Universität Bremen

**FB3: Mathematik/Informatik**

M. Buchem, N. Megow, A. Lindermayr, Z. Liu

**Sommer 2024**

# Hands-on Tutorial on Optimization

Exercise Sheet: Cheese Empire (26.09.2024)

## Exercise 1

We pay another visit to Baron Chaese von Due, who has used your optimization method to perform a series of successful raids in the cellars of cheese collectors worldwide. Being not only a cheese gourmet but also a clever businessman and a brilliant inventor, von Due is by no means resting on his laurels! One night, while indulging in the sight of his tremendous spoils, it occurred to him how he could let the whole country take part in his delight (and incidentally make a ton of money at the same time). A few years have passed and von Due has put his plan into action. Through a complicated process that involves heating the cheese and adding wine and other ingredients, von Dues factories produce a viscid mixture. At first, people were sceptical to buy the product, but as soon as the idea of dipping pieces of bread into the mixture had spread, von Due was literally overwhelmed by the incoming orders. He has since scaled up his business and opened several factories near his most important customers. These factories, in turn, are supplied with cheese from a number of different sources (the legality of which we shall not call into question). Unfortunately, von Due has slightly lost track of his quickly growing enterprise and he asks your for help in optimizing his production plans.

(a) Von Due needs your help in determining how much of his mixture every one of his factories should produce per month to minimize the total production cost. Note that a factory can only supply customers at the same location.

(b) As soon as von Due is back on track with his production planning, he strives for improvements. As you will have noticed, some of his factories can produce the cheese mixture at significantly lower cost than others. To exploit these advantages, von Due has developed the KäMaPi (short for Käsemassepipeline), a special pipeline that allows him to transport his cheese mixture across long distances. This sounds simpler than it is, as the mixture would immediately solidify in any regular pipe, which would stop the flow. The pipeline hence has to be heated in order to keep the mixture flowing. This works remarkably well, but the design is not flawless: Somehow, the pipelines are not completely tight and a certain fraction of the produce leaks and is lost during transportation. Von Due has asked you to find out, between which locations the construction of a KäMaPi would be profitable. Von Due is unsure about the durability of the KäMaPis, so he asks you to start by planning only two years ahead. In order to save cost, all KäMaPis should be built in a straight line without detours between the locations. (Thanks to his good relations with local politics, von Due does not expect any difficulties with obtaining planning permission for the constructions.) The technical details of the KäMaPis are as follows:

| | |
|---|---|
| capacity: | 3 tons/month |
| construction cost: | length [km] $*$ 0.2 Gulden |
| heating cost: | length [km] $*$ transported cheese mixture (without leakage) [tons] $*$ 0.02 Gulden |
| leakage: | (length [km] $\cdot$ 0.1) % of the transported cheese mixture |

(c) Von Dues good relations to politics pay off once again: Through a (admittedly highly dubious) subsidy programme, he can sell some of his mixture to the government in addition to his own production: If a factory that produces $p$ tons of cheese mixture for von Dues customers produces an additional $\alpha \cdot p$ tons for the government, the additional mixture will be picked up by the government at the production site, the production cost will be reimbursed and von Due can receive an additional amount of $K - \frac{K}{1+\alpha}$ Gulden (depending on the capacity $K$ of the respective factory). Note that this subsidy is optional, so in particular von Due will not opt for it if the amount $K - \frac{K}{1+\alpha}$ is negative.

Integrate the new government programme into your model and compute an optimal production plan for von Due.