Quantifying Circuit Overhaed/Involvement Implementation

A complementary Part of: Circuit Transpilation and Overhead Analysis

Using Python & Qiskit

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In this part I will not write too much, and I will use python comments. As this notebook is meant to be just implementation and results showing of the Idea discussed in the main notebook. there is some redundancy in cell numbering as I restart the kernel for each case of the 4 cases. Because Qiskit State Tomography cannot work after using it many times. (BrokenProcessPool: A process in the process pool was terminated abruptly while the future was running or pending.) (I failed in my search for this problem solution).

I use tomography for the sake of generalization, as this will also work for real fault gates. (Initially, I had this idea to make some analysis about quantum error, and to detect some pattern)

In [4]:
```
#First import needed packages
from qiskit import *
from qiskit.ignis.verification import state_tomography_circuits,StateTomogr
from qiskit import QuantumCircuit, execute, Aer, QuantumRegister, Classical
import numpy as np
from numpy import linalg as LA
import matplotlib.pyplot as plt
```

Function 1 of 2: Density trip in the Quantum Circuit

Given a quantum circuit this function will compute the density matrix for the resulting state from just applying 1^{st} gate of the circuit, then from applying 1^{st} and 2^{ed}, then 1^{st}, 2^{ed} and 3^{ed}, and so on till applying all the gates in the circuit(circuit itself).

In [5]:
```
#I will impelement that in reversed manner, I.E get density of the circuit
def qc_trip(qc):
    qcc=qc.copy()   #a mirror circuit to pop from
    n=qcc.num_qubits #number of quantum registers
    densities=[]
    while 1:
        tom_circ=state_tomography_circuits(qcc,list(range(n)))   #preparing
        tom_res=execute(tom_circ, Aer.get_backend('qasm_simulator')) #execu
        #figuring out the density matrix
        stf=StateTomographyFitter(tom_res.result(), tom_circ, meas_basis='P
        density=stf.fit()

        #components=len(qcc.data)

        densities=[density]+densities  #adding this in the right order incl
        if len(qcc.data)>0:
            qcc.data.pop()
        else:
                break

    return densities
```

Function 2 of 2: Trace Distance

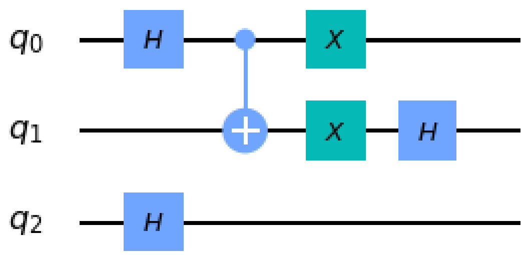
Just compute the trace distance between 2 density matrices.

In [6]:
```
def trace_distance(rho1, rho2):

    A = rho1 - rho2
    eigA, vecA = LA.eig(A)

    t = 0
    for j in range(len(eigA)):
        if (eigA[j] > 0):
            t = t + eigA[j]

    return t
```

Implementation on different cases

In [7]:
```
#original circuit
qc1=QuantumCircuit(3)
qc1.h(0)
qc1.cx(0,1)
qc1.x([0,1])
qc1.h([1,2])

qc1.draw("mpl")
```

Out[7]:



In [4]:

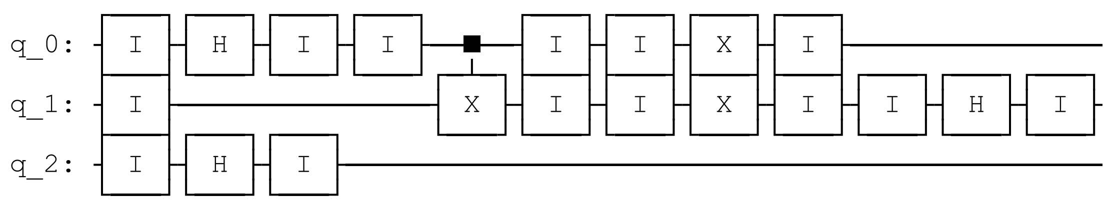
```python
#transpiled circuit
pi=np.pi
qc2=QuantumCircuit(3)
qc2.rz(pi/2,0)
qc2.rx(pi/2,0)
qc2.rz(pi/2,0)
qc2.rz(pi/2,1)
qc2.rx(pi/2,1)
qc2.rz(pi/2,1)
qc2.cz(0,1)
qc2.rz(pi/2,1)
qc2.rx(pi/2,1)
qc2.rz(pi/2,1)
qc2.rz(pi/2,[0,1])
qc2.rx(pi,[0,1])
qc2.rz(pi/2,[0,1])
qc2.rz(pi/2,[1,2])
qc2.rx(pi/2,[1,2])
qc2.rz(pi/2,[1,2])
qc2.draw()   #if I draw("mpl") the tomography error appear, have no idea!
```

