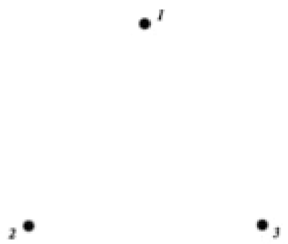


Figure 1

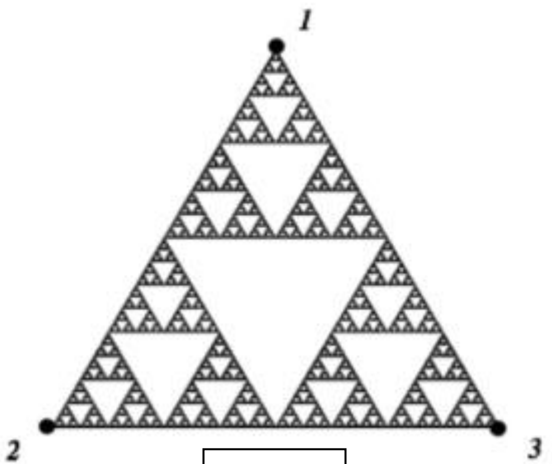


Figure 2

Chaos concerns deterministic systems whose behavior can in principle be predicted. Chaotic systems are predictable for a short time then became unpredictable . The amount of time that the behavior of a chaotic system can be effectively predicted depends on three factors: How much uncertainty can be tolerated in the forecast, how accurately its current state can be measured, And the Lyapunov time, about 1 millisecond in electrical circuits, and the most interest cases of chaotic dynamical system takes place as attractor.

Lorenz attractor is one of the simplest attractor that appears in weather forecasting with equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

These three equation when solved using softwares yields an osilating (noisy) curves of x,y,z versus time (figure 3), but one good way to check the sensitivity of initial conditions is to graph x vursus y versus z which yealds apretty graph (figure 4).

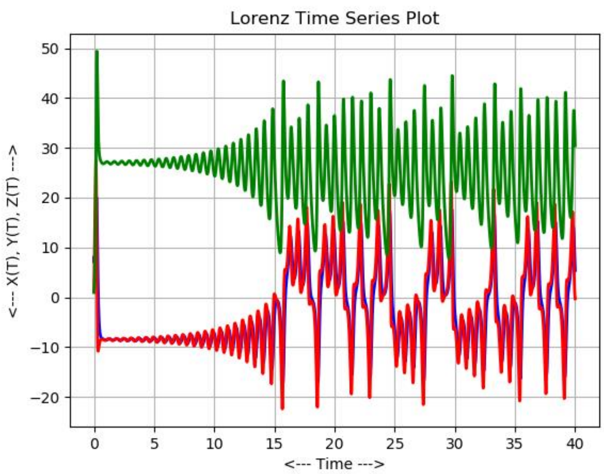


Figure 3

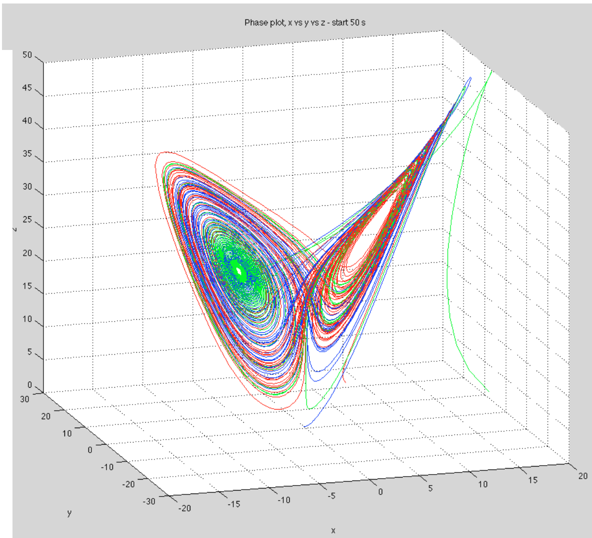


Figure 4

3.. Purpose (Purpose and theory replace include methodology)

Our purpose is to study chaotic behavior in circuits to have a piece of chaos from universe in our hand.

