Linear classifiers: prediction equations

LINEAR CLASSIFIERS IN PYTHON

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Dot Products

```
x = np.arange(3)
x
```

```
array([0, 1, 2])
```

```
y = np.arange(3,6)
y
```

```
array([3, 4, 5])
```

```
x*y
```

```
array([0, 4, 10])
```

```
np.sum(x*y)
```

```
14
```

```
x@y
```

```
14
```

- `x@y` is called the dot product of `x` and `y`, and is written $x \cdot y$.

Linear classifier prediction

- $\text{raw model output} = \text{coefficients} \cdot \text{features} + \text{intercept}$
- Linear classifier prediction: compute raw model output, check the **sign**
  - if positive, predict one class
  - if negative, predict the other class
- This is the same for logistic regression and linear SVM
  - `fit` is different but `predict` is the same

How LogisticRegression makes predictions

$$\text{raw model output} = \text{coefficients} \cdot \text{features} + \text{intercept}$$

```
lr = LogisticRegression()

lr.fit(X,y)

lr.predict(X)[10]
```

```
0
```

```
lr.predict(X)[20]
```

```
1
```

How LogisticRegression makes predictions (cont.)

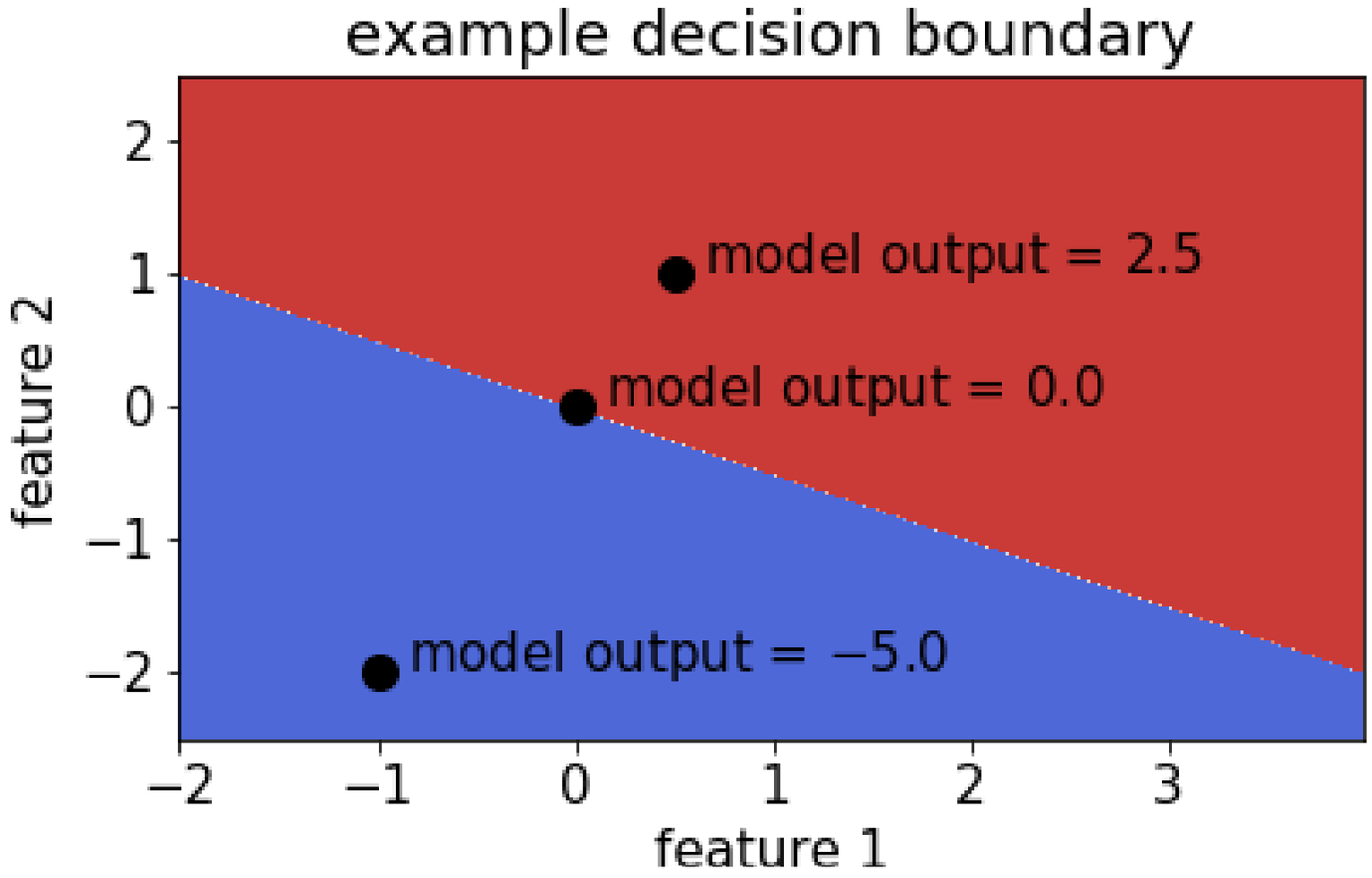
```
lr.coef_ @ X[10] + lr.intercept_ # raw model output
```

```
array([-33.78572166])
```

