

Software Foundations of Security & Privacy

Digest Notes

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These notes are a condensed companion to the official course lecture notes. Each lecture is summarized as: a goal, one-page map, core definitions, key proof patterns/recipes, common pitfalls, and a short “check yourself” set. Full rule lists and semantic definitions are collected in the appendices and referenced from the relevant lecture.

Lecture 2 (Jan 11, 2026): Propositional Logic and Proof

Goal: Use sequent calculus to (1) precisely define validity of formulas/sequents and (2) build/check formal derivations; understand soundness/completeness/decidability via invertibility + termination.

One-page map

- **Objects:** propositional formulas F built from variables and connectives; sequents $\Gamma \vdash \Delta$.
- **Semantics:** $\Gamma \vdash \Delta$ is valid iff “all Γ true implies some Δ true”.
- **Proof system:** right rules decompose goals; left rules decompose assumptions; identity closes branches.
- **Meta-properties:** soundness (derivable \Rightarrow valid), completeness (valid \Rightarrow derivable), decidability (procedure to decide validity).

Core definitions

- **Definition.** *Formula*: built from p using $\wedge, \vee, \rightarrow, \neg, \top, \perp$.
- **Definition.** *Valid formula*: true under every truth assignment.
- **Definition.** *Satisfiable formula*: true under some truth assignment.
- **Definition.** *Sequent* $\Gamma \vdash \Delta$: assumptions Γ and goals Δ (multi-succedent, disjunctive).
- **Definition.** *Valid sequent*: if all formulas in Γ are true then at least one formula in Δ is true.
- **Definition.** *Derivation*: tree built bottom-up using inference rules; leaves typically close by identity.

Key rules / theorems (high level)

- **Key idea.** Right/Left rules eliminate a top-level connective from a goal/assumption.
- **Key idea.** Disjunction needs multi-succedent sequents for completeness.
- **Theorem (informal).** Soundness: if $\Gamma \vdash \Delta$ is derivable then it is valid.
- **Theorem (informal).** Completeness + decidability: all rules are invertible and reductive, so proof search terminates and succeeds iff sequent is valid.

Proof-search recipe (sequent calculus)

- **Step 1:** Apply invertible rules bottom-up whenever possible (decompose connectives).
- **Step 2:** Stop at variable-only leaves $p_1, \dots, p_n \vdash q_1, \dots, q_m$.
- **Step 3:** Leaf closes iff some $p_i = q_j$ (identity). Otherwise it is a countermodel seed.

Pitfalls

- **Common pitfall.** Confusing formula implication $F \rightarrow G$ with sequent entailment $F \vdash G$: they coincide in validity but are different syntactic objects.
- **Common pitfall.** Using a two-rule disjunction right-introduction ($\vee R_1 / \vee R_2$) breaks completeness for classical truth-table validity. In general, avoid applying more than one rule at a time.

Check yourself

1. **Check yourself.** Prove $p \vee (p \rightarrow q)$ is valid using sequent calculus.
2. **Check yourself.** Given a failed leaf $p, q \vdash r$, what truth assignment witnesses invalidity?

Reference: Full propositional sequent-calculus rules are in Appendix A.

Lecture 3 (Jan 13, 2026): Dynamic Logic

Goal: Extend sequent reasoning to talk about programs; express safety-style properties as postconditions; derive rules for program constructs (if/assignment/sequence/while).

One-page map

- **New syntax:** programs α (assign/seq/if/while) and modalities $[\alpha]Q$ (box) and $\langle\alpha\rangle Q$ (diamond).
- **Intuition:** $[\alpha]Q$ means “every terminating run of α ends in a state satisfying Q ”.
- **Theme:** Rules should reduce properties of compound programs to properties of smaller programs.

Core definitions

- **Definition.** *Trace*: (possibly infinite) sequence of states/events during execution.
- **Definition.** *Safety property*: “nothing bad happens” (prefix-closed).
- **Definition.** *Liveness property*: “something good eventually happens”.
- **Definition.** *Box/diamond*: $[\alpha]Q$ (all terminating runs end in Q) and $\langle\alpha\rangle Q$ (some terminating run ends in Q). Since our language has no nondeterminism, $\langle\alpha\rangle Q$ is equivalent to “ α terminates and $[\alpha]Q$ ”.

Key rules (as “recipes”)

- **Key idea. If:** split on the guard.
- **Key idea. Sequential Composition:** rewrite $[\alpha; \beta]Q$ as $[\alpha]([\beta]Q)$.
- **Key idea. Assignment:** do *not* assume $x = e$ with the same x on both sides of the state change; introduce fresh x' (post-state value) and substitute.
- **Key idea. While:** unfolding is sound but not reductive; loop invariants are the scalable rule.

