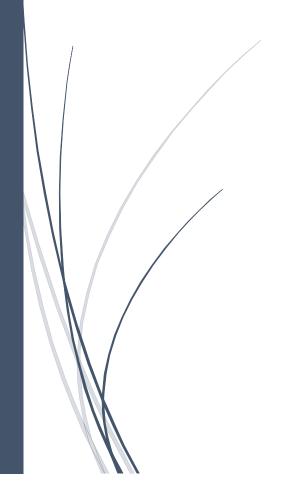


August/2025

**FFT** 

project



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FFT

# Introduction

This is the final project of the internship @ Analog Devices.

We are going to implement the FFT with radix-2 and N=8 with MATLAB and then follow the hardware steps (digital flow)

Having sized all signals in the design, you are now ready to begin the digital design flow:

- Microarchitecture design.
- RTL code.
- Verification.
- Backend.

We will not cover the Verification and Backend steps till now.

# **FFT**

It's "fast fourier transform" an algorithm that computes the discrete fourier transform "DFT" of a sequence, or its inverse.

A fourier transform converts a signal from its original domain to a representation in the frequency domain and vice versa.

FFT manages to reduce the complexity of computing the DFT from  $O(n^2)$  to O(nxlog n) where n is the data size.

The difference in speed can be enormous, especially for long data sets where n may be in the thousands or millions.

#### For Radix-2:

• The input size N must be a power of 2 (here, 8).



 The algorithm recursively splits the signal into even and oddindexed parts, computes FFT of both, and combines using twiddle factors.

# **MATLAB Part**

#### **Design Steps**

- Isolate Core Algorithm.
- 2. Prepare for Instrumentation.
- 3. Fixed-Point Designer.
- 4. Create Types Table.
- 5. Finalize Design Parameters.
- Add Fixed-Point Types to the Table.
- 7. Optimize Algorithm.

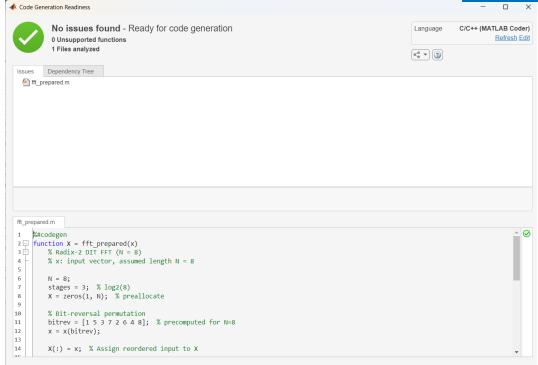
# 1<sup>st</sup> step I isolated the function file from the test files

```
function X = FFT(x)
                                                                          % test_fft.m
                                                            \bigcirc
           N = length(x);
                                                                          clc; clear;
           % Base case
                                                                          % Input vector
 5
                                                                          x = rand(1, 8); % You can change to complex: x = rand(1, 8)
           if N == 1
 6
               X = X;
                                                                          % Run custom FFT
               % Recursive FFT on even and odd indices
 8
                                                                  8
                                                                          X_{custom} = FFT(x);
               X_{even} = FFT(x(1:2:end));
                                                                  9
10
               X_{odd} = FFT(x(2:2:end));
                                                                 10
                                                                          % Run MATLAB built-in FFT
11
                                                                 11
                                                                          X_{builtin} = fft(x);
12
               % Twiddle factors
                                                                 12
13
               W = \exp(-2i * pi * (0:N/2-1) / N);
                                                                 13
                                                                          % Display results
                                                                          disp("Custom FFT Output:")
15
                                                                 15
               X = [X_even + W .* X_odd, ...
                                                                          disp("MATLAB FFT Output:")
17
                    X_even - W .* X_odd];
                                                                          disp(X_builtin)
18
                                                                 18
19
                                                                 19
                                                                          % Difference
20
                                                                 20
                                                                          err = norm(X custom - X builtin);
21
                                                                 21
                                                                          fprintf("Difference Norm = %.6e\n", err);
22
                                                                 22
                                                                          % Plot results
23
                                                                 23
```

