Benha University
1st Term 2019/2020 -- Final
Class: 3rd Year Students (Security)
Subject: Algorithms
Course Code: FCS313

Faculty of Computers & AI
Date: 5/1/2020
Time: 3 hours
Total Marks: 50 Marks
Examiner(s): Dr. Karam Gouda

Answer the following questions:

Question No. 1 [8 Marks]

a- Define what does an "Algorithm" mean, its correctness ?

b- Why do we study the course "Design and Analysis of Computer Algorithms?

c- How many iterations of the following nested loops:

```
i=1;
loop (i <= N)
        j = 1;
        loop (j <= N)
                j = j * 2;
        i = i + 1;
```

d- How many iterations of the following nested loops:

```
i=1;
loop (i <= N)
        j = 1;
        loop (j <= i)
                j = j + 1;
        i = i + 1;
```

Question No. 2 [9 Marks]

■ State whether the following are **true** or **false**.

1- The same algorithm can be represented in only one way.
2- There may exist several algorithms for solving the same problem.
3- Algorithms for the same problem can be based on very different ideas and can solve the problem with dramatically different speeds.
4- Divide & Conquer design technique breaks the problem into overlapping sub-problems.
5- Greedy design technique repeatedly do what is the best now.
6- Dynamic Programming design technique breaks the problem into distinct sub-problems.
7- Time complexity computes how much memory is taken by the algorithm to run.
8- CPU speed can affect the running time of an algorithm.
9- In sorting algorithms, the number of items to be sorted detemines the input size of the algorithm.

1-	2-	3-	4-	5-
6-	7-	8-	9-	

Question No. 3 [3 Marks]

Define the Greatest Common Divisor (GCD) of two nonnegative integers m and n. Write an algorithm in the pseudo-code format to calculate GCD(m, n) of m and n, using the following property GCD(m, n) = GCD(n, m mod n) ?

Question No. 4 [5 Marks]

- Define the big oh (o) notation and directly apply this definition to prove that $f(n) = 3n + 7 = O(n)$.
- Prove that any binary tree of height h has at most 2h leaves.

Question No. 5 [5 Marks]

Define: Complete and full binary trees, min-heap structure. Simulate the HEAPSORT algorithm on the data array $A(1:8) = \{20; 1; 9; 10; 5; 25; 13; 13\}$. Note: Sort in decreasing order.

Question No. 6 **[9 Marks]**

Write the pseudocode of Merge Sort and prove that its worst-case time is O(n lg n) by using Recursion tree method. Then simulate the Merge Sort algorithm on the data array A(1 : 10) = {2; 10; 1; 20; 15; 30; 1; 19; 25; 45}. (Sort in decreasing order).

Question No. 7 [11 Marks]

1. Consider the following graph, given by its adjacency list, and followed by edge weights.

aList[1] = [2,3], weights: 6, 12
aList[2] = [1,3,5,6], weights: 6, 5, 14, 8
aList[3] = [1,2,4,7], weights: 12, 5, 9, 7
aList[4] = [3], weights: 9
aList[5] = [2,6], weights: 14, 3
aList[6] = [2,5,7], weights: 8, 3, 10
aList[7] = [3,6,8], weights: 7, 10, 15
aList[8] = [7], weights: 15

(a) Draw the equivalent graph. [2 Marks]
(b) Write its equivalent adjacency matrix. [1 Mark]
(c) What is the shortest path between node 3 and node 5. [1 Mark]
(d) What is the degree of node 1 and node 3. [1 Mark]
(e) List two spanning trees. [2 Marks]
(f) Is the graph connected? [1 Mark]
(g) Simulate the Kruskal algorithm on that graph to find a minimum spanning tree. [3 Marks]

Benha University | Third year
Faculty of Computer & Informatics | Final Exam, Jan. 2011
Subject: ALgorithms | Time: 3 hours

Solve as much as you can. (Total Grade: 65 points) The Exam in two pages

1. For the following pair of functions determine the smallest integer value of $n \geq 0$ for which the first function becomes greater than or equal to the second function. (4 points)

