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M Novello[1], E Goulart[1], J M Salim[1] and S E Perez Bergliaffa[2]

[1] Instituto de Cosmologia Relatividade Astrofisica (ICRA-Brasil/CBPF), Rua Dr Xavier Sigaud, 150, CEP 22290-180, Rio de Janeiro, Brazil
[2] Departamento de Fisica Teorica, Universidade do Estado do Rio de Janeiro, R. São Francisco Xavier, 524, Maracanã, CEP 20559-900, Rio de Janeiro, Brazil

Abstract
It will be shown that a given realization of nonlinear electrodynamics, used as a source of Einstein's equations, generates a cosmological model with interesting features, namely a phase of current cosmic acceleration, and the absence of an initial singularity, thus pointing to a way of solving two important problems in cosmology.

PACS number: 98.80.$-$k

1. Introduction

In spite of the success of the standard cosmological model (SCM), several problems are still to be solved, most notably the cause of the current acceleration stage, and the initial singularity. The observation that the universe is undergoing a phase of accelerated expansion has sparked intense activity towards investigating the possible candidates that fuel the acceleration. One possibility is the modification of the gravitational action by the addition to the Einstein–Hilbert action of terms that depend on some power of the curvature. The simplest of these models [1] with action given by

$$S = \frac{M_{\text{Pl}}^2}{2} \int \sqrt{-g} \left(R - \frac{\alpha^4}{R} \right) \mathrm{d}^4 x,$$

has been shown to disagree with solar system observations [2]. Later it was proved that more general models [3, 4], such as those given by the Lagrangian

$$\mathcal{L} = \sum_n c_n R^n, \tag{1}$$

can describe a phase of accelerated expansion (controlled by the negative powers of n) and produce other modifications in a strong regime (where terms with $n > 0$ are dominant).

A second possibility relates the acceleration to the matter sector of the theory, either by incorporating a cosmological constant or by postulating the existence of matter fields with peculiar properties. In the latter case, the most popular choice is a scalar field in the presence

of a potential [5], or with a nontrivial Lagrangian (see [6] and references therein). Yet another possibility is a vector field, as discussed in [7]. Although the addition of these fields to the action yields acceleration (and in some cases dark matter as a byproduct [6]), the need of (yet to be observed) matter with unusual properties is certainly a hindrance.

Another problem in the description furnished by the SCM is that of the initial singularity. It is a well known fact that under certain assumptions, the SCM unavoidably leads to a singular behaviour of the curvature invariants in what has been termed the Big Bang. This is a highly distressing state of affairs, because the presence of a singularity entails the breakdown of all known physical theories. There are hopes that quantum gravity will solve this deficiency but, since this theory is still incomplete, one is obliged to explore alternative routes in order to avoid the initial singularity.

We shall show in this paper that nonlinear electrodynamics (NLED) can be useful in the discussion of possible solutions to the two problems of the SCM. Regarding the accelerated expansion, an alternative to the two possibilities presented above was given in the model introduced in [9], where the action for the electromagnetic field was modified by the addition of a new term, namely

$$S = \int \sqrt{-g} \left(-\frac{F}{4} + \frac{\gamma}{F} \right) \mathrm{d}^4 x, \tag{2}$$

with $F \equiv F_{\mu\nu}F^{\mu\nu}$. Through a suitable averaging process, this action yields an accelerated expansion phase for the evolution of the universe in the weak-EM-field regime. It also correctly describes the electric field of an isolated charge [9] without the need of averaging.

Scenarios that avoid the initial singularity have been intensely studied over the years. As examples of the latest realizations we can quote the pre-big-bang universe [10] and the ekpyrotic universe [11]. While these models are based on deep changes to known physics (involving new entities as scalar fields or branes) the model we present here relies instead on a unique entity, the electromagnetic field. The new ingredient we introduce is a modification of the dynamics, which differs from that of Maxwell in certain regimes. Specifically, the Lagrangian we will work with is given by

$$\mathcal{L} = \alpha F^2 - \frac{1}{4}F + \frac{\gamma}{F}. \tag{3}$$

The dimensional constants α and γ are to be determined by observation. We shall see that in the Friedmann–Lemâitre–Robertson–Walker (FLRW) geometry the first term dominates in very early epochs, the Maxwell term dominates in the radiation era, and the last term is responsible for the accelerated phase.

It will be shown in this paper that the Lagrangian given in equation (3) yields a unified scenario to describe both the acceleration of the universe (for weak fields, as discussed above) and the avoidance of the initial singularity as a consequence of its properties in the strong-field regime (see also [8]). The plan of the paper is as follows. In section 2, we present some consequences of applying the Tolman average procedure to a nonlinear EM field configuration in the FLRW geometry; we describe its energy content and show how nonlinearities change the equation of state of the electromagnetic field. In section 3, we present the notion of magnetic universe, analyse the related energy density and pressure and show the unexpected result that the dependence of the magnetic field on the scale factor $a(t)$ is the same irrespectively of the form of the Lagrangian. Section 4 deals with the conditions of the existence of a bouncing and the acceleration for a generic fluid. In section 5, we present a simple model of a bouncing universe dominated by the nonlinear properties of the averaged magnetic field, as well as a discussion about a cyclic universe and its corresponding properties in this model. Section 6 presents the modifications that nonlinearities of the electromagnetic field in the

weak regime produce for a late phase of the universe. Several interesting topics related to nonlinear electromagnetism (such as the field of a point charge and causality) are discussed in the appendices.