2<sup>nd</sup> step check for codegen

We wrote the fft in a loop based and bit-reversed iterative style





# 3<sup>rd</sup> step

- ▶ Once we have prepared our algorithm for code generation, we can now use Fixed-Point Designer to instrument and accelerate it.
- ► To generate an instrumented MEX file for an algorithmic function, use the buildInstrumentedMex function as follows:

buildInstrumentedMex function\_name -args {functionInputs}

► The -args option tells MATLAB the data type of each input to be passed to the function by passing an example argument.

```
K>> buildInstrumentedMex fir_trans -args {x, b}
```

Run this from the MATLAB command window:

This version logs min and max for all variables

```
matlab

buildInstrumentedMex fft_prepared -args {rand(1,8)}

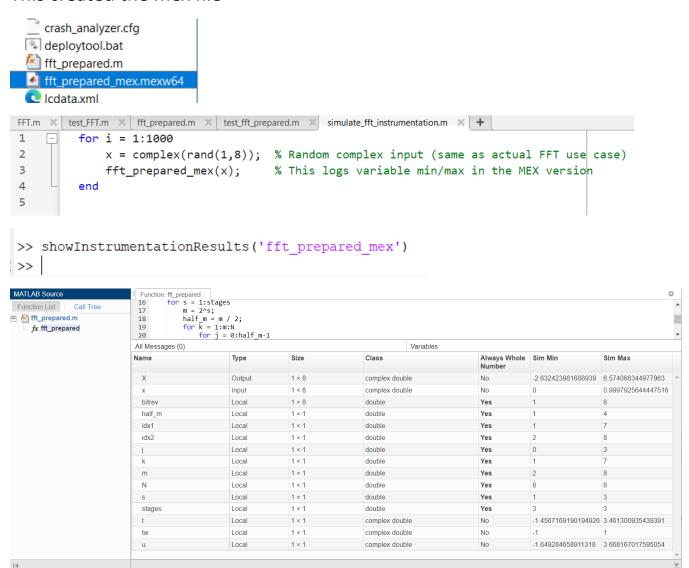
★ What this does:

Tells MATLAB to simulate your function using input type double(1x8)

Generates an instrumented MEX version: fft_prepared_mex
```



#### This created the mex file



# 4<sup>th</sup> , 5<sup>th</sup> ,6<sup>th</sup> step

# Making type table

```
X test_FFT.m X fft_prepared.m X test_fft_prepared.m X simulate_fft_instrumentation.m X fft_prepared_fixpt.m X test_fft_prepared_fixpt.m
1 -
       function T = fft_type()
                                                                                                                             \odot
 2
       % Define fixed-point types with 12 total bits (4 integer, 8 fractional)
 3
 4
           WL = 12; % Total word length
 5
           FL = 8;
                       % Fraction length
 6
 7
           T.x = fi(0, 1, WL, FL);
                                        % Input
 8
           T.X = fi(0, 1, WL, FL);
                                        % Output/intermediate
 9
           T.W = fi(0, 1, WL, FL);
                                        % Twiddle factor
10
           T.t = fi(0, 1, WL, FL);
                                        % Temporary result
11
           T.u = fi(0, 1, WL, FL);
                                       % Intermediate variable
12
13
```



```
FFT.m X test_FFT.m X fft_prepared.m X test_fft_prepared.m X simulate_fft_instrumentation.m X fft_type.m X
                                                                                             fft prepared fixpt.m × test fff
 1 🖃
         function X = fft_prepared_fixpt(x)
 2 -
        % Fixed-point Radix-2 FFT (N = 8)
 3
        % Uses fixed-point types defined in fft_type.m
 4
        % Input: x must be a complex fi vector (like T.x)
 5
 6
             %#codegen
 7
             N = 8;
             stages = 3;
 8
 9
10
             % Load fixed-point type definitions
11
             T = fft_type();
12
13
             \% Initialize output with same type as T.X
14
             X = repmat(T.X, 1, N);
15
16
             % Bit-reversal permutation
17
             bitrev = [1 5 3 7 2 6 4 8];
             x = x(bitrev);
18
                                                            % Reorder input
19
             X(:) = cast(x, 'like', T.X);
                                                            % Store into output
20
21
             % Precompute twiddle factors once
22
             persistent W:
23
             if isempty(W)
                  W = cast(exp(-2i * pi * (0:N/2 - 1) / N), 'like', T.W);
24
25
26
27
             % FFT Computation (Butterfly)
28 -
             for s = 1:stages
29
                 m = 2^s:
30
                 half_m = m / 2;
31 [
                 for k = 1:m:N
32
                     for j = 0: half_m - 1
33
                          idx1 = k + j;
                          idx2 = k + j + half_m;
34
35
                         tw = W(mod(j * (N / m), N/2) + 1);
t = cast(tw * X(idx2), 'like', T.t);
u = cast(X(idx1), 'like', T.u);
                                                                         % Twiddle
36
37
                                                                         % Twiddle product
38
                                                                         % Copy
39
                          X(idx1) = cast(u + t, 'like', T.X);
X(idx2) = cast(u - t, 'like', T.X);
40
                                                                         % Butterfly +
41
                                                                         % Butterfly -
42
43
                 end
44
45
        end
46
```

