



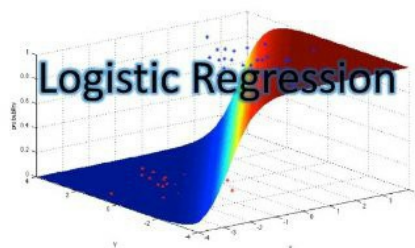
Chapter 2.0 : Logistic Regression with Math.



Madhu Sanjeevi (Mady) in Deep Math Machine learning.ai

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Sep 26, 2017 · 5 min read

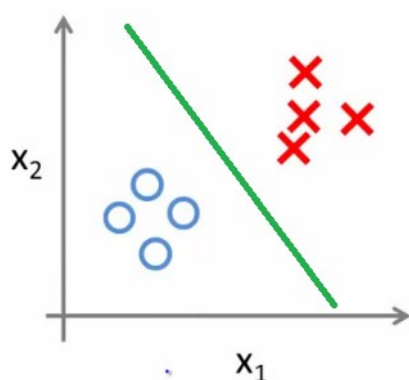


In the previous story we talked about **Linear Regression** for solving regression problems in machine learning , This story we will talk about Logistic Regression for classification problems.

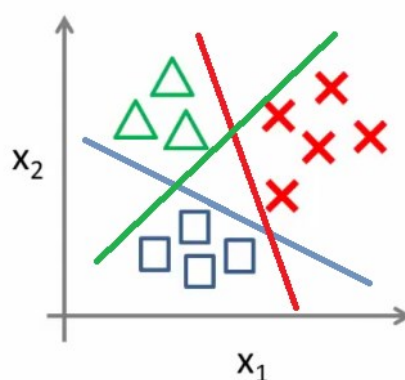
You may be wondering why the name says regression if it is a classification algorithm, well, It uses the regression inside to be the classification algorithm.

Classification : Separates the data from one to another.

Binary classification:



Multi-class classification:



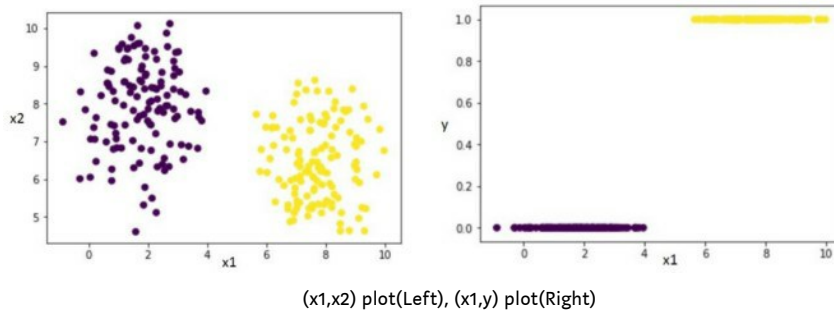
One Vs All method (Right)

This story we talk about **binary classification** (0 or 1) Here target variable is either 0 or 1

Goal is to find that green straight line (which separates the data at best)

so we use regression for drawing the line , makes sense right?

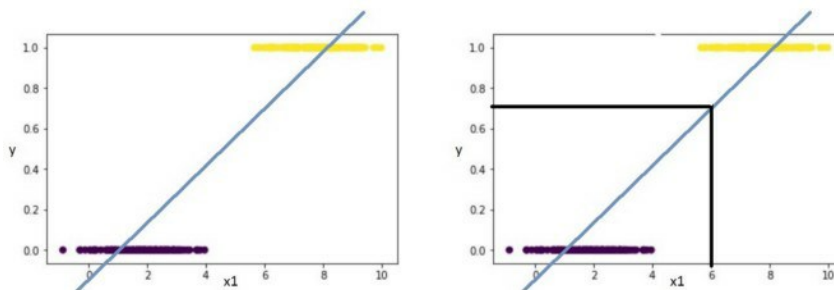
Lets take a random dataset and see how it works,