Section 7 presents a complete scenario in which both problems—bouncing and acceleration—are treated in a single and a unified theory.

2. The average procedure and the fluid representation

The effects of a nonlinear electromagnetic theory in a cosmological setting have been studied in several articles [13]. In particular, it has been shown that under certain assumptions (as for instance the existence of a compactified space) a Born–Infeld field as a source can model the accelerated expansion of the universe [13], although this is not possible in the original formulation of the theory, as we will show in the appendix. Note that due to the isotropy of the spatial hyper-surfaces of the FLRW geometry, an average procedure is needed if electromagnetic fields are to act as a source of gravity. Given a generic gauge-independent Lagrangian $\mathcal{L} = \mathcal{L}(F, G)$, written in terms of the two invariants $F \equiv F_{\mu\nu}F^{\mu\nu}$ and $G \equiv F^{\mu\nu}F^*_{\mu\nu}$, the associated energy–momentum tensor, defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}\sqrt{-g}}{\delta g^{\mu\nu}}, \tag{4}$$

reduces to

$$T_{\mu\nu} = -4\mathcal{L}_F F_\mu{}^\alpha F_{\alpha\nu} + (G\mathcal{L}_G - \mathcal{L})g_{\mu\nu}. \tag{5}$$

Following a standard procedure [14] we define the volumetric spatial average of a quantity X at the time t by

$$\overline{X} \equiv \lim_{V \to V_0} \frac{1}{V} \int X\sqrt{-g}\, \mathrm{d}^3 x, \tag{6}$$

where $V = \int \sqrt{-g}\, \mathrm{d}^3 x$ and V_0 is a sufficiently large time-dependent 3-volume. In this notation, for the electromagnetic field to act as a source for the FLRW model we need to impose that

$$\overline{E_i} = 0, \qquad \overline{H_i} = 0, \qquad \overline{E_i H_j} = 0, \tag{7}$$

$$\overline{E_i E_j} = -\tfrac{1}{3}E^2 g_{ij}, \qquad \overline{H_i H_j} = -\tfrac{1}{3}H^2 g_{ij}. \tag{8}$$

With these conditions, the energy–momentum tensor of the EM field associated with $\mathcal{L} = \mathcal{L}(F, G)$ can be written as that of a perfect fluid,

$$T_{\mu\nu} = (\rho + p)v_\mu v_\nu - pg_{\mu\nu}, \tag{9}$$

where

$$\rho = -\mathcal{L} + G\mathcal{L}_G - 4\mathcal{L}_F E^2, \tag{10}$$

$$p = \mathcal{L} - G\mathcal{L}_G - \tfrac{4}{3}(2H^2 - E^2)\mathcal{L}_F, \tag{11}$$

and $\mathcal{L}_A \equiv \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}A}$, $A = F, G$. In the present work, we shall restrict to the case of nonlinear theories defined by $\mathcal{L} = \mathcal{L}(F)$. In such a case, the energy–momentum is diagonal, with

$$\rho = -\mathcal{L} - 4E^2\mathcal{L}_F, \qquad p = \mathcal{L} + \tfrac{4}{3}(E^2 - 2H^2)\mathcal{L}_F. \tag{12}$$

3. Magnetic universe

A particularly interesting case of the scenario outlined in the previous section occurs when only the average of the magnetic part is different from zero. We shall call this case a *magnetic universe*. This situation turns out to be relevant in cosmology, because the electric field is screened by the charged primordial plasma, while the magnetic field lines are frozen [16]. In spite of this fact, we shall devote some attention to the mathematically interesting case in which $E^2 = \sigma^2 H^2 \neq 0$ in appendix A.

Before discussing the magnetic universe, we shall show that when the dynamics of the EM field is given by the following series

$$\mathcal{L} = \sum_k c_k F^k, \tag{13}$$

(where k takes values on the integers) the fluid can be interpreted as composed of k non-interacting fluids. In order to show that this property is indeed valid, we shall work with the standard form of the FLRW geometry in Gaussian coordinates

$$ds^2 = dt^2 - a(t)^2 (dr^2 + \chi^2 \, d\Omega^2). \tag{14}$$

The conservation of the energy–momentum tensor projected in the direction of the co-moving velocity $v^\mu = \delta_0^\mu$ yields

$$\dot{\rho} + (\rho + p)\theta = 0, \tag{15}$$

where $\theta \equiv v^\mu_{;\mu} = 3\frac{\dot{a}}{a}$.