## Test

```
X test_FFT.m X fft_prepared.m X test_fft_prepared.m X simulate_fft_instrumentation.m X fft_type.m X fft_prepared_fixpt.m X test_fft_prepared_fixpt.m X
                                                                                                                                            A
        function test fft prepared fixpt()
        \% Testbench for fft_prepared_fixpt using fft_type.m definitions
3
 4
 5
             T = fft_type(); % Load types
 6
 7
             % Create a test input using the same fixed-point format as T.x
8
             x = cast(complex(rand(1,8)), 'like', T.x);
 9
10
             % Run the fixed-point FFT
             y_fixed = fft_prepared_fixpt(x);
11
12
13
             % Compare with floating-point FFT
14
             y_float = fft_prepared(double(x));
15
16
             % Calculate norm difference
17
             diff = norm(double(y_fixed) - y_float);
18
             % Display result
20
             fprintf("Fixed-point error norm = %.3e\n", diff);
21
22
             % Optional: plot comparison
23
             subplot(2,1,1); \; stem(abs(y\_fixed), \; 'filled'); \; title('Fixed-point \; FFT \; |Y|'); \\ subplot(2,1,2); \; stem(abs(y\_float), \; 'filled'); \; title('Floating-point \; FFT \; |Y|'); \\
24
25
26
27
```



This table will be used in step 6 to cast all variables to fixed-point versions

And maximum -ve is 1000.00000000 = -8

```
From fft_type.m:

matlab

WL = 12;
FL = 8;

→ That gives integer bits = 12 - 8 = 4 bits, including the sign bit.

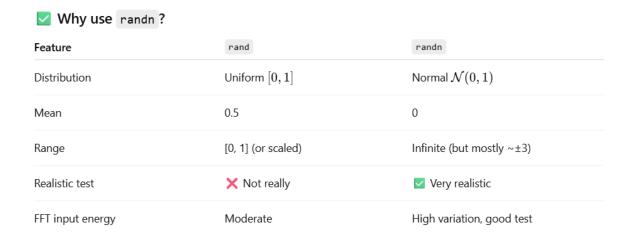
+ Fixed-point range (signed):

• Min: -2<sup>3</sup> = -8

• Max: +2<sup>3</sup> - 2<sup>-8</sup> = 7.9961

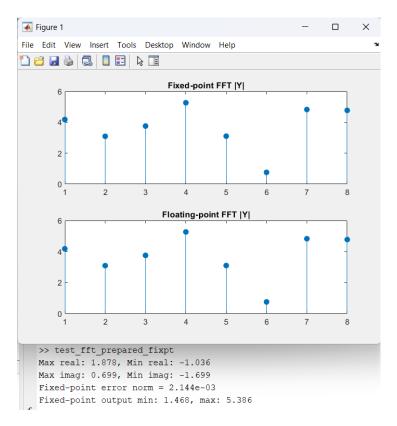
So any value you use in testing must lie between -8 and +8, or you risk overflow.
```

