```
import torch
import torch.nn as nn
import torch.optim as optim
from torch.utils.data import DataLoader, TensorDataset
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import kagglehub
from sklearn.model_selection import train_test_split
from google.colab import files
files.upload()
```

Choose Files No file chosen Upload widget is only available when the cell has been executed in the current browser session. Please rerun this cell to enable.

```
Saving kaggle.json to kaggle.json
```

```
!mkdir -p ~/.kaggle
!mv kaggle.json ~/.kaggle/
!chmod 600 ~/.kaggle/kaggle.json
```

```
!kaggle competitions download -c digit-recognizer
```

```
Downloading digit-recognizer.zip to /content
  0% 0.00/15.3M [00:00<?, ?B/s]
100% 15.3M/15.3M [00:00<00:00, 1.33GB/s]
```

```
!unzip digit-recognizer.zip -d mnist_data
```

```
Archive:  digit-recognizer.zip
  inflating: mnist_data/sample_submission.csv
  inflating: mnist_data/test.csv
  inflating: mnist_data/train.csv
```

```
train_df = pd.read_csv('mnist_data/train.csv')

print(train_df.shape)
```

```
(42000, 785)
```

```
X = train_df.drop('label', axis=1).values / 255.0
t = train_df['label'].values

X_train, X_temp, t_train, t_temp = train_test_split(
    X, t, test_size=0.4, random_state=42, stratify=t
)

X_val, X_test, t_val, t_test = train_test_split(
    X_temp, t_temp, test_size=0.5, random_state=42, stratify=t_temp
)
```

Converting into tensors

```
X_train = torch.tensor(X_train, dtype=torch.float32)
t_train = torch.tensor(t_train, dtype=torch.long)

X_val = torch.tensor(X_val, dtype=torch.float32)
t_val = torch.tensor(t_val, dtype=torch.long)

X_test = torch.tensor(X_test, dtype=torch.float32)
t_test = torch.tensor(t_test, dtype=torch.long)
```

Creation of Neural Network

```
class NeuralNetwork(nn.Module):
    def __init__(self,input_size,h1_size,h2_size,h3_size,out_size):
        super(NeuralNetwork, self).__init__()
        self.flatten = nn.Flatten()
        self.l1 = nn.Linear(input_size,h1_size)
        self.l2 = nn.Linear(h1_size,h2_size)
        self.l3 = nn.Linear(h2_size,h3_size)
        self.l4 = nn.Linear(h3_size,out_size)
        self.relu = nn.ReLU()
        for layer in self.modules():
            if isinstance(layer, nn.Linear):
                nn.init.kaiming_normal(layer.weight, nonlinearity='relu')
                nn.init.zeros_(layer.bias)
    def forward(self, x):
        out = self.flatten(x)
        out = self.l1(out)
        out = self.relu(out)
        out = self.l2(out)
        out = self.relu(out)
        out = self.l3(out)
        out = self.relu(out)
        out = self.l4(out)
        return out
```

Trainning Function

```
def train_model(nn_model, epochs=20,batch_size=64):
    train_losses = []
    val_losses = []
    train_accuracies = []
    val_accuracies = []
    train_loader = DataLoader(TensorDataset(X_train, t_train), batch_size=batch_size, shuffle=True)
    val_loader = DataLoader(TensorDataset(X_val, t_val), batch_size=batch_size, shuffle=False)
    test_loader = DataLoader(TensorDataset(X_test, t_test), batch_size=batch_size, shuffle=False)
    for epoch in range(epochs):
        nn_model.train()
        train_loss = 0
        correct_preds = 0
        total_preds = 0

        for X_batch, t_batch in train_loader:
            X_batch = X_batch.to(device)
            t_batch = t_batch.to(device)
            optimizer.zero_grad()
            y = nn_model(X_batch)
            loss = criteration(y, t_batch)
            loss.backward()
            optimizer.step()
            train_loss += loss.item()
            _, y_pred = torch.max(y, 1)
            correct_preds += (y_pred == t_batch).sum().item()
            total_preds += t_batch.size(0)

        avg_epoch_train_loss = train_loss / len(train_loader)
        train_losses.append(avg_epoch_train_loss)
        train_acc = correct_preds / total_preds * 100
        train_accuracies.append(train_acc)

        nn_model.eval()
        correct_pred = 0
        total_pred = 0
        val_loss = 0

        with torch.no_grad():
            for X_batch, t_batch in val_loader:
                X_batch = X_batch.to(device)
                t_batch = t_batch.to(device)
                y_val = nn_model(X_batch)
                loss = criteration(y_val, t_batch)
                val_loss += loss.item()
                _, y_pred = torch.max(y_val, 1)
                correct_pred += (y_pred == t_batch).sum().item()
                total_pred += t_batch.size(0)

        avg_epoch_val_loss = val_loss / len(val_loader)
        val_losses.append(avg_epoch_val_loss)
        val_accuracy = correct_pred / total_pred * 100
        val_accuracies.append(val_accuracy)
        train_acc_mean = np.array(train_accuracies)
        val_acc_mean = np.array(val_accuracies)
        train_acc_std = np.std(train_accuracies, axis=0)
        val_acc_std = np.std(val_accuracies, axis=0)
        loss_diff = np.diff(train_losses)

        print(f'Epoch {epoch + 1}/{epochs}, Train Loss: {avg_epoch_train_loss:.4f}, Val Loss: {avg_epoch_val_loss:.4f}')

    return train_losses, val_losses, train_accuracies, val_accuracies, train_acc_mean, val_acc_mean, train_acc_std, val_acc_stc
```

Initialzing an instance

```
input_size = 784
h1_size = 256
h2_size = 128
h3_size = 64
out_size = 10
nn_model = NeuralNetwork(input_size,h1_size,h2_size,h3_size,out_size)
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
nn_model.to(device)
```

```
/tmp/ipython-input-2953671239.py:12: FutureWarning: `nn.init.kaiming_normal` is now deprecated in favor of `nn.init.kaiming_norm
  nn.init.kaiming_normal(layer.weight, nonlinearity='relu')
NeuralNetwork(
  (flatten): Flatten(start_dim=1, end_dim=-1)
  (l1): Linear(in_features=784, out_features=256, bias=True)
  (l2): Linear(in_features=256, out_features=128, bias=True)
  (l3): Linear(in_features=128, out_features=64, bias=True)
  (l4): Linear(in_features=64, out_features=10, bias=True)
  (relu): ReLU()
)
```

Loss function

```
criteration = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(nn_model.parameters(), lr=0.01)
```