Using the values of energy density and pressure given in equation (12) along with equation (13) we obtain that

$$\rho = \sum_k \rho_k, \qquad p = \sum_k p_k, \tag{16}$$

where, for the magnetic universe, we have

$$\rho_k = -c_k 2^k H^{2k}, \qquad p_k = c_k 2^k H^{2k} \left(1 - \frac{4k}{3}\right). \tag{17}$$

From the conservation equation (15) we obtain

$$\mathcal{L}_F \left[(H^2)\dot{} + 4H^2 \frac{\dot{a}}{a} \right] = 0. \tag{18}$$

The important result that follows from this equation is that the dependence on the specific form of the Lagrangian appears as a multiplicative factor. This property allows us to obtain the dependence of the field with the scale factor independently of the particular form of the Lagrangian, since equation (18) yields

$$H = \frac{H_0}{a^2}. \tag{19}$$

This property implies that it is possible to associate with each power k an independent fluid with energy density ρ_k and pressure p_k, in such a way that the corresponding equation of state, which is preserved throughout the expansion of the universe, is given by

$$p_k = \left(\frac{4k}{3} - 1\right) \rho_k. \tag{20}$$

Before proceeding to examine the consequences of these fluids in the expansion of the universe, let us turn to the analysis of the conditions needed for a bounce and for an accelerated expansion.

We shall begin with the acceleration. From Einstein's equations, the acceleration of the FRWL universe is related to its matter content by

$$3\frac{\ddot{a}}{a} = -\frac{1}{2}(\rho + 3p).$$ (21)

Thus, in order to have an accelerated expansion, the source must satisfy the constraint $(\rho + 3p) < 0$. In terms of the quantities defined in equation (12), this constraint translates into

$$\mathcal{L}_F > \frac{\mathcal{L}}{4H^2}.$$ (22)

Any nonlinear electromagnetic theory that admits a configuration satisfying this inequality will yield accelerated expansion. Referring now to the particular Lagrangian given in equations (13), (20) shows that if the k-component with $k \leqslant 0$ dominates the evolution of the universe, there will be accelerated expansion (assuming that $\rho_k > 0$).

The conditions required to have a bouncing universe follow from Raychaudhuri's equation, which states that the expansion factor must obey the equation

$$\dot{\theta} + \tfrac{1}{3}\theta^2 = \tfrac{1}{2}(\rho + 3p).$$ (23)

In addition to this equation, the energy content of the fluid must satisfy the constraint

$$\rho = \frac{1}{3}\theta^2 + \frac{3\epsilon}{a^2},$$ (24)

which comes from the $0 - 0$ component of Einstein equation in the FLRW framework, and $\epsilon = 0, \pm 1$. Thus two conditions must be fulfilled for the bounce to be a minimum of the scale factor $(\rho + 3p)_b > 0$ and $\rho_b = \frac{3\epsilon}{a_b^2}$.[3]

Having established the conditions for accelerated expansion and for a bounce, we shall show next that the magnetic universe with the Lagrangian given in equation (3) satisfies both conditions.

4. A bouncing universe

Let us briefly review the scenario proposed in [8], where a nonsingular universe was presented, based on the Lagrangian

$$L = -\tfrac{1}{4}F + \alpha F^2,$$ (25)

where α has the dimension of (length)^4. Note that Maxwell's electrodynamics can be obtained from equation (25) by setting $\alpha = 0$, or recovered in the limit of small fields. Using the average process introduced in section (2), it follows that

$$\rho = \tfrac{1}{2}H^2(1 - 8\alpha H^2),$$ (26)

$$p = \tfrac{1}{6}H^2(1 - 40\alpha H^2).$$ (27)

It was shown in section (3) that $H = H_0/a^2$, where H_0 is a constant. Inserting this result into Friedmann's equation through equation (26) leads to

$$\dot{a}^2 = \frac{H_0^2}{6a^2}\left(1 - \frac{8\alpha H_0^2}{a^4}\right) - \epsilon.$$ (28)

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[3] We assume here that the cosmological constant is zero. See [17] for the analysis with $\Lambda \neq 0$.

Since the right-hand side of this equation cannot be negative, it follows that, regardless of the value of ϵ, for $\alpha > 0$ the scale-factor $a(t)$ cannot be arbitrarily small. The solution of equation (28) is implicitly given as

$$t = \pm \int_{a_0}^{a(t)} \frac{\mathrm{d}z}{\sqrt{\frac{H_0^2}{6z^2} - \frac{8\alpha H_0^4}{6z^6} - \epsilon}}, \tag{29}$$

where $a(0) = a_0$. For the Euclidean section, equation (29) can be easily solved to yield

$$a^2 = H_0\sqrt{\tfrac{2}{3}t^2 + 8\alpha}. \tag{30}$$

An upper bound for α can be derived by imposing that the expression for the scale factor coincides with that of radiation in the standard cosmological model at the time of nucleosynthesis, thus yielding $\alpha \ll 10^{-3}$ seg^2.

From equation (19), the average strength of the magnetic field H evolves with time as

$$H^2 = \frac{3}{2} \frac{1}{t^2 + 12\alpha}. \tag{31}$$

Expression (30) is singular for $\alpha < 0$, since there exists a time $t = \sqrt{-12\alpha}$ for which $a(t)$ is arbitrarily small. For $\alpha > 0$ the radius of the universe attains a minimum value at $t = 0$, given by

$$a_{\min}^2 = H_0\sqrt{8\alpha}. \tag{32}$$

Therefore, the actual value of $a_{\min}$ depends on H_0, which turns out to be the sole free parameter of this model. The energy density ρ given by equation (26) reaches its maximum value $\rho_{\max} = 1/64\alpha$ at the instant $t = t_0$, where

$$t_0 = \sqrt{12\alpha}. \tag{33}$$

For smaller values of t, the energy density decreases, vanishing at $t = 0$, while the pressure becomes negative. Only for values of t such that $t \lesssim \sqrt{4\alpha}$, the nonlinear effects are relevant for the solution. Note that the solution given in equation (30) reduces to that corresponding to Maxwell's Lagrangian, namely the scale factor for the radiation era, for large t.

