Revisiting Supernova Project data

Mohamed Elashri

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Abstract

In this project, we are going to analyze the supernova data-set to extract important physical quantities of great interest which are Hubble's constant and percentage of matter vs dark energy densities in the universe. We also investigate different cosmological models to see which model fits best the data. We are going to use data analysis tools and fit techniques to study this data and produce graphs represent the physics beyond our data set. We have a data set of supernova survey project which contains the apparent magnitude and red shifts of the large set of Type Ia supernovas which acted as standard candles. From these data we run statistical analysis to measure some of the important cosmological parameters which are Hubble constant and density of matter and dark energy in the universe. We test different model to fit the data and the significance of the parameters measurement in different data ranges.

1 Theoretical Background

Supernova type Ia are standard candles (or can be made so), so can be used to measure the contents of the Universe . Standard candles are a class of astrophysical objects, such as supernovae or variable stars, which have known luminosity due to some characteristic quality possessed by the entire class of objects.

In cosmology, a given cosmology model, with a given set of density parameters, σ_i which are the densities of universe components , will give us a function $D_L(z)$. This function is called luminosity distance which can be written based on some physical reasoning in terms of red-shifts as follows

$$D_L(z) = (1 + z)x \tag{1}$$

The idea behind using SN data is that we know from local measurements that the luminosity of SNe have a well known time dependence: They rise, reach a maximum luminosity L_{max}. then fade back again. L_{max} is found to be the same for all SNe (up to some corrections,) which make SNe good candidates to be considered standard candles. Therefore, if we measure the red-shift z of a SN, then we can predict its flux $F = \frac{L}{4\pi\ D_L^2(z|\sigma i)}$ and compare it with the measured flux. If they match for many SNe in a range of redshifts, then we know we have chosen the right density parameters σ_i. The question then becomes to quantify the phrase "if they match." And that is the

use of statistics! We will use the χ^2 statistics to quantify how closely our data match with our model.
We are also interested in estimating the matter and dark energy density parameters based on this real data. also we want to model data and see the best fit model.

2 analysis and discussion

In this project, using supernova project data [1] we Incorporated the data analysis techniques of minimization of χ^2 to test models and how this statistically affects the physics of interest. we obtained the data then we try to read in in a useful form. we have a range of red shift values from 0.01 to around 1.5 and a much wider range for distance modulus m-M. So we used vertical log plot for convenient representation for data which shown in plot 1 (with error bars from error estimation in distance modulus included in data).