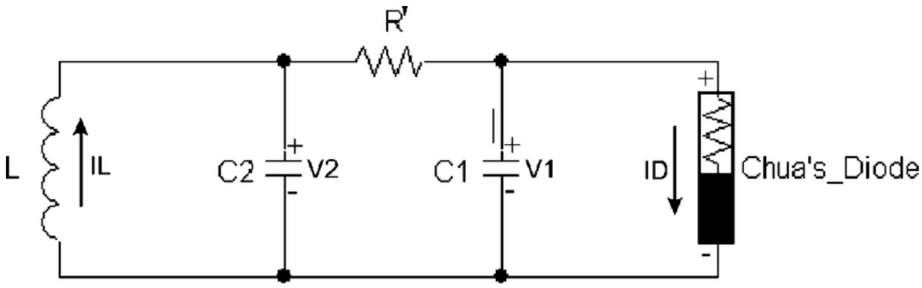
we intend to:

- Using OrCAD software to simulate the Chua circuit itself
- simulate the circuit chaotic behavior using MATLAB.
- using a method to control the chaotic behavior and see if it was effective.

4.. Theory

A)) Chua circuit

Chua circuit looks like:



And by applying Kirchhoff laws and neglect the internal inductor resistance, we can get equations that governs this circuit:

$$C_1 \frac{dv_1}{dt} = \frac{v_2 - v_1}{R} - i_d(v_1)$$
$$C_2 \frac{dv_2}{dt} = \frac{(v_1 - v_2)}{R} - i_l$$
$$L \frac{di_L}{dt} = -v_2$$

"Id" is piece wise differentiable function (abrupt changing slopes) and that is the non linear element in the circuit(figure 5).

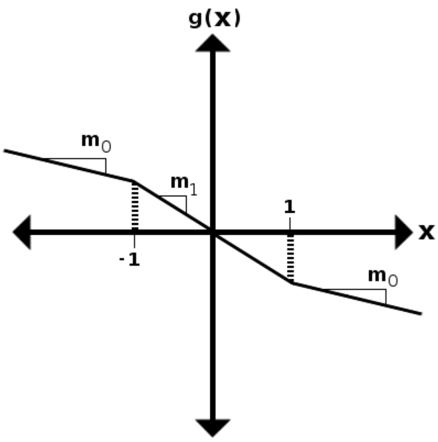


Figure 5

B)) Chaos control in Chua circuit

The chaotic behavior in general may be useful as one may want to make a system show a very different behavior with small changes, but the problem is that this change can exceed the desired behavior, so controlling the chaotic behavior of the system is very important.

In general if we want to control a system of linear ODEs we will add terms to each equation and then we determine these unknown terms by forcing the error dynamics as:

$$\dot{E} + KE = 0$$

Where E is the error or the difference between the solution of DE with unknown terms(Y) and the desired solution we want the system converge ore oscillate around (Y^*).

$$(\dot{Y} - \dot{Y}^*) + K(Y - Y^*) = 0$$

$$Y = Y_{old} + Control$$

You can conclude now that I created a new system Y by adjusting the old system and this edit will happen so that the solution Y will settle down exponentially (whatever the IC) to certain desired solution Y^*.

Now we are ready to apply these general method to control our Chua circuit:

First: we can write the old system as,

$$\dot{v}_1 = \frac{1}{C_1}\left[\frac{v_2 - v_1}{R} - g(v_1)\right]$$

$$\dot{v}_2 = \frac{1}{C_2}\left[\frac{v_1 - v_2}{R} + i_3\right]$$

$$\dot{i}_3 = \frac{1}{L}[-v_2 - R_0 i_3]$$

Where g(v1) can written as:

$$g(v1) = G_b v_1 + \frac{1}{2}(G_a - G_b)[|v_1 + B_p| + |v_1 - B_p|]$$

Where: Ga and Gb are the slopes of first and last line, Bp is the slope of middle line.

