Carthage University
INSAT
Department of Mathematics & Computer Sciences

Exercises sheet: Convergence of random variables and confidence intervals

Exercise 1: Consider the probability space $\big(\Omega = [0,1], \mathbb{P} = d\omega\big)$ which means that $\mathbb{P}\big\{[\omega_1, \omega_2]\big\} = \omega_2 - \omega_1$ for all $0 \leq \omega_1 < \omega_2 \leq 1$. And let the sequence of random variables $\{X_n\}_{n\geq 1}$ defined as follows

$$X_n = \begin{cases} 1 & 0 \leq \omega \leq \dfrac{n+1}{2n} \\ 0 & \text{otherwise} \end{cases}$$

and let X be defined as follows

$$X = \begin{cases} 1 & 0 \leq \omega \leq \dfrac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Check that $X_n \xrightarrow[n\to+\infty]{a.s.} X$.

Exercise 2: Let $\{X_n\}_{n\geq 1}$ be a sequence of random variables such that $X_n \sim \mathcal{E}(n)$ for all $n \geq 1$. Show that X_n converges in probability to zero.

Exercise 3: Let $\{X_n\}_{n\geq 1}$ be a sequence of random variables such that $X_n \sim \mathcal{G}\left(\dfrac{\lambda}{n}\right)$ for $\lambda > 0$ and all n large enough.
And let $Y_n = \dfrac{X_n}{n}$ for all n large enough. Show that X_n converges in distribution to $\mathcal{E}(\lambda)$.

Exercise 4: A scientific paper publish the results of a recent study on the mercury contamination of a lake fishes in Florida. The records of mercury concentration in the muscle tissue of 32 fishes chosen at random (in ppm: part per million) are below:

1.23	0.49	0.49	1.08	0.59	0.28	0.18	0.1
0.94	1.33	0.19	1.16	0.98	0.34	0.34	0.19
0.21	0.4	0.04	0.83	0.05	0.63	0.34	0.75
0.04	0.86	0.43	0.044	0.81	0.15	0.56	0.84

1. Compile the data above and represent graphically.
2. Compute its parameters.
3. Let μ be the population mean of the mercury concentration in the muscle tissue of all fishes of the lake, if we suppose that the variance is $\sigma = 0,12$ give a confidence interval for μ of confidence level 95%.

Exercise 5:
We measure the charge of cut-off (in kilogram) of a sample of 20 plastic cable. The results are listed below:

19.41	16.69	17.71	18.15
21.07	20.54	19.52	18.49
18.69	18.88	19.59	15.10
20.21	20.14	20.14	21.92
22.64	21.20	17.16	19.75

1. Check if the statistical variable charge of cut-off is likely normally distributed.
2. Find out a confidence interval, of the charge of cut-off for this kind of plastic cables, of level 95%.

Exercise 6:
We measure, for a sample of 100 portion of fresh water of a natural source, the concentration (milligram per liter) of calcium. A confidence interval of the population mean of level 95% is: $0.49 \leq \mu \leq 0.82$.

1. A confidence interval of level 99% is larger or sharper?
2. Find out an estimation of the sample size n for which the width of the confidence interval of level 99% is les or equal to 0.3.