PAGE
DATE

Handwritten Explanation

objective function:-

$$f(x) = x^T \Sigma x - \lambda \mu^T x$$

$x^T \Sigma x \rightarrow$ portfolio variance "RISK TERM"

$-\lambda \mu^T x \rightarrow$ Expected return "Return Term"

$\lambda \rightarrow$ Tradeoff

Constraints:

- $\sum_{i=1}^{n} x_i = 1$ "sum weights add to 1"
- $x_i \geq 0 \ \forall i$ " weights are non-negative"
- $x_i \leq \mu_i$ " to not exceed Maximum percentage for asset i"

First derivative:-

$\because \frac{d}{dx}(x^T A x) = (A + A^T) x \rightarrow \nabla (x^T \Sigma x) = (\Sigma + \Sigma^T) x$

$\because \frac{d}{dx}(b^T x) = b \rightarrow \nabla(-\lambda \mu^T x) = -\lambda \mu$

equavilant to 2Σ as "Σ" is symmetric $\therefore \Sigma + \Sigma^T = 2\Sigma$

$\therefore \nabla f(x) = 2\Sigma x - \lambda \mu$

Second Derivative:

$\because \Sigma$ is constant, $-\lambda\mu$ is constant vector that doesn't depend on x

$$\therefore \nabla^2 f(x) = 2\Sigma$$

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PAGE
DATE

Convexity Check:

$\because$ A quadratic function $x^T \Sigma x$ is convex iff Σ is PSD

$\therefore$ if Σ is PSD $\rightarrow$ then 2Σ is PSD $\rightarrow$ $\therefore f(x)$ is convex

in our Project:

Minimum eigenvalue $= 5.29 \times 10^{-6} \geq 0$

$\therefore \Sigma$ is PSD $\quad \therefore f(x)$ is convex

as all eigenvalue of Σ must be ≥ 0 to be PSD

* Note:-

$\lambda \mu^T x \rightarrow$ is an affine term

$\Rightarrow$ any affine function is convex

$\therefore$ adding convex to convex ~~Rre~~ preserves the convexity

$$\underbrace{f(x) = \underbrace{x^T \Sigma x}_{\text{convex}} + \underbrace{(-\lambda \mu^T x)}_{\text{convex}}}_{\text{convex}}$$