



Top highlight

Chapter 3: Support Vector machine with Math.



Madhu Sanjeevi (Mady)

[Follow](#)

Sep 29, 2017 · 5 min read

Last story we talked about **Logistic Regression** for classification problems, This story I wanna talk about one of the main algorithms in machine learning which is **support vector machine**. *Lets get started!*

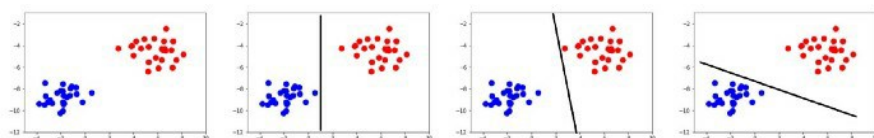
SVM can be used to solve both classification and regression problems, in this story we talk about classification only(binary classification and linear data).

Before we dive into the story I wanna point this

Why SVM???

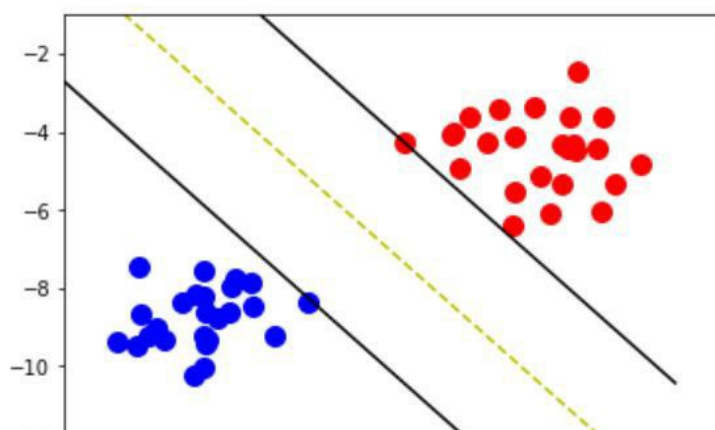
We have couple of other classifiers there, so why do we have to choose SVM over any other ??

Well! It does a pretty good job at classification than others. for example observe the below image.



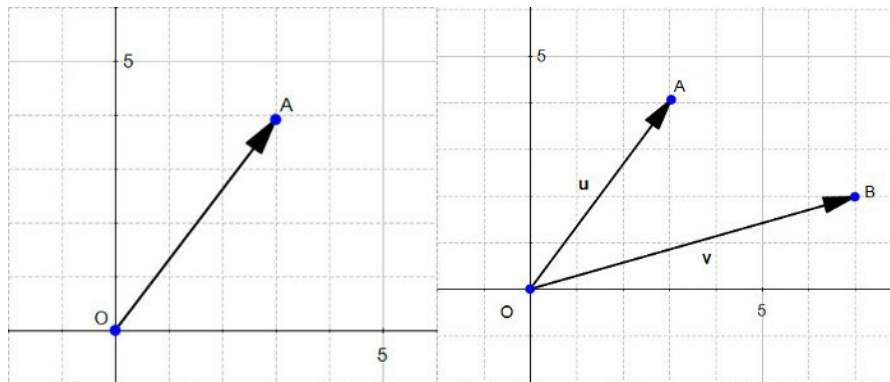
We have a data-set which is linearly separable from +’s and — ’s , we can separate the data using logistic regression (or other) we may get something like above (which is reasonable).

This is how SVM does



First I want you to be familiar with the above words so let's complete that

Vector, it's a n dimensional object which has magnitude(length) and direction, it starts from origin(0,0).

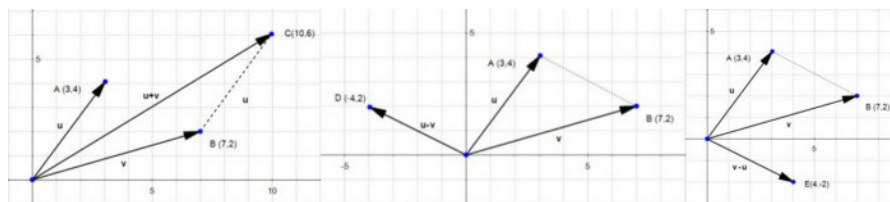


u and v are vectors

The magnitude or length of a vector u (O-A distance) is written $\|u\|$ and is called its norm.