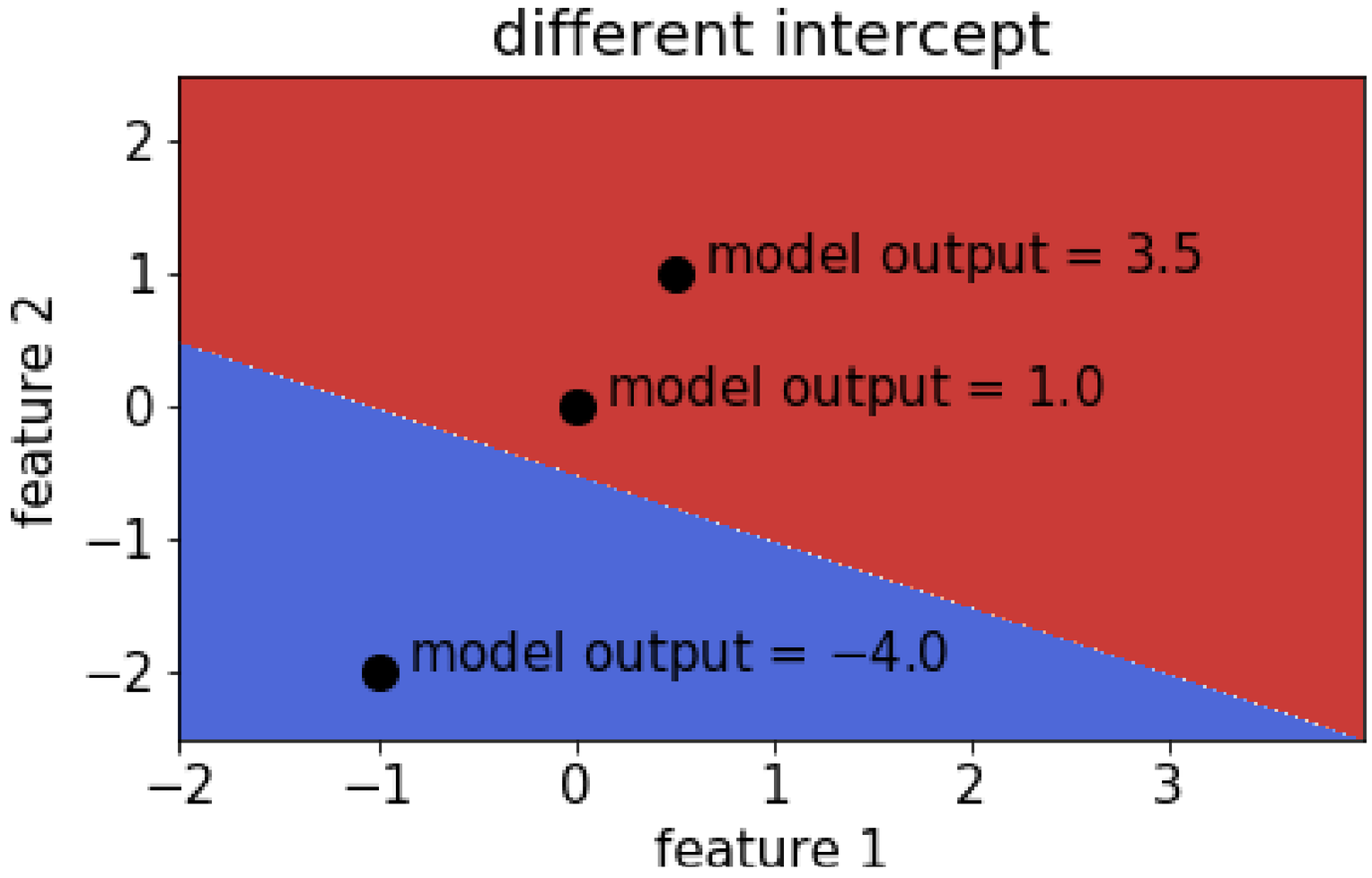
```
lr.coef_ @ X[20] + lr.intercept_ # raw model output
```

```
array([ 0.08050621])
```