Second: we adjust the old system by adding controlling terms,

$$\frac{dv_1}{dt} = \frac{1}{C_1}[\frac{v_2 - v_1}{R} - g(v_1) + i_{u1}]$$

$$\frac{dv_2}{dt} = \frac{1}{C_2}[\frac{v_1 - v_2}{R} + i_3 + i_{u2}]$$

$$\frac{di_3}{dt} = \frac{1}{L}[-v_2 - R_0 i_3] + i_{u3}$$

Third: just make some substitutions to look good, let

$$x=\frac{v_1}{B_p}, y=\frac{v_2}{B_P}, z=\frac{i_3R}{B_p}, \tau=+\frac{t}{C_2R}, u_1=\pm\frac{R}{B_p}i_{u1,}i_{u2}=u_{2,}$$

$$i_{u3}=u_3, k=1, m_0=G_aR, m1=G_bR, \alpha=\frac{C_2}{C_1}, \beta=\frac{C_2R^2}{L}, \gamma=\frac{C_2R_0}{L}$$

Then the equations will look like:

$$\dot{x}=\alpha(y-x-f(x))+u_1$$
$$\dot{y}=x-y+z+u_2$$
$$\dot{z}=-\beta y-\gamma z+u_3$$

Where $f(x)=m_1x+0.5(m_0-m_1)\big[|x+1|-|x-1|\big]$

Fourth: substitute in forced error,

$$\dot{E}+KE=0$$

$$E=(X-X^*),\quad X=[x \quad y \quad z]^T$$

$$\begin{bmatrix} \dot{x}-\dot{x}^* \\ \dot{y}^*-\dot{y}^* \\ \dot{z}-\dot{z}^* \end{bmatrix}+\begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}\begin{bmatrix} x-x^* \\ y-y^* \\ z-z^* \end{bmatrix}=0$$

$$\begin{bmatrix} \alpha(y-x-f(x))+u_1-\dot{x}^* \\ x-y+z+u_2-\dot{y}^* \\ -\beta y-\gamma z+u_3-\dot{z}^* \end{bmatrix}+\begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}\begin{bmatrix} x-x^* \\ y-y^* \\ z-z^* \end{bmatrix}=0$$

Finally: get the controlling terms

$$u_1=-\alpha y+\alpha x+\alpha f(x)+\dot{x}^*-k_1x+k_1x^*$$
$$u_2=-x+y-z+\dot{y}^*-k_2y+k_2y^*$$
$$u_3=\beta y+\gamma z+\dot{z}^*-k_3z+k_3z^*$$

5.. Predictions

It is predicted when we solve Chua circuit equations in MATLAB:

A)) Before adding control

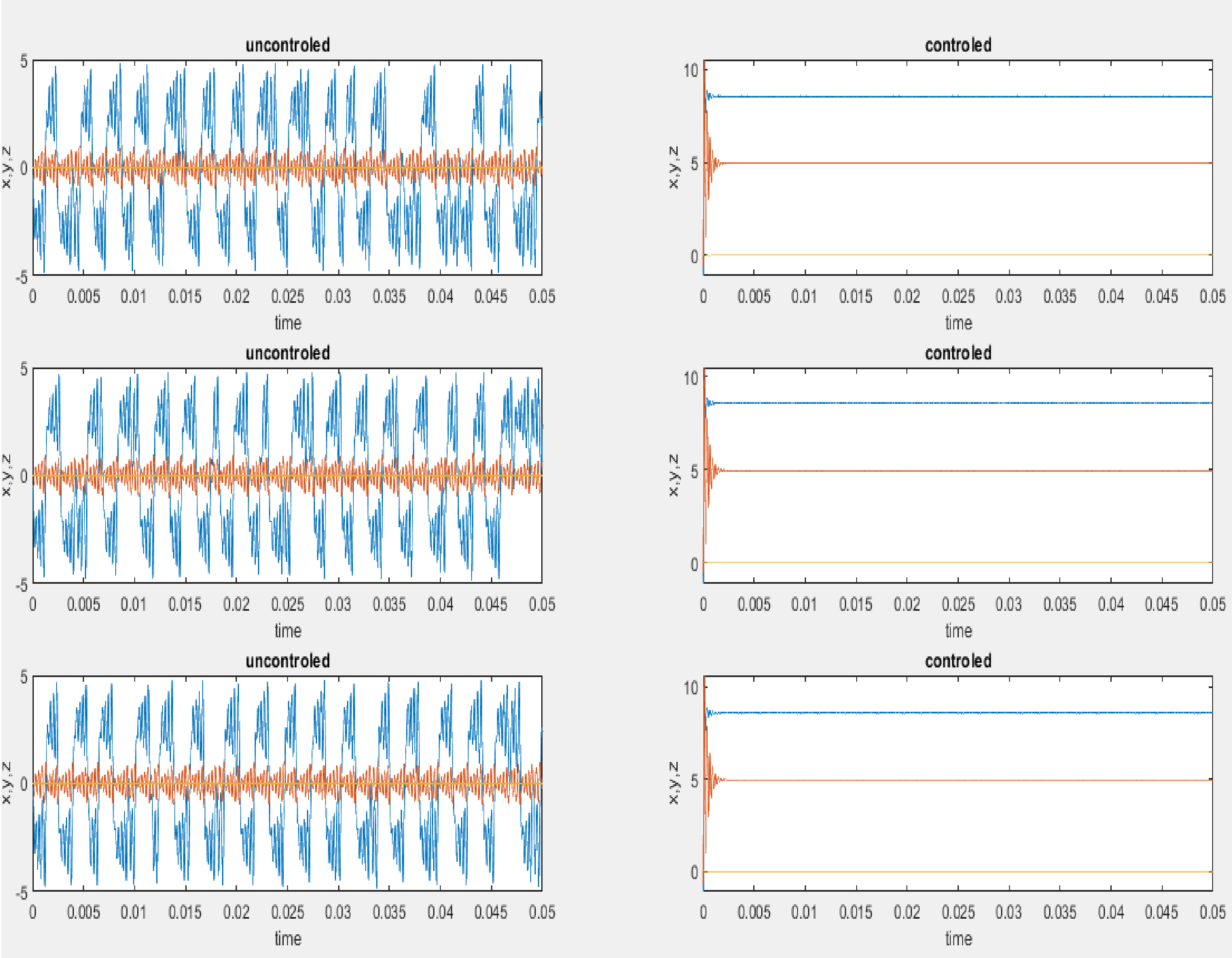
- oscillation graphs of V1,V2 and IL verses time, just like attractor chaos in atmosphere.
- double attractor graph when plotting V1 vs V2 vs IL.

B)) After adding control

- oscillation of V1 and V2 and IL will settle down to one single value, and this value will be the same whatever the IC.
- attractor orbits of different IC will converge to one point.

6.. Results and discussion

result 1:



Note:

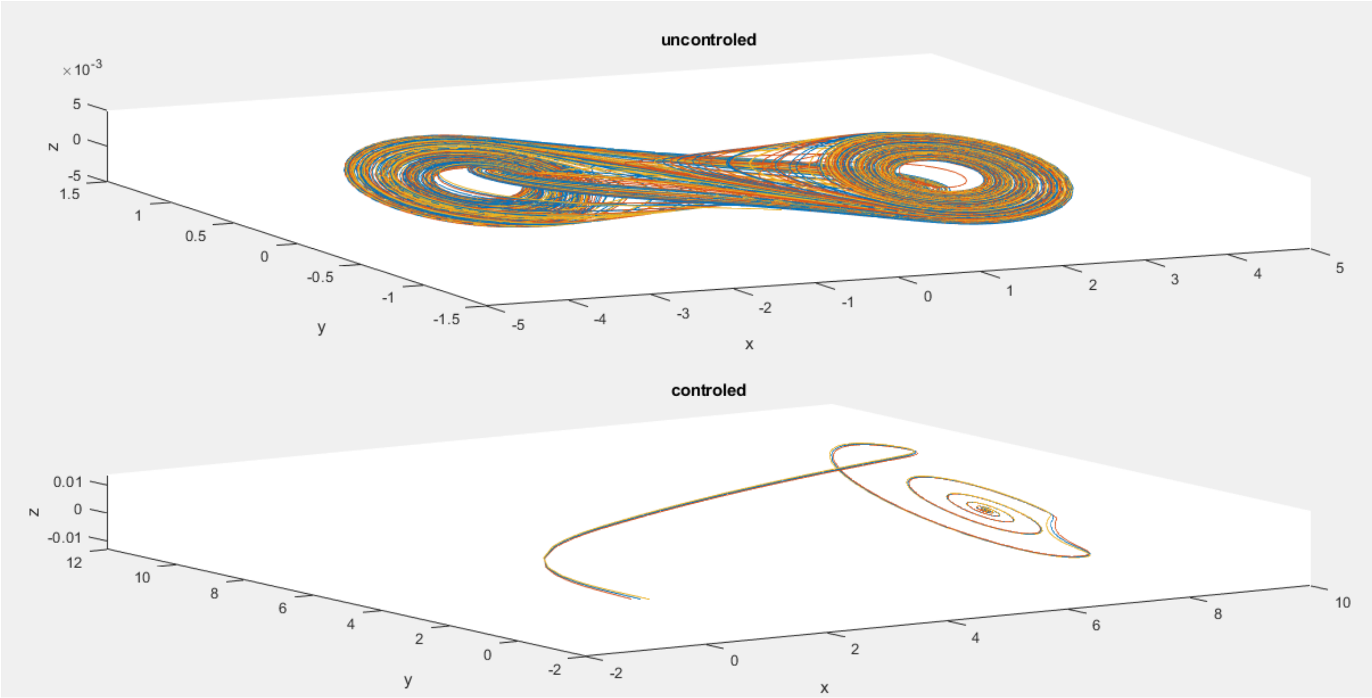
- Different row of plots corresponds to different initial conditions
- Different color corresponds to a solution of one equation of the chua system

discussion

As predicted before adding the control the Chua parameter made oscillation with time these oscillations differ from one IC to another.

After controlling, the oscillation of x,y,t or V1,V2,IL settled down at certain desired points which means that the chaos is limited for only first little time, and these point are the same fore different IC in contrast with the uncontrolled plots.

result 2:



Note

- Different colors corresponds to different initial conditions.

discussion

As predicted Chua parameters V1,V2,IL when plotted together it showed some chaotic behavior just like Lorentz attractor, however this curve called double scroll attractor.

After controlling, curves of different initial conditions are more coherent, they are far at first but at the end (the most inner part of the spiral) the will be at the same point and for the end of time.

7.. applications

One of the important applications of chaos is in secure communication. The block diagram of the communication system is shown in (Figure 6(a)). The circuit implementation is shown in (Figure 6(d)) and the simulation is shown in (Figure 6(e)). One of the problems faced in the implementation of the communication system was that of synchronization. Efforts are in progress to perfectly match the chaotic systems at the transmitter and the receiver.

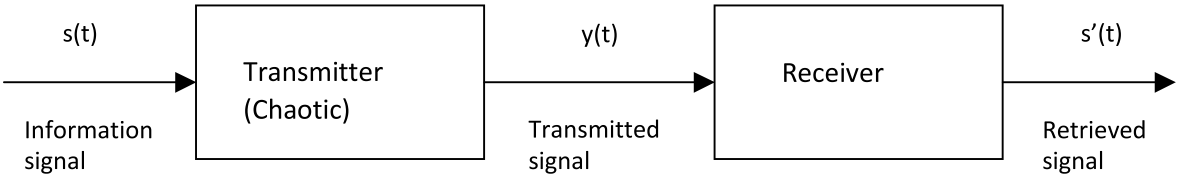


Fig. 6(a) Block Diagram of Communication System

Transmitter

The internal block diagram of the transmitter is shown in Fig. 2(b). The transmitter uses a summer to mask the information signal s(t) with the chaotic signal $V_{c1}(t)$ generated by the chua's circuit to produce the resultant signal r(t). The buffer is used to get signal without attenuation and the inverter is used transmit the resultant signal without any phase shift.