Out[4]:

```
     ┌──────────┐┌──────────┐┌──────────┐   ┌──────────┐ ┌────────┐ »
q_0: ┤ RZ(pi/2) ├┤ RX(pi/2) ├┤ RZ(pi/2) ├─■─┤ RZ(pi/2) ├─┤ RX(pi) ├─»
     ├──────────┤├──────────┤├──────────┤ │ ├──────────┤┌┴────────┴┐»
q_1: ┤ RZ(pi/2) ├┤ RX(pi/2) ├┤ RZ(pi/2) ├─■─┤ RZ(pi/2) ├┤ RX(pi/2) ├»
     ├──────────┤├──────────┤├──────────┤   └──────────┘└──────────┘»
q_2: ┤ RZ(pi/2) ├┤ RX(pi/2) ├┤ RZ(pi/2) ├───────────────────────────»
     └──────────┘└──────────┘└──────────┘                           »
«     ┌──────────┐
«q_0: ┤ RZ(pi/2) ├──────────────────────────────────────────────────────
«     ├──────────┤┌──────────┐┌────────┐┌──────────┐┌──────────┐┌──────────
«q_1: ┤ RZ(pi/2) ├┤ RZ(pi/2) ├┤ RX(pi) ├┤ RZ(pi/2) ├┤ RZ(pi/2) ├┤ RX(pi/2)
«     └──────────┘└──────────┘└────────┘└──────────┘└──────────┘└──────────
«q_2: ──────────────────────────────────────────────────────────────────
«
«
«q_0: ────────────
«     ┌──────────┐
«q_1: ┤ RZ(pi/2) ├
«     └──────────┘
«q_2: ────────────
«
```

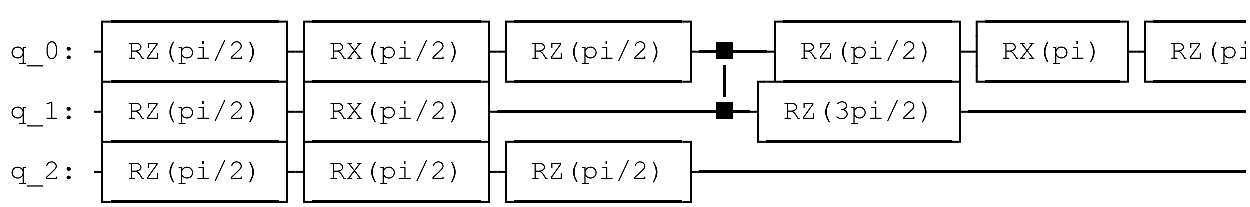
In [4]:
```
#dummy transpiled circuit (non significant gates)
qc3=QuantumCircuit(3)
qc3.i(0)
qc3.h(0)
qc3.i(0)
qc3.i([0,1])
qc3.cx(0,1)
qc3.i([0,1])
qc3.i([0,1])
qc3.x([0,1])
qc3.i([0,1])
qc3.i([1,2])
qc3.h([1,2])
qc3.i([1,2])

qc3.draw()
```

Out[4]:



In [4]:
```
#transpiled reduced circuit
pii=np.pi
qc4=QuantumCircuit(3)
qc4.rz(pii/2,0)
qc4.rx(pii/2,0)
qc4.rz(pii/2,0)
qc4.rz(pii/2,1)
qc4.rx(pii/2,1)
qc4.cz(0,1)
qc4.rz(3*pii/2,1)
qc4.rz(pii/2,0)
qc4.rx(pii,0)
qc4.rz(pii/2,0)
qc4.rz(pii/2,2)
qc4.rx(pii/2,2)
qc4.rz(pii/2,2)
qc4.draw()
```

Out[4]:



computing the circuit trip for each

In [8]:
```
a=qc_trip(qc1)
```

In [5]:
```
b=qc_trip(qc2)
```

In [5]:
```
c=qc_trip(qc3)
```

In [5]:
```
d=qc_trip(qc4)
```

then compute the State trip as defined in the main notebook.