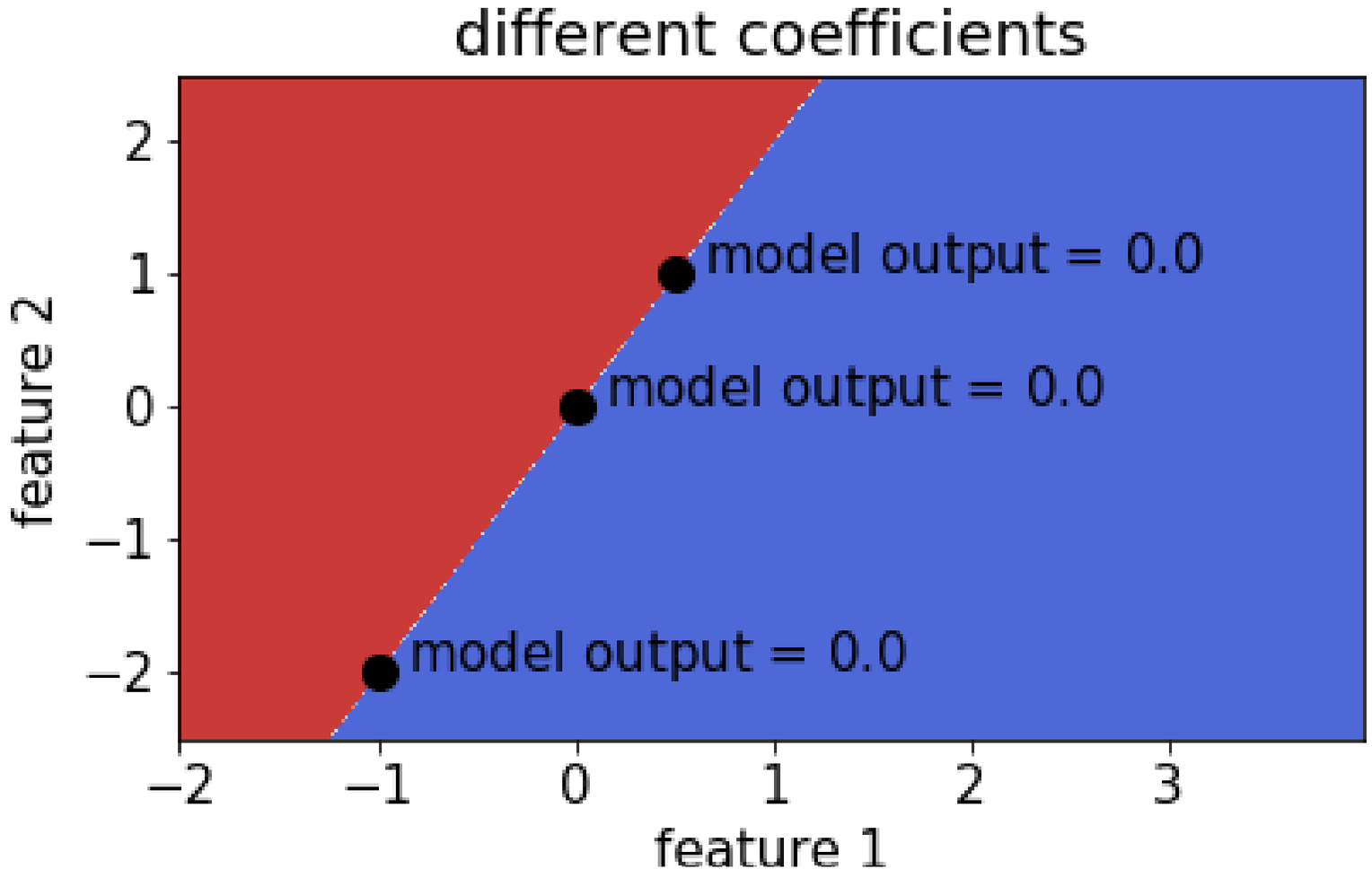
The raw model output



The raw model output



The raw model output



Let's practice!

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What is a loss function?

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Least squares: the squared loss

- scikit-learn's `LinearRegression` minimizes a loss:

$$\sum_{i=1}^{n}(\text{true } i\text{th target value} - \text{predicted } i\text{th target value})^2$$

- Minimization is with respect to coefficients or parameters of the model.
- Note that in scikit-learn `model.score()` isn't necessarily the loss function.

Classification errors: the 0-1 loss

- Squared loss not appropriate for classification problems (more on this later).
- A natural loss for classification problem is the number of errors.
- This is the **0-1 loss**: it's 0 for a correct prediction and 1 for an incorrect prediction.
- But this loss is hard to minimize!

Minimizing a loss

```
from scipy.optimize import minimize
```

```
minimize(np.square, 0).x
```

```
array([0.])
```

```
minimize(np.square, 2).x
```

```
array([-1.88846401e-08])
```

Let's practice!

