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SYNTHESIS LECTURES ON CONTROL AND MECHATRONICS #1

Lecture #1

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ABSTRACT

This monograph describes the Reaction Wheel Pendulum, the newest inverted-pendulum-like device for control education and research. We discuss the history and background of the reaction wheel pendulum and other similar experimental devices. We develop mathematical models of the reaction wheel pendulum in depth, including linear and nonlinear models, and models of the sensors and actuators that are used for feedback control. We treat various aspects of the control problem, from linear control of the motor, to stabilization of the pendulum about an equilibrium configuration using linear control, to the nonlinear control problem of swingup control. We also discuss hybrid and switching control, which is useful for switching between the swingup and balance controllers. We also discuss important practical issues such as friction modeling and friction compensation, quantization of sensor signals, and saturation. This monograph can be used as a supplement for courses in feedback control at the undergraduate level, courses in mechatronics, or courses in linear and nonlinear state space control at the graduate level. It can also be used as a laboratory manual and as a reference for research in nonlinear control.

KEYWORDS

feedback control, inverted pendulum, modeling, dynamics, nonlinear control, stabilization, friction compensation, quantization, hybrid control.

Contents

CHAPTER 1

Introduction

"I THINK," shrilled Erjas, "that this is our most intriguing discovery on any of the worlds we have yet visited!"

— PENDULUM by Ray Bradbury and Henry Hasse, Super Science Stories, 1941

1.1 THE REACTION WHEEL PENDULUM

This monograph is concerned with modeling and control of a novel inverted pendulum device, called the *Reaction Wheel Pendulum*, shown in Figure 1.1. This device, first introduced in [18], is perhaps the simplest of the various pendulum systems in terms of its dynamic properties, consequently, its controllability properties. At the same time, the Reaction Wheel Pendulum exhibits several properties, such as underactuation and nonlinearity,[1] that make it an attractive and useful system for research and advanced education. As such, the Reaction Wheel Pendulum is ideally suited for educating university students at virtually every level, from entering freshman to advanced graduate students.

From a mechanical standpoint, the Reaction Wheel Pendulum is a *simple pendulum* with a rotating wheel, or bob, at the end. The wheel is attached to the shaft of a 24-Volt, permanent magnet DC-motor and the coupling torque between the wheel and pendulum can be used to control the motion of the system. The Reaction Wheel Pendulum may be thought of as a simple pendulum in parallel with a torque-controlled inertia (and therefore a double integrator).

This monograph can be used as a supplemental text and laboratory manual for either introductory or advanced courses in feedback control. The level of background knowledge assumed is that of a first course in control, together with some rudimentary knowledge of dynamics of physical systems. Familiarity with Matlab is also useful, as Matlab is used throughout as a programming environment. In subsequent chapters, we describe the dynamic modeling, identification, and control of the Reaction Wheel Pendulum and include suggested laboratory exercises illustrating important concepts and problems in control. Some

[1]We will define these terms and discuss them in detail in subsequent chapters.

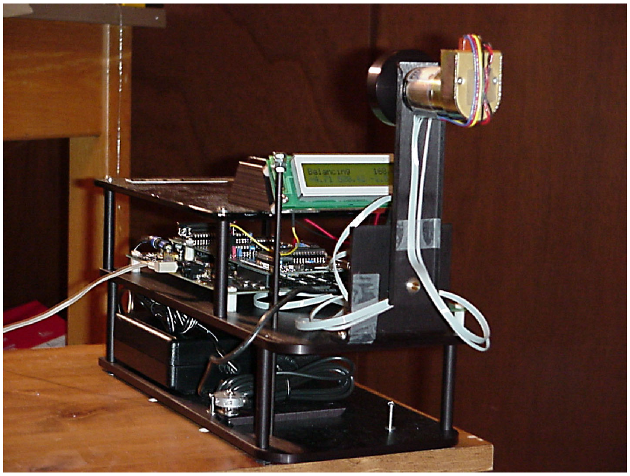

FIGURE 1.1: The reaction wheel pendulum.

examples of the problems that can be easily illustrated using the Reaction Wheel Pendulum are

- Modeling
- Identification
- Simple motor control experiments: velocity and position control
- Nonlinear control of the pendulum
- Stabilization of the inverted pendulum
- Friction compensation
- Limit Cycle Analysis
- Hybrid control—swingup and balance of the pendulum.

1.2 THE PENDULUM PARADIGM

Taking its name from the Latin *pendere*, meaning to hang,[2] the pendulum is one of the most important examples in dynamics and control and has been studied extensively since the time of Galileo. In fact, Galileo's empirical study of the motion of the pendulum raised important questions in mechanics that were answered only with Newton's formulation of the laws of motion and later work of others. Galileo's careful experiments noted that a pendulum nearly returns to its released height and eventually comes to rest with lighter ones coming to rest

[2]Other cognates include suspend (literally, to hang below) and depend (literally, to hang from).

