Série II :

Exercice 1 :

x_i	0	1	2	3
y_i	2	1	2	3

$P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

$$\begin{cases} x=0 & a_0 + a_1 \cdot 0 + a_2 \cdot 0 + a_3 \times 0 = 2 \\ x=1 & a_0 + a_1 + a_2 + a_3 = 1 \\ x=2 & a_0 + 2a_1 + 4a_2 + 8a_3 = 2 \\ x=3 & a_0 + 3a_1 + 9a_2 + 27a_3 = 3 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

- Méthode direct :

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 2 \\ 1 & 3 & 9 & 27 & 3 \end{array}\right)$$

$L_2 \leftarrow L_2 - L_1$, $L_3 \leftarrow L_3 - L_1$, $L_4 \leftarrow L_4 - L_1$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 2 & 4 & 8 & 0 \\ 0 & 3 & 9 & 27 & 1 \end{array}\right)$$

$L_3 \leftarrow L_3 - 2L_2$, $L_4 \leftarrow L_4 - 3L_2$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 2 & 6 & 2 \\ 0 & 0 & 6 & 24 & 4 \end{array}\right)$$

$L_4 \leftarrow L_4 - 3L_3$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 2 & 6 & 2 \\ 0 & 0 & 0 & 6 & -2 \end{array}\right)$$

Donc :

$$\begin{cases} 6a_3 = -2 \Leftrightarrow a_3 = \boxed{-\frac{1}{3}} \\ 2a_2 = 2 + 6 \times \frac{1}{3} \Leftrightarrow a_2 = \boxed{2} \\ a_1 = \boxed{-8/3} \\ a_0 = \boxed{2} \end{cases}$$

Donc

$$\boxed{P(x) = 2 - \frac{8}{3}x + 2x^2 - \frac{1}{3}x^3}$$

- Méthode de Lagrange :

On a $P(x) = \sum_{i=0}^{3} L_i(x) \cdot y_i$.

$(0,2)$ $(1,1)$ $(2,2)$ $(3,3)$

avec $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{3} \frac{(x - x_j)}{(x_i - x_j)}$.

$P(x) = 2L_0(x) + L_1(x) + 2L_2(x) + 3L_3(x)$.

- $L_0(x) = \frac{(x-1)(x-2)(x-3)}{-1 \times (-2) \times (-3)}$.

$= \frac{(x-1)(x-2)(x-3)}{-6}$

- $L_1(x) = \frac{x(x-2)(x-3)}{1 \times (-1) \times (-2)} = \frac{x(x-2)(x-3)}{2}$

- $L_2(x) = \frac{x(x-1)(x-3)}{2 \times 1 \times (-1)} = \frac{x(x-1)(x-3)}{-2}$

- $L_3(x) = \frac{x(x-1)(x-2)}{3 \times 2 \times 1} = \frac{x(x-1)(x-2)}{6}$

$P(x) = \frac{(x-1)(x-2)(x-3)}{-3} + \frac{x(x-2)(x-3)}{2}$

$- x(x-1)(x-3) + \frac{x(x-1)(x-2)}{2}$

$P(x) = (x-1)(x-3)\left[\frac{x-2}{-3} - \frac{3x}{3}\right]$

$+ x(x-2)/2 \times \left[x - 3 + x - 1\right]$

$$P(x) = (x-1)(x-3)\left(\frac{-4x+2}{3}\right) + x(x-2)(x-2)$$

$$= -\frac{2}{3}\left[(x-1)(x-3)(2x-1)\right] + x(x-2)^2$$

$$= -\frac{2}{3}\left[2x^3 - 9x^2 + 7x - 3\right] + (x^3 - 4x^2 + 4x)$$

$$= -\frac{x^3}{3} + 2x^2 + \frac{8}{3}x + 2.$$

$$P(x) = 2 + \frac{8}{3} + 2x^2 - \frac{1}{3}x^3$$