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Loss function diagrams

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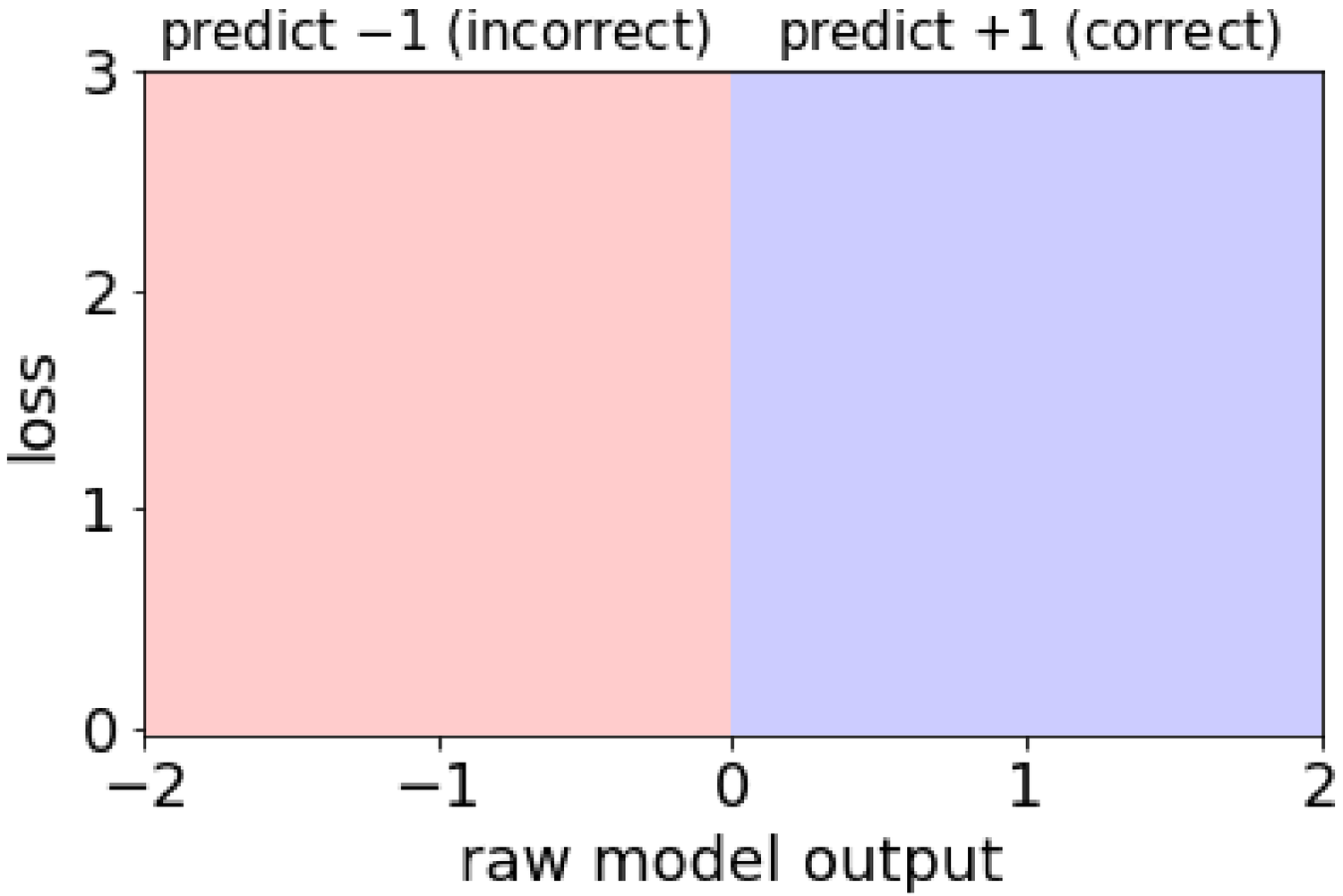
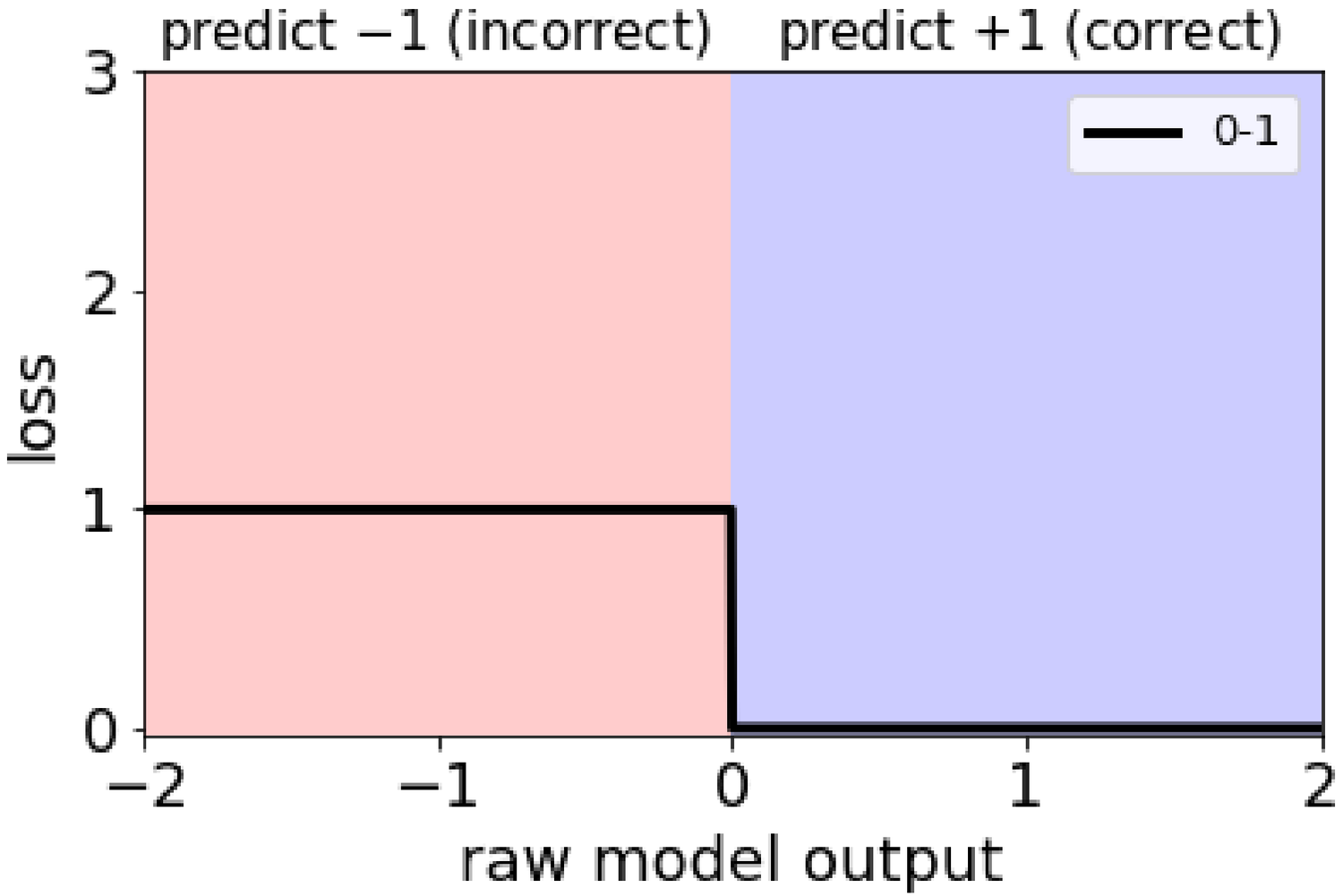
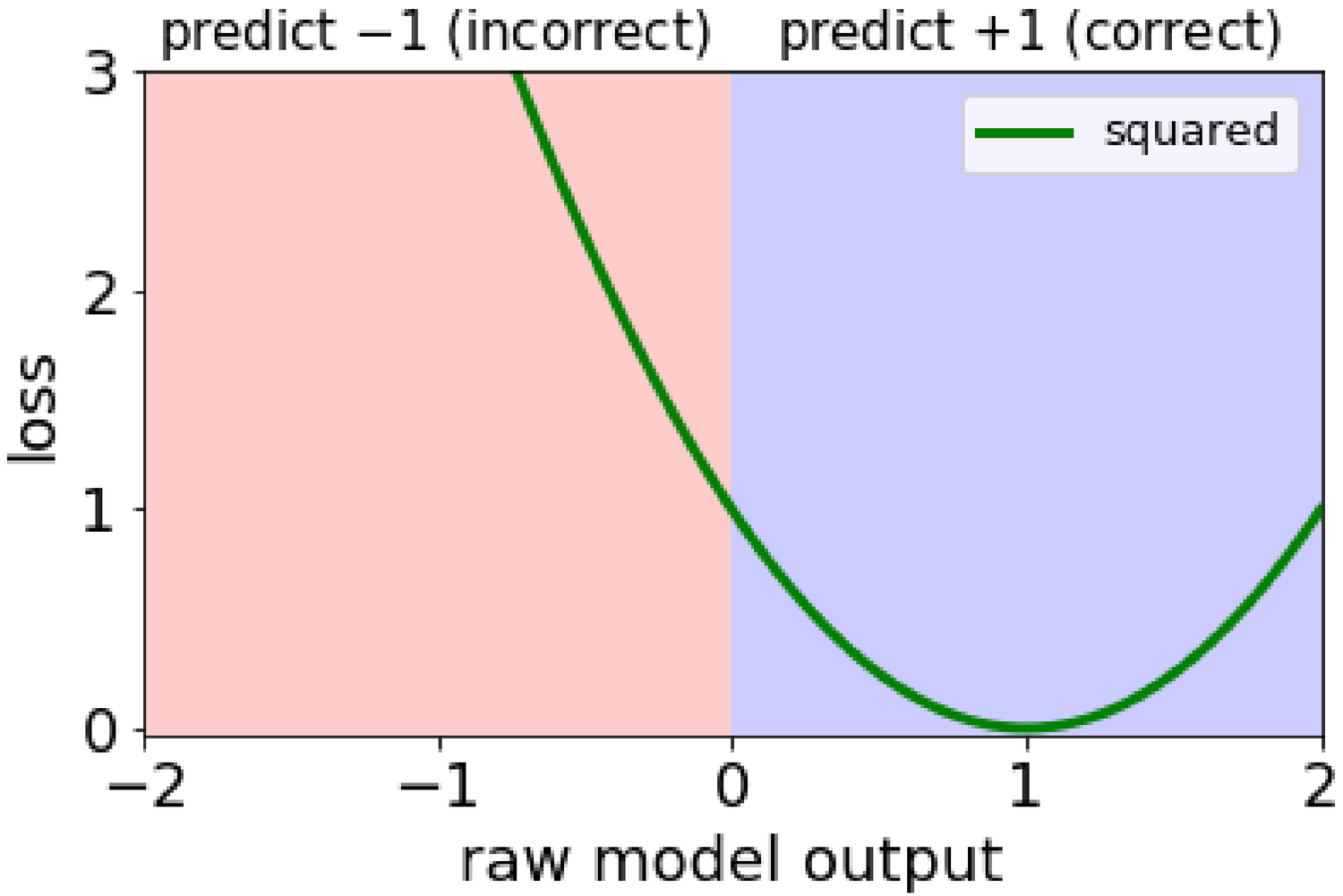
The raw model output