Next we shall show that the Lagrangian given in equation (25) yields a cyclic universe. The qualitative properties of the evolution of the scale factor can be obtained by interpreting equation (28) as describing the motion of a particle with constant energy $-\epsilon$ and position $a(t)$ under the influence of the potential V given by

$$V = -\frac{H_0^2}{6a^2}\left(1 - \frac{8\alpha H_0^2}{a^4}\right). \tag{34}$$

It follows from the analysis of V that the scale factor $a(t)$ bounces between its minimum $a_b(t)$ and its maximum $a_B(t)$ which are provided by the two real solutions of the cubic equation

$$\epsilon x^3 - \frac{H_o^2}{6}x^2 + \frac{4}{3}\alpha H_0^4 = 0, \tag{35}$$

where $x \equiv a^2$.

To close, let us add that the cosmological perturbations of the model discussed in this section were shown to be stable in [15].

5. An accelerated universe

As discussed in the previous section, a positive power of the invariant F in the Lagrangian yields a nonsingular FLRW cosmology. In fact, were higher values of k present, they would dominate the early evolution. Let us see what happens when there are negative powers of F in the Lagrangian by analysing the example given in equation (3) [9]. We restrict our analysis to the magnetic universe and set $\overline{E^2} = 0$, with a residual magnetic field, for the reasons already explained. In this case,

$$\rho = \frac{H^2}{2} + \frac{\mu^8}{2} \frac{1}{H^2},$$

where we have set $\gamma \equiv -\mu^8$.[4] Since $H = H_0/a^2$, the evolution of the density with the scale factor is

$$\rho = \frac{H_0^2}{2} \frac{1}{a^4} + \frac{\mu^8}{2H_0^2} a^4. \tag{36}$$

For small a it is the ordinary radiation term that dominates. The $1/F$ term takes over only after $a = \sqrt{H_0}/\mu$, and grows without bound afterwards. In fact, the curvature scalar is

$$R = T^\mu_\mu = \rho - 3p = \frac{4\mu^8}{H_0^2} a^4.$$

Using the density given in equation (36) equation (21) gives

$$3\frac{\ddot{a}}{a} + \frac{H_0^2}{2} \frac{1}{a^4} - \frac{3}{2} \frac{\mu^8}{H_0^2} a^4 = 0.$$

To get a regime of accelerated expansion, we must have

$$\frac{H_0^2}{a^4} - 3\frac{\mu^8}{H_0^2} a^4 < 0,$$

which implies that the universe will accelerate for $a > a_c$, with[5]

$$a_c = \left(\frac{H_0^4}{3\mu^8}\right)^{1/8}.$$

The examples of positive and negative powers of F analysed in the two previous sections suggest that it is worth analysing a complete scenario constructed by combining these cases in a single phenomenological scenario. This will be done in the next section.

6. A complete scenario

We shall analyse here the evolution of the toy model generated by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F + \alpha F^2 - \frac{\mu^8}{F} \tag{37}$$

with the dependence of the magnetic field on the scale factor given by $H = H_0/a^2$. In figure 1, we plot the Lagrangian as a function of the value of the field. As seen from the plot, the derivative dL/dF has three zeros. In these points $\rho + p$ vanishes. In the case of the pure magnetic universe the value of F is always positive.

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[4] Note that on dimensional grounds, $\gamma = \hbar^2 \times \mu^8$, where μ is a fundamental constant with dimension of length^{-1}, which should be constrained by observation. An approximate value for γ was derived in [9]: $\sqrt{|\gamma|} \approx 4 \times 10^{-28} \text{g cm}^{-3}$.
[5] If instead of considering a toy model, in which the universe goes from the standard radiation era to the NLED phase, we consider the transition from a matter-dominated era to the NLED phase, the result would be $a_c^7 = \Omega_{m0}\rho_{c0}H_0^4/(3\mu^8)$, where Ω_{m0} is the fraction of matter density, and ρ_{c0} is the critical density.
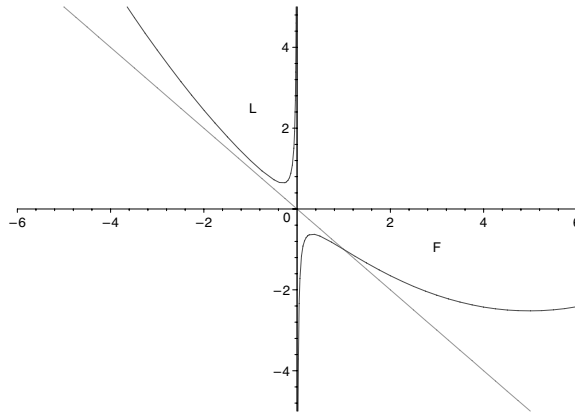
Figure 1. The Lagrangian given in equation (37) as a function of *F*.