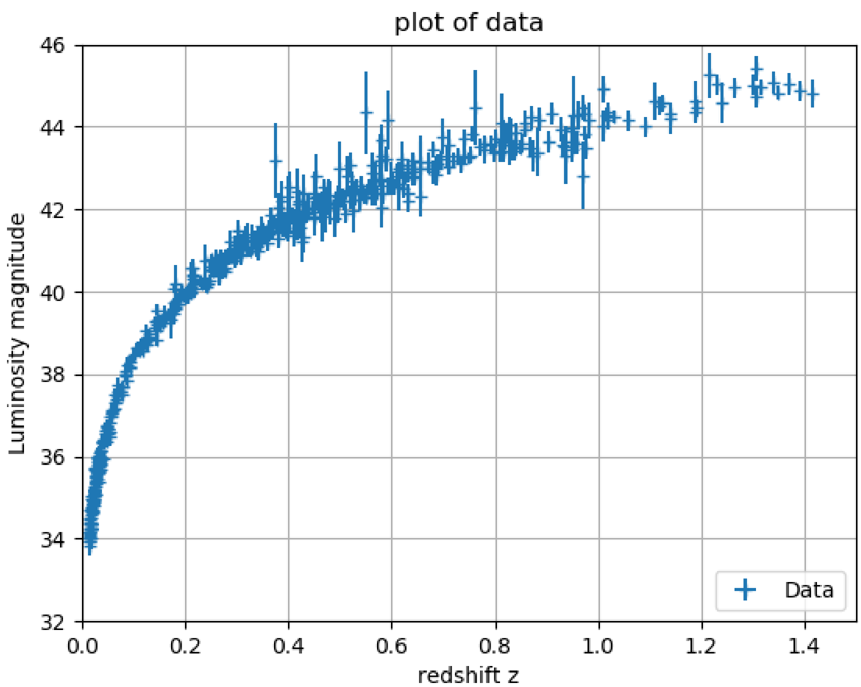

Figure 1: Plot of distance modulus vs red-shifts for our data

Now we want to test our theoretical models against this data, based on the theoretical background discussion for the relation between modulus distance and luminosity distance which have several models to calculate in terms of red shift z. we are test three different models in this work.

$$D_1 = \frac{cz}{H_0}$$
$$D_2 = \frac{cz}{H_0}\sqrt{1+z}$$
$$D_3 = \frac{cz\,(\sqrt{1+z})}{H_0\,\sqrt{\Omega_m(1+z)^3\,\Omega_L}(1+z)}\,dz$$

Now we have one parameter to estimate in model one (Linear model) and model two (Non Linear model) which is the Hubble's constant H_0 and two parameters in Model three (FRW model) which is the matter and dark energy densities in the universe Ω_m and Ω_L respectively. We will use the H_0 estimation from linear and non-linear models when we are working on FRW model. The reason for that is shown in figure 2 , this plot is a fit plot for the three models against the data and we see that for small z, the three models fit the data and working fine so we can get estimation for H_0 from this range. Tables 1 and 2 summarize the values for our estimation based on different red-shifts ranges.

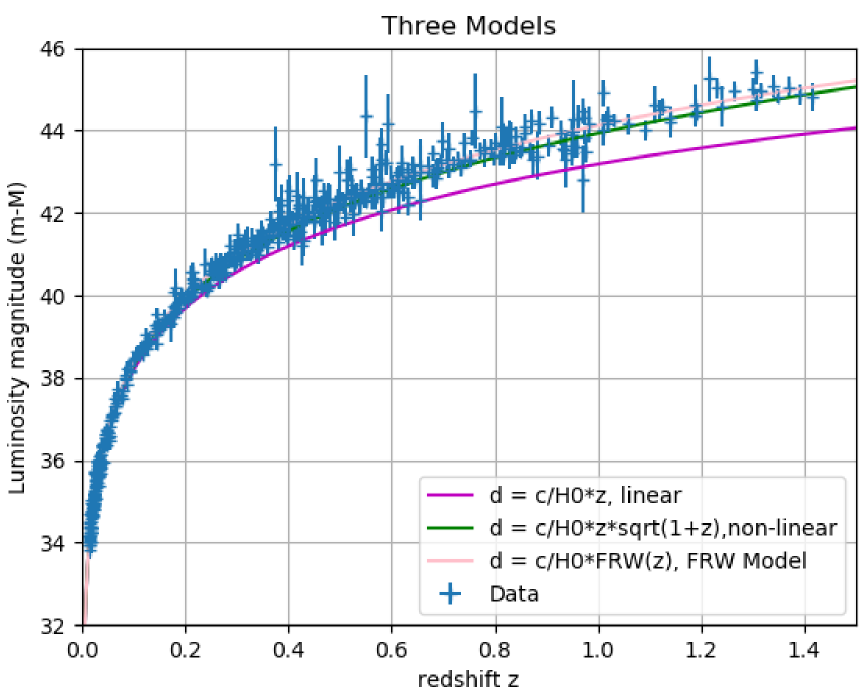

Figure 2: Plot of distance modulus vs red-shifts for our data with the three models

Table 1: Table 1 For Linear model

redshift range	H_0	χ^2
z < 0.05	68.394	4.043
z < 0.09	68.288	4.005
z < 0.15	67.557	3.756
z < 0.2	66.961	3.569
z < 0.25	66.188	3.353