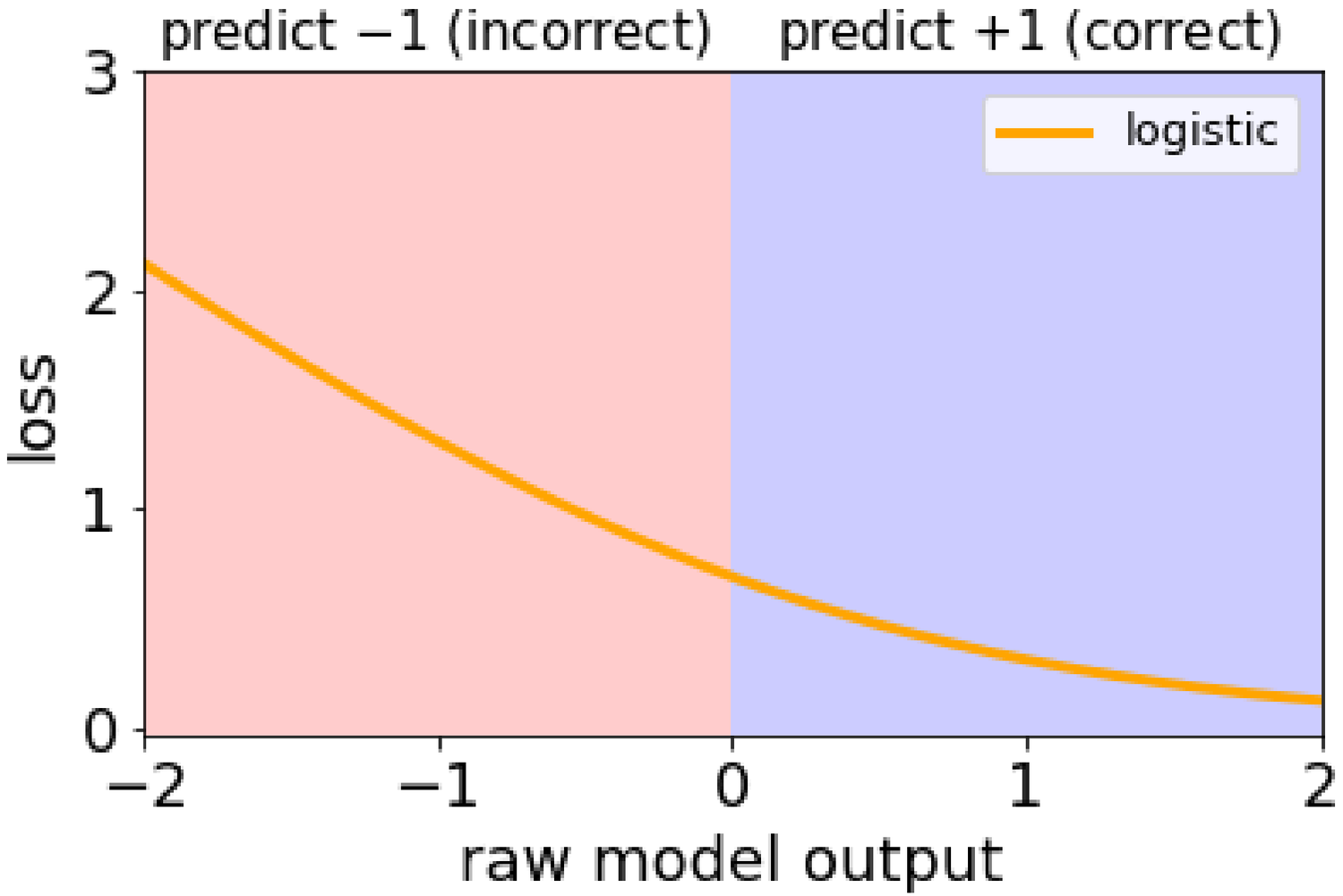
0-1 loss diagram



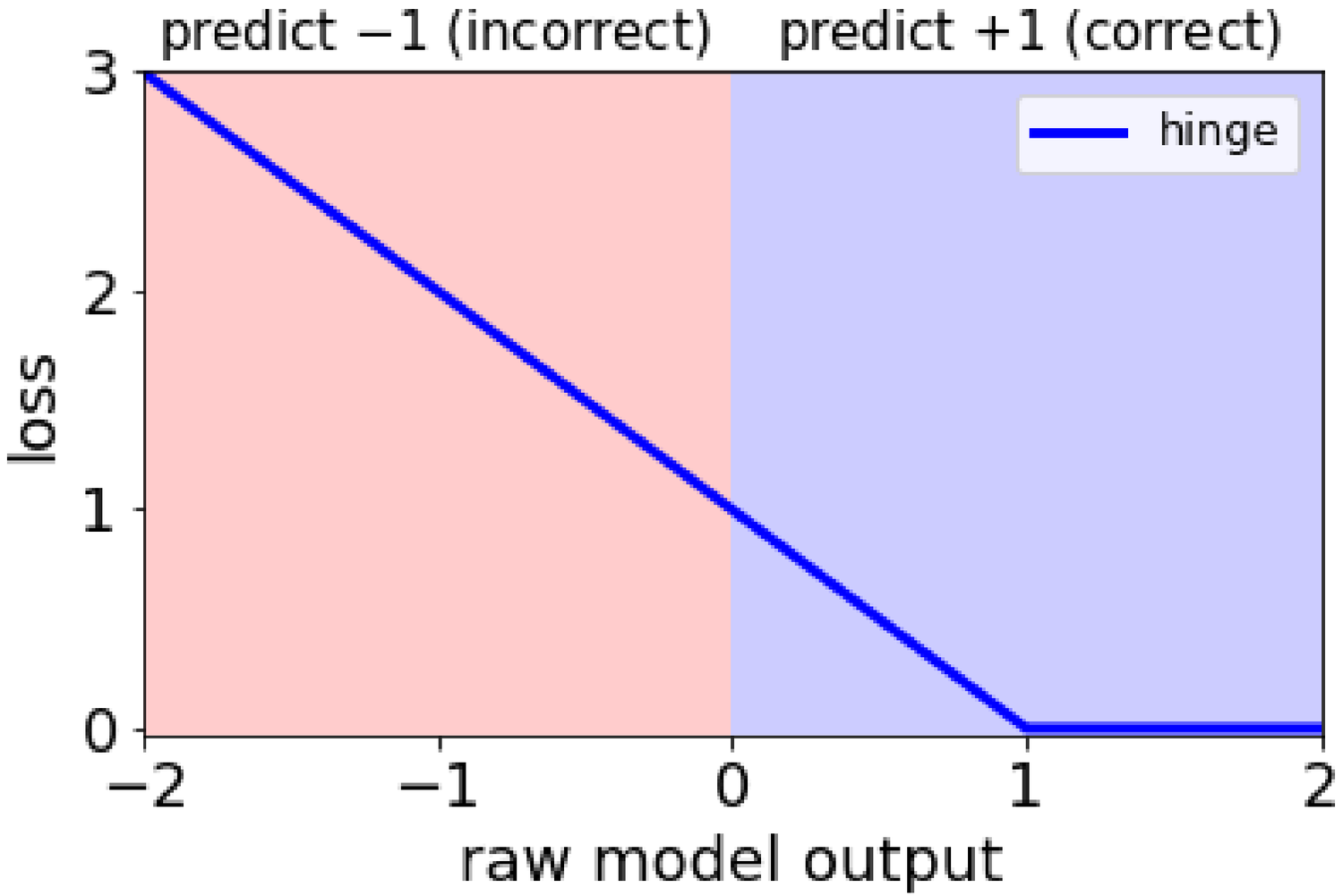