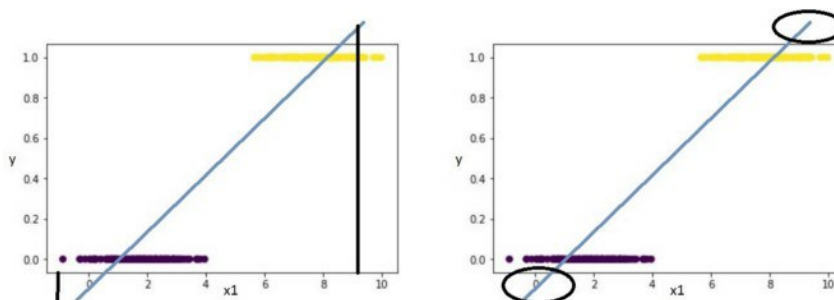
if we observe the *right* picture we have our independent variable (X) and dependent variable(y) so this is the graph we should consider for the classification problem

Given X or (Set of x values) we need to predict whether it's 0 or 1 (Yes/No).

If we apply Linear regression for above data we get something like this,



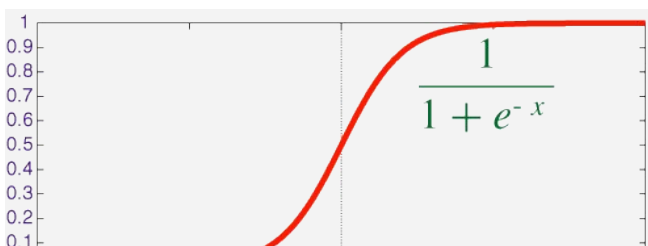
Given X value 6 we can say y is 0.7 (close to 1), that's cool but wait, What if I give negative X value or greater X value??? The output is this



We only accept the values between 0 and 1 (We don't accept other values) to make a decision (Yes/No)

| so how do we proceed further?

There is an awesome function called **Sigmoid** or **Logistic function** , we use to get the values between 0 and 1



This function squashes the value (any value) and gives the value between 0 and 1

How??? and what is 'e' ???



e here is 'exponential function'
the value is **2.71828**

this is how the value is always between 0 and 1.

$$\begin{aligned}
 2.71828^{+x} &= \text{Positive value} \\
 2.71828^{-x} &= \frac{1}{\text{Positive value}} = \text{Value between (0 and 1)} \\
 \boxed{\frac{1}{1 + \text{Positive value}} \quad \text{or} \quad \frac{1}{1 + \frac{1}{\text{Positive value}}}} &= \frac{1}{1 + e^{-x}}
 \end{aligned}$$

Sigmoid Function

So far we know that we first apply the linear equation and apply Sigmoid function for the result so we get the value which is between 0 and 1.

The hypothesis for *Linear regression* is **$h(X) = \theta_0 + \theta_1 * X$**

The hypothesis for this algorithm is

$$\frac{1}{1 + e^{-(\theta_0 + \theta_1 * X)}}$$

Logistic function for Logistic regression.

| How does it work??

1. First we calculate the **Logit function**, what the heck is that??

logit = $\theta_0 + \theta_1 * X$ (hypothesis of linear regression)

2. We apply the above Sigmoid function (Logistic function) to logit.

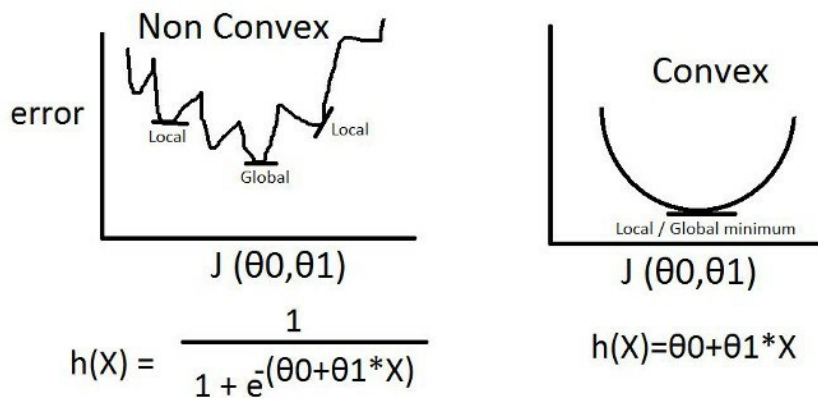
3 we calculate the error , Cost function (Maximum log-Likelihood)

Cost function for linear regression is

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Cost function

here it does not work as $h(x)$ hypothesis gives non convex function for $J(\theta_0, \theta_1)$ so we are not guaranteed that we reach best minimum.