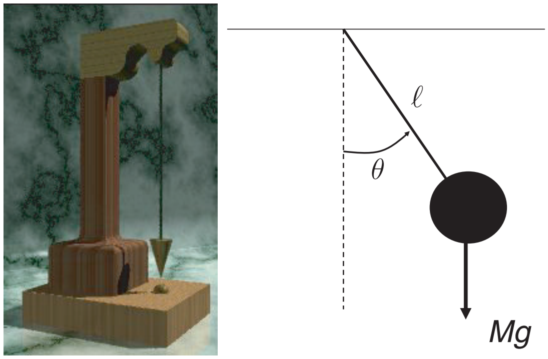

FIGURE 1.2: The simple pendulum.

faster. He discovered that the period of oscillation of a pendulum is independent of the bob weight, depending only on the pendulum length, and that the period is nearly independent of amplitude (for small amplitudes). All of these properties are now easily derived from Newton's laws and the equations of motion, discussed below.

In physics courses, therefore, the simple pendulum is often introduced to illustrate basic concepts like periodic motion and conservation of energy, while more advanced concepts like chaotic motion are illustrated with the forced pendulum and/or double pendulum.

1.2.1 The Simple Pendulum

To begin, let us consider a simple pendulum as shown in Figure 1.2 and discuss some of its elementary properties. Here, θ represents the angle that the pendulum makes with the vertical, ℓ and M are the length and mass, respectively, of the pendulum and g is the acceleration of gravity (9.8 m/sec^2 at the surface of the earth). The equation of motion of the simple pendulum is[3]

$$\ddot{\theta} + \frac{g}{\ell}\sin(\theta) = 0 \tag{1.1}$$

Notice that the ordinary differential equation (1.1) is nonlinear due to the term $\sin(\theta)$. If we approximate $\sin(\theta)$ by θ, which is valid for small values of θ, we obtain the linear system

$$\ddot{\theta} + \omega^2\theta = 0 \tag{1.2}$$

where we have defined $\omega^2 := g/\ell$. Equation (1.2) is called the *simple harmonic oscillator*. One can verify by direct substitution that the above equation has the general solution

$$\theta(t) = A\cos(\omega t) + B\sin(\omega t) \tag{1.3}$$

[3] Consult any introductory physics text.

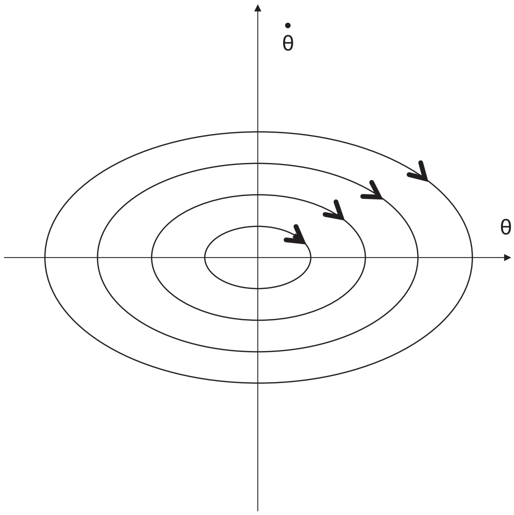

FIGURE 1.3: Phase portrait of the simple harmonic oscillator.

where A and B are constants determined by the initial conditions. In fact, it is an easy exercise to show that $A = \theta(0)$ and $B = \dot{\theta}(0)/\omega$. Moreover, the rate of change (velocity) of the harmonic oscillator is given by

$$\dot{\theta}(t) = B\omega\cos(\omega t) - A\omega\sin(\omega t) \tag{1.4}$$

and a directly calculation shows that

$$\theta^2(t) + \frac{1}{\omega^2}\dot{\theta}^2(t) = r^2 \tag{1.5}$$

where $r^2 = A^2 + B^2$. Equation (1.5) is the equation of an ellipse parameterized by time. This parameterized curve is called a *trajectory* of the harmonic oscillator system (see Figure 1.3). The totality of all such trajectories, one for each pair of initial conditions (A, B), is called the *phase portrait* of the system. Note that the period of oscillation of the simple harmonic oscillator (ω) is independent of the amplitude. It is surprising that, unlike the equation for the simple harmonic oscillator, which we easily solved, there is no closed form solution of the simple pendulum equation (1.1) analogous to Equation (1.3). A solution can be expressed in terms of so-called *elliptic integrals* but that subject is beyond the scope of this text. The phase portrait of the simple pendulum can be generated by numerical simulation as shown in Figure 1.4. We can gain added insight into this phase portrait by considering the scalar function

$$E = \frac{1}{2}\dot{\theta}^2 + \frac{g}{\ell}(1 - \cos(\theta)) \tag{1.6}$$

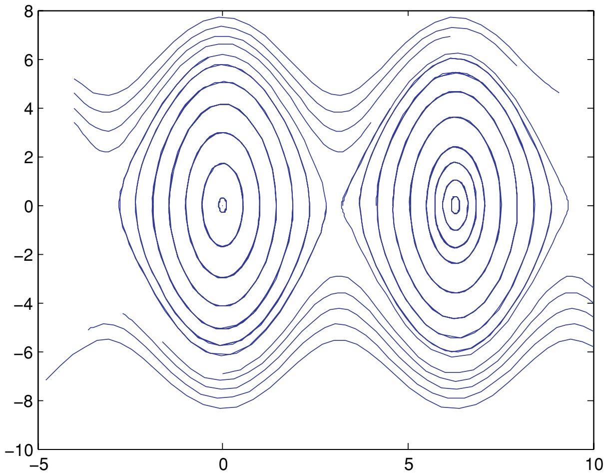

FIGURE 1.4: Phase portrait of the simple pendulum.