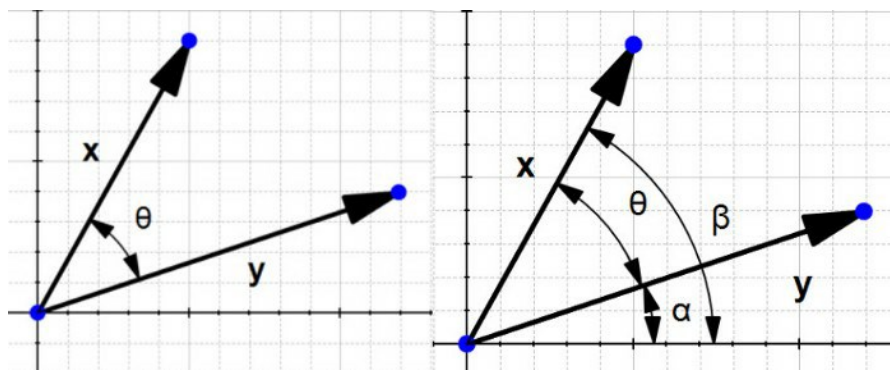
The direction of the vector u is defined by the angle θ with respect to the horizontal axis, and with the angle α with respect to the vertical axis.

Addition and subtraction of Vectors



Addition(Left) Sub(Center and Right)

Dot product of vectors



Dot product of x and y (finding the θ)

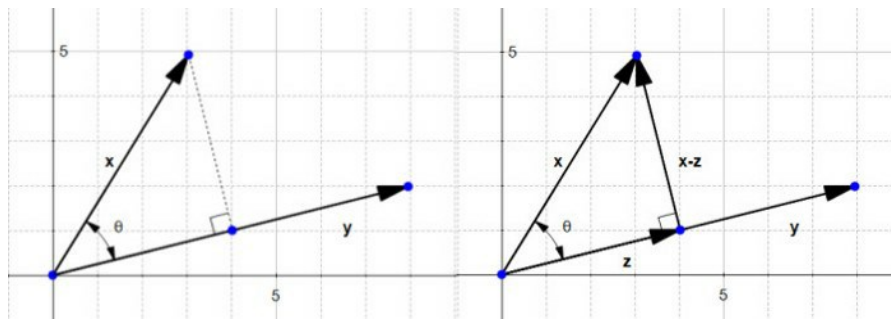
$$\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + x_2 y_2 = \sum_{i=1}^2 (x_i y_i)$$

If we work on a bit we get this

$$\|\mathbf{x}\| \|\mathbf{y}\| \cos(\theta) = x_1 y_1 + x_2 y_2$$

$$\|\mathbf{x}\| \|\mathbf{y}\| \cos(\theta) = \mathbf{x} \cdot \mathbf{y}$$

The orthogonal projection of a vector.



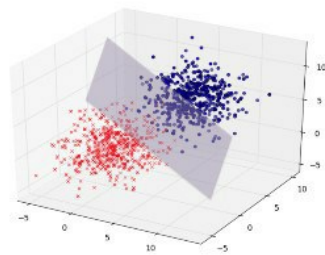
Vector \mathbf{x} onto vector \mathbf{y} (Left) ,new vector \mathbf{z} (right)

Why are we interested by the orthogonal projection ? Well , it allows us to compute the distance between \mathbf{x} and the line which goes through \mathbf{y} ($\mathbf{x}-\mathbf{z}$).

The SVM hyperplane

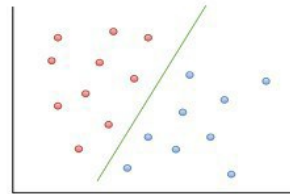
$$\mathbf{w}^T \mathbf{x} = 0$$

Hyperplane



$$y = ax + b$$

Line



The line equation and hyperplane equation — same, its a different way to express the same thing,

It is easier to work on more than two dimensions with the hyperplane notation.

so now we know how to draw a hyperplane with the given dataset,

so what's next??



We have a data-set , we want to draw a hyper plane something like above (which separates the data well).