In [9]:
```
state_trip_a=[]   #a list of StateTrips for all circuit components
for i in range(len(a)):
    state_trip_a.append(trace_distance(a[0],a[i]))
state_trip_a=np.round(np.real(state_trip_a),4)   #just make the number form
```

In [6]:
```
state_trip_b=[]
for i in range(len(b)):
    state_trip_b.append(trace_distance(b[0],b[i]))
state_trip_b=np.round(np.real(state_trip_b),4)
```

In [6]:
```
state_trip_c=[]
for i in range(len(c)):
    state_trip_c.append(trace_distance(c[0],c[i]))
state_trip_c=np.round(np.real(state_trip_c),4)
```

In [6]:
```
state_trip_d=[]
for i in range(len(d)):
    state_trip_d.append(trace_distance(d[0],d[i]))
state_trip_d=np.round(np.real(state_trip_d),4)
```

then compute the gate trip as defined in the main notebook.

In [10]:
```
gate_trip_a=[]
for i in range(len(a)):
    if i==0:
        gate_trip_a.append(0)
    else:
        gate_trip_a.append(trace_distance(a[i-1],a[i]))

gate_trip_a=np.round(np.real(gate_trip_a),4)
```

In [7]:
```
gate_trip_b=[]
for i in range(len(b)):
    if i==0:
        gate_trip_b.append(0)
    else:
        gate_trip_b.append(trace_distance(b[i-1],b[i]))

gate_trip_b=np.round(np.real(gate_trip_b),4)
```

In [7]:
```
gate_trip_c=[]
for i in range(len(c)):
    if i==0:
        gate_trip_c.append(0)
    else:
        gate_trip_c.append(trace_distance(c[i-1],c[i]))

gate_trip_c=np.round(np.real(gate_trip_c),4)
```

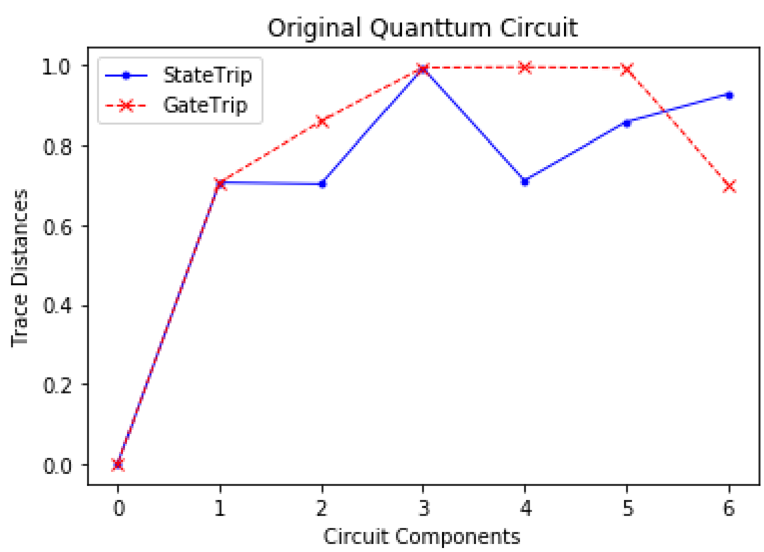
In [7]:
```
gate_trip_d=[]
for i in range(len(d)):
    if i==0:
        gate_trip_d.append(0)
    else:
        gate_trip_d.append(trace_distance(d[i-1],d[i]))

gate_trip_d=np.round(np.real(gate_trip_d),4)
```

Now, let's plot the state trip and Gate trip.

This will give an intuition why this method work.