Assignment gotcha (why freshness matters)

- **Common pitfall.** The tempting rule “assume $x = e$ after $x := e$ ” is *unsound* because it confuses the value of x *before* and *after* the assignment.
- **Lecture example:** with the unsound rule you could (wrongly) justify $x = 2 \vdash [x := 1]x = 3$ by reducing it to the premise $x = 2, x = 1 \vdash x = 3$, which is valid only because the antecedents are contradictory.
- **Key idea.** Fix (as in the notes): introduce a fresh post-state variable x' and replace the postcondition $Q(x)$ with $Q(x')$. Rule shape:
 $\Gamma \vdash [x := e]Q(x), \Delta$ reduces to $\Gamma, x' = e \vdash Q(x'), \Delta$ (with x' fresh).

Loop-invariant recipe

- Pick an invariant J and prove these sequents (turnstile, not implication):
 - **Init:** $\Gamma \vdash J$
 - **Preserved:** $J, P \vdash [\alpha]J$
 - **Post:** $J, \neg P \vdash Q$
- **Important:** Γ (facts true only initially) is intentionally dropped from the preserved/post obligations.

Check yourself

1. **Check yourself.** Explain why $[\alpha]Q$ is vacuously true when α does not terminate. What property is this capturing?
2. **Check yourself.** For the swap program (using + and -), what must the precondition include to avoid undefined behavior in bounded-integer languages?

Reference: Full dynamic-logic language summary is in Appendix [B](#), and full rule summary is in Appendix [C](#).

Lecture 4 (Jan 20, 2026): Semantics of Programs

Goal: Pin down the meaning of expressions, programs, and formulas so rule soundness proofs become “expand definitions and chase the quantifiers”.

One-page map

- **State:** $\omega(x)$ is the value of variable x .
- **Expression meaning:** $\omega[e] \in \mathbb{Z}$.
- **Program meaning:** relation $\omega[\alpha]\nu$ between pre/post states.
- **Formula meaning:** $\omega \models P$.
- **Reusable move:** prove equivalences (e.g. $[\alpha; \beta]Q \leftrightarrow [\alpha][\beta]Q$) and use them as rewrite rules.

Core clauses (keep only the high-yield ones)

- **Definition.** Assignment updates state: $\nu = \omega[x \mapsto \omega[e]]$.
- **Definition.** Sequence composes via an intermediate state: $\exists \mu$.
- **Definition.** While termination is “there exists an n iterations” (bounded-iteration semantics).

Reusable proof pattern: “unwind definitions”

- **To show** $P \leftrightarrow Q$ is valid: fix an arbitrary state ω , assume $\omega \models P$, expand definitions until you can derive $\omega \models Q$ (and vice versa).
- **Key idea.** This justifies using valid equivalences as both left- and right-rules (sound + invertible).

Pitfalls

- **Common pitfall.** Quantification over program states is subtle: naive “substitute a constant for x ” can fail when x is read across many loop states; semantics uses state update $\omega[x \mapsto c]$.

Check yourself

1. **Check yourself.** Prove (informally, by definitions) $\models [\alpha; \beta]Q \leftrightarrow [\alpha][\beta]Q$.
2. **Check yourself.** Give a simple program α and postcondition Q where $\omega \models [\alpha]Q$ holds because α does not terminate.

Reference: Full semantics (expressions, programs, formulas) are summarized in Appendix D.

Appendices

A Propositional sequent calculus (full rule list)

Sequents: $\Gamma \vdash \Delta$ where Γ, Δ are (multi)sets of formulas. **Validity:** all Γ true implies some Δ true.

Identity

$$\frac{}{\Gamma, F \vdash F, \Delta} \text{id}$$

Logical connectives

True	$\frac{}{\Gamma \vdash \top, \Delta} \top R$	$\frac{\Gamma \vdash \Delta}{\Gamma, \top \vdash \Delta} \top L$
False	$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \perp, \Delta} \perp R$	$\frac{}{\Gamma, \perp \vdash \Delta} \perp L$
Negation	$\frac{\Gamma, F \vdash \Delta}{\Gamma \vdash \neg F, \Delta} \neg R$	$\frac{\Gamma \vdash F, \Delta}{\Gamma, \neg F \vdash \Delta} \neg L$
Conjunction	$\frac{\Gamma \vdash F, \Delta \quad \Gamma \vdash G, \Delta}{\Gamma \vdash F \wedge G, \Delta} \wedge R$	$\frac{\Gamma, F, G \vdash \Delta}{\Gamma, F \wedge G \vdash \Delta} \wedge L$
Disjunction	$\frac{\Gamma \vdash F, G, \Delta}{\Gamma \vdash F \vee G, \Delta} \vee R$	$\frac{\Gamma, F \vdash \Delta \quad \Gamma, G \vdash \Delta}{\Gamma, F \vee G \vdash \Delta} \vee L$
Implication	$\frac{\Gamma, F \vdash G, \Delta}{\Gamma \vdash F \rightarrow G, \Delta} \rightarrow R$	$\frac{\Gamma \vdash F, \Delta \quad \Gamma, G \vdash \Delta}{\Gamma, F \rightarrow G \vdash \Delta} \rightarrow L$
NOR (\downarrow)	$\frac{\Gamma, F \vdash \Delta \quad \Gamma, G \vdash \Delta}{\Gamma \vdash F \downarrow G, \Delta} \downarrow R$	$\frac{\Gamma \vdash F, G, \Delta}{\Gamma, F \downarrow G \vdash \Delta} \downarrow L$