   (a) n^2, $10n$

   (b) 2^n, $2n^3$

2. Given the recurrence $T(n) = 4T(n/2)+n$. Prove that $T(n) = O(n^2)$ by using Substitution method. (10 points)

3. Write the set definition of the O-notation. What the meaning of $n^2 + O(n) = O(n^2)$. (4 points)

4. Write the pseudocode of Merge Sort and prove that its worst-case time is $\Theta(n\ lg\ n)$ by using Recursion tree method. Then simulate the Merge Sort algorithm on the data array $A(1:10) = \{2, 10, 1, 20, 15, 30, 1, 19, 25, 45\}$. (Sort in decreasing order). (15 points)

5. Prove that If $A(n) = a_m n^m + \ldots + a_1 n + a_0$ is a polynomial of degree m then $A(n) = O(n^m)$. (5 points)

6. Prove that any binary tree of height h has at most 2^h leaves. (5 points)

7. Write an algorithm to calculate the Fibonacci number F_n in time $\Theta(lg\ n)$ and prove its correctness. (Hint: Use Recursive squaring) (10 points)

8. Consider the following graph, given by its adjacency list, and followed by edge weights.

   aList[1] = [2,3], weights: 6, 12
   aList[2] = [1,3,5,6], weights: 6, 5, 14, 8
   aList[3] = [1,2,4,7], weights: 12, 5, 9, 7
   aList[4] = [3], weights: 9
   aList[5] = [2,6], weights: 14, 3
   aList[6] = [2,5,7], weights: 8, 3, 10
   aList[7] = [3,6,8], weights: 7, 10, 15
   aList[8] = [7], weights: 15

   (a) Draw the equivalent graph. (2 points)

   (b) Write its equivalent adjacency matrix. (2 points)

   (c) What is the shortest path between node 3 and node 5. (2 points)

(d) What is the degree of node 1 and node 3. (2 points)

(e) List two spanning trees. (2 points)

(f) Is the graph connected? (2 points)

(g) Simulate the Prim's algorithm on that graph to find a minimum spanning tree. (8 points)

9. Given two sequences X and Y, write a formula to calculate the length of a longest-common subsequence of X and Y in terms of their prefixes' lengths. Then simulate the Dynamic Programming algorithm on computing all Longest Common Subsequence of the two sequences $ABCBDAB$ and $BDCABA$. (8 points)

Benha University
Faculty of Science
Computer Science Section

Subject: Algorithms
Year: 4th, Jan. 2011
Time: 2 hours

Solve as much as you can. (Total Grade: 80 points)

1. Determine the frequency counts for all statements in the following two algorithm segments: (6 points)

```
for i = 1 to n do
    for j = 1 to i do
        for k = 1 to j do
            x = x + 1
```

```
i = 1
while i ≤ n do
    x = x + 1
    i = i + 1
```

2. Given the recurrence $T(n) = T(n/4) + T(n/2) + n^2$. Prove that $T(n) = \Theta(n^2)$ by using Recursion-tree method. (10 points)

3. Write the set definition of the O-notation. What the meaning of $n^2 + O(n) = O(n^2)$. (6 points)

4. Write the pseudocode of Insertion Sort and prove that its worst-case time is $\Theta(n^2)$. Then simulate the Insertion Sort algorithm on the following data array: $A(1:10) = \{2, 10, 1, 20, 15, 30, 1, 19, 25, 45\}$. Note: Sort in decreasing order. (20 points)

5. Prove that any binary tree of height h has at most 2^h leaves. (10 points)

6. Given two arrays $A(1:n)$ and $B(1:m)$ of numbers sorted in nondecreasing order. Write an algorithm which merges them into an array $C(1:n+m)$ in time $O(n+m)$, where the numbers in C are sorted in nondecreasing order. (10 points)

7. Write an algorithm to calculate the Fibonacci number F_n in time $\Theta(lg\ n)$ and prove its correctness. (Hint: Use Recursive squaring) (15 points)

8. Given a graph $G = (V, E)$, Let T be a Minimum Spanning Tree (MST) of G, and let $A \subsetneq V$. Prove that if $(u, v) \in E$ is the least-weight edge connecting A to $V \setminus A$. Then, $(u, v) \in T$. (10 points)

9. Discus the two key ingredients that a problem must have for dynamic programming to be a viable solution technique. (6 points)

10. Given two sequences X and Y, write a formula to calculate the length of a longest-common subsequence of X and Y in terms of their prefixes' lengths. Then simulate the Dynamic Programming algorithm on computing all Longest Common Subsequence of the two sequences $ABCBDAB$ and $BDCABA$. (10 points)

Benha University	Fina Exam, March. 2011
Faculty of Computer & Informatics	3-rd Year
Subject: ALgorithms	Time: 3 hours

Solve as much as you can questions.