The function E is, in fact, proportional to the total energy, kinetic plus potential, of the simple pendulum. Computing the derivative of E yields

$$\dot{E} = \dot{\theta}\ddot{\theta} + \frac{g}{\ell}\sin(\theta)\dot{\theta} \tag{1.7}$$

$$= \dot{\theta}\left(\ddot{\theta} + \frac{g}{\ell}\sin(\theta)\right) \tag{1.8}$$

It follows that $\dot{E} = 0$ along solutions of the simple pendulum equation. This means that the function E is constant along solution trajectories, i.e., the level curves of E are trajectories. In classical mechanics, such a function is called a *first integral of the motion*. Each trajectory in Figure 1.4 is, therefore, a level curve of the function E. This is intuitively clear as we know from elementary physics that, without friction, the energy of the pendulum is constant. We will have much more to say about the pendulum dynamics in the chapters that follow. Since angles are typically given modulo 2π we obtain a nice representation by introducing cuts at $\pm\pi$ and glueing the parts together to form a cylinder. Such a representation, which is called a *manifold*, is useful when we do not have to take the number of rotations into account explicitly. Notice that for the particular implementation where the encoder is connected to wires it may be of interest to keep track of the number of revolutions so that we are not tearing the wires.

1.2.2 The Pendulum in Systems and Control

The simple pendulum system is interesting in its own right to study problems in dynamics and control. However, its importance is more than academic as many practical engineering

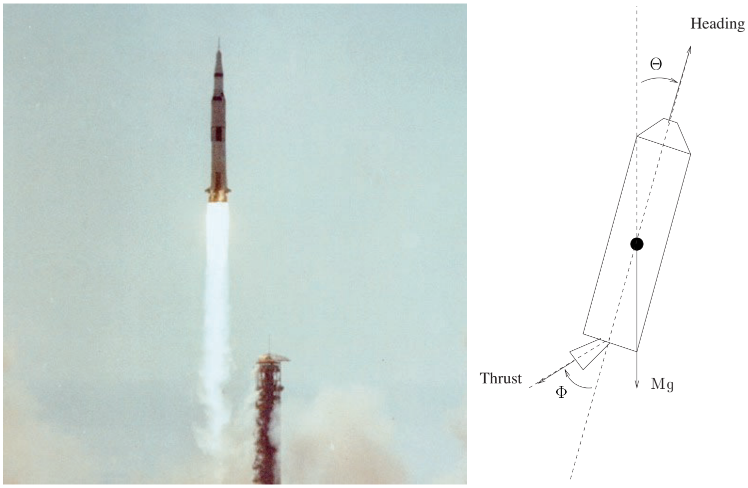

FIGURE 1.5: Liftoff of the Apollo 11 lunar mission. The rocket acts like an inverted pendulum balanced at the end of the thrust vectored motor.

systems can be approximately modeled as pendulum systems. In this section, we discuss several interesting examples and applications that can be modeled as pendulum systems.

Figure 1.5 shows the liftoff of a Saturn V rocket.[4] Active control is required to maintain proper attitude of the Saturn V rocket during ascent. Figure 1.5 also shows a diagram of a rocket whose pitch angle Θ can be controlled during ascent by varying the angle, Ψ, of the thrust vector. The pitch dynamics of the rocket can be approximated by a controlled simple pendulum.

In Biomechanics, the pendulum is often used to model bipedal walking. Figure 1.6 shows the Honda Asimo humanoid robot. In bipedal robots the stance leg in contact with the ground is often modeled as an inverted pendulum while the *Swing Leg* behaves as a freely swinging pendulum, suspended from the hip. In fact, studies of human postural dynamics and locomotion have found many similarities with pendulum dynamics. Measurements of muscle activity during walking indicate that the leg muscles are active primarily during the beginning of the swing phase after which they shut off and allow the leg to swing through like a pendulum. Nature has thus taught us to exploit the natural tendency of the leg to swing like a pendulum during walking, which may partially accounts for the energy efficiency of walking.

Likewise, quiet standing requires control of balance. So-called *Postural Sway* results from stretch reflexes in the muscles, which are a type of local feedback stabilization of the inverted pendulum dynamics involved in standing.

[4]This particular photo is of the Apollo 11 launch carrying astronauts Neil Armstrong, Buzz Aldrin, and Michael Collins on their historic journey to the moon.