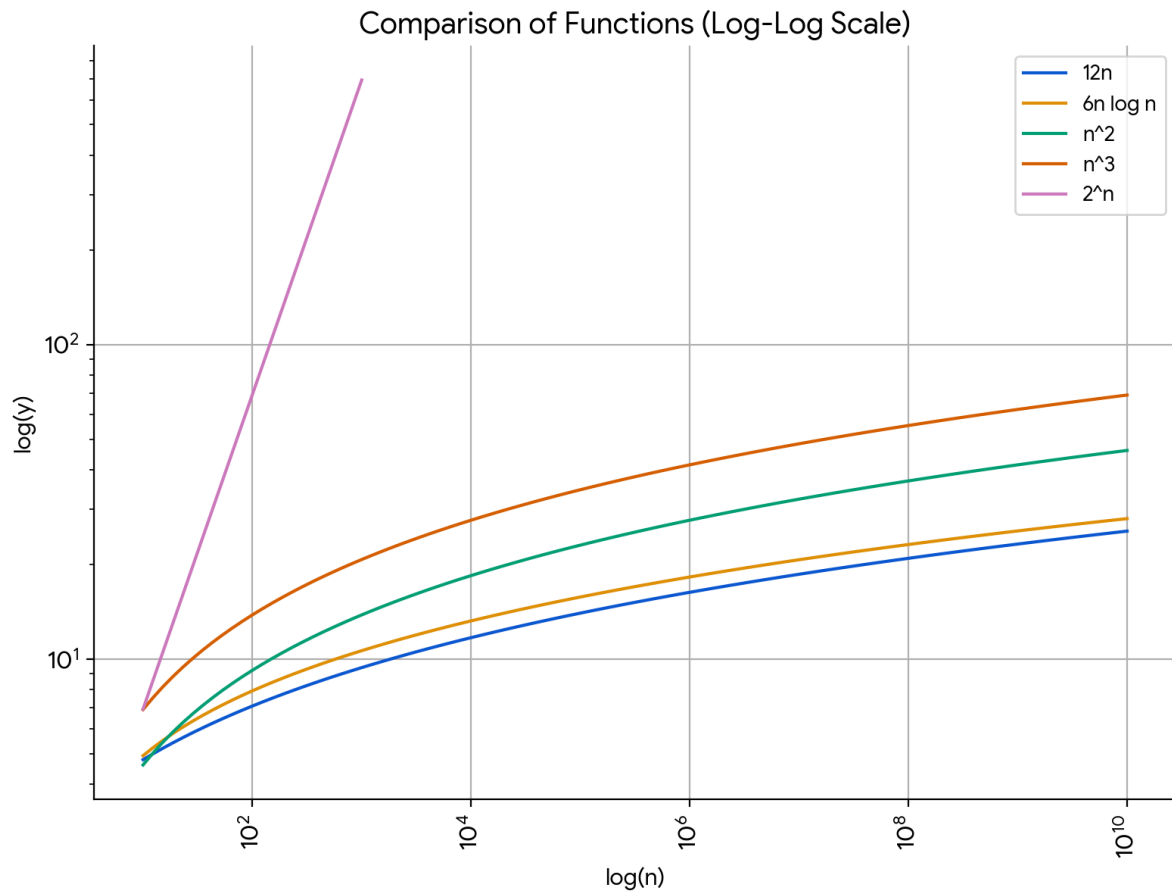


R-1.1



R1.2

To find the value  $n_0$  where Algorithm A becomes more efficient than Algorithm B, we need to solve the inequality:

$$10n \log n < n^2$$

This inequality is difficult to solve analytically. However, we can use numerical methods or plotting techniques to find an approximate value for  $n_0$ .

By plotting the two functions  $10n \log n$  and  $n^2$  on a graph, we can visually determine the intersection point, which will give us an approximate value for  $n_0$ .

R1.6

To order functions by their big-O notation, we need to compare their growth rates as  $n$  approaches infinity. Here's the ordered list:

1. **Constant functions:**  $1/n$ ,  $2$
2. **Logarithmic functions:**  $\log \log n$ ,  $4 \log n$
3. **Polynomial functions:**  $n$ ,  $n \log n$ ,  $n^2 \log n$ ,  $n^3$
4. **Exponential functions:**  $2^n$ ,  $4^n$ ,  $2^n \log n$

**R-1.10 Give a big-O characterization, in terms of  $n$ , of the running time of the Loop1 method below:**

Algorithm Loop1( $n$ )

$s = 0$

for  $i = 1$  to  $n$  do

$s = s + i$

The loop iterates  $n$  times. In each iteration, a constant-time operation is performed. Therefore, the total running time is  **$O(n)$** .

**R-1.14 Perform a similar analysis for method Loop5 below:**

Algorithm Loop5( $n$ )

$s = 0$

for  $i = 1$  to  $n$  do

for  $j = 1$  to  $i$  do

$s = s + i$

The outer loop iterates  $n$  times. For each iteration of the outer loop, the inner loop iterates  $i$  times. The total number of iterations is:

$$1 + 2 + 3 + \dots + n = n(n+1)/2$$

This is approximately  **$O(n^2)$** .

**Prove:  $\log_b(x^a) = a \log_b(x)$**

We can use the properties of logarithms to prove this:

Let  $y = \log_b(x^a)$ . Then, by the definition of logarithms:

$$b^y = x^a$$

Taking the logarithm of both sides with base  $b$ :

$$\log_b(b^y) = \log_b(x^a)$$

Using the property  $\log_b(b^x) = x$ :

$$y = a \log_b(x)$$

Therefore,  $\log_b(x^a) = a \log_b(x)$ .