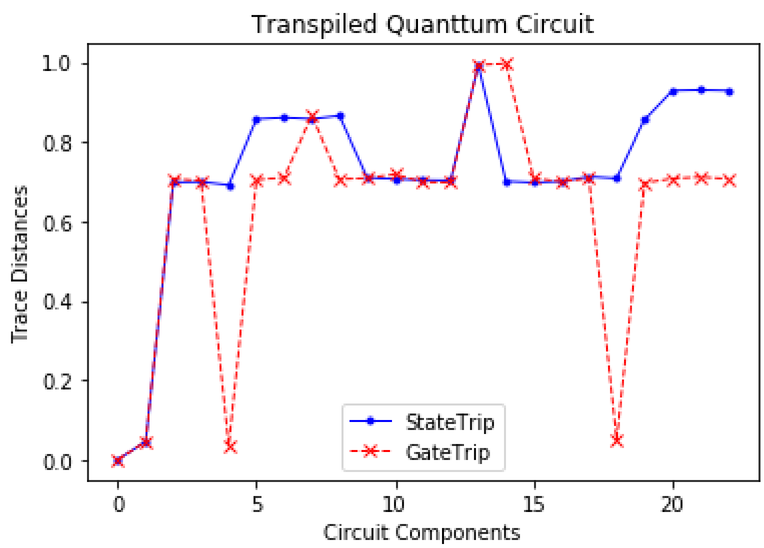
In [11]:
```
fig, ax = plt.subplots()
ax.plot(list(range(len(state_trip_a))), state_trip_a, linewidth=1, linestyl
ax.plot(list(range(len(gate_trip_a))), gate_trip_a, linewidth=1, linestyle=
ax.legend()
ax.set_title("Original Quanttum Circuit")
ax.set_xlabel("Circuit Components")
ax.set_ylabel("Trace Distances")
```

Out[11]:
```
Text(0, 0.5, 'Trace Distances')
```



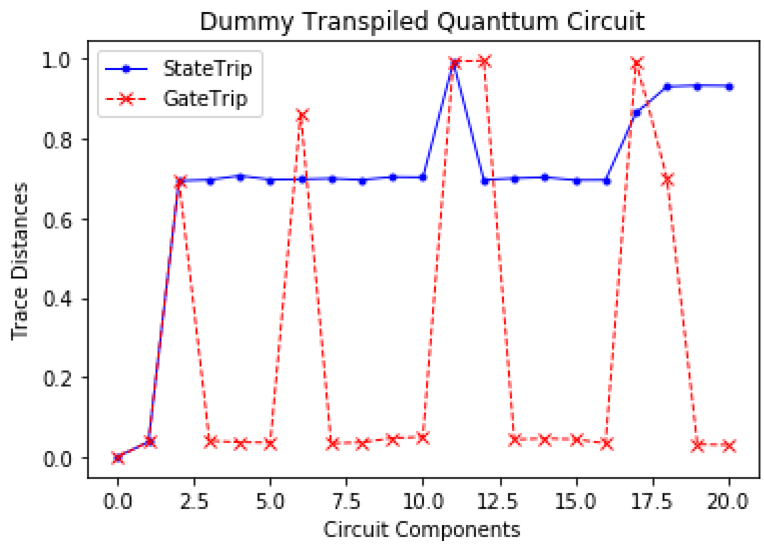
In [9]:
```
fig, ax = plt.subplots()
ax.plot(list(range(len(state_trip_b))), state_trip_b, linewidth=1, linestyl
ax.plot(list(range(len(gate_trip_b))), gate_trip_b, linewidth=1, linestyle=
ax.legend()
ax.set_title("Transpiled Quanttum Circuit")
ax.set_xlabel("Circuit Components")
ax.set_ylabel("Trace Distances")
```