I changed my code part that randomize the input to be more general using randn





## Ex of simulation in matlab



# the full codes will be in separate files .m

Fixed-Point FFT De	sign Summary (Steps 1–6)
Step	What Was Done
1. Isolate Core Algorithm	Created fft_prepared.m and fft_prepared_fixpt.m to isolate FFT logic.
2. Prepare for Instrumentation	Ensured code is vectorized, uses #codegen , and is MEX-compatible.
3. Fixed-Point Designer Setup	Created fft_type.m to define reusable fixed-point types.
4. Create Types Table	Defined complex fixed-point templates ( $fi(complex(0,0),)$ ) for input/output/intermediate signals.
5. Finalize Design Parameters	Chose word length = 12 bits, fraction length = 8 bits.
6. Add Fixed-Point Types	Applied casting using 'like', T.x', and verified accuracy:  • Error norm ≈ 1.9e-3  • FFT output range: min ≈ 1.4, max ≈ 7.2  • Verified correctness using floating-point reference & plots.

Result: Functional, portable, and validated fixed-point FFT model ready for RTL and ASIC flow.

1



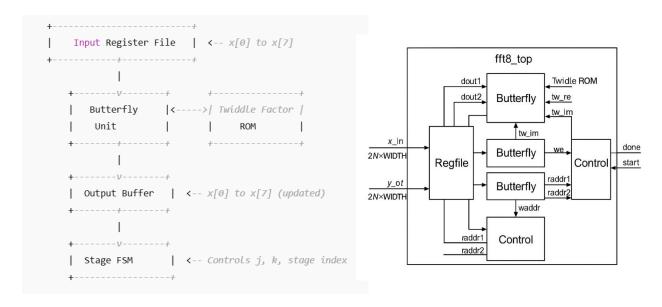
# **Arch design**

#### common hardware architectures to consider:

Architecture	Description	Pros	Cons
Iterative	One butterfly reused across time steps	Low area	Slowest (needs full control FSM)
In-place	Reuse the same memory for updates (matches your MATLAB logic)	Compact	Needs careful memory/control
Pipelined	One stage per clock cycle, full unrolling	Highest throughput	High area

# We will choose iterative in place architecture

## RTL blocks:





Signal	Width	Description
X[7:0]	2×12 bits (real & imag) each	Internal FFT data buffer
W[3:0]	2×12 bits	Twiddle factors ROM (4 values for 8-point FFT)
u , t	2×12 bits	Intermediate values in butterfly
idx1 , idx2	3 bits	Indices for FFT pairs
stage	2 bits	Stage 0 to 2 (3 stages)
j, k	3 bits	Butterfly loop control

**Flow of the design :** first we inputs the 8 samples x(0) x(1) ... x(7) real and img with size 12-bit to the regfile and then passed it to the butterfly inputs (U,V) and then the butterfly make the calculations to multiply the V with the W "twiddle factor" and the U multiplied by 1 and after that butterfly outputs (out1,out2) out1=U+V.W and out2=U-V.W, and after calculation we then rewrite the regfile with the result.

The FSM control the flow and provide the W values for the butterfly and provide the address to the regfile and tell what stage are we in now and the butterfly index.

