$$U_n \leq 2 - \frac{1}{n}$$

Hint : $\dfrac{1}{(n+1)^2} \leq \dfrac{1}{n} - \dfrac{1}{n+1}$

3) Deduce that the sequence $(U_n)_n$ is convergent.

Solution.

1) $B = \left\{ \frac{1}{2} + \frac{1}{n}, \frac{1}{2} - \frac{1}{n}, n \in \mathbb{N}^* \right\}$

$= \left\{ \frac{1}{2} + \frac{1}{n}, n \in \mathbb{N}^* \right\} \cup \left\{ \frac{1}{2} - \frac{1}{n}, n \in \mathbb{N}^* \right\} = B_1 \cup B_2$

then, $\sup B = \max(\sup B_1, \sup B_2)$, and

$\inf B = \min(\inf B_1, \inf B_2)$, therefore.

- $\sup B_1 = \sup\left\{\frac{1}{2}\right\} + \sup\left\{\frac{1}{n}, n \in \mathbb{N}^*\right\}$

$= \frac{1}{2} + 1 = \frac{3}{2}$ 0.5

- $\sup B_2 = \sup\left\{\frac{1}{2}\right\} + \sup\left\{-\frac{1}{n}, n \in \mathbb{N}^*\right\}$

$= \frac{1}{2} + \inf\left\{\frac{1}{n}, n \in \mathbb{N}^*\right\}$ 0.5

$= \frac{1}{2} + 0 = \frac{1}{2}$

Scanné avec CamScanner

Test N°1

m, the infimum,

n of the previous

}.

on 1

)] + 4

ed by 3

is increasing.

$_n$ is convergent.

$\ldots n \in \mathbb{N}^* \} = B_1 \cup B_2$

$\sup B_2) , and$

$\inf B_2)$ therefore,

$\frac{1}{n}, n \in \mathbb{N}^* \}$

- So $\sup B = \max \{ \frac{3}{2}, \frac{1}{2} \} = \frac{3}{2}$ 0,5

- $\inf B_1 = \frac{1}{2} + \inf \{ \frac{1}{n}, n \in \mathbb{N}^* \} = \frac{1}{2} + 0 = \frac{1}{2}$ 0,5

- $\inf B_2 = \frac{1}{2} - \sup \{ \frac{1}{n}, n \in \mathbb{N}^* \} = \frac{1}{2} - 1 = -\frac{1}{2}$ 0,5

So $\inf B = \min (\frac{1}{2}, -\frac{1}{2}) = -\frac{1}{2}$ 0,5

moreover, $\frac{3}{2} = \frac{1}{2} + \frac{1}{1} \in B$ and $-\frac{1}{2} = \frac{1}{2} - \frac{1}{1} \in B$ 0,5 0,5

Then $\max B = \frac{3}{2}$, and $\min B = -\frac{1}{2}$

02/ $f(x) = 2 [\frac{1}{5} (x+2)] + 4$, where $f(x) = 22$

$\Leftrightarrow 2 [\frac{1}{5} (x+2)] + 4 = 22 \Leftrightarrow [\frac{1}{5} (x+2)] = 9$

$\Leftrightarrow 9 \leq \frac{1}{5} (x+2) < 10$

$\Leftrightarrow 43 \leq x < 48$ 1

$\Leftrightarrow x \in [43, 48[$.

Task 02

01/ we have $U_{n+1} - U_n = \frac{1}{(n+1)^2} \geq 0$ 1

Then $(U_n)_n$ is increasing.

02/ $P(n) : U_n \leq 2 - \frac{1}{n}$, $n \geq 1$ 1

Step 1: $P(1) : U_1 \leq 2 - \frac{1}{1} \Leftrightarrow 1 \leq 1$ true

Step 2: Assume $P(n)$ is true and we prove that $P(n+1)$ is true

$U_{n+1} = U_n + \frac{1}{(n+1)^2} \leq 2 - \frac{1}{n} + \frac{1}{n} - \frac{1}{(n+1)^2}$

$= 2 - \frac{1}{(n+1)^2}$

Then $P(n+1)$ is true, by Induction $P(n)$ is true.

03) by the above question 1

$\forall n \geq 1$, $U_n \leq 2$, then the sequence

U_n increasing and bounded

above so $(U_n)_n$ is convergent