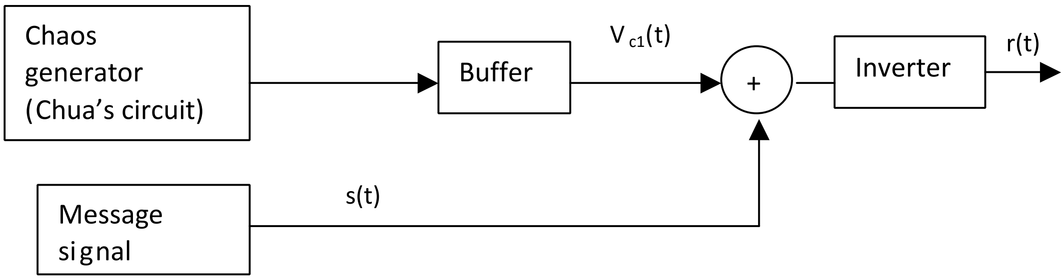


Fig. 6(b) Internal Block Diagram of Transmitter

Receiver

The internal block diagram of the receiver is shown in Fig. 2(c). The receiver consists of a chua's circuit similar to the one at the transmitter to generate a chaotic signal $V_{c1}(t)$ that is perfectly matched with the chaotic signal generated at the transmitter. The signal r(t) from the transmitter and the chaotic signal $V_{c1}(t)$ generated from the receiver chaotic system (chua's circuit) are passed through a subtractor, the output of which is $s'(t) = r(t) - V_{c1}(t)$, which is the same as the message signal. The buffer is used for coupling to make sure that the signal is not attenuated.