We take **log(hypothesis)** to calculate the cost function

Logistic regression cost function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_{\theta}(x^{(i)}), y^{(i)})$$

$$= -\frac{1}{m} \left[\sum_{i=1}^m y^{(i)} \log h_{\theta}(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_{\theta}(x^{(i)})) \right]$$

$$P(y=1 | x; \theta) = h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Taken from Prof. Andrew Ng's Coursera ML course

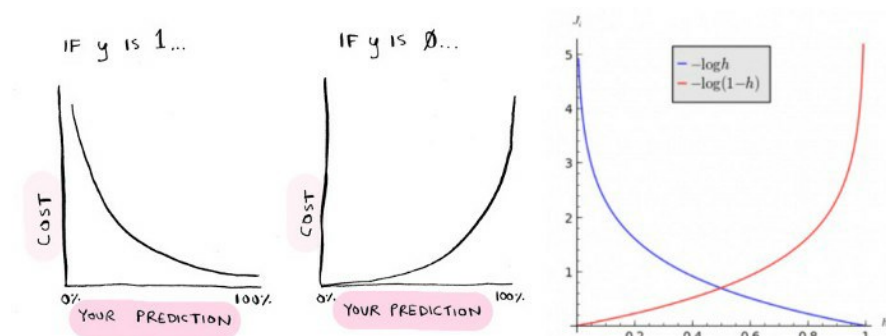
If it does not make sense , let me make it sense to you

usually error is what?? (predicted — actual)**2 right??

```
so if predicted = 1 and actual= 1
error = 0
so if predicted = 1 and actual= 0
error = 1
so if predicted = 0 and actual= 1
error = 1
so if predicted = 0 and actual= 0
error = 0
```

Note: predicted can be 0.5 and so on... also
So every time we get the error between 0 and 1 which is not useful.

just take a look at this picture and observe something..



From Left picture

If actual $y = 1$ and predicted $= 0$ the cost goes to infinity and If actual $y = 1$ and predicted $= 1$ the cost goes to minimum.

If actual $y = 0$ and predicted $= 1$ the cost goes to infinity and If actual $y = 0$ and predicted $= 0$ the cost goes to minimum.

From Right picture

if we apply **log** to **hypothesis (predicted)** we get some values (cost) which is useful to estimate the overall error.

Here is the final picture.

Logistic regression cost function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_{\theta}(x^{(i)}), y^{(i)})$$

$\overset{0}{\uparrow}$ If actual $y=1$

$$= -\frac{1}{m} \left[\sum_{i=1}^m y^{(i)} \log h_{\theta}(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_{\theta}(x^{(i)})) \right]$$



$\overset{0}{\downarrow}$ If actual $y=0$

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_{\theta}(x^{(i)}), y^{(i)})$$

$$\text{Cost}(h_{\theta}(x), y) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\theta}(x)) & \text{if } y = 0 \end{cases}$$

Note: $y = 0$ or 1 always

that's it. based on the actual **y** values we calculate different functions.

4. Next step is to apply **Gradient descent** to change the **θ** values in our hypothesis (I already covered check this [link](#)).

That's it We are done!

we got the Logistic regression ready, we can now predict new data with the model we just built.

Predicting new data, remember?? we give new X values we get the predicted y values how does it work ??

$$\frac{e^{-(\theta_0 + \theta_1 * X)}}{1 + e^{-(\theta_0 + \theta_1 * X)}}$$

Bam!!!!

we get the probability score(s).

So That's it for this story , In the next story I will code this algorithm from scratch and also using Tensorflow and scikitlearn.

See ya!

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456 claps



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