1. Given a graph $G = (V, E)$, Let T be a Minimum Spanning Tree (MST) of G, and let $A \subseteq V$. Prove that if $(u, v) \in E$ is the least-weight edge connecting A to $V \setminus A$. Then, $(u, v) \in T$.

2. Define the big oh (O) notation and prove that
If $A(n) = a_m n^m + \ldots + a_1 n + a_0$ is a polynomial of degree m then $A(n) = o(n^m)$. What the meaning of $n^2 + O(n) = O(n^2)$.

3. Given two arrays $A(1 : n)$ and $B(1 : m)$ of numbers sorted in nondecreasing order. Write an algorithm which merges them into an array $C(1 : n + m)$ in time $O(n + m)$, where the numbers in C are sorted in nondecreasing order.

4. Write an algorithm to calculate the Fibonacci number F_n in time $\Theta(lg\ n)$ and prove its correctness. (Hint: Use Recursive squaring)

5. Write the pseudocode of insertion sort and prove that its worst-case time is $\Theta(n^2)$.

6. For the following pair of functions determine the smallest integer value of $n \geq 0$ for which the first function becomes greater than or equal to the second function.
(a) n^2, $10n$
(b) $2n$, $2n^3$

7. Given two sequences X and Y , write a formula to calculate the length of a longest-common subsequence of X and Y in terms of their prefixes' lengths. Then simulate the Dynamic Programming algorithm on computing all Longest Common Subsequence of the two sequences $ABCBDAB$ and $BDCABA$.

Benha University **Final Exam: 3rd year**
Faculty of Computers and Informatics
Subject: Algorithms **First semester (2008-2009)**

Solve 4 questions.

1. Solve each of the following

   (a) Define: Optimization problem, complete and full binary trees, spanning tree, min-heap structure, hight and depth of the tree, Adjacency matrix representation of a graph $G = (V_G, E_G)$. (12 points)

   (b) Define the big oh (o) notation and prove that
   If $A(n) = a_m n^m + \ldots + a_1 n + a_0$ is a polynomial of degree m then $A(n) = o(n^m)$. (8 points)

2. Define binary trees and prove that (20 points)

   (a) Any binary tree of height h has at most 2^h leaves.

   (b) The maximum number of nodes on level i is 2^{i-1}.

3. Given a sequence of numbers. Show how the Divide and Conquer technique is used to sort this sequence in decreasing order. Analyze the resulting algorithm. (20 points)

4. Simulate the HEAPSORT algorithm on the data array
   $A(1:8) = \{20, 1, 9, 10, 5, 25, 13, 13\}$. Note: Sort in decreasing order. (20 points)

5. Given n sorted sequences $X_1, \ldots, X_n$. These sequences can be recursively merged together in pairs to obtain one sorted sequence. Show how the greedy technique is used to obtain the optimal 2-way merge pattern. What is the time bound of the resulting algorithm. (20 points)

Faculty of Science-Tanta Final Exam.-January 2004

Department of Mathematics 3-rd Year( )

Subject: Algorithms and Number System

Solve 4 questions.

1. The Fibonacci sequence is given as $1, 1, 2, 3, 5, 8, 13, \ldots$. Give its recursive definition and write a recursive algorithm which returns the nth Fibonacci number.

2. Define binary trees and prove that

   (a) any binary tree of height h has at most 2^h leaves.

   (b) The maximum number of nodes on level i is 2^{i-1}.

3. Simulate the HEAPSORT algorithm on the data array $A(1:8) = \{20, 1, 9, 10, 5, 25, 13, 13\}$. Note: Sort in decreasing order.

4. Write an algorithm which searches an array $A(1:n)$ for the element x. If x occurs, then set j to its position in the array else set j to zero.

5. Define optimization problems. Show how the greedy technique works for them.