As shown in appendix C, the theory described by equation (37) applied to the case of a point charge gives results that are in agreement with observation. In the cosmological case, it implies the existence of three distinct epochs according to the value of the field. In the bouncing era the value of the curvature scalar is small and the volume of the universe attains its minimum. The energy density and the pressure are dominated by the terms coming from the quadratic Lagrangian F^2 and are approximately given by

$$\rho \approx \frac{H^2}{2}(1 - 8\alpha^2 H^2), \qquad p \approx \frac{H^2}{6}(1 - 40\alpha^2 H^2).$$

In the radiative era, the standard Maxwellian term dominates. Due to the dependence on a^{-2} of the field, this phase is defined by $H^2 \gg H^4$ yielding the approximation

$$\rho \approx \frac{H^2}{2}, \qquad p \approx \frac{H^2}{6}.$$

Finally, the accelerating era takes place when the universe becomes large, and the $1/F$ term dominates. In such a case,

$$\rho \approx \frac{1}{2}\frac{\mu^8}{H^2}, \qquad p \approx -\frac{7}{6}\frac{\mu^8}{H^2}.$$

We can analyse the whole evolution using the effective potential, given by

$$V(a) = \frac{A}{a^6} - \frac{B}{a^2} - Ca^6. \tag{38}$$

The constants in $V(a)$ are given by

$$A = 4\alpha H_0^4, \qquad B = \frac{1}{6}H_0^2, \qquad C = \frac{\mu^8}{2H_0^4},$$

and are all positive.

The analysis of $V(a)$ and its derivatives implies solving polynomial equations in a, which can be reduced to cubic equations through the substitution $z = a^4$. The existence and features of the roots of such equations are discussed in [18]. A key point to the analysis is the sign of D, defined as follows. For a general cubic equation
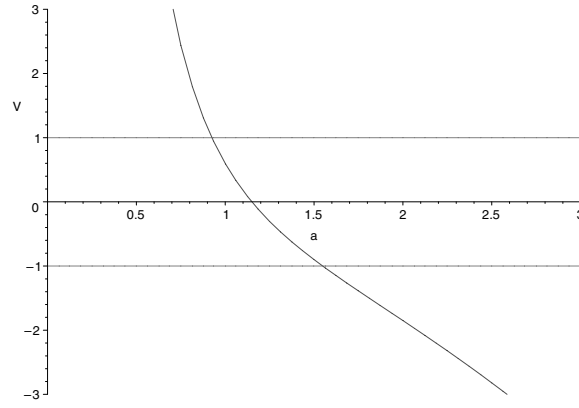
$$x^3 + px = q,$$

Figure 2. Plot of the effective potential for $D_V > 0$, $D_{V'} > 0$.
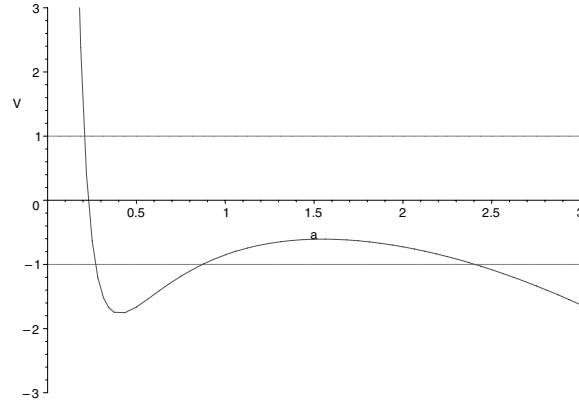

Figure 3. Plot of the effective potential for $D_V > 0$, $D_{V'} = 0$.

the discriminant D is given by

$$D = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2 .$$

We will denote by D_V the discriminant corresponding to the potential and $D_{V'}$ that of the derivative of V. From the behaviour of the potential and its derivatives for $a \to 0$ and $a \to \infty$ we see that only one or three zeros of the potential are allowed. In the case of one zero (described by $D_V > 0$, $D_{V'} > 0$), there must be an inflection point, and the qualitative plot of the potential is given in figure 2. The position of the inflection point depends on the values of the parameters of the model. We see that this model is nonsingular for any value of ϵ, but it does not display a transition from an accelerated regime to a non-accelerated one, so we shall move on to the other possibility (given by $D_V > 0$, $D_{V'} = 0$), displayed in figure 3. This figure shows the qualitative behaviour of the potential for typical values of the parameters. Again, the model is nonsingular for any value of ϵ. The maximum and the minimum of V are always well above the line $\epsilon = 1$ so there is no transition in the acceleration for this value of ϵ. For $\epsilon = 0$ or 1, the model displays a 'coasting period' about the transition point.

7. Causality and NLED

Before going to the concluding remarks, we would like to call the attention of the reader to a salient feature of nonlinear electrodynamics in a cosmological setting. Most of our description of the universe is based on the behaviour of light in a gravitational field. In a FLRW scenario the existence of a horizon inhibits the exchange of information between arbitrary parts of the universe. The observation of the high degree of isotropy of the cosmic microwave background radiation (CMBR) generated a conflict with causality: different parts of the universe in the standard FLRW geometry could not have enough time to homogenize. The inflationary scenario was engineered precisely to solve this problem. But it is worth mentioning that nonlinear theories of electrodynamics also introduce a new study in causality, which we will very briefly overview next.