Train using normal parameters

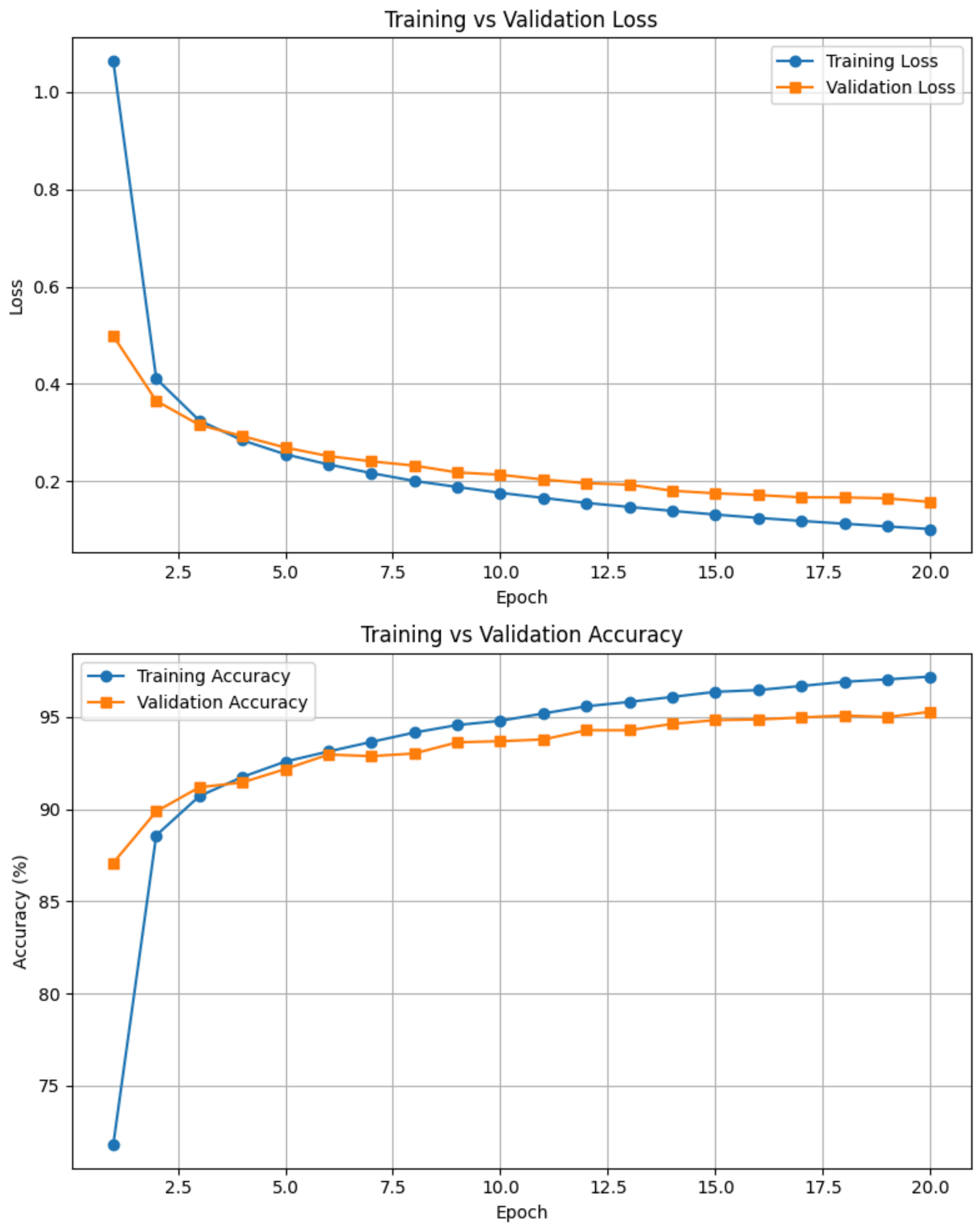
```
epochs = 20
train_losses, val_losses, train_accuracies, val_accuracies, train_acc_mean, val_acc_mean, train_acc_std, val_acc_std, loss_diff
```

```
Epoch 1/20, Train Loss: 1.0637, Val Loss: 0.4985
Epoch 2/20, Train Loss: 0.4107, Val Loss: 0.3651
Epoch 3/20, Train Loss: 0.3247, Val Loss: 0.3156
Epoch 4/20, Train Loss: 0.2841, Val Loss: 0.2927
Epoch 5/20, Train Loss: 0.2552, Val Loss: 0.2690
Epoch 6/20, Train Loss: 0.2344, Val Loss: 0.2516
Epoch 7/20, Train Loss: 0.2166, Val Loss: 0.2407
Epoch 8/20, Train Loss: 0.2004, Val Loss: 0.2320
Epoch 9/20, Train Loss: 0.1880, Val Loss: 0.2179
Epoch 10/20, Train Loss: 0.1759, Val Loss: 0.2132
Epoch 11/20, Train Loss: 0.1656, Val Loss: 0.2033
Epoch 12/20, Train Loss: 0.1552, Val Loss: 0.1958
Epoch 13/20, Train Loss: 0.1469, Val Loss: 0.1927
Epoch 14/20, Train Loss: 0.1389, Val Loss: 0.1803
Epoch 15/20, Train Loss: 0.1313, Val Loss: 0.1750
Epoch 16/20, Train Loss: 0.1245, Val Loss: 0.1714
Epoch 17/20, Train Loss: 0.1182, Val Loss: 0.1669
Epoch 18/20, Train Loss: 0.1126, Val Loss: 0.1665
Epoch 19/20, Train Loss: 0.1069, Val Loss: 0.1646
Epoch 20/20, Train Loss: 0.1016, Val Loss: 0.1572
```

Plotting training vs validation errors

```
plt.figure(figsize=(8, 10))
print("Validation Accuracy" , max(val_accuracies))
# --- Plot 1: Training vs Validation Loss ---
plt.subplot(2, 1, 1)
plt.plot(range(1, epochs + 1), train_losses, label='Training Loss', marker='o')
plt.plot(range(1, epochs + 1), val_losses, label='Validation Loss', marker='s')
plt.title('Training vs Validation Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.legend()
plt.grid(True)

# --- Plot 2: Training vs Validation Accuracy ---
plt.subplot(2, 1, 2)
plt.plot(range(1, epochs + 1), train_accuracies, label='Training Accuracy', marker='o')
plt.plot(range(1, epochs + 1), val_accuracies, label='Validation Accuracy', marker='s')
plt.title('Training vs Validation Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')
plt.legend()
plt.grid(True)

plt.tight_layout()
plt.show()
```

```
Validation Accuracy 95.27380952380953
```