Linear regression loss diagram



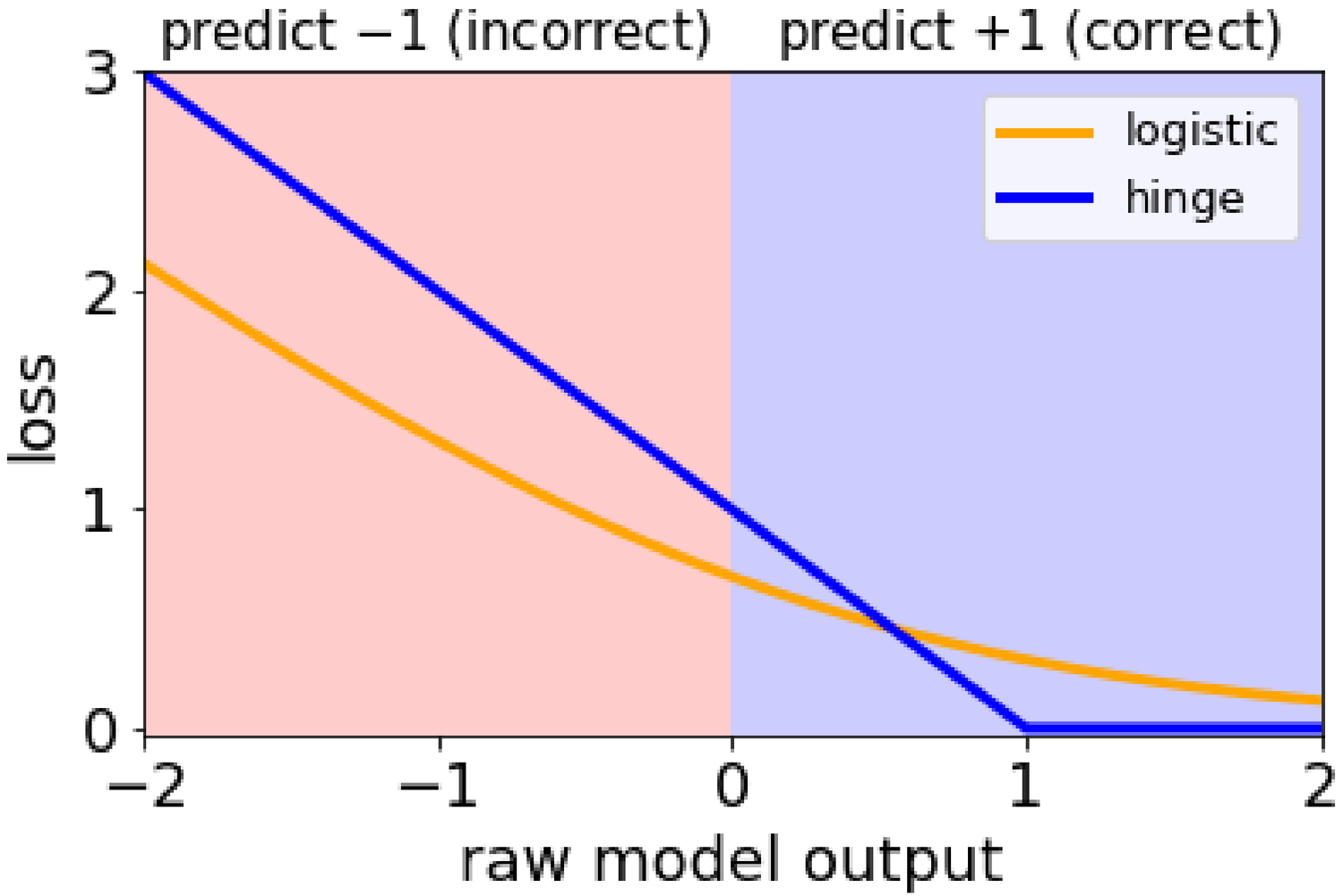
Logistic loss diagram



Hinge loss diagram



Hinge loss diagram



Let's practice!

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