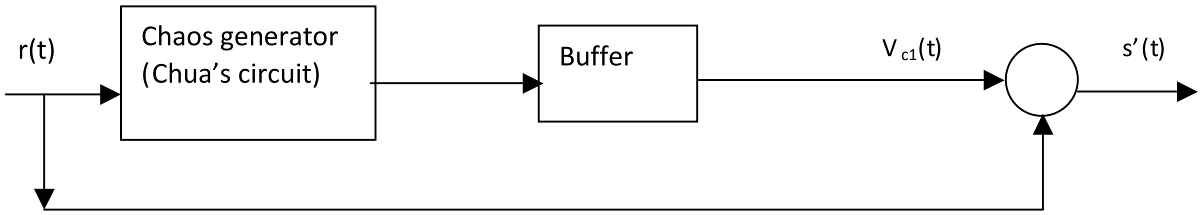


Fig. 6(b) Internal Block Diagram of Receiver

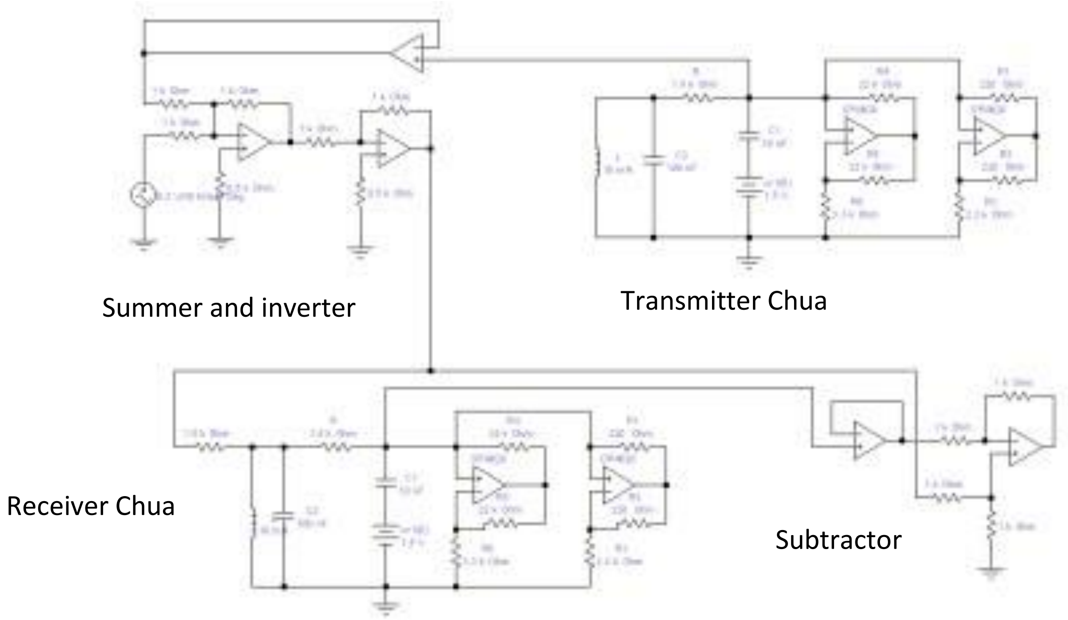


Fig. 6(d) Circuit Implementation of Secure Communication System

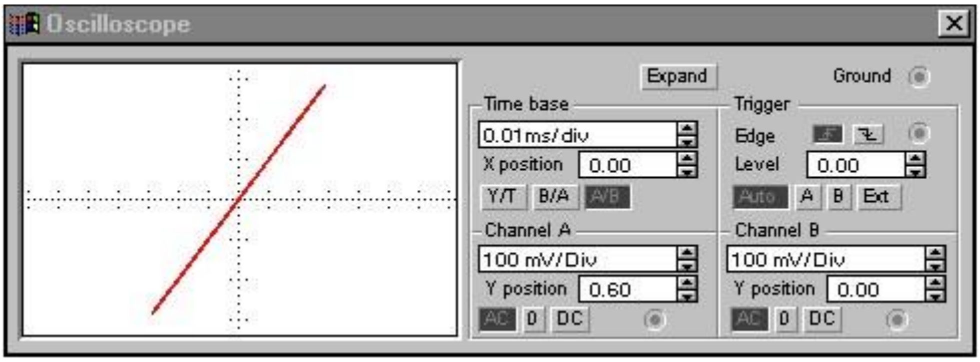


Fig. 6(e) Phase Portrait of

7.. Conclusion

In conclusion Chua circuit is a piece that contain some chaos of universe of it and after studying it was found that it shows double scroll attractor chaotic behavior and it can be controlled by adding extra components act as controllers as it may be more useful if the chaos controlled to some extent. The Chua circuit have many applications in secret telecommuting and cryptography.

8.. reference

- Gaurav Gandhi and Tamas Roska Bharathwaj Muthuswamy. “Chua’s Circuit for High School Students”. In: International Journal of Bifurcation and Chaos (2007).
- Moumita Mujherjee, Suman Hadler, “Stabilization and Control of Chaos Based on Nonlinear Dynamic Inversion”, 1st International Conference on Power Engineering, Computing and CONtrol, PECCON, 4 March 2017.
- Valentin Siderskiy. Chua’s circuit diagrams, equations, simlulations and how to build. 2016. url: http://Chuacircuits.com.
- O'Connell, Ryan Ann, "An Exploration of Chaos in Electrical Circuits" (2016). Senior Projects Spring 2016. Paper 326. http://digitalcommons.bard.edu/senproj_s2016/326

