University M'Hamed Bougara of Boumerdes

Faculty of Sciences
Department of Mathematics
First-Year C S and C E

Academic year : 2023/2024
Date: 18 / 01 / 2024
Duration: 1h30

Exam of Analysis 01

Exercise 1 (4 marks). *Let A be nonempty subset of $\mathbb{R}$, defined by*

$$A = \left\{ a_n = \frac{n}{n+1} + 1,\ n \in \mathbb{N} \right\}.$$

1. *Recall the characterization of the supremum theorem.*
2. *Show that A is a bounded set.*
3. *Find, if there exist, the supremum, the infimum, the maximum, and the minimum of A. Justify your answer.*

Exercise 2 (8 marks). *Let the sequence $(u_n)_n$ be recursively defined by:*

$$\begin{cases} u_0 = a,\ a \in \mathbb{R}^+, \\ u_{n+1} = \dfrac{1}{2} u_n (u_n + 1), \quad n \in \mathbb{N}. \end{cases}$$

I) Suppose that $a = \frac{1}{2}$.

1. *Prove that: $\forall n \in \mathbb{N};\ 0 < u_n < 1$.*
2. *Study the monotonicity of $(u_n)_n$.*
3. *Deduce that $(u_n)_n$ is convergent and compute its limit.*

II) Suppose that $a > 1$.

1. *Prove that: $\forall n \in \mathbb{N};\ u_n > 1$.*
2. *Show that $(u_n)_n$ is strictly increasing and $u_n > \left(\dfrac{1+a}{2}\right)^n,\ \forall n \in \mathbb{N}$.*
3. *Deduce the nature of $(u_n)_n$.*

Exercise 3 (4 marks). *Let f be a function defined by:*

$$f(x) = \begin{cases} e^x\left(-x + \frac{4}{\pi}\right), & x \le 0; \\ \dfrac{\sin(x) - \cos(x)}{x - \frac{\pi}{4}}, & 0 < x < \frac{\pi}{4}; \\ \sqrt{2} + \frac{\pi}{4} - x, & x > \frac{\pi}{4}. \end{cases}$$

1. *Determine the set $\mathcal{D}_f$.*
2. *Study the continuity and the differentiability of f on $\mathcal{D}_f$.*
3. *Does f accept a continuous extension? If yes, set it.*

Exercise 4 (4 marks).

1. *State the mean value (finite increment's) theorem on $[a, b]$.*
2. *Show that:*

$$\forall x > 0;\ \frac{1}{x+1} < \ln(x+1) - \ln(x) < \frac{1}{x}.$$

3. *By using the second question, conclude the following limit:*

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x.$$


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Note...
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