# Twiddle factor & radix-2 algorithm

# After assuming W<sub>N</sub><sup>m</sup>=e<sup>-j2pi(m/N)</sup>

Here, we consider about the property of the twiddle factor. The twiddle factor has the periodic property as be shown in equation (2.2)

$$\begin{pmatrix} W_N^{k+N} = W_N^k \\ W_N^{k+\frac{N}{2}} = -W_N^k \end{pmatrix}$$

#### Equation 4-20

$$X(m) = \sum_{n=0}^{(N/2)-1} x(2n) W_{N/2}^{nm} + W_N^m \sum_{n=0}^{(N/2)-1} x(2n+1) W_{N/2}^{nm} ,$$

and

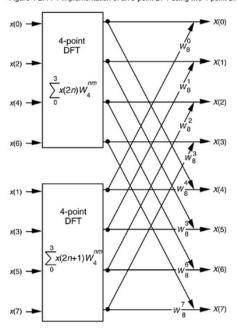
#### Equation 4-20'

$$X(m+N/2) = \sum_{n=0}^{(N/2)-1} x(2n)W_{N/2}^{nm} - W_N^m \sum_{n=0}^{(N/2)-1} x(2n+1)W_{N/2}^{nm}.$$

So here we are. We need not perform any sine or cosine multiplications to get X(m+N/2). We just change the sign of the twiddle factor  $W_N^m$  and use the results of the two summations from X(m) to get X(m+N/2). Of course, m goes from 0 to (N/2)-1 in Eq. (4-20) which means, for an N-point DFT, we perform an N/2-point DFT to get the first N/2 outputs and use those to get the last N/2 outputs. For N = 8, Eqs. (4-20) and (4-20') are implemented as shown in Figure 4-2.

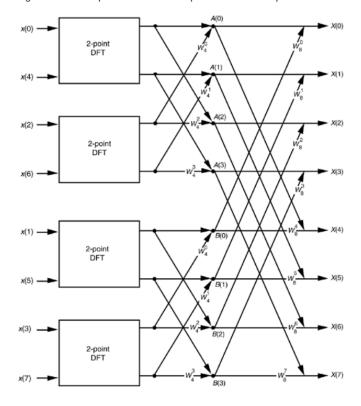


Figure 4-2. FFT implementation of an 8-point DFT using two 4-point DFTs.



# We can make it 4 2-points DFT

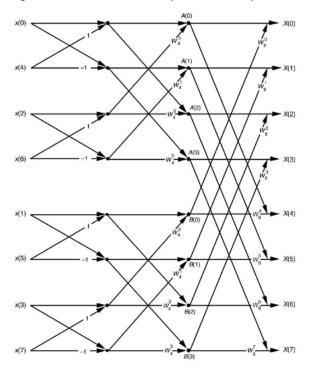
Figure 4-3. FFT implementation of an 8-point DFT as two 4-point DFTs and four 2-point DFTs.



Replacing the 2-point DFT by butterfly



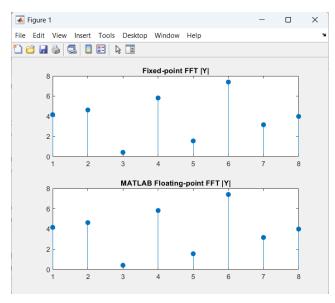
Figure 4-5. Full decimation-in-time FFT implementation of an 8-point DFT.



RTL files will be in separate folder.

