# M way tree time complexity

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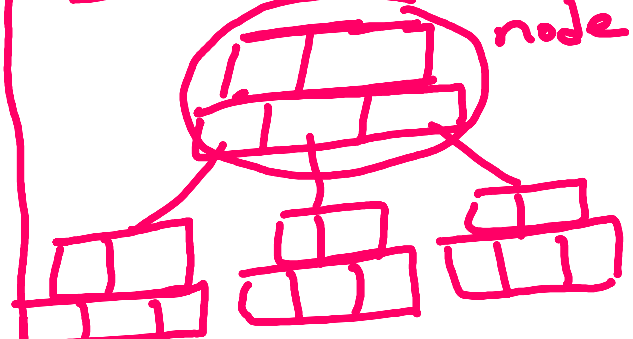
Time Complexity of Search, insert & delete operations in M-way search tree is $O(h)$. Where '$h$' is height of M-way search tree.

## To find lower bound of '$h$'

Let '$n$' be the total no. of keys in '$M$' way search tree.
Maximum number of keys that can be stored in M-way search tree is found as follows.

| Level | No. of nodes |
|---|---|
| 1 | 1 |
| 2 | $m$ |
| 3 | $m^2$ |
| ⋮ | |
| | $h-1$ |

Example $m=3$



$h$ $m$ $1$

$\therefore$ Total maximum nodes $= 1 + m + m^2 + \cdots + m^{h-1}$

$$= \frac{m^h - 1}{m - 1}$$

Total maximum keys $= \frac{m^h - 1}{\cancel{m-1}} \times \cancel{m-1}$ (each node $m-1$ keys)

$$n = m^h - 1$$

$$\Rightarrow m^h = n + 1$$

$$\Rightarrow h \log_m m = \log_m n+1 \text{ (Taking log)}$$

$$\Rightarrow h = \log_m n+1$$

$\therefore$ Best Case time complexity

$$h = O(\log_m n+1)$$

<u>To find upper bound of h</u>

Minimum no. of keys that can be stored in M-way

can be ...
Search tree is found as follows.

In each level minimum one node.

| Level | No. of nodes |
|---|---|
| 1 | 1 |
| 2 | 1 |
| $\vdots$ | |
| h | 1 |

$\therefore$ Total no. of nodes $\left.\right\} = 1 + 1 + \cdots h$ times

$= h$

$\therefore n = h.$