Q1: what is the meaning of an Algorithm?

An *algorithm* is a sequence of unambiguous instructions for solving a problem, i.e., for obtaining a required output for any legitimate input in a finite amount of time.

Q2: For the following pair of functions, determine the smallest integer value of n≥0 for which the first function becomed greater than or equal to the second function.

A: n^2 , $10n$
B: 2^n , $2n^3$

A:

n	10n	n^2
0	0	0
1	10	1
2	20	4
3	30	9
4	40	16
5	50	25
6	60	36
7	70	49
8	80	64
9	90	81
10	100	100
11	110	121
12	120	144

B:

n	2^n	$2n^3$
0	1	0
1	2	2
2	4	16
3	8	54
4	16	128
5	32	250
6	64	432
7	128	686
8	265	1024
9	512	1458
10	1024	2000
11	2048	2662
12	4096	3456
13	8192	4394
14	16384	5488

n = 10   n = 12

Q3: analyze the Time Complexity for the following algorithm:

```
i = 1
      Loop (i <= 10)
            j=1
            Loop (j <= i)
                  k=1
                  Loop (k<= j)
                        Application code
                        k= k+1
            j= j+1
      i= i+1
```

method 1

Outer loop:
Outer loop iterations = n times

First inner loop : the inner loop is dependent on the outer loop for one of its factors, the number of iterations in the body of the inner loop is 1+2+3+4+……+8+9+10 = 55 , the average of this loop is 5.5 (55/10), this can be written as (N+1)/2.

Second inner loop : the inner loop is dependent on the first inner loop for one of its factors, the number of iterations in the body of the inner loop is 1+3+6+10+……+36+45+55 = 220 , the average of this loop is 22 (220/10), this can be written as (N+1)*2.

#Iterations = #loop iterations * #inner loop iterations

$T(N) = n * ((N+1)/2) * ((N+1)*2)$
$T(N) = n^3 + n^2 + 2n$

method 2

$$= \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{i} j$$

$$= \sum_{i=1}^{n} \frac{i(i+1)}{2}$$

$$= \frac{1}{2} \left(\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i \right)$$

$$= \frac{1}{2} \left(\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right)$$

$$= \frac{1}{12} \left(n(n+1)(2n+1) + 3n(n+1) \right)$$

$$= \frac{n(n+1)}{12} (2n+4)$$

$$= \frac{1}{6} n(n+1)(n+2)$$

T(N) = (n³ + 3n² +2n) / 6

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method 3

T(N) = (n³ + 3n² +2n) / 6