Table 2: Table 2 For Non-linear model

redshift range	H_0	χ^2
z < 0.05	69.322	1.051
z < 0.09	69.463	1.045
z < 0.15	69.119	1.031
z < 0.2	68.973	1.018
z < 0.25	68.973	0.995

Based on these values, we see that the value of Hubble's constant decreases for both models when we increase the range of red shift that we include in the calculation. We also see from the the second figure that the best place is the range 0 to 0.05 red shift so we will use the value for this range in our estimation. We also notice with increasing z the Hubble's constant in the linear model begin to decrease more quickly than non-linear one. This is consistent because we see from the plot that the non-linear model has a larger range of fit for data than the linear model. So when we go a little beyond the first few points we are getting a worse estimation for both models but the non-linear model is better in this case. But we can stick to the first few points as they reveals the same physics of Hubble's constant as they are from nearby supernovas. We see that Linear model has a χ^2 decreasing more than the non-linear model (which has smaller value overall) which reveals the fact that the non-linear is a better fit for these ranges.

The figures 3 and 4 gives shows the fit and Hubble's constant values for the linear and non-linear models. We also try to using interpolation to calculate the χ^2 for this two models. Although that we don't see the linear model of great interest we expect to get high χ^2 which is true and we get a value of 4.043 at the range 0 to 0.05 z.