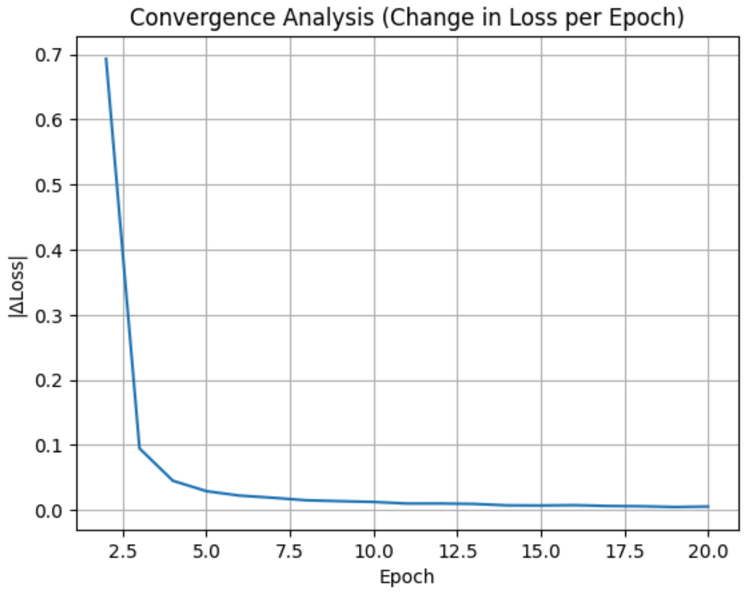


```
epochs = 20
n_runs = 5

all_train_accuracies = []
all_val_accuracies = []

for run in range(n_runs):
    nn_model = NeuralNetwork(input_size,h1_size,h2_size,h3_size,out_size)
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    nn_model.to(device)
    criteration = nn.CrossEntropyLoss()
    optimizer = torch.optim.SGD(nn_model.parameters(), lr=0.01)
    train_losses, val_losses, train_accuracies, val_accuracies, *_ = train_model(nn_model, epochs)
    all_train_accuracies.append(train_accuracies)
    all_val_accuracies.append(val_accuracies)

all_train_accuracies = np.array(all_train_accuracies)
all_val_accuracies = np.array(all_val_accuracies)

train_acc_mean = all_train_accuracies.mean(axis=0)
train_acc_std = all_train_accuracies.std(axis=0)
val_acc_mean = all_val_accuracies.mean(axis=0)
val_acc_std = all_val_accuracies.std(axis=0)

plt.errorbar(range(1, epochs + 1), train_acc_mean, yerr=train_acc_std, fmt='-o', capsize=3, label='Train Accuracy')
plt.errorbar(range(1, epochs + 1), val_acc_mean, yerr=val_acc_std, fmt='-s', capsize=3, label='Validation Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.legend()
plt.show()
```

```
/tmp/ipython-input-2871887219.py:12: FutureWarning: `nn.init.kaiming_normal` is now deprecated in favor of `nn.init.kaiming_no
  nn.init.kaiming_normal(layer.weight, nonlinearity='relu')
Epoch 1/20, Train Loss: 1.1368, Val Loss: 0.5160
Epoch 2/20, Train Loss: 0.4194, Val Loss: 0.3691
Epoch 3/20, Train Loss: 0.3267, Val Loss: 0.3246
Epoch 4/20, Train Loss: 0.2830, Val Loss: 0.2919
Epoch 5/20, Train Loss: 0.2550, Val Loss: 0.2703
Epoch 6/20, Train Loss: 0.2334, Val Loss: 0.2623
Epoch 7/20, Train Loss: 0.2141, Val Loss: 0.2384
Epoch 8/20, Train Loss: 0.1980, Val Loss: 0.2309
Epoch 9/20, Train Loss: 0.1843, Val Loss: 0.2226
Epoch 10/20, Train Loss: 0.1718, Val Loss: 0.2107
Epoch 11/20, Train Loss: 0.1608, Val Loss: 0.2049
Epoch 12/20, Train Loss: 0.1516, Val Loss: 0.1939
Epoch 13/20, Train Loss: 0.1426, Val Loss: 0.1928
Epoch 14/20, Train Loss: 0.1345, Val Loss: 0.1833
Epoch 15/20, Train Loss: 0.1270, Val Loss: 0.1791
Epoch 16/20, Train Loss: 0.1204, Val Loss: 0.1741
Epoch 17/20, Train Loss: 0.1136, Val Loss: 0.1699
Epoch 18/20, Train Loss: 0.1083, Val Loss: 0.1691
Epoch 19/20, Train Loss: 0.1025, Val Loss: 0.1669
Epoch 20/20, Train Loss: 0.0974, Val Loss: 0.1600
Epoch 1/20, Train Loss: 1.1697, Val Loss: 0.5337
Epoch 2/20, Train Loss: 0.4308, Val Loss: 0.3720
Epoch 3/20, Train Loss: 0.3345, Val Loss: 0.3201
Epoch 4/20, Train Loss: 0.2893, Val Loss: 0.2942
Epoch 5/20, Train Loss: 0.2597, Val Loss: 0.2778
Epoch 6/20, Train Loss: 0.2368, Val Loss: 0.2528
Epoch 7/20, Train Loss: 0.2187, Val Loss: 0.2411
Epoch 8/20, Train Loss: 0.2007, Val Loss: 0.2393
Epoch 9/20, Train Loss: 0.1877, Val Loss: 0.2188
Epoch 10/20, Train Loss: 0.1749, Val Loss: 0.2133
Epoch 11/20, Train Loss: 0.1635, Val Loss: 0.2072
Epoch 12/20, Train Loss: 0.1532, Val Loss: 0.1948
Epoch 13/20, Train Loss: 0.1440, Val Loss: 0.1875
Epoch 14/20, Train Loss: 0.1360, Val Loss: 0.1831
Epoch 15/20, Train Loss: 0.1284, Val Loss: 0.1820
Epoch 16/20, Train Loss: 0.1217, Val Loss: 0.1752
Epoch 17/20, Train Loss: 0.1150, Val Loss: 0.1690
Epoch 18/20, Train Loss: 0.1092, Val Loss: 0.1680
Epoch 19/20, Train Loss: 0.1033, Val Loss: 0.1623
Epoch 20/20, Train Loss: 0.0983, Val Loss: 0.1634
Epoch 1/20, Train Loss: 1.1156, Val Loss: 0.5091
```

```
plt.plot(range(2, epochs + 1), np.abs(loss_diff))
plt.title('Convergence Analysis (Change in Loss per Epoch)')
plt.xlabel('Epoch')
plt.ylabel('|ΔLoss|')
plt.grid(True)
plt.show()
```

```
Epoch 9/20, Train Loss: 0.1839, Val Loss: 0.2190
```



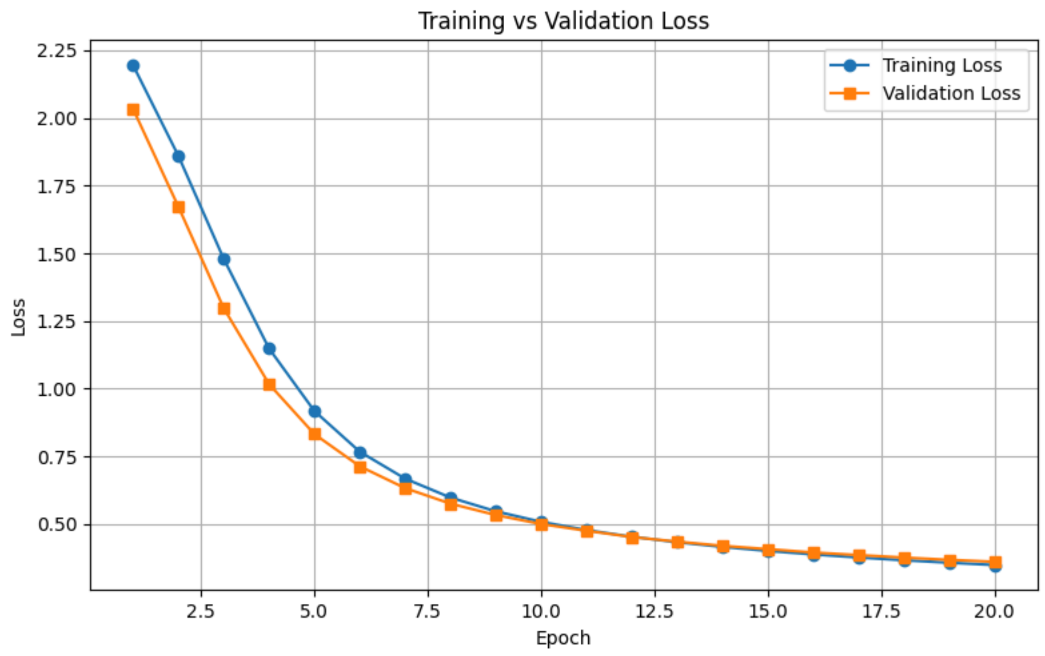
```
Epoch 17/20, Train Loss: 0.1143, Val Loss: 0.1718
Epoch 18/20, Train Loss: 0.1087, Val Loss: 0.1659
Epoch 19/20, Train Loss: 0.1033, Val Loss: 0.1597
```

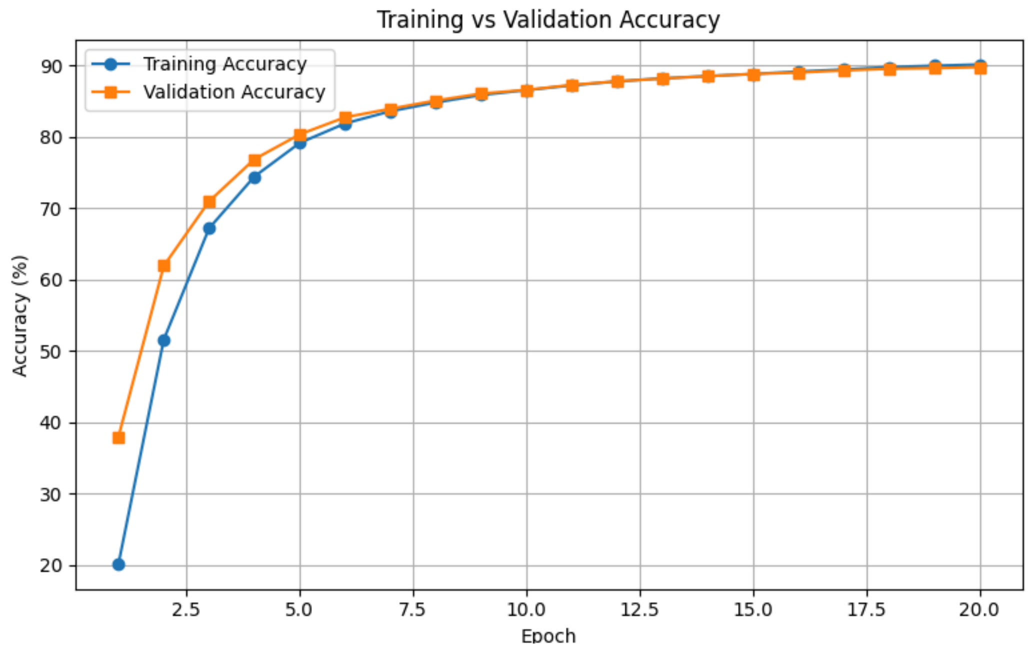
Test Different Learning Rate

```
learning_rates = [0.001, 0.01, 0.1, 1.0]
epochs = 20
```

