Key answer

Quiz 1 Phys 380

I) $B_{\text{top wire}} = 0 \qquad (\sin\theta = 0)$

$$B_{arc} = \frac{\mu_0 I}{4\pi R}\left(\frac{3\pi}{2}\right) = \frac{3}{4} \times \frac{\mu_0 I}{2R}$$

The botton straight segment produces contribution with $\theta_1 = 0$ at the far left end and $\theta_2 = \frac{\pi}{2}$ at the right end directly below the center of curvature.

$$\Rightarrow B_{segment} = \frac{\mu_0 . I}{4\pi R}\left(\cos 0 - \cos\frac{\pi}{2}\right) = \frac{\mu_0 I}{4\pi R}$$

which is half the field of the infinite wire.

Total field at the center of curvature:

$$B = B_{arc} + B_{segment} = \left(\frac{3}{4} + \frac{1}{2\pi}\right)\frac{\mu_0 I}{2R}$$

II) $I_{encl.} = \int \vec{J} . d\vec{A} = \int (\alpha r'^2)(2\pi r')\, dr'$

$$= \int 2\pi\alpha\, r'^3\, dr'$$

For $r < R$: $I_{enc.} = \int_0^r 2\pi\alpha\, r'^3\, dr' = \frac{\pi\alpha r^4}{2}$

Applying Ampere's law:

$$B(2\pi r) = \mu_0 \frac{\pi\alpha r^4}{2} \longrightarrow B = \frac{\mu_0 \alpha}{4} r^3$$

For $r > R$: $I_{encl.} = \int_0^R 2\pi\alpha r'^3 dr' = \frac{\pi\alpha R^4}{2}$

Applying Ampere's law: $B(2\pi r) = \mu_0 \frac{\pi\alpha R^4}{2} \rightarrow B = \frac{\mu_0 \alpha R^4}{4r}$

Key answer

Quiz 2 phys 380

(a) Charge on a ring of radius r and thickness dr is just $dq = (2\pi r\,dr)\sigma$ where $\sigma = \frac{Q}{\pi R^2}$

(surface charge density of the uniformly distributed charge.

$$dI = \frac{dq}{dt} = dq\frac{\omega}{2\pi}$$

area of the ring is $A = \pi r^2$

$$\therefore \quad dm_z = dI \cdot A = 2\pi r\,dr\,\frac{Q}{\pi R^2} \cdot \pi r^2 \cdot \frac{\omega}{2\pi}$$

$$m_z = \int_0^R \frac{Q}{R^2}\,\omega r^3 dr = \frac{Q}{4} R^2 \omega$$

(b) $\times\left(\frac{M}{M}\right) \longrightarrow m_z = \frac{Q}{4M} M R^2 \omega$

$$= \frac{Q}{2M}\left(\frac{1}{2} M R^2 \omega\right)$$

$$= \frac{Q}{2M}\left(I\omega\right) = \mu L_z$$

$$\mu = \frac{Q}{2M}$$

(c) Total charge in the ring goes around exactly one time in one period of its revolution

$$\Rightarrow \quad I = \frac{Q}{T} = \frac{Q\omega}{2\pi}$$

$$m_z = IA = \frac{Q\omega}{2\pi}\pi R^2 = \frac{Q\omega}{2} R^2$$