B Dynamic logic: Language summary

Variables	x, y, z
Constants	$c ::= \dots, -1, 0, 1, \dots$
Expressions	$e ::= c \mid x \mid e_1 + e_2 \mid e_1 * e_2 \mid \dots$
Programs	$\alpha, \beta ::= x := e \mid \alpha ; \beta \mid \text{if } P \text{ then } \alpha \text{ else } \beta \mid \text{while } P \alpha$
Formulas	$P, Q ::= e_1 \leq e_2 \mid e_1 = e_2 \mid \dots$ $P \wedge Q \mid P \vee Q \mid P \rightarrow Q \mid P \leftrightarrow Q \mid \neg P \mid \top \mid \perp$ $[\alpha]Q \mid \langle \alpha \rangle Q$

C Dynamic logic: rule summary (box modality)

Reading: $[\alpha]Q$ is partial correctness (all *terminating* runs end in Q).

$$\begin{array}{c} \frac{\Gamma, P \vdash [\alpha]Q, \Delta \quad \Gamma, \neg P \vdash [\beta]Q, \Delta}{\Gamma \vdash [\text{if } P \text{ then } \alpha \text{ else } \beta]Q, \Delta} [\text{if}]R \\ \frac{\Gamma, x' = e \vdash Q(x'), \Delta}{\Gamma \vdash [x := e]Q(x), \Delta} [=]R^{x'} \\ \frac{\Gamma \vdash [\alpha](\beta]Q), \Delta}{\Gamma \vdash [\alpha ; \beta]Q, \Delta} [:]R \\ \frac{\Gamma, P \vdash [\alpha]([\text{while } P \alpha]Q), \Delta \quad \Gamma, \neg P \vdash Q, \Delta}{\Gamma \vdash [\text{while } P \alpha]Q, \Delta} [\text{unfold}]R \\ \frac{\Gamma \vdash J, \Delta \quad J, P \vdash [\alpha]J \quad J, \neg P \vdash Q}{\Gamma \vdash [\text{while}_J P \alpha]Q, \Delta} [\text{while}]R \end{array}$$

D Semantics cheat sheet

States: total maps $\omega : \{x, y, \dots\} \rightarrow \mathbb{Z}$.

Expressions

$$\omega[\llbracket c \rrbracket] = c \quad \omega[\llbracket x \rrbracket] = \omega(x) \quad \omega[\llbracket e_1 + e_2 \rrbracket] = \omega[\llbracket e_1 \rrbracket] + \omega[\llbracket e_2 \rrbracket]$$

(and similarly for other operators).

State update

$$(\omega[x \mapsto c])(x) = c \quad (\omega[x \mapsto c])(y) = \omega(y) \text{ for } y \neq x$$

Programs as relations Write $\omega[\llbracket \alpha \rrbracket]\nu$ for “executing α can take prestate ω to poststate ν ”.

$$\begin{aligned} \omega[\llbracket x := e \rrbracket]\nu &\text{ iff } \nu = \omega[x \mapsto \omega[\llbracket e \rrbracket]] \\ \omega[\llbracket \alpha ; \beta \rrbracket]\nu &\text{ iff } \exists \mu. \omega[\llbracket \alpha \rrbracket]\mu \wedge \mu[\llbracket \beta \rrbracket]\nu \\ \omega[\llbracket \text{if } P \text{ then } \alpha \text{ else } \beta \rrbracket]\nu &\text{ iff } (\omega \models P \wedge \omega[\llbracket \alpha \rrbracket]\nu) \text{ or } (\omega \not\models P \wedge \omega[\llbracket \beta \rrbracket]\nu) \\ \omega[\llbracket \text{while } P \alpha \rrbracket]\nu &\text{ iff } \omega[\llbracket \text{while } P \alpha \rrbracket]^n\nu \text{ for some } n \in \mathbb{N} \end{aligned}$$

with the bounded-iteration clauses:

$$\begin{aligned} \omega[\llbracket \text{while } P \alpha \rrbracket]^0\nu &\text{ iff } (\omega \not\models P \wedge \omega = \nu) \\ \omega[\llbracket \text{while } P \alpha \rrbracket]^{n+1}\nu &\text{ iff } (\omega \models P \wedge \exists \mu. \omega[\llbracket \alpha \rrbracket]\mu \wedge \mu[\llbracket \text{while } P \alpha \rrbracket]^n\nu) \end{aligned}$$

Formulas

$$\omega \models [\alpha]Q \text{ iff } \forall \nu. (\omega[\llbracket \alpha \rrbracket]\nu \Rightarrow \nu \models Q) \quad \omega \models \langle \alpha \rangle Q \text{ iff } \exists \nu. (\omega[\llbracket \alpha \rrbracket]\nu \wedge \nu \models Q)$$