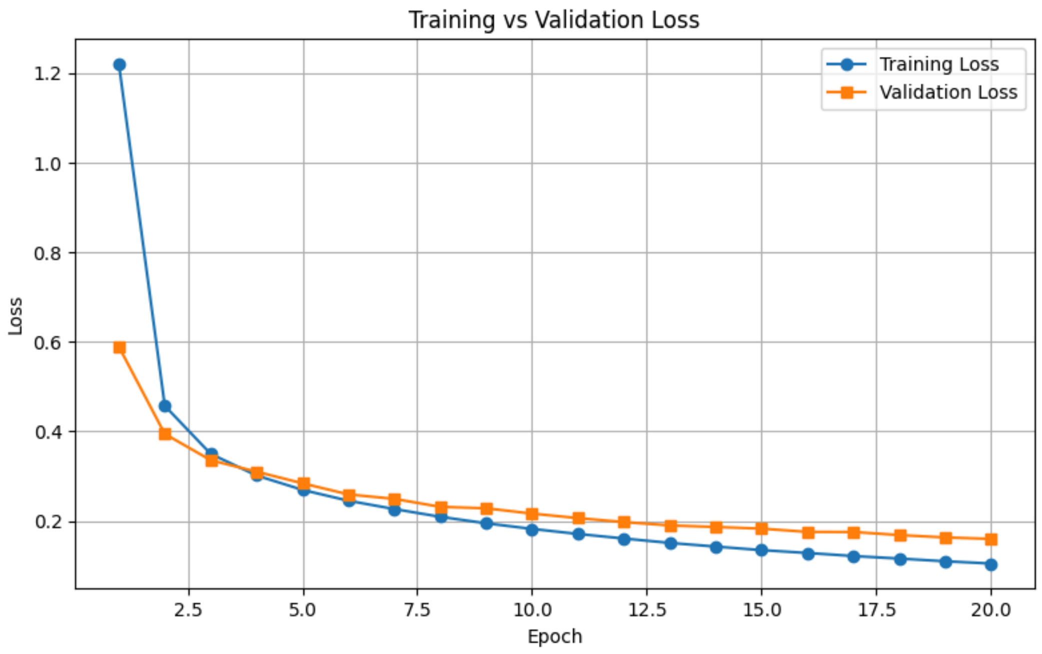
```
for lr in learning_rates:
    nn_model = NeuralNetwork(input_size, h1_size, h2_size, h3_size, out_size)
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    nn_model.to(device)
    optimizer = torch.optim.SGD(nn_model.parameters(), lr=lr)
    criteration = torch.nn.CrossEntropyLoss()

    train_losses, val_losses, train_accuracies, val_accuracies, train_acc_mean, val_acc_mean, train_acc_std, val_acc_std, loss_

    print(f"\nFinished run lr={lr}")
    print(f"Train Loss final: {train_losses[-1]:.4f}, Val Loss final: {val_losses[-1]:.4f}")
    print(f"Train Acc final: {train_accuracies[-1]:.2f}%, Val Acc final: {val_accuracies[-1]:.2f}%")


    # --- Analyze stability ---
    target_acc = 90

    reached_epochs = [i+1 for i, acc in enumerate(val_accuracies) if acc >= target_acc]

    if reached_epochs:
        print(f"Converged to {target_acc}% validation accuracy at epoch {reached_epochs[0]}")
    else:
        print(f"Did not reach {target_acc}% validation accuracy")

    unstable = any((val_accuracies[i-1] - val_accuracies[i]) > 1 for i in range(1, len(val_accuracies)))
    if unstable:
        print("Unstable — validation accuracy fluctuated by more than 1% between epochs")
    else:
        print("Training appears stable")


    plt.figure(figsize=(8, 10))
    # --- Plot 1:
    plt.subplot(2, 1, 1)
    plt.plot(range(1, epochs + 1), train_losses, label='Training Loss', marker='o')
    plt.plot(range(1, epochs + 1), val_losses, label='Validation Loss', marker='s')
    plt.title('Training vs Validation Loss')
    plt.xlabel('Epoch')
    plt.ylabel('Loss')
    plt.legend()
    plt.grid(True)

    # --- Plot 2:
    plt.subplot(2, 1, 2)
    plt.plot(range(1, epochs + 1), train_accuracies, label='Training Accuracy', marker='o')
    plt.plot(range(1, epochs + 1), val_accuracies, label='Validation Accuracy', marker='s')
    plt.title('Training vs Validation Accuracy')
    plt.xlabel('Epoch')
    plt.ylabel('Accuracy (%)')
    plt.legend()
    plt.grid(True)

    plt.tight_layout()
    plt.show()
```

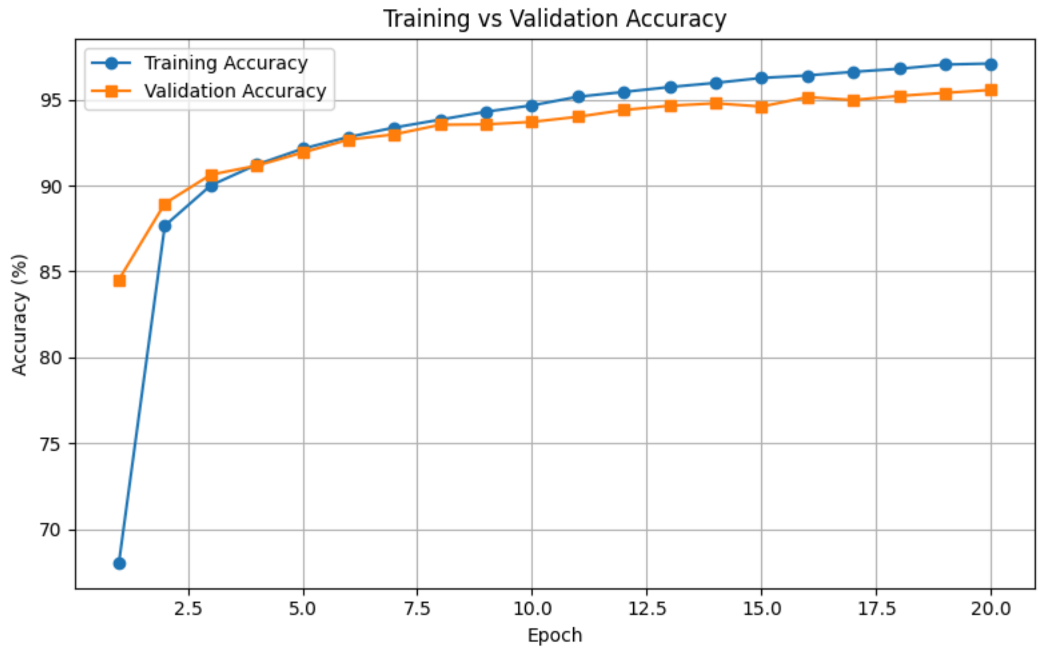
```
/tmp/ipython-input-2871887219.py:12: FutureWarning: `nn.init.kaiming_normal` is now deprecated in favor of `nn.init.kaiming_no
  nn.init.kaiming_normal(layer.weight, nonlinearity='relu')
Epoch 1/20, Train Loss: 2.1954, Val Loss: 2.0337
Epoch 2/20, Train Loss: 1.8600, Val Loss: 1.6723
Epoch 3/20, Train Loss: 1.4794, Val Loss: 1.2971
Epoch 4/20, Train Loss: 1.1478, Val Loss: 1.0164
Epoch 5/20, Train Loss: 0.9170, Val Loss: 0.8317
Epoch 6/20, Train Loss: 0.7667, Val Loss: 0.7128
Epoch 7/20, Train Loss: 0.6671, Val Loss: 0.6321
Epoch 8/20, Train Loss: 0.5972, Val Loss: 0.5747
Epoch 9/20, Train Loss: 0.5463, Val Loss: 0.5325
Epoch 10/20, Train Loss: 0.5075, Val Loss: 0.4993
Epoch 11/20, Train Loss: 0.4771, Val Loss: 0.4745
Epoch 12/20, Train Loss: 0.4524, Val Loss: 0.4514
Epoch 13/20, Train Loss: 0.4320, Val Loss: 0.4342
Epoch 14/20, Train Loss: 0.4148, Val Loss: 0.4188
Epoch 15/20, Train Loss: 0.3999, Val Loss: 0.4066
Epoch 16/20, Train Loss: 0.3871, Val Loss: 0.3941
Epoch 17/20, Train Loss: 0.3757, Val Loss: 0.3846
Epoch 18/20, Train Loss: 0.3655, Val Loss: 0.3754
Epoch 19/20, Train Loss: 0.3563, Val Loss: 0.3670
Epoch 20/20, Train Loss: 0.3480, Val Loss: 0.3599

Finished run lr=0.001
Train Loss final: 0.3480, Val Loss final: 0.3599
Train Acc final: 90.15%, Val Acc final: 89.77%
Did not reach 90% validation accuracy
✅ Training appears stable
```



