1 Question 1

$z_1^{(2)} = \sum_{j \in N(1)} \alpha_{1j}{}^{(2)} W^{(2)} z_j{}^{(1)} = \alpha_{12}{}^{(2)} W^{(2)} z_2{}^{(1)} + \alpha_{13}{}^{(2)} W^{(2)} z_3{}^{(1)}$
$z_4^{(2)} = \sum_{j \in N(4)} \alpha_{4j}{}^{(2)} W^{(2)} z_j{}^{(1)} = \alpha_{42}{}^{(2)} W^{(2)} z_2{}^{(1)} + \alpha_{43}{}^{(2)} W^{(2)} z_3{}^{(1)} + \alpha_{45}{}^{(2)} W^{(2)} z_5{}^{(1)} + \alpha_{46}{}^{(2)} W^{(2)} z_6{}^{(1)}$
We also know that : $z_2(1) = z_6(1)$ and $z_3(1) = z_5(1)$ then :
$z_4^{(2)} = (\alpha_{42}{}^{(2)} + \alpha_{46}{}^{(2)}) W^{(2)} z_2{}^{(1)} + (\alpha_{43}{}^{(2)} + \alpha_{45}{}^{(2)}) W^{(2)} z_3{}^{(1)}$.
And using the formula of $\alpha_{ij}^{(2)}$; we find that $\alpha_{42}{}^{(2)} = \frac{1}{2}\alpha_{12}{}^{(2)}$ and $\alpha_{46}{}^{(2)} = \frac{1}{2}\alpha_{12}{}^{(2)}$.
Also; $\alpha_{43}{}^{(2)} = \frac{1}{2}\alpha_{13}{}^{(2)}$ and $\alpha_{45}{}^{(2)} = \frac{1}{2}\alpha_{13}{}^{(2)}$.
Then we conclude that $z_1^{(2)} = z_4^{(2)}$.

2 Question 2

No, the model likely won't achieve high accuracy. Identical features mean the node features provide no discriminative information. The GNN will rely solely on the graph structure (adjacency matrix) to differentiate nodes, limiting its ability to learn meaningful representations for classification.

3 Question 3

$$\mathbf{z} = \begin{pmatrix} 2.2 & -0.6 & 1.4 \\ 0.2 & 1.8 & 1.5 \\ 0.5 & 1.1 & -1.0 \\ 0.7 & 0.1 & 1.3 \\ 1.2 & -0.9 & 0.3 \\ 2.2 & 0.9 & 1.2 \\ -0.7 & 1.8 & 1.5 \\ -0.4 & 1.8 & 0.1 \\ 2.2 & -0.6 & 1.5 \end{pmatrix}$$

(i) : Sum :

$$\mathbf{z_{G_1}} = \begin{pmatrix} 2.9 & 2.3 & 1.9 \end{pmatrix}$$
$$\mathbf{z_{G_2}} = \begin{pmatrix} 3.4 & 1.9 & 4.3 \end{pmatrix}$$
$$\mathbf{z_{G_3}} = \begin{pmatrix} 1.8 & 1.2 & 1.6 \end{pmatrix}$$

(ii): Mean :

$$\mathbf{z_{G_1}} = \begin{pmatrix} 0.97 & 0.77 & 0.63 \end{pmatrix}$$
$$\mathbf{z_{G_2}} = \begin{pmatrix} 0.85 & 0.47 & 1.1 \end{pmatrix}$$
$$\mathbf{z_{G_3}} = \begin{pmatrix} 0.9 & 0.6 & 0.8 \end{pmatrix}$$

(iii) : Max:

$$\mathbf{z_{G_1}} = \begin{pmatrix} 2.2 & 1.8 & 1.5 \end{pmatrix}$$
$$\mathbf{z_{G_2}} = \begin{pmatrix} 2.2 & 1.8 & 1.5 \end{pmatrix}$$
$$\mathbf{z_{G_3}} = \begin{pmatrix} 2.2 & 1.8 & 1.5 \end{pmatrix}$$

If we compute the pairwise euclidean distances $||z_{G_1} - z_{G_2}||$ and $||z_{G_1} - z_{G_3}||$ and $||z_{G_2} - z_{G_3}||$, we observe that the sum method produces larger distances between graphs, which means it is able to distinguish these graphs the best.

4 Question 4:

The relationship will be $z_{G_2} = 2z_{G_1}$ .In our model, we use sum as a readout function; and it is proportional to the number of nodes, and since both graphs have features initialized to 1, the sum for G_2 (8 nodes) will be twice that of G_1 (4 nodes).