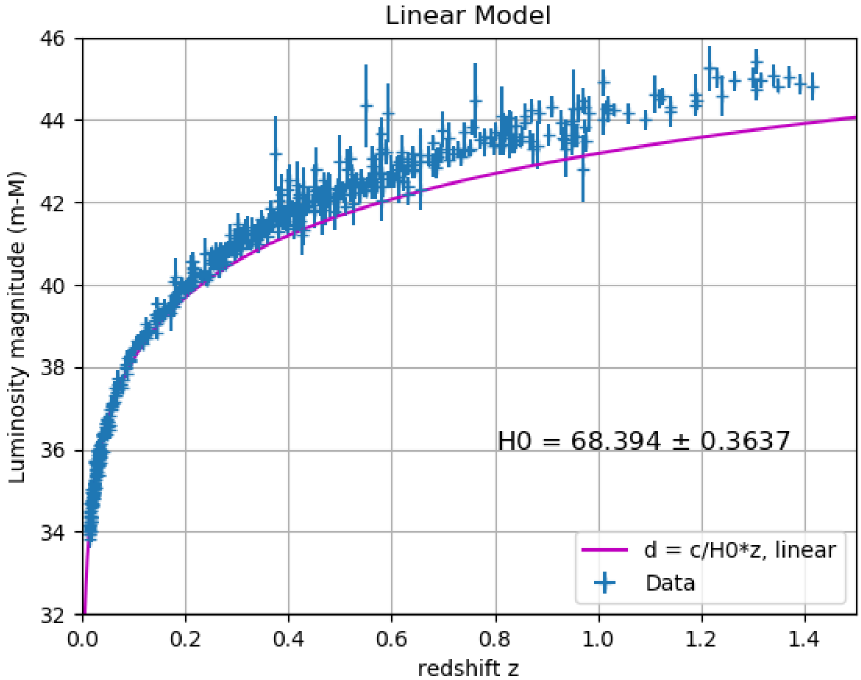

Figure 3: Linear Model

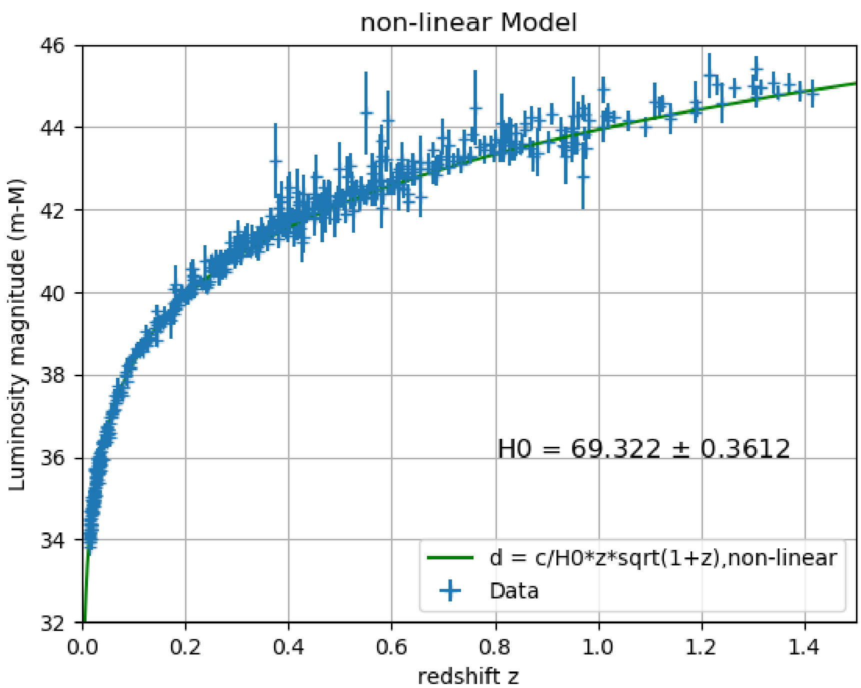

Figure 4: Non-linear Model

Now we are interesting in FRW model which are the general equation of luminosity distance in standard cosmology theory. We are going to minimize the χ^2 to estimate the density ratio assuming that we only have matter with dark energy (flat universe). So our basic assumption is $\Omega_m + \Omega_L = 1$. So this reduces the problem to estimating only one parameter which will be Ω_m and then we can get the value of Ω_L using our constrain. by interpolating our model and define χ_2 we are going to plot the χ_2 per degree of freedom against Ω_m to see which value of the matter density corresponding to minimum χ^2 instead of doing it by iteration and put it in table (we have too many values actually). Doing this will give us figure 5.

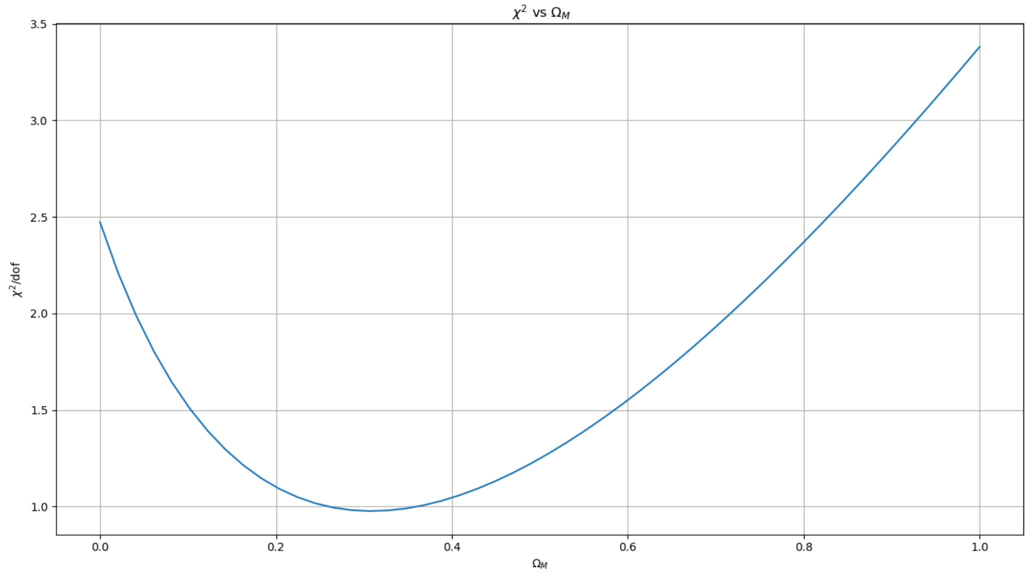

Figure 5: Plot of χ^2 vs Ω_m

The minimum χ^2 value is 0.9768 which can be read directly from this graph or the plot software can help read it off. This value corresponding to Ω_m of 0.3093 and Ω_L of 0.6907. These values are consistent with the other measurements [2,3]