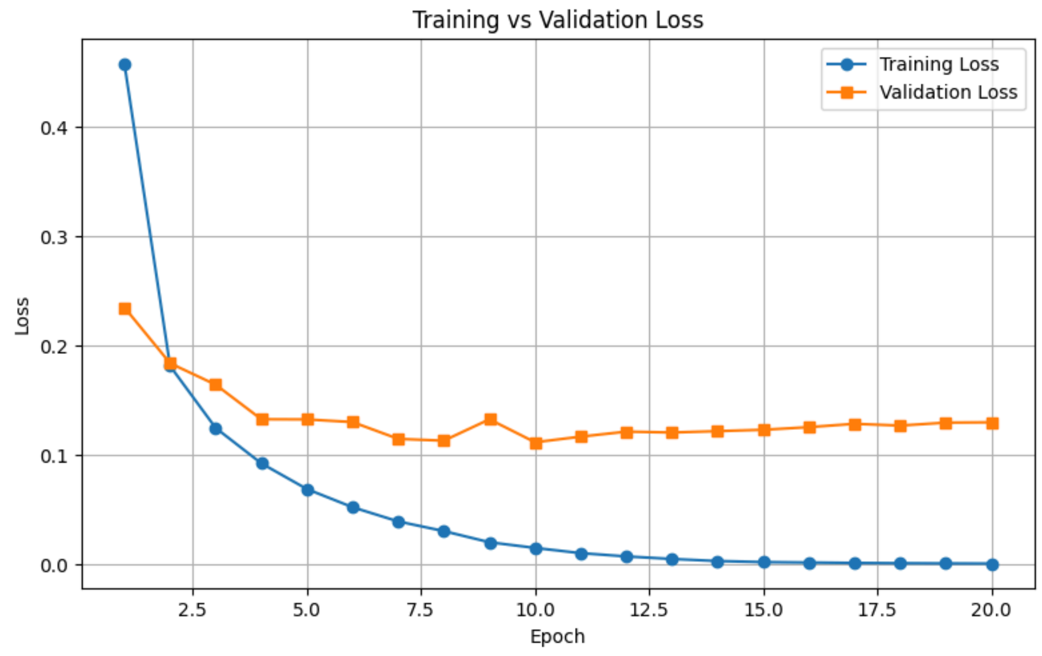


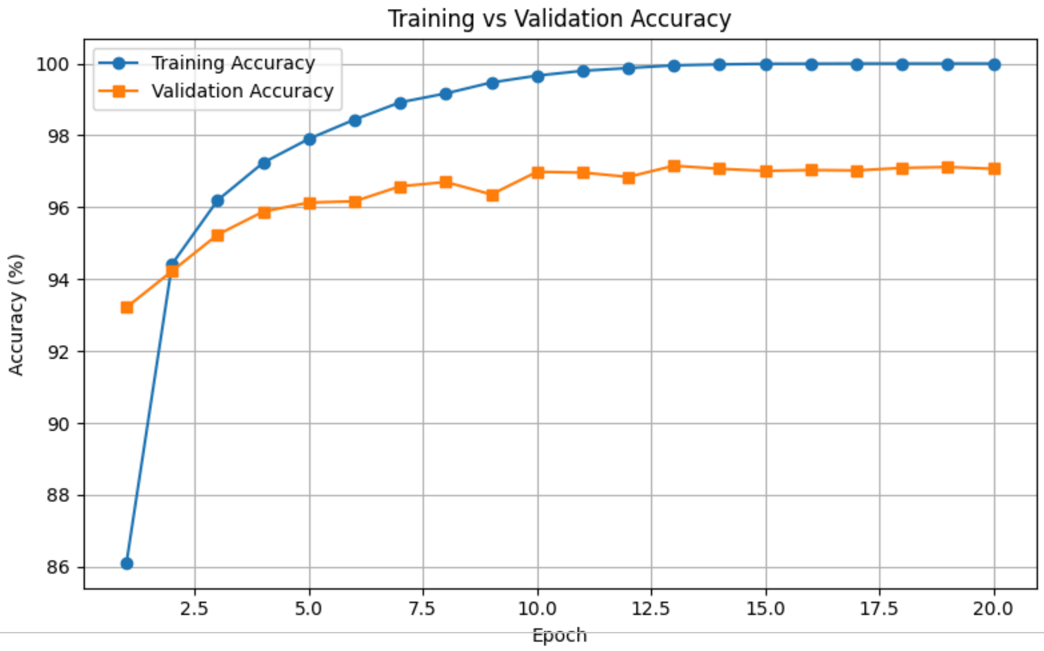
```
Epoch 1/20, Train Loss: 1.2207, Val Loss: 0.5892
Epoch 2/20, Train Loss: 0.4572, Val Loss: 0.3945
Epoch 3/20, Train Loss: 0.3500, Val Loss: 0.3359
Epoch 4/20, Train Loss: 0.3016, Val Loss: 0.3101
Epoch 5/20, Train Loss: 0.2695, Val Loss: 0.2842
Epoch 6/20, Train Loss: 0.2453, Val Loss: 0.2594
Epoch 7/20, Train Loss: 0.2267, Val Loss: 0.2495
Epoch 8/20, Train Loss: 0.2094, Val Loss: 0.2317
Epoch 9/20, Train Loss: 0.1949, Val Loss: 0.2284
Epoch 10/20, Train Loss: 0.1824, Val Loss: 0.2165
Epoch 11/20, Train Loss: 0.1711, Val Loss: 0.2065
Epoch 12/20, Train Loss: 0.1609, Val Loss: 0.1976
Epoch 13/20, Train Loss: 0.1511, Val Loss: 0.1901
Epoch 14/20, Train Loss: 0.1429, Val Loss: 0.1866
Epoch 15/20, Train Loss: 0.1350, Val Loss: 0.1830
Epoch 16/20, Train Loss: 0.1286, Val Loss: 0.1756
Epoch 17/20, Train Loss: 0.1217, Val Loss: 0.1755
Epoch 18/20, Train Loss: 0.1161, Val Loss: 0.1686
Epoch 19/20, Train Loss: 0.1101, Val Loss: 0.1633
Epoch 20/20, Train Loss: 0.1050, Val Loss: 0.1600

Finished run lr=0.01
Train Loss final: 0.1050, Val Loss final: 0.1600
Train Acc final: 97.10%, Val Acc final: 95.56%
Converged to 90% validation accuracy at epoch 3
✅ Training appears stable
```





```
Epoch 1/20, Train Loss: 0.4577, Val Loss: 0.2352
Epoch 2/20, Train Loss: 0.1817, Val Loss: 0.1843
Epoch 3/20, Train Loss: 0.1251, Val Loss: 0.1646
Epoch 4/20, Train Loss: 0.0927, Val Loss: 0.1331
Epoch 5/20, Train Loss: 0.0692, Val Loss: 0.1329
Epoch 6/20, Train Loss: 0.0526, Val Loss: 0.1305
Epoch 7/20, Train Loss: 0.0397, Val Loss: 0.1151
Epoch 8/20, Train Loss: 0.0308, Val Loss: 0.1135
Epoch 9/20, Train Loss: 0.0206, Val Loss: 0.1330
Epoch 10/20, Train Loss: 0.0154, Val Loss: 0.1120
Epoch 11/20, Train Loss: 0.0107, Val Loss: 0.1172
Epoch 12/20, Train Loss: 0.0077, Val Loss: 0.1218
Epoch 13/20, Train Loss: 0.0054, Val Loss: 0.1210
Epoch 14/20, Train Loss: 0.0035, Val Loss: 0.1222
Epoch 15/20, Train Loss: 0.0025, Val Loss: 0.1234
Epoch 16/20, Train Loss: 0.0021, Val Loss: 0.1258
Epoch 17/20, Train Loss: 0.0017, Val Loss: 0.1289
Epoch 18/20, Train Loss: 0.0015, Val Loss: 0.1273
Epoch 19/20, Train Loss: 0.0013, Val Loss: 0.1300
Epoch 20/20, Train Loss: 0.0012, Val Loss: 0.1303

Finished run lr=0.1
Train Loss final: 0.0012, Val Loss final: 0.1303
Train Acc final: 100.00%, Val Acc final: 97.07%
Converged to 90% validation accuracy at epoch 1
✅ Training appears stable
```