Let us take as an example the action[6]

$$S = \int \sqrt{-\gamma} L(F) \, \mathrm{d}^4 x \tag{39}$$

(γ is the determinant of the background metric). Using either the traditional perturbation method or the method of the discontinuities devised by Hadamard (see [8] and references therein), it can be shown that nonlinear photons do not move on the light cone of the background metric. Instead they move along the null surfaces of an effective metric given by

$$g^{\mu\nu} = L_F \gamma^{\mu\nu} - 4 L_{FF} F^{\mu\alpha} F_\alpha{}^\nu. \tag{40}$$

Note that this effective metric is generated solely by the self-interaction of the electromagnetic field since the metric coincides with the background geometry when the theory is linear.

For Lagrangians that depend also on F^*, an analogous effective geometry appears, although there may be some special cases in which the propagation coincides with that dictated by the background metric [8]. Another feature of the more general case $L = L(F, F^*)$ is that birefringence is present. That is, the two polarization states of the photon propagate in a different way. In some special cases, there is also bimetricity (one effective metric for each state). In some special cases of nonlinear theories (such as Born–Infeld electrodynamics) a single metric can appear [8].

Let us apply the effective metric to the case in which the electromagnetic field rests on its fundamental state. In the case of the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F - \frac{\mu^8}{F}, \tag{41}$$

the fundamental state has the particular solution

$$F^2 = 4\mu^8,$$

which corresponds to an energy–momentum tensor equivalent to a fluid distribution characterized by the condition $\rho + p = 0$ and generates a de Sitter geometry for the background metric $\gamma_{\mu\nu}$ as seen by all forms of matter and energy content, as long as we neglect the gravitational influence of such remaining matter and energy.

For the Lagrangian given above, the effective metric tensor takes the form:

$$g^{\mu\nu}_{(\mathrm{eff})} = \left(-\frac{1}{4} + \frac{\mu^8}{F^2} \right) g^{\mu\nu} + \frac{8\mu^8}{F^3} F^{\mu\alpha} F_\alpha{}^\nu,$$

which in the case of the fundamental state reduces to

$$g^{\mu\nu}_{(\mathrm{eff})} = \pm \frac{1}{\mu^4} F^{\mu\alpha} F_\alpha{}^\nu.$$

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[6] We could have considered $L = L(F, G)$ instead, where $G \equiv F^*_{\mu\nu} F^{\mu\nu}$. This case is studied in detail in [8].
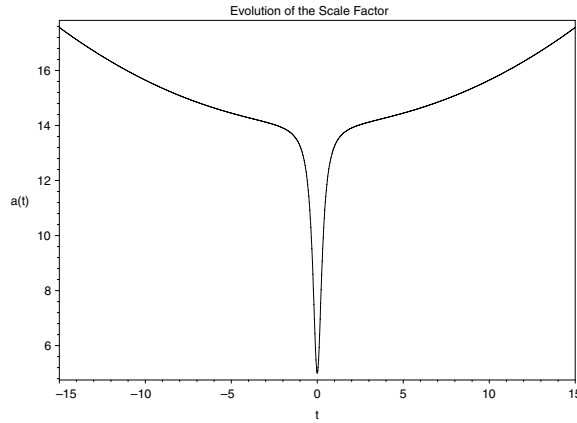
Figure 4. Qualitative plot of the scale factor as a function of t.

This is a very peculiar and interesting situation: the fundamental state of the theory described by the inverse symmetric Lagrangian given in equation (41) generates a de Sitter universe as seen by all existing matter with one exception, namely the photons, which follows geodesics in the above anisotropic geometry $g_{(\text{eff})}^{\mu\nu}$.

8. Concluding remarks

The standard cosmological model furnishes a rather complete picture of our universe, but it does not yet solve some important problems. Among these, the most notable are the initial singularity, and the accelerated expansion. We have shown here that a specific realization of nonlinear electrodynamics yields a unified description of the evolution of the universe, in the sense that with this single matter source we obtain a qualitative description of three of the main phases: bounce (instead of singularity), radiation and acceleration. The description of the matter content with a single field has the advantage, when compared to multi-field models, that in principle less parameters and initial conditions are needed. The above-mentioned features are shown in the plot of the scale factor in terms of the cosmological time (see figure 4). In order to obtain a more accurate description of the time evolution of the scale factor, matter should be added in an intermediate phase between the radiation and the accelerated era. The details will be presented elsewhere, but let us show here that the presence of matter can be used to obtain a relation between the parameters of the theory. The total fractional density of the universe evolves as

$$\Omega = \Omega_{m0} \left(\frac{a_0}{a}\right)^3 + \Omega_{de0} \left(\frac{a_0}{a}\right)^{3(1+\lambda)},$$

where Ω_{m0} is the (dark + normal) fractional matter density, and Ω_{de0} is the fractional dark energy density, both evaluated today. Current observations seem to be consistent with $\Omega_{m0} \approx 0.3$ and $\Omega_{de0} \approx 0.7$ [12]. In our model, dark energy is represented by the NLEM contribution characterized by $p = \lambda\rho$, with $\lambda = -7/3$. Hence from

$$\Omega_{de0} = \frac{\mu^8}{2H_0\rho_{c0}} \approx 0.7,$$

we obtain that $\mu^8 = 1.4H_0^4\rho_{c0}$.