Out[9]:
```
Text(0, 0.5, 'Trace Distances')
```



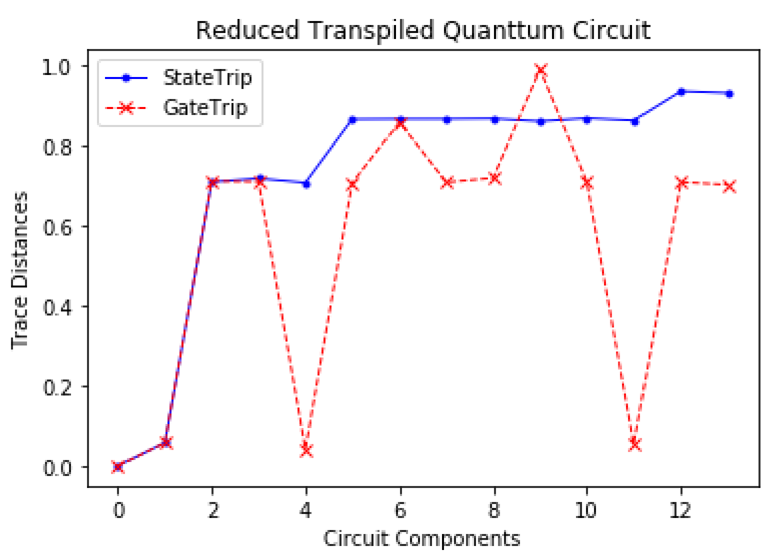
In [8]:
```
fig, ax = plt.subplots()
ax.plot(list(range(len(state_trip_c))), state_trip_c, linewidth=1, linestyl
ax.plot(list(range(len(gate_trip_c))), gate_trip_c, linewidth=1, linestyle=
ax.legend()
ax.set_title("Dummy Transpiled Quanttum Circuit")
ax.set_xlabel("Circuit Components")
ax.set_ylabel("Trace Distances")
```

Out[8]: Text(0, 0.5, 'Trace Distances')



In [8]:
```
fig, ax = plt.subplots()
ax.plot(list(range(len(state_trip_d))), state_trip_d, linewidth=1, linestyl
ax.plot(list(range(len(gate_trip_d))), gate_trip_d, linewidth=1, linestyle=
ax.legend()
ax.set_title("Reduced Transpiled Quanttum Circuit")
ax.set_xlabel("Circuit Components")
ax.set_ylabel("Trace Distances")
```

Out[8]: Text(0, 0.5, 'Trace Distances')



Now lets Compute the Involvement (as defined in the main notebook) for each.

In [12]:
```
from scipy.integrate import simps  #any integration will work for these sim
```

In [13]:
```
#involvement of the original circuit
involvement_a=simps(state_trip_a)-np.abs(simps(state_trip_a)-simps(gate_tri
print("Involvement of the original circuit: ", involvement_a)
```

```
Involvement of the original circuit:  4.260900000000001
```

In [11]:
```
#involvement of the transpiled circuit
involvement_b=simps(state_trip_b)-np.abs(simps(state_trip_b)-simps(gate_tri
print("Involvement of the transpiled circuit: ", involvement_b)
```

```
Involvement of the transpiled circuit:  14.3159
```

In [11]:
```
#involvement of the dummy transpiled circuit
involvement_c=simps(state_trip_c)-np.abs(simps(state_trip_c)-simps(gate_tri
print("Involvement of the dummy transpiled circuit: ", involvement_c)
```

```
Involvement of the dummy transpiled circuit:  5.386966666666666
```

In [11]:
```
#involvement of the reduced transpiled circuit
involvement_d=simps(state_trip_d)-np.abs(simps(state_trip_d)-simps(gate_tri
print("Involvement of the reduced transpiled circuit: ", involvement_d)
```

```
Involvement of the reduced transpiled circuit:  7.337333333333333
```

As we see, these results capture what I stated in the main notebook.

Finally, calculate the OverHead (as defined in the main notebook) between the 2 circuits and see how it changed.

In [12]:
```
#I just save the values of involvement here
involvement_a= 4.2609    #the original circuit
involvement_b= 14.3159      #the transpiled circuit
involvement_c= 5.386966666666666     #the dummy transpiled circuit
involvement_d= 7.337333333333333     #the reduced transpiled circuit
```

In [13]:
```
#transpiled circuit overhead from the original circuit
overhead_ba=involvement_b - involvement_a
print("transpiled circuit overhead from the original circuit: ", overhead_b
```

```
transpiled circuit overhead from the original circuit:  10.03620000000000
1
```

In [14]:
```
#reduced transpiled circuit overhead from the original circuit
overhead_da=involvement_d - involvement_a
print("reduced transpiled circuit overhead from the original circuit: ", ov
```

```
reduced transpiled circuit overhead from the original circuit:  3.0576333
333333343
```

It Works Smoothly and as expected!