# **Results**

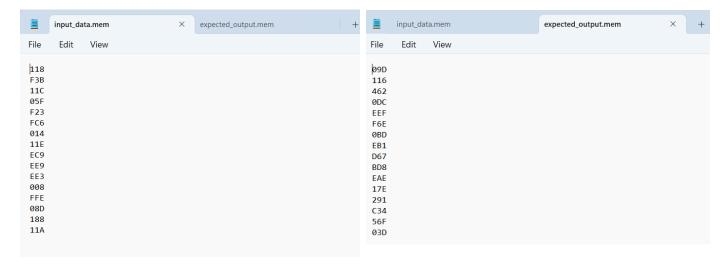
First the **MATLAB** randomize 8-sample input and then do the FFT and generate the output and compute the **AVG. SQNR** and also the error between my FFT model and internal MATLAB FFT and make 2 files one for the inputs and other for the outputs to pass them to the testbench done to test also the RTL design.





```
>> test_fft_prepared_fixpt
      Max real: 1.438, Min real: -2.944
      Max imag: 0.628, Min imag: -0.865
      Fixed-point error norm = 2.686e-03
      SQNR vs MATLAB FFT = 73.340 dB
      Fixed-point output min: 0.422, max: 7.396
      Fixed-point input samples:
           Columns 1 through 6
               1.0938 \ -\ 0.7695 i \quad 1.1094 \ +\ 0.3711 i \quad -0.8633 \ -\ 0.2266 i \quad 0.0781 \ +\ 1.1172 i \quad -1.2148 \ -\ 1.0898 i \quad -1.1133 \ +\ 0.0312 i \quad -0.0863 i \quad -0.0863
           Columns 7 through 8
           -0.0078 + 0.5508i 1.5312 + 1.1016i
                                     DataTypeMode: Fixed-point: binary point scaling
                                          Signedness: Signed
                                           WordLength: 12
                              FractionLength: 8
      Stored integers (hex):
      X[1] re=118 im=F3B
      X[2] re=11C im=05F
      X[3] re=F23 im=FC6
      X[4] re=014 im=11E
      X[5] re=EC9 im=EE9
      X[6] re=EE3 im=008
      X[7] re=FFE im=08D
fx X[8] re=188 im=11A
        Fixed-point FFT output samples:
            Columns 1 through 6
               Columns 7 through 8
                 2.5664 - 3.7969i 5.4336 + 0.2383i
                                      DataTypeMode: Fixed-point: binary point scaling
                                            Signedness: Signed
                                            WordLength: 12
                                FractionLength: 8
        Stored integers (hex):
        Y[1] re=09D im=116
        Y[2] re=462 im=0DC
        Y[3] re=EEF im=F6E
        Y[4] re=0BD im=EB1
        Y[5] re=D67 im=BD8
        Y[6] re=EAE im=17E
        Y[7] re=291 im=C34
        Y[8] re=56F im=03D
fx >>
```

# The 2 generated files input, output





#### **QUESTASIM** do file

```
# compile_dut_tb_cov.do
# ===== COMPILE DUT WITH COVERAGE =====
vlog -cover bcst fft8_top.v
vlog -cover bcst fft8 ctrl.v
vlog -cover bcst regfile fft.v
vlog -cover bcst butterfly.v
vlog -cover bcst twiddle rom.v
# ===== COMPILE TESTBENCH WITHOUT COVERAGE ======
vlog fft_testbench.v
# ===== LOAD SIMULATION WITH COVERAGE ======
vsim -coverage -voptargs=+acc work.fft8_tb
# ===== ADD WAVES =====
# ===== ADD WAVES =====
# ===== ADD WAVES =====
add wave -position insertpoint \
sim:/fft8_tb/N \
sim:/fft8_tb/WIDTH \
sim:/fft8_tb/FRACTION \
sim:/fft8_tb/clk \
sim:/fft8_tb/rst \
sim:/fft8_tb/start \
                                                          add wave -position insertpoint \
                                                          sim:/fft8_tb/dut/regs/mem_re \
sim:/fft8_tb/load \
                                                          sim:/fft8_tb/dut/regs/mem_im
sim:/fft8_tb/load_addr \
sim:/fft8_tb/out_addr \
sim:/fft8_tb/data_in_re \
sim:/fft8_tb/data_in_im \
                                                          # ===== RUN SIMULATION ======
sim:/fft8 tb/done \
                                                          run -all
sim:/fft8_tb/data_out_re
sim:/fft8_tb/data_out_im \
                                                          # ===== SAVE COVERAGE REPORT ======
sim:/fft8_tb/input_data \
                                                          coverage save fft8 cov.ucdb
                                                          coverage report -details -output fft8_cov_report.txt
sim:/fft8_tb/expected_data \
sim:/fft8_tb/bit_rev_map \
sim:/fft8_tb/i \
sim:/fft8_tb/idx
```