Note that in spite of the fact that the fluid as a whole would have a time-dependent equation of state, only one component of the fluid dominates in each of the epochs, leading to an effective equation of state with a constant parameter.

We have presented here a toy model which shows that nonlinear electrodynamics could be used to build a complete qualitative scenario of the evolution of the universe. We leave the matter of the agreement between the values of the equation of state parameters used here and those that follow from observation to a future publication. Eventual modifications of the model by the addition of new terms in the nonlinear Lagrangian, following the same one-field phenomenological approach used here, will also be considered, if suggested by observation.

Acknowledgments

Appendix A. Almost magnetic universe (AMU)

Although the main goal of this paper is the study of the pure magnetic universe, it is worth making some comments on the case in which there is an electric component such that

$$E^2 = \sigma^2 H^2,$$

where $0 < \sigma^2 < 1$. Let us assume that the Lagrangian is given by the power series in equation (13), and study the relation between the energy density and the pressure. It follows from equations (11) and (13) that

$$\rho = -\sum_k c_k F^k \left(1 + \frac{2\sigma^2}{1 - \sigma^2} k\right), \qquad p = \sum_k c_k F^k \left(1 + \frac{2}{3}\frac{\sigma^2 - 2}{1 - \sigma^2} k\right). \tag{A.1}$$

Setting $p_k = \lambda_k \rho_k$ we obtain

$$\lambda_k = \frac{1}{3}\frac{2k(\sigma^2 - 2) - 3(\sigma^2 - 1)}{\sigma^2 - 1 - 2\sigma^2 k}. \tag{A.2}$$

Thus, the equation of state depends on both the power k and on the partition σ^2, the dependence on these parameters being caused by the nonlinearity of the theory. Note that in the linear case (that is, for $k = 1$) the value of the ratio p/ρ reduces to the σ-independent value $\frac{1}{3}$, as is clear by inspection of equation (A.2). On the other hand, when there is 'equipartition', such that $\sigma^2 = 1$ then, independently of the value of the power k, it follows that $\lambda = \frac{1}{3}$. Note that for $k < 0$ the equipartition is not possible, the value of the partition is restricted to the open domain $0 < \sigma^2 < 1$.

In two special cases (i.e., $k = 1$ or $\sigma^2 = 0$) the dependence of the field with $a(t)$ is the same. In the general case a straightforward calculation reduces the conservation equation, equation (15), in the case in which σ is a constant, to the form

$$\sum_k c_k k F^k \left[\left(1 + \frac{2k\sigma^2}{1 - \sigma^2}\right)\frac{\dot{F}}{F} + 4\frac{1 + \sigma^2}{1 - \sigma^2}\frac{\dot{a}}{a}\right] = 0. \tag{A.3}$$

It follows that different parts of the fluid interact, so it is not possible to treat the fluid as composed by distinct non-interacting parts. The solution of equation (A.3) provides the implicit dependence of the field in terms of the scalar of curvature and the constant σ^2. Let us see this by an example. In the case of

$$\mathcal{L} = -\frac{F}{4} + \frac{\gamma}{F}, \tag{A.4}$$

the solution of equation (A.3) is given by

$$F^{\frac{3\sigma^2-1}{\sigma^2+1}}(F^2 - \mu^8)^{-1}a^{-4} = \text{constant}. \tag{A.5}$$

The energy density takes the form

$$\rho = \frac{\sigma^2+1}{2}H^2 - \frac{3\sigma^2-1}{2(\sigma^2-1)^2}\mu^8 H^{-2}, \tag{A.6}$$

and for the pressure we find

$$p = \frac{\sigma^2+1}{6}H^2 + \frac{5\sigma^2-7}{6(\sigma^2-1)^2}\mu^8 H^{-2}. \tag{A.7}$$

We see from these expressions that although one could separate the fluid in two parts by defining $\rho = \rho_1 + \rho_2$ and $p = p_1 + p_2$ the relations between these quantities depend on the equipartition constant. Indeed, we have

$$p_1 = \tfrac{1}{3}\varrho_1, \tag{A.8}$$

and

$$p_2 = \frac{5\sigma^2-7}{3(1-3\sigma^2)}\varrho_2. \tag{A.9}$$

Thus, there is an interaction between the fluids.

Appendix B. Born–Infeld electrodynamics

Although the final version proposed by Born and Infeld of nonlinear electrodynamics involves both the invariants F and G, in this section we shall use the simplified form $L = L(F)$. In this case,

$$L_{\text{BI}} = \beta^2(1 - \sqrt{X}), \tag{B.1}$$

where

$$X \equiv 1 + \frac{1}{2\beta^2}F. \tag{B.2}$$

Following Born and Infeld, we have added a constant term into the Lagrangian in order to eliminate a sort of cosmological constant and to set the value of the field to zero at infinity. We will come back to this question later on.