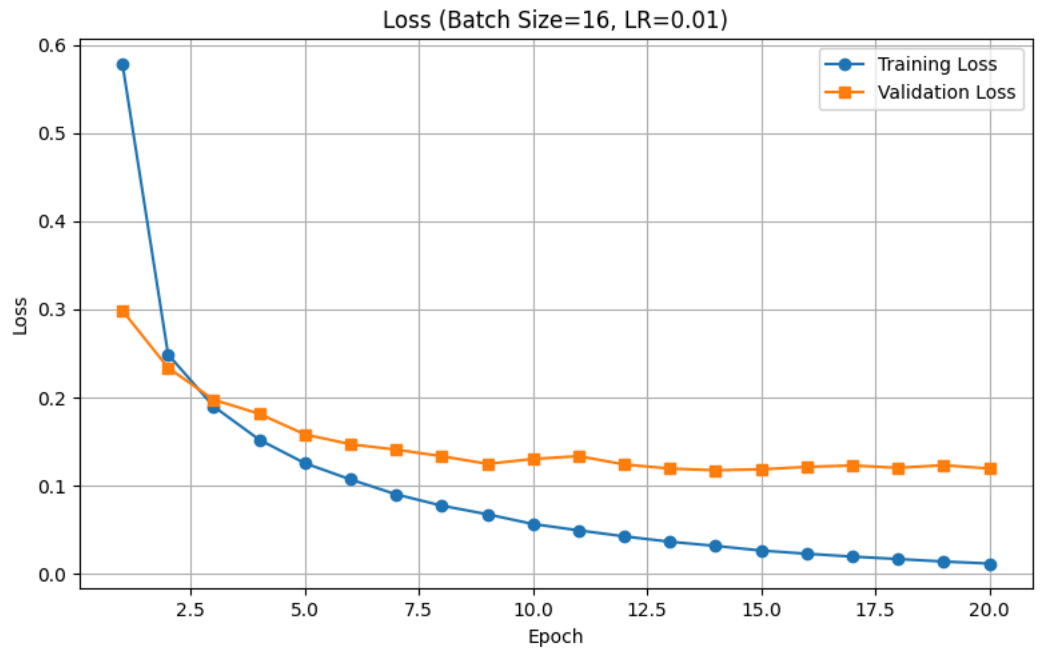
```
Epoch 1/20, Train Loss: 2.3151, Val Loss: 2.3033
Epoch 2/20, Train Loss: 2.3057, Val Loss: 2.3036
Epoch 3/20, Train Loss: 2.3052, Val Loss: 2.3057
Epoch 4/20, Train Loss: 2.3056, Val Loss: 2.3032
Epoch 5/20, Train Loss: 2.3052, Val Loss: 2.3073
Epoch 6/20, Train Loss: 2.3055, Val Loss: 2.3044
Epoch 7/20, Train Loss: 2.3049, Val Loss: 2.3063
Epoch 8/20, Train Loss: 2.3050, Val Loss: 2.3038
Epoch 9/20, Train Loss: 2.3050, Val Loss: 2.3022
Epoch 10/20, Train Loss: 2.3049, Val Loss: 2.3049
Epoch 11/20, Train Loss: 2.3051, Val Loss: 2.3041
Epoch 12/20, Train Loss: 2.3050, Val Loss: 2.3058
Epoch 13/20, Train Loss: 2.3049, Val Loss: 2.3079
Epoch 14/20, Train Loss: 2.3050, Val Loss: 2.3052
```

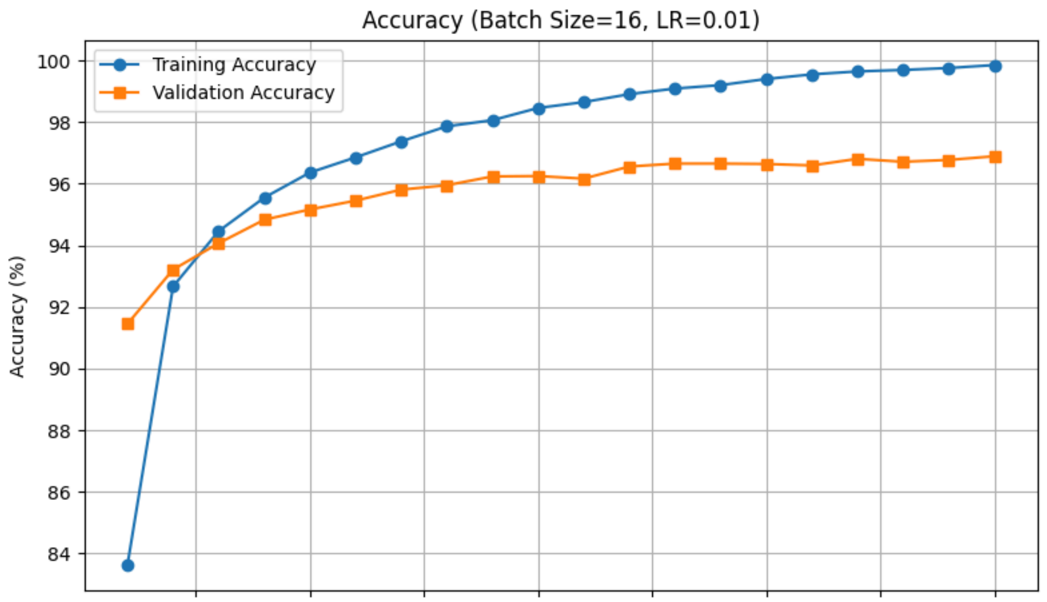
After analyzing the effect of different learning rates on model performance, the learning rate of 0.1 achieved the best overall accuracy (97.1%) while also providing the balance between convergence speed and stability. It enabled the model to converge significantly faster than smaller rates (0.01 or 0.001), which showed slower yet smoother progress, and remained stable compared to higher rates (1.0), which caused oscillations or divergence.

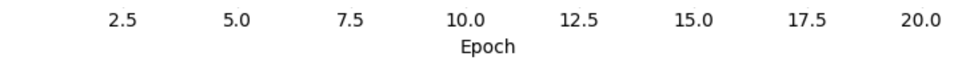
Batch size analysis

```
import time

batch_sizes = [16, 32, 64, 128]
lr = 0.01
epochs = 20

summary = []

for batch_size in batch_sizes:
    print(f"\nTraining with batch_size={batch_size}")
    print("=============================")

    nn_model = NeuralNetwork(input_size, h1_size, h2_size, h3_size, out_size)
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    nn_model.to(device)
    optimizer = torch.optim.SGD(nn_model.parameters(), lr=lr)
    criteration = torch.nn.CrossEntropyLoss()

    start_time = time.time()
    train_losses, val_losses, train_accuracies, val_accuracies, train_acc_mean, val_acc_mean, train_acc_std, val_acc_std, loss_
        nn_model, epochs=epochs, batch_size=batch_size
    )
    elapsed_time = time.time() - start_time

    print(f"\nFinished run with batch_size={batch_size}, lr={lr}")
    print(f"Final Train Loss: {train_losses[-1]:.4f}, Val Loss: {val_losses[-1]:.4f}")
    print(f"Final Train Acc: {train_accuracies[-1]:.2f}%, Val Acc: {val_accuracies[-1]:.2f}%")
    print(f"Time taken: {elapsed_time:.2f} seconds")

    summary.append({
        "batch_size": batch_size,
        "val_acc": val_accuracies[-1],
        "time_sec": elapsed_time
    })

    plt.figure(figsize=(8, 10))

    plt.subplot(2, 1, 1)
    plt.plot(range(1, epochs + 1), train_losses, label='Training Loss', marker='o')
    plt.plot(range(1, epochs + 1), val_losses, label='Validation Loss', marker='s')
    plt.title(f'Loss (Batch Size={batch_size}, LR={lr})')
    plt.xlabel('Epoch')
    plt.ylabel('Loss')
    plt.legend()
    plt.grid(True)

    plt.subplot(2, 1, 2)
    plt.plot(range(1, epochs + 1), train_accuracies, label='Training Accuracy', marker='o')
    plt.plot(range(1, epochs + 1), val_accuracies, label='Validation Accuracy', marker='s')
    plt.title(f'Accuracy (Batch Size={batch_size}, LR={lr})')
    plt.xlabel('Epoch')
    plt.ylabel('Accuracy (%)')
    plt.legend()
    plt.grid(True)

    plt.tight_layout()
    plt.show()

print("\n=== Summary: Batch Size Comparison ===")
for s in summary:
    print(f"Batch Size={s['batch_size']:>3} | Val Acc={s['val_acc']:.2f}% | Time={s['time_sec']:.2f}s")
```

