Carthage University
INSAT
Department of Mathematics & Computer Sciences

Practical works: Markov Chain

Practice#1
Consider the function

$$\varphi(x) = \frac{1}{2}\left(x + \frac{a}{x}\right).$$

i. Show that φ_a is a contraction on $]0, +\infty[$, for all $a > 0$, and determine its contraction coefficient.

ii. Show that the recursive sequence defined below

$$\begin{cases} x_0 & > & 0 \\ x_{n+1} & = & \varphi_a(x_n) \end{cases}$$

is convergent to $\sqrt{a}$ for all given initial value x_0.

iii. Write a code that generate the n^{th} term of the sequence above for a given initial value x_0 and a given positive number $a > 0$.

iv. Plot the dynamics of your recursive sequence, and compare it with the value of $\sqrt{a}$.

Practice#2
Given a Markov Chain with three states "red", "green", "blue", defined by its transition matrix $\mathbb{T}$ and its initial states' distribution $\mathbb{P}_{X_0}$.

i. Write a program that generates the n first distributions states.

ii. Visualize your the dynamics of the underlying MC, where at each step you plot a simulation of the corresponding states' distribution. You should get something similar to the next figure