Introduction

14 September 2020 20:03

Amortized Analysis

- Seq of operations

↓ time

average over all operations

- Probability is not involved

- "average performance of each operation in worst case"

1. Aggregate analysis

seq. of 'n'

T(n) - Total time complexity 'n' operations worst case

amortized cost per = $\frac{T(n)}{}$

operations) n

Stack operations

Push(S, 'x')

pop(S)

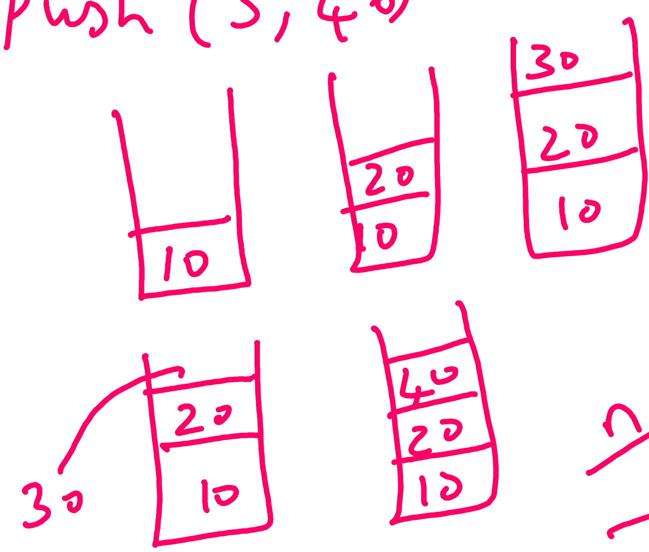
Actual Cost of

push - $O(1)$

pop - $O(1)$

Seq of operations

Push(S, 10), Push(S, 20)
Push(S, 30), Pop(S),
Push(S, 40)



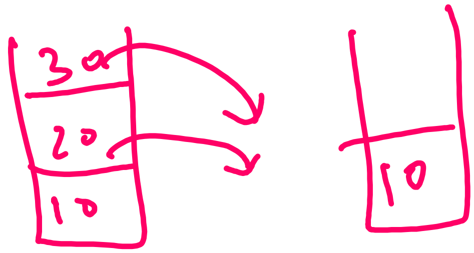
n = 5
'n' operations

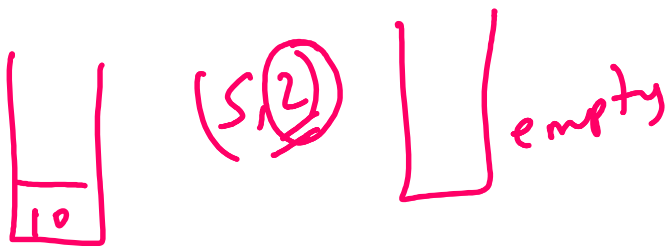
Total cost of push & pop $\} = \underline{\underline{O(n)}}$

3) multipop (S, 'k')

k - elements are popped

multipop (S, 2)





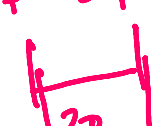
multipop(S, k) = min(s, k)

s

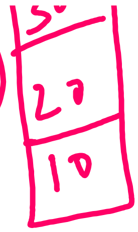
no. of elements in stack

Total cost of multipop 'single' = min(s, k)

Worst case total cost of multipop = O(s)



(S, 2)



~~$s = 3$~~

$s = n$

multipop(s, ...

$= 2$

multipop(s, 3) $= 3$

$= \boxed{O(n)}$

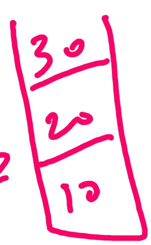
$\overset{O(n)}{\text{multipop}(s, k)}$, $\overset{O(n)}{\text{multipop}(s, k)}$ ..... n operations

Total cost of 'n' seq of multipop $= O(n^2)$

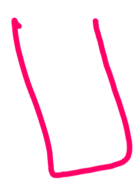
$= n \times O(n)$

$= O(n^2)$? Not tight

$n = 3$



multipop(s, 3)



multipop(,) ×

Closest upper bound $= O(n)$

Total cost $\left.\begin{matrix}\text{Seq of operations}\\ \text{push, pop, multipop}\end{matrix}\right\} = O(n)$

$T(n) = O(n)$

$\left.\begin{matrix}\text{Amortized}\\ \text{cost of}\\ \text{push or pop}\\ \text{or multipop}\end{matrix}\right\} = \dfrac{O(n)}{n} = O(1)$

2) Accounting Method

Seq. operations

Operation → cost is assigned.

'Charge'

'Credit'

more credit

less credit

amortized cost

'credit

'more cre...

'less' credit

Stack operation

C_i - Actual cost of i^{th} operation

$\hat{C}_i$ - Amortized cost of i^{th} operation

$$\sum_{i=1}^{n} \hat{C}_i \geq \sum_{i=1}^{n} C_i$$

Amor cost Act. cost

Total credits $= \sum_{i=1}^{n} \hat{C}_i - \sum_{i=1}^{n} C_i$

≥ 0

Stack	Actual
Push	$O(1)$
Pop	$O(1)$
MultiPop	$\min(s, k)$

$s = n$

$O(n)$

Multipop min(s,k)

Amortized cost

push	2
pop	0
multipop	0

constant

following seq

		Amo. cost	Act. cost	Credit
1	push(S,10)	2	1	2-1=1
2	Push(S,20)	2	1	4-2=2
3	Push(S,30)	2	1	6-3=3
4	Pop(S)	0	1	3-1=2
5	Pop(S)	0	1	2-1=1
6	multipop(S,3)	0	min(1,3) = 1	1-1=0
		6=n	6=n	

n = 6

Total amortized cost = O(n)

= Total act. cost