# Digital Circuits

## Sheet 1: Number Systems

### Decimal System

Base (radix) = 10 determine the quantity that a number represents.
Symbols: 0,1,2,3,4,5,6,7,8,9

**Examples:**

$45.67 = 7*10^{-2} + 6*10^{-1} + 4*10^{1} + 5*10^{0}$
$724.5 =$ ---------- $75*10^{-1} + 4*10^{0} + 2*10^{1} + 7*10^{2}$

### Binary System

Base (radix) = 2
Symbols: 0,1

**Examples:**

$10111.01 = 1*2^{-2} + 0*2^{-1} + 1*2^{0} + 1*2^{1} + 1*2^{2}\ 0*2^{3} + 1*2^{4} = (23.25)_{10}$
$11011.101 =$ ----------

$$1*2^{-3} + 0*2^{-2} + 1*2^{-1} + 1*2^{0} + 1*2^{1}\ 0*2^{2} + 1*2^{3} + 1*2^{4} = (27.625)_{10}$$

### Octal System

Base (radix) = 8
Symbols: 0,1,2,3,4,5,6,7

**Examples:**
$(137.4)_8 = 4*8^{-1} + 7*8^{0} + 3*8^{1} + 1*8^{2} = (95.5)_{10}$
$(736.4)_8 =$ ----------

$$4*8^{-1} + 6*8^{0} + 3*8^{1} + 17*8^{2}$$

### Hexadecimal System

Base (radix) = 16
Symbols: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

**Examples:**
$(F3)_{16} =$ ---------- $(243)_{10}$
$(AB.4)_{16} =$ ---------- $(171.25)_{10}$

## Converting from decimal to any system

**Example:**

a) Convert $(5.125)_{10}$ to binary
"Separate the integer part from the fraction part"

**Fraction part:**
$0.12510 = (\text{----------})_2$

| | integer | fraction |
|---|---|---|
| 0.125× 2 | 0.25 | 0 |
| 0.25×2 | 0.5 | 0 |
| 0.5×2 | 0.0 | 1 ↓ |

Result = $(0.001)_2$

**Integer part:**
$510 = (\text{----------})_2$

| | | reminder |
|---|---|---|
| 5 \| | 2 | 1 ↑ |
| 2 \| | 2 | 0 |
| 1 \| | 2 | 1 |
| 0 \| | | |

Result= $(101)_2$

b) $(286.85)_{10} = (\text{--------})_8$ 436.66314

NB: *Take care of the stopping case: reaching zero or repeated fraction*

## Converting between Binary and Octal or Hexadecimal

**Example:**

$107.54_8 = (\text{----------})_2$

| 1 | 0 | 7 | . | 5 | 4 |
|---|---|---|---|---|---|
| **001** | **000** | **111** | . | **101** | **100** |

$(AB.C8)_{16} = (\text{----------})_2$ 10101011.11001

## Converting between Octal and Hexadecimal

$(AB.C8)_{16} = (\text{---------})_8$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Step 1: | A | | B | . | C | | 8 |
| Step 2: | 1010 | | 1011 | . | 1100 | | 1000 |
| Step 3: | 010 | 101 | 011 | . | 110 | 010 | |
| Step 4: | 2 | 5 | 3 | . | 6 | 2 | |

## Binary Table:

| | $2^3$ 8 | $2^2$ 4 | $2^1$ 2 | $2^0$ 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 |
| 6 | 0 | 1 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 |
| 8 | 1 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 |
| (A) 10 | 1 | 0 | 1 | 0 |
| (B) 11 | 1 | 0 | 1 | 1 |
| (C) 12 | 1 | 1 | 0 | 0 |
| (D) 13 | 1 | 1 | 0 | 1 |
| (E) 14 | 1 | 1 | 1 | 0 |
| (F) 15 | 1 | 1 | 1 | 1 |

1. Convert the following binary numbers to decimal:
   - 101110
   - 1110101
   - 110110100.

2. Convert the following numbers with the indicated bases to decimal:
   - $(12121)_3$
   - $(4310)_5$
   - $(50)_7$
   - $(198)_{12}$

3. Convert the following decimal numbers to binary:
   - 1231
   - 673
   - 1998.

4. Convert the following decimal numbers to the basis indicated:
   - 7562 to octal
   - 1938 to hexadecimal
   - 213.375 to binary
   - 726.4 to octal
   - 35.203 to octal

5. Convert the following binary numbers to the basis indicated:
   - 1100.011 to decimal
   - 1110110101.0011 to octal
   - 1101001110111101.101101 to hexadecimal

6. Convert the following numbers to decimal:
   - $(472)_8$ = (----------)$_{10}$
   - $(153.48)_8$ = (--------)$_{10}$
   - (21 FB )$_{16}$ = (---------)$_{10}$

7. Convert the following numbers according to the radix indicated:
   - (1FA6.2D )$_{16}$ = (-------)$_8$
   - $(360)_8$ = (---------)$_{16}$
   - $(368170.AB)_{16}$ = (--------)$_8$

**Check your answers from here:** http://coderstoolbox.net/number/