## Questa result

```
# === FFT Output Comparison ===
# Y[0] DUT: 157 + j278 | Expected: 157 + j278
   -> MATCH
# Y[1] DUT: 1121 + j220 | Expected: 1122 + j220
   -> MISMATCH
# Y[2] DUT: -273 + j-146 | Expected: -273 + j-146
   -> MATCH
# Y[3] DUT: 189 + j-335 | Expected: 189 + j-335
    -> MATCH
 Y[4] DUT: -665 + j-1064 | Expected: -665 + j-1064
   -> MATCH
# Y[5] DUT: -337 + j382 | Expected: -338 + j382
    -> MISMATCH
# Y[6] DUT: 657 + j-972 | Expected: 657 + j-972
   -> MATCH
# Y[7] DUT: 1391 + j61 | Expected: 1391 + j61
   -> MATCH
 ** Note: $stop
                  : fft_testbench.v(222)
  Time: 335 ns Iteration: 0 Instance: /fft8_tb
# Break in Module fft8_tb at fft_testbench.v line 222
```

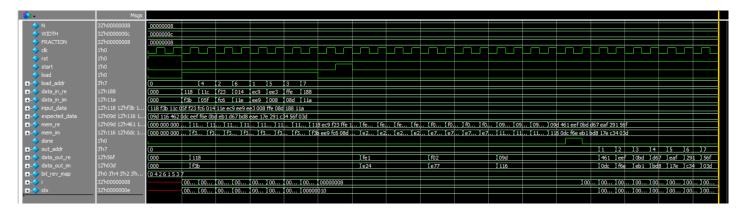
It's written in decimal but easily divided by 256 to see it in format Q4.8

#### Just like matlab

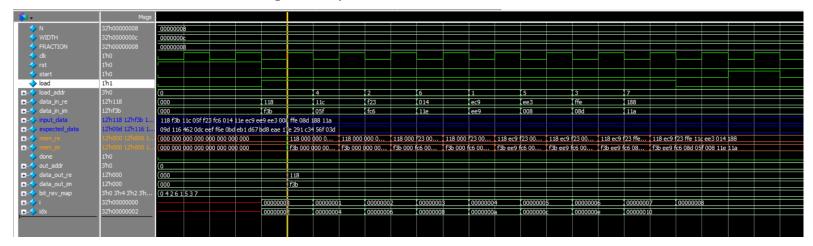


# Wave form trace & latency calculation

The full wave form



# Zoomed in for loading the input data

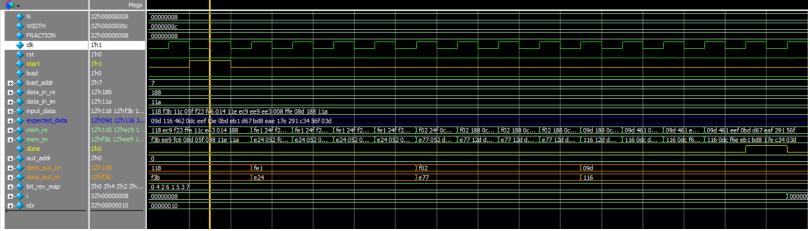


As we see here the **blue** signals is the input/output file from matlab so after the **load signal** is high we begin to take the inputs in 8 cycles so after **8 cycles** from load the **orange** signal is the internal memory in the design that hold the inputs data.

After that we have start signal it goes 1 when loading all input and then the calculation starts and after all calculations end then done signal go high.

We need to calculate the latency "number of cycles from start of the calculations to the first output". If we need execution time it will be "number of cycles from start of the calculations to the done=1".