How can we find the optimal hyperplane(yellow line) ?

if we maximize the margin(distance) between two hyperplanes then divide by 2 we get the decision boundary.

how do we maximize the margin??

lets take only 2 dimensions, we get the equation for hyper line is

$w \cdot x + b = 0$ which is same as $w \cdot x = 0$ (which has more dimensions)

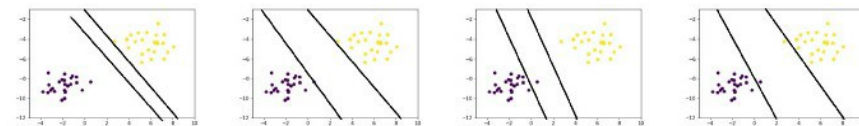
For each vector x_i either :

$$w \cdot x_i + b \geq 1 \text{ for } x_i \text{ having the class } 1$$

or

$$w \cdot x_i + b \leq -1 \text{ for } x_i \text{ having the class } -1$$

Rules for separating dataset



First two are following our Rules while others ain't

if $w \cdot x + b = 0$ then we get the decision boundary

→The yellow dashed line

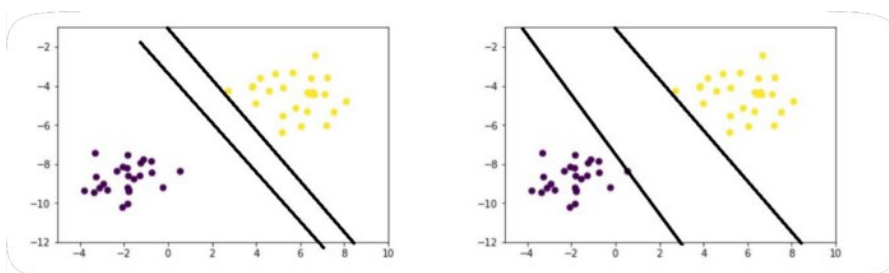
if $w \cdot x + b = 1$ then we get (+)class hyperplane

for all positive(x) points satisfy this rule ($w \cdot x + b \geq 1$)

if $w \cdot x + b = -1$ then we get (-)class hyperplane

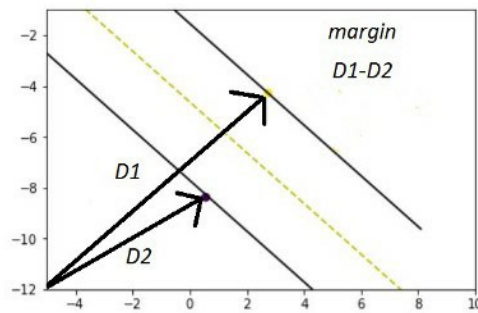
for all negative(x) points satisfy this rule ($w \cdot x + b \leq -1$)

Observe this picture.



Above both are following our rules so how do I pick the max margin one???

Answer : pick one which has **minimum Magnitude of w**



$$D1 = w^T x + b = 1 \quad w^T x + b - 1 = 0$$

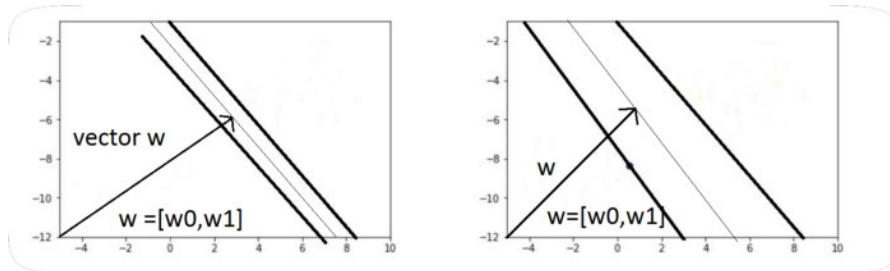
$$D2 = w^T x + b = -1 \quad w^T x + b + 1 = 0$$

$$w^T x + b - 1 - w^T x + b + 1$$

↓ Solve algebraically

$$\frac{2}{|w|}$$

so to increase the margin, minimize the $|w|$ $\frac{1}{2} |w|^2$



$||w||$ should be minimum.