Using expressions (12) we obtain

$$\rho = \frac{\beta^2}{\sqrt{X}}\left(1 - \sqrt{X} + \frac{H^2}{\beta^2}\right), \tag{B.3}$$

$$p = \frac{\beta^2}{\sqrt{X}}\left(\sqrt{X} - 1 - \frac{1}{3}\frac{H^2}{\beta^2}\right). \tag{B.4}$$

It follows that

$$\rho + 3p = \frac{2\beta^2}{\sqrt{X}}(\sqrt{X} - 1) \tag{B.5}$$

is a positive-definite quantity. Hence the Born–Infeld Lagrangian defined in equation (B.1) cannot accelerate the universe (see, however, [13]).
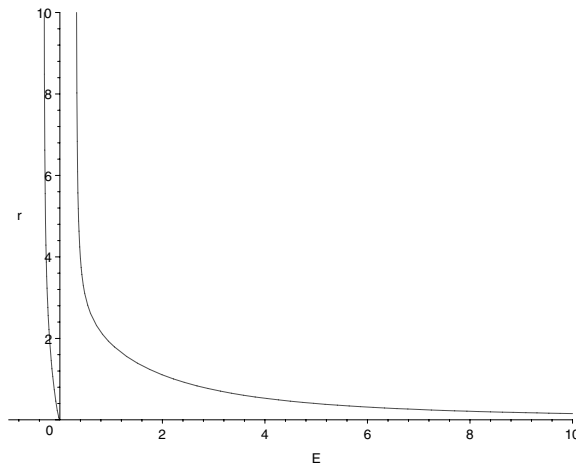
Figure 5. Plot of $r = r(E)$ for a positive charge. The plot for negative q is the mirror image of this plot.

Appendix C. Static and spherically symmetric electromagnetic solution and the asymptotic regime

In this paper, we have presented a modification of Maxwell's electromagnetism that can describe, in a unified way, several phases of the evolution of the universe. The least we can ask of this modification is that it agrees with conventional electromagnetism when the electric field generated by a point-charged particle is considered. For a general nonlinear Lagrangian $L = L(F)$, Maxwell's equation for a point charge reduces to

$$r^2 L_F E(r) = \text{const.} \tag{C.1}$$

In the case of the Lagrangian given in equation (3) we get

$$E(r) \left(4\alpha^2 E(r)^2 - \frac{\gamma}{4E(r)^2} + \frac{1}{4} \right) = \frac{q}{r^2}. \tag{C.2}$$

The polynomial in E that follows from this equation cannot be solved exactly, but to study the dependence of E with r we can plot from equation (C.2) the function $r = r(E)$ (see figure 5). Although the plot displays two branches both for positive and negative q, the unphysical branches can be discarded without further consequences. It also follows from the plot that E goes to a constant value for $r \rightarrow \infty$. By taking derivatives of equation (C.2) it can be shown that the function $E(r)$ has no extrema[7]. Hence, the modulus of the electric field decreases monotonically with increasing r, from an infinite value at the origin to a constant (nonzero but small) value at infinity. Equation (C.2) then shows that $E_\infty = \pm\sqrt{\gamma}$. This situation is akin to that in the theory defined by the action

$$S = \frac{M_P^2}{2} \int \sqrt{-g} \left(R - \frac{\alpha^4}{R} \right) d^4 x.$$

It was shown in [1] that the static and spherically symmetric solution of this theory does not approach Minkowski asymptotically: it tends instead to (anti)-de Sitter spacetime.

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[7] Note that $E = 0$ is not a solution of equation (C.2).

Regarding the behaviour of the field for small values of r, if we compare the term corresponding to Maxwell's case in equation (C.2) with the other two, we get that for the field to be Maxwell-like we need

$$\sqrt{\gamma} \ll E^2 \ll \frac{1}{16\alpha^2}.$$

With the explicit dependence for the field given by $E(r) = q/r^2$, it would be possible to set a value for α in agreement with the observation by

$$\alpha^2 \ll \frac{r_0^4}{16q^2},$$

where r_0 is a reference value set by the experiment.

Let us finish this appendix with two comments. For very weak fields we can neglect the quadratic term in the Lagrangian, and the energy–momentum tensor reduces to the form

$$T_{\mu\nu} = -L\eta_{\mu\nu} - 4L_F F_{\mu\alpha} F_\nu^\alpha, \tag{C.3}$$

which in the case of a point charge ($F_{01} = E(r)$) is

$$T_0^0 = T_1^1 = \frac{1}{2E^2}(E^4 - 3\mu^8), \qquad T_2^2 = T_3^3 = -\frac{1}{2E^2}(E^4 + \mu^8). \tag{C.4}$$

In the asymptotic regime we can set $E \approx \mu^2$, obtaining for the energy–momentum tensor

$$T_0^0 = T_1^1 = T_2^2 = T_3^3 = -\mu^4, \tag{C.5}$$

which has the form of a cosmological constant. By adding an extra term in the Lagrangian (as was done in the case of Born–Infeld) we could eliminate the residual constant field at infinity. Such ambiguity does not arise in the case of Maxwell's electrodynamics due to the linearity of the equations. However, for nonlinear electromagnetic theories, the geometrical structure at infinity is a matter of choice: the field is a constant that can be different from zero. Such a property can be translated in a formal question: what is the asymptotic regime of the geometry of space-time: Minkowski or de Sitter? In linear electrodynamics the answer to that question is unique, but if nonlinearities are present, the possibility of a de Sitter structure must be considered. In theories in which a solution different from zero for the equation $L_F = 0$ exists, such a question has to be investigated combined with cosmology[8].

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