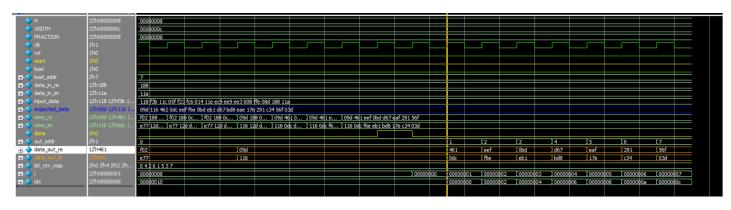
As we see from the start to the first data\_out\_re , data\_out\_im is 9 cycles and from the start to the done is 13 cycles but from the start to the all calculations end mem\_re and mem\_im being filled with the output is 12 cycles that because the done state take one cycle from the end of calculation to go to 1 since it's sequential.

We can conclude that the values saved in mem\_re , mem\_im are the same as stored in the expected\_data.

Latency=9 clock cycles.

Execution=12 clock cycles.

# Reading output after done





## SQNR eqn

For N output samples:

$$ext{SQNR} = 10 \log_{10} \left( rac{\sum_{n=0}^{N-1} |y_{ ext{float}}(n)|^2}{\sum_{n=0}^{N-1} |y_{ ext{float}}(n) - y_{ ext{fixed}}(n)|^2} 
ight) \, ext{dB}$$

- Numerator → total signal power of the ideal output.
- Denominator → total error power caused by fixed-point quantization.
- Higher SQNR  $\rightarrow$  less quantization noise  $\rightarrow$  fixed-point result is closer to floating-point.
- Typical scale:
  - $> 60 \text{ dB} \rightarrow \text{excellent (error barely noticeable)}$
  - $40-60 \text{ dB} \rightarrow \text{acceptable for most DSP}$
  - $< 30 \text{ dB} \rightarrow \text{large visible distortion}$

#### **Coverage Report**

Code coverage (bcst) branch, conditional, statement, toggle

# fft\_ctrl module

Why it's not 100% because of the case default we can remove it since we covered all 4 cases



Condition Coverage: Enabled Coverage Conditions	Bins  3	Covered	
Statement Coverage: Enabled CoverageStatements	Bins  32	Hits  32	
Toggle Coverage: Enabled Coverage Toggles	Bins  182	Hits  58	Misses Coverage  124 31.86%

Why toggle don't reach high coverage % ,because we used only one test case. For more we can run it 100 times for example.

## twiddle rom module

#### Also because of the case

```
File twiddle rom.v
-----CASE Branch------
 22
23 1
27 1
31 1
                   9 Count coming in to CASE
                          4
  35 1
39 1
Branch totals: 4 hits of 5 branches = 80.00%
Statement Coverage:
   Enabled Coverage
                      Bins Hits Misses Coverage
   Statements
                        11 9 2 81.81%
                      61 1 1 6 1 11
Toggle Coverage:
  Enabled Coverage
                       Bins Hits Misses Coverage
                              44 8 84.61%
  Toggles
                        52
```



# reg\_file module

nch Coverage:	=========			
Enabled Coverage	Bins	Hits	Misses	Coverage
Branches	8	8	0	100.00%
tement Coverage:				
Enabled Coverage	Bins	Hits	Misses	Coverage
Statements	18	18	0	100.00%
gle Coverage:				
Enabled Coverage	Bins	Hits	Misses	Coverage
Toggles	398	330	68	82.91%

# butterfly module

```
=== Design Unit: work.butterfly
_____
Statement Coverage:
              Bins Hits Misses Coverage
---- 8 8 0 100.00%
                           8 0 100 cm
  Enabled Coverage
  -----
  Statements
Toggle Coverage:
  Enabled Coverage
                      Bins
                            Hits Misses Coverage
                      ____
                             ----
                                  _____
  Toggles
                       384
                             364
                                    20
                                        94.79%
```

# fft8\_top module



## The total coverage is

Total Coverage By Instance (filtered view): 91.75%

# **ASIC** phase

For the physical flow design we will use OpenLane opensource tool with technology and std cell used skywater 130nm, sky130\_fd\_sc\_hd.

We will run flow.tcl and then the reports will created if the flow run successfully for all steps including synthesis and routing and power and pnr and cts .....

But we will do it later.

# Thank You