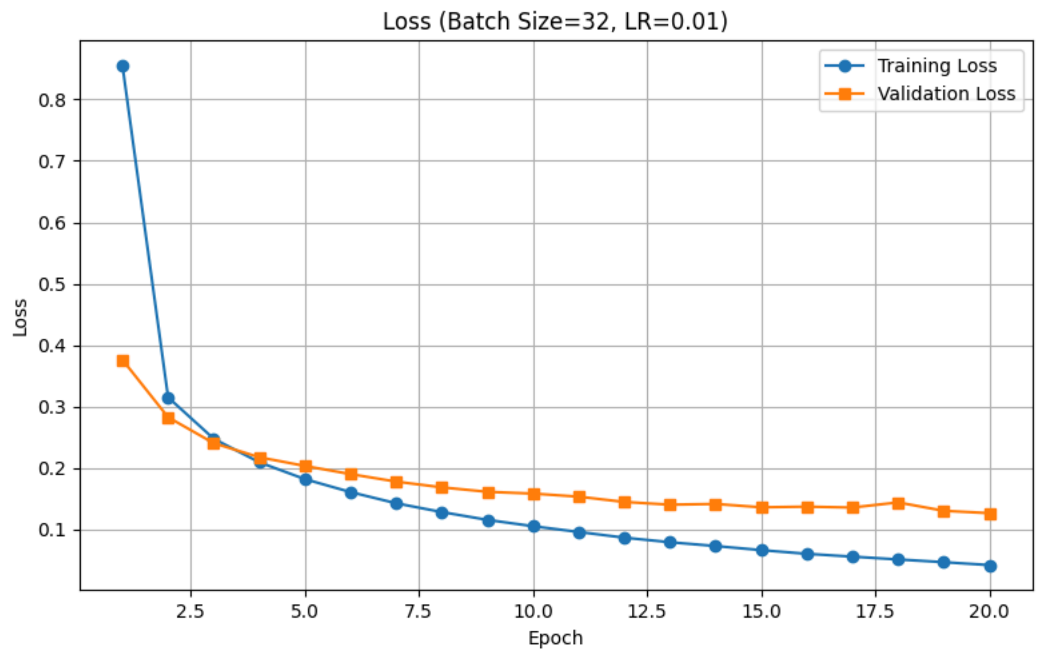
```
Training with batch_size=16
==============================
/tmp/ipython-input-2871887219.py:12: FutureWarning: `nn.init.kaiming_normal` is now deprecated in favor of `nn.init.kaiming_no
  nn.init.kaiming_normal(layer.weight, nonlinearity='relu')
Epoch 1/20, Train Loss: 0.5781, Val Loss: 0.2986
Epoch 2/20, Train Loss: 0.2489, Val Loss: 0.2338
Epoch 3/20, Train Loss: 0.1900, Val Loss: 0.1976
Epoch 4/20, Train Loss: 0.1523, Val Loss: 0.1817
Epoch 5/20, Train Loss: 0.1256, Val Loss: 0.1583
Epoch 6/20, Train Loss: 0.1072, Val Loss: 0.1472
Epoch 7/20, Train Loss: 0.0903, Val Loss: 0.1410
Epoch 8/20, Train Loss: 0.0775, Val Loss: 0.1336
Epoch 9/20, Train Loss: 0.0678, Val Loss: 0.1249
Epoch 10/20, Train Loss: 0.0567, Val Loss: 0.1304
Epoch 11/20, Train Loss: 0.0495, Val Loss: 0.1337
Epoch 12/20, Train Loss: 0.0427, Val Loss: 0.1243
Epoch 13/20, Train Loss: 0.0366, Val Loss: 0.1195
Epoch 14/20, Train Loss: 0.0319, Val Loss: 0.1177
Epoch 15/20, Train Loss: 0.0267, Val Loss: 0.1187
Epoch 16/20, Train Loss: 0.0230, Val Loss: 0.1215
Epoch 17/20, Train Loss: 0.0198, Val Loss: 0.1231
Epoch 18/20, Train Loss: 0.0170, Val Loss: 0.1204
Epoch 19/20, Train Loss: 0.0142, Val Loss: 0.1234
Epoch 20/20, Train Loss: 0.0119, Val Loss: 0.1196

Finished run with batch_size=16, lr=0.01
Final Train Loss: 0.0119, Val Loss: 0.1196
Final Train Acc: 99.85%, Val Acc: 96.89%
Time taken: 62.17 seconds
```