This is the final picture designed by me

$$\begin{aligned} \text{if } w^T x + b = 1 & \quad y_+ \\ \text{if } w^T x + b = -1 & \quad y_- \end{aligned}$$

$$\begin{aligned} (w^T x + b) y_+ &= (1) y_+ \\ (w^T x + b) y_- &= (-1) y_- \end{aligned}$$

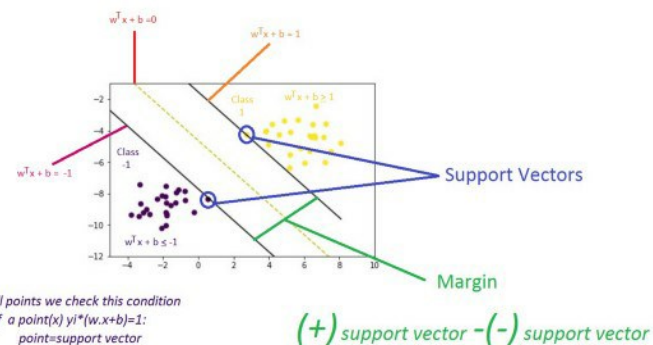
$$y_+ = 1, y_- = -1$$

$$\begin{aligned} (w^T x + b) y_+ &= (1)(1) \\ (w^T x + b) y_- &= (-1)(-1) \end{aligned}$$

$$y_i (w^T x + b) = 1$$

$$y_i = \{y_+, y_-\}$$

for all points we check this condition
if a point(x) $y_i(w \cdot x + b) = 1$:
point= support vector
classified correctly save parameters
else if > 1 :
classified correctly save parameters
else:
classified incorrectly adjust parameters



so either we save the w and b values and keep going or we adjust the parameter (w and b) and keep going. Another optimization problem, SVM.

Adjusting parameters? Sounds like **Gradient descent** right? Yes!!!

It is a convex optimization problem which surely gives us global minimum value.

Once its optimized we are done!

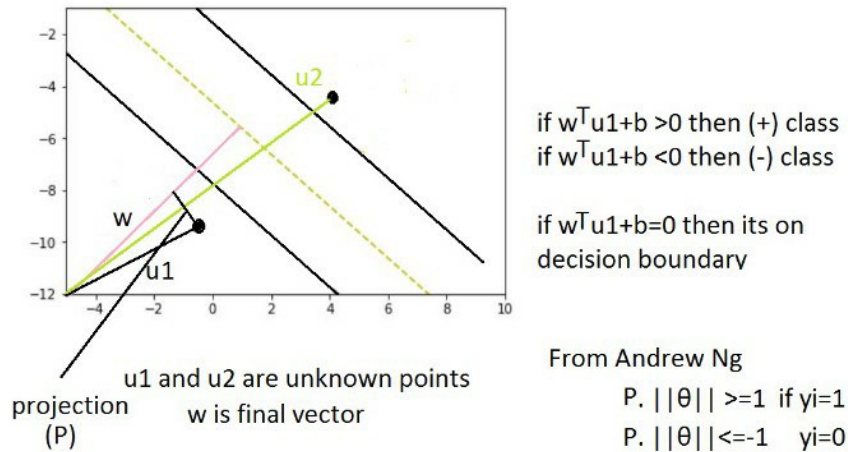
| so what's next??

Predicting!





Now we give a unknown point and we want to predict whether it belongs to positive class or negative class.



That's it for this story. hope you enjoyed and learned something.

we just talked about half of SVM only , the other half is about Kernals (we will talk about it later)

In the next story we will code this algorithm from scratch (without using any ML libraries).

Until then

See ya!

Machine Learning

Support Vector Machine

Classification

Supervised Learning

Svm



1.4K claps



WRITTEN BY

Madhu Sanjeevi (Mady)

Follow

Writes about Technology (AI, ML, DL) | Writes about Human Mind and Computer Mind. interested in ||Programming|| Science || Psychology || NeuroScience || Math



Deep Math Machine learning.ai



Following 

This is all about machine learning and deep learning (Topics cover Math, Theory and Programming)

[See responses \(4\)](#)

More From Medium

Related reads

Introduction to Machine Learning Algorithms: Linear Regression



Rohith Gandhi in Towards Data Science
May 27, 2018 · 7 min read



1.92K



Related reads

Gradient Descent in Python



Sagar Mainkar in Towards Data Science
Aug 24, 2018 · 7 min read



841



Also tagged Classification

Logistic Regression- Derived from Intuition



Soumalya Nandi in Towards Data Science
Jul 17 · 7 min read ★



18