9.. Apendix

THE MATLAB Function (Chuacircuits.com, Control method of 2017 paper is exclusively added to this function in this report(IN RED))

```
%----------RealChua.m----------
function out = ControlRealChua(t,in)

x = in(1); %v_1
y = in(2); %v_2
z = in(3); %i_L



C1  = 10*10^(-9);    %10nF
C2  = 100*10^(-9);   %100nF
R = 1800;            %1.8k Ohms
G = 1/R;

%Chua Diode*************************************
R1 = 220;
R2 = 220;
R3 = 2200;
R4 = 22000;
R5 = 22000;
R6 = 3300;

Esat = 9; %9V batteries
E1 = R3/(R2+R3)*Esat;
E2 = R6/(R5+R6)*Esat;

m12 = -1/R6;
m02 = 1/R4;
m01 = 1/R1;
m11 = -1/R3;
```

```
m1 = m12+m11;

if(E1>E2)
m0 = m11 + m02;
else
m0 = m12 + m01;
end

mm1 = m01 + m02;
Emax = max([E1 E2]);
Emin = min([E1 E2]);


if abs(x) < Emin
   g = x*m1;
elseif abs(x) < Emax
    g = x*m0;
    if x > 0
    g = g + Emin*(m1-m0);
    else
    g = g + Emin*(m0-m1);
    end

elseif abs(x) >= Emax
    g = x*mm1;
    if x > 0
        g = g + Emax*(m0-mm1) + Emin*(m1-m0);
    else
        g = g + Emax*(mm1-m0) +  Emin*(m0-m1);
    end
end

%end Chua Diode************************************

%Gyrator*********************************
R7  = 100;  %100 Ohms
R8  = 1000; %1k Ohms
R9  = 1000; %1k Ohms
R10 = 1800;
C   = 100*10^(-9); %100nF
L = R7*R9*C*R10/R8; %18mH


%end Gyrator******************************
% Chua's Circuit Equations
k1=0.001;
k2=0.01;
k3=0.0007;
xref=30;
yref=20;
zref=10;
R0=0;
C1=10*10^(-9);
C2  = 100*10^(-9);  %100nF

xdot = (1/C1)*(G*(y-x)-g)+(1/10*10^(-9))*((C2/C1)*(G*(x-y)+g)-
k1*(x*G)+G*k1*xref);
ydot = (1/C2)*(G*(x-y)+z)+(1/mm1*C2)*(y-x-(z/G)-k2*y+k2*yref);
zdot  = -(1/L)*(y+R0*z)+((C2*y/L*G)+(C2*y*R0/L)-k3*z+k3*zref)*(1/(G*mm1));

out = [xdot ydot zdot]';
end
```

MATLAB Results Code

```
[t,y1] = ode45(@RealChua,[0 0.05],[-0.3 -0.6 0]);
[t,y2] = ode45(@RealChua,[0 0.05],[-0.4 -0.5 0]);
[t,y3] = ode45(@RealChua,[0 0.05],[-0.2 -0.7 0]);
[t,y4] = ode45(@ControlRealChua,[0 0.05],[-0.3 -0.6 0]);
[t,y5] = ode45(@ControlRealChua,[0 0.05],[-0.4 -0.5 0]);
[t,y6] = ode45(@ControlRealChua,[0 0.05],[-0.2 -0.7 0]);

figure(1)
clf
subplot(2,1,1)
t = linspace(0,0.05,length(y1));
plot3(y1(:,1),y1(:,2),y1(:,3), y2(:,1),y2(:,2),y2(:,3),
y3(:,1),y3(:,2),y3(:,3))
title('uncontroled')
xlabel('x')
ylabel('y')
zlabel('z')
subplot(2,1,2)
t = linspace(0,0.05,length(y4));
plot3(y4(:,1),y4(:,2),y4(:,3), y5(:,1),y5(:,2),y5(:,3),
y6(:,1),y6(:,2),y6(:,3))
title('controled')
xlabel('x')
ylabel('y')
zlabel('z')

figure(2)
clf
subplot(2,1,1)
t = linspace(0,0.05,length(y1));
plot(t,y1)
title('uncontroled')
xlabel('time')
ylabel('x,y,z')
subplot(2,1,2)
t = linspace(0,0.05,length(y4));
plot(t,y4)
title('controled')
xlabel('time')
ylabel('x,y,z')

figure(3)
clf
subplot(3,2,1)
t = linspace(0,0.05,length(y1));
plot(t,y1)
title('uncontroled')
xlabel('time')
ylabel('x,y,z')
subplot(3,2,2)
t = linspace(0,0.05,length(y4));
plot(t,y4)
title('controled')
xlabel('time')
ylabel('x,y,z')
subplot(3,2,3)
t = linspace(0,0.05,length(y2));
plot(t,y2)
title('uncontroled')
xlabel('time')
ylabel('x,y,z')
subplot(3,2,4)
t = linspace(0,0.05,length(y5));
plot(t,y5)
title('controled')
```

```
xlabel('time')
ylabel('x,y,z')
subplot(3,2,5)
t = linspace(0,0.05,length(y3));
plot(t,y3)
title('uncontroled')
xlabel('time')
ylabel('x,y,z')
subplot(3,2,6)
t = linspace(0,0.05,length(y6));
plot(t,y6)
title('controled')
xlabel('time')
ylabel('x,y,z')
```