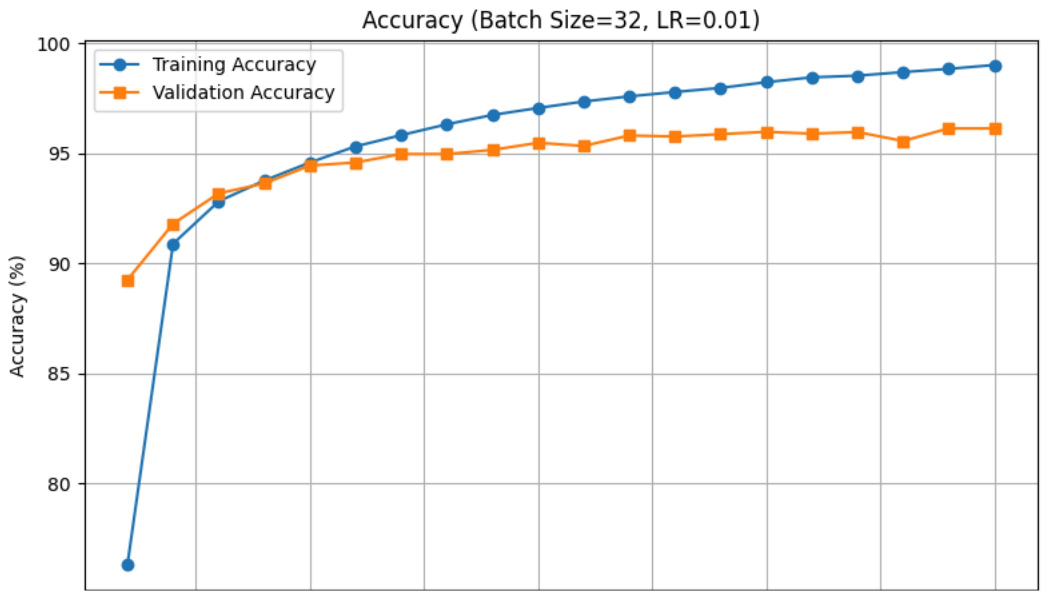


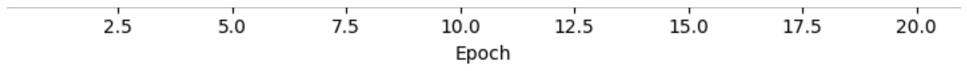




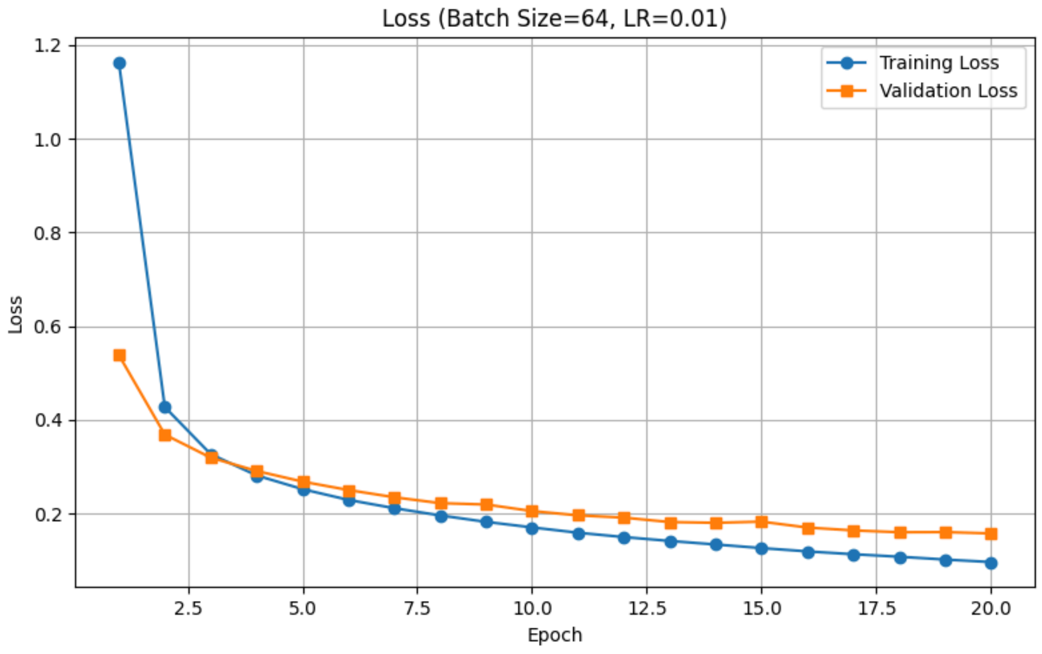
```
Training with batch_size=32
==============================
Epoch 1/20, Train Loss: 0.8550, Val Loss: 0.3763
Epoch 2/20, Train Loss: 0.3152, Val Loss: 0.2829
Epoch 3/20, Train Loss: 0.2479, Val Loss: 0.2407
Epoch 4/20, Train Loss: 0.2096, Val Loss: 0.2178
Epoch 5/20, Train Loss: 0.1821, Val Loss: 0.2036
Epoch 6/20, Train Loss: 0.1610, Val Loss: 0.1903
Epoch 7/20, Train Loss: 0.1428, Val Loss: 0.1781
Epoch 8/20, Train Loss: 0.1286, Val Loss: 0.1687
Epoch 9/20, Train Loss: 0.1158, Val Loss: 0.1615
Epoch 10/20, Train Loss: 0.1056, Val Loss: 0.1586
Epoch 11/20, Train Loss: 0.0962, Val Loss: 0.1536
Epoch 12/20, Train Loss: 0.0869, Val Loss: 0.1450
Epoch 13/20, Train Loss: 0.0795, Val Loss: 0.1406
Epoch 14/20, Train Loss: 0.0733, Val Loss: 0.1417
Epoch 15/20, Train Loss: 0.0667, Val Loss: 0.1363
Epoch 16/20, Train Loss: 0.0606, Val Loss: 0.1374
Epoch 17/20, Train Loss: 0.0560, Val Loss: 0.1359
Epoch 18/20, Train Loss: 0.0515, Val Loss: 0.1443
Epoch 19/20, Train Loss: 0.0470, Val Loss: 0.1306
Epoch 20/20, Train Loss: 0.0424, Val Loss: 0.1268

Finished run with batch_size=32, lr=0.01
Final Train Loss: 0.0424, Val Loss: 0.1268
Final Train Acc: 99.02%, Val Acc: 96.14%
Time taken: 34.37 seconds
```

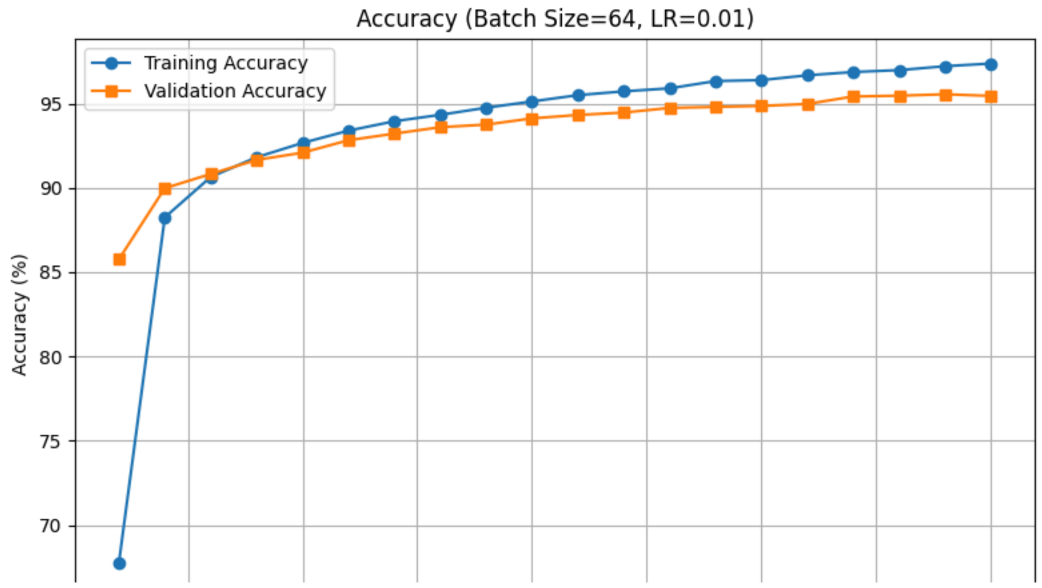






```
Training with batch_size=64
==============================
Epoch 1/20, Train Loss: 1.1626, Val Loss: 0.5382
Epoch 2/20, Train Loss: 0.4274, Val Loss: 0.3686
Epoch 3/20, Train Loss: 0.3270, Val Loss: 0.3195
Epoch 4/20, Train Loss: 0.2815, Val Loss: 0.2910
Epoch 5/20, Train Loss: 0.2522, Val Loss: 0.2683
Epoch 6/20, Train Loss: 0.2292, Val Loss: 0.2503
Epoch 7/20, Train Loss: 0.2115, Val Loss: 0.2349
Epoch 8/20, Train Loss: 0.1961, Val Loss: 0.2223
Epoch 9/20, Train Loss: 0.1823, Val Loss: 0.2194
Epoch 10/20, Train Loss: 0.1706, Val Loss: 0.2054
Epoch 11/20, Train Loss: 0.1592, Val Loss: 0.1962
Epoch 12/20, Train Loss: 0.1499, Val Loss: 0.1914
Epoch 13/20, Train Loss: 0.1416, Val Loss: 0.1819
Epoch 14/20, Train Loss: 0.1341, Val Loss: 0.1803
Epoch 15/20, Train Loss: 0.1264, Val Loss: 0.1828
Epoch 16/20, Train Loss: 0.1193, Val Loss: 0.1702
Epoch 17/20, Train Loss: 0.1133, Val Loss: 0.1641
Epoch 18/20, Train Loss: 0.1079, Val Loss: 0.1602
Epoch 19/20, Train Loss: 0.1019, Val Loss: 0.1605
Epoch 20/20, Train Loss: 0.0966, Val Loss: 0.1577

Finished run with batch_size=64, lr=0.01
Final Train Loss: 0.0966, Val Loss: 0.1577
Final Train Acc: 97.37%, Val Acc: 95.45%
Time taken: 20.27 seconds
```


Loss (Batch Size=64, LR=0.01)
Training Loss
Validation Loss
Loss
Epoch


Accuracy (Batch Size=64, LR=0.01)
Training Accuracy
Validation Accuracy
Accuracy (%)