Assignement 15

A. Show that the MST decision problem is polynomial-time reducible to the Subset Sum problem.

MST (Minimum Spanning Tree) Decision Problem:

- Given a graph G=(V,E)G = (V, E), the goal is to determine if there exists a spanning tree for GG whose total weight is less than or equal to some given value WW.

Subset Sum Problem:

- Given a set of integers and a target value SS, determine if there is a subset of the integers that adds up to SS.

Polynomial-time Reduction:

1. **Convert MST to Subset Sum**:
   - For the MST problem, the set of edge weights can be treated as the set of integers in the Subset Sum problem.
   - The target sum SS in the Subset Sum problem corresponds to the given weight WW in the MST problem.
2. **Set Construction**:
   - Take the weights of the edges in the graph as the set of numbers.
   - The target sum SS is set to WW.
   - If there exists a subset of these edge weights (representing a spanning tree) such that the total weight is less than or equal to WW, then there is a solution to the MST decision problem.
3. **Verification**:
   - This reduction is polynomial-time because we simply map the edge weights of the graph to a set of integers and check for a subset sum solution in the same time complexity as solving the MST problem. Since both are solvable in polynomial time, this reduction works.

Thus, the MST decision problem is reducible to the Subset Sum problem in polynomial time.

B. Show the Shortest Path decision problem is polynomial-time reducible to the MST decision problem.

Shortest Path Decision Problem:

- Given a graph GG, two nodes uu and vv, and a value dd, the decision version asks: "Is there a path from uu to vv with a total weight less than or equal to dd?"

Reduction Steps:

1. **Convert Shortest Path to MST**:
   - Transform the shortest path problem into a decision problem by asking if there is a path between uu and vv with total weight ≤d\leq d.
2. **Constructing MST**:
   - Now, reduce this shortest path problem to an MST problem by considering the subgraph of GG that includes only the nodes and edges related to the path from uu to vv.
   - The shortest path from uu to vv can be thought of as part of a spanning tree that minimizes the total weight between the nodes in the subgraph.
3. **Reduction**:
   - If the MST for this subgraph has a weight ≤d\leq d, then the shortest path decision problem is solvable. If not, then no such path exists within the limit dd.

This reduction is efficient because the transformation between the shortest path and MST problems involves building the subgraph and checking its spanning tree, which can be done in polynomial time.