The last thing in this analysis is that we want to plot a contour plot for Ω_m - Ω_L plane which contain the χ^2 confidence level. This is important for physical interpretation. this puts the limits of the models and under the assumption of big bang theory with a flat universe we can get information about the structure of the universe. This shows up in figure 6.

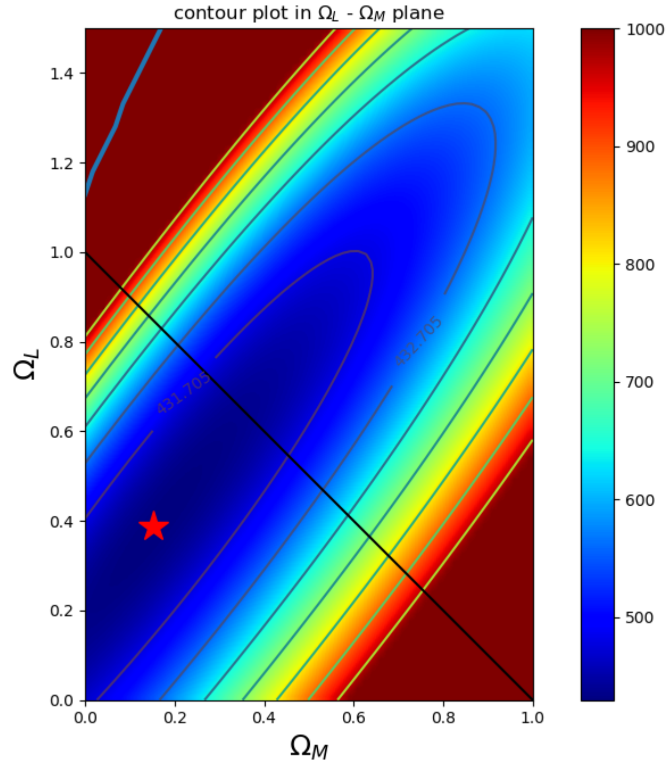

Figure 6: contour plot in Ω_m - Ω_L plane

figure 6 shows that the confidence intervals in the Ω_m - Ω_L plane are consistent with the other work for the supernova project [4] and High-z supernova [5] teams. We had somehow a tighter constrains than the High-z supernova project. But from this figure we can rule out the possibility for the case of open universe where (Ω_m,Ω_L) = (1,0). because we had the confidence levels rule out this. However we should mention that in this analysis there are main sources of systematic error that we should consider when discussing how significance this result. We had estimated the Hubble's constant using the low z values. But in reality there are intrinsic differences between type Ia supernova at high and low red-shifts. with taking into account this we estimated that this effect does not make a significance effect as we saw in table 2 for non-linear model. we have used the estimation for Hubble's constant from this model as it has the least χ^2 change across the red shift different ranges.

There are also some problems that need more time to be done, is the statistical study of correlation between the properties of high and low red-shifts supernova and our parameter estimation. Although we find the effect is small but for a larger set of data we need to understand this effect more. Also we can incorporate models that contain curvature of universe so there is another Ω_k but this lead to a less constrained conditions for parameter estimation that we could not handle in this short period of time.

References

[1] http://supernova.lbl.gov/union/

[2] arXiv:1303.5062 [astro-ph.CO]

[3] arXiv:1807.06209 [astro-ph.CO]

[4] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Nature 391, 51 (1998) [astro-ph/9712212]

[5] A.G. Riess et al. [Hi-Z Supernova Team Collaboration], Astron. Journ. 116, 1009 (1998) [astro-ph/9805201].