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Nonlinear electrodynamics and cosmology

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Abstract. Nonlinear electrodynamics (NLED) generalizes Maxwell's theory for strong fields. When coupled to general relativity NLED presents interesting features like the non-vanishing of the trace of the energy-momentum tensor that leads to the possibility of violation of some energy conditions and of acting as a repulsive contribution in the Raychaudhuri equation. This theory is worth to study in cosmological and astrophysical situations characterized by strong electromagnetic and gravitational fields.

1. Introduction

Seventy-five years ago Max Born and Leopold Infeld proposed a new theory, a fully relativistic and gauge invariant nonlinear electrodynamics (1934) [1]. The origins of the nonlinear electrodynamics of Born and Infeld (BI) can be traced to the work of Gustav Mie (1909), who made the first attempt to construct a purely electromagnetic theory of charged particles. But the question why a charged particle does not explode under the repulsion of the Coulomb forces acting between its constituents had been asked even before.

The aim of BI was to cure the divergence of the field and energy of the point charge at the charge position. BI proposed a Lagrangian that depended in nonlinear way of the two electromagnetic invariants. It is the simplest possible Lagrangian: the square root of the determinant of a second rank covariant tensor. Besides some reasonable physical requirements and so constructed to preserve the finiteness of fields, the proposal was arbitrary.

The development of quantum electrodynamics (QED) soon after (1936) showed that nonlinear effects arise in the interaction of electromagnetic fields with vacuum. In fact, the effective Lagrangian that takes into account the vacuum polarization by a constant electric field, to first order in the radiative contributions, has the same form as the BI Lagrangian [2], [3].

A renewed interest in BI theory arose in connection with string theory and p-branes. It turns out that Lagrangians with determinants, equivalent to the BI Lagrangian, frequently appear in these theories [4].

When BI theory is coupled to general relativity (GR), interesting features arise that make worth to apply it to systems with both strong electromagnetic and gravitational fields. The Weyl anomaly is one of this features that is due to the non-vanishing of the trace of the energy-momentum tensor. It leads to the violation of some energy conditions as well as to avoid the focussing of neighbouring geodesics [5].

Another effect is that photon trajectories are modified due to the presence of the nonlinear electromagnetic field. The consequence is that light trajectories are not null geodesics of the background metric but null geodesics of an effective metric that depends on the nonlinear electromagnetic field [6].

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2. Born Infeld's theory and quantum electrodynamics

The form of the Lagrangian that BI proposed, as the theory that generalized Maxwell's, was inspired in a finiteness principle for the electromagnetic field magnitude, analogous to the special relativity theory that assumed an upper limit to the velocity of light. For a maximal field strength b, the proposed Lagrangian that depend non-linearly on the electromagnetic invariants, $F = \frac{1}{2}(B^2 - E^2)$ and $G = E{\cdot}B$, has the form

$$L_{BI} = b^2\sqrt{1 - \frac{(B^2 - E^2)}{b^2} + \frac{(E{\cdot}B)^2}{b^4}} - b^2. \tag{1}$$

2.1. The static spherically symmetric solution in BI theory

One of the achievements of BI theory is that it solves the point charge singularity. Solving for the static spherically symmetric case, without magnetic field, the BI equations are

$$\begin{aligned}
&\vec{B} = \vec{H} = 0, \quad \nabla \times \vec{E} = 0, \quad \nabla \cdot \vec{D} = 0,\\
&D_r = \frac{e}{r^2}, \quad E_r = -\phi_{,r}(r), \quad \phi(r) = \frac{e}{r_0^2} f(\frac{r}{r_0}), \quad r_0^2 = \frac{e}{b},\\
&f(x) = \int_x^\infty \frac{dy}{\sqrt{1+y^4}}.
\end{aligned} \tag{2}$$

The electrostatic potential $\phi(r)$ is finite everywhere since it is given in terms of an elliptic integral that is finite at the origin. This makes the difference from the Coulombian potential that diverges as one approaches the origin. Moreover, using the charge and classical radius of the electron (r_0) BI obtained an estimation for the maximum field strength: $b = \frac{e}{r_0^2} = 1.8 \times 10^{18}$ V/cm.

For fields smaller than b, $E < b, \quad B < b$, expanding the square root in Eq. (1) and keeping terms up to order $O(b^{-2})$, we obtain

$$L_{BI} \approx \frac{(E^2 - B^2)}{2} + \frac{(E{\cdot}B)^2}{2b^2} - \frac{(E^2 - B^2)^2}{8b^2} + \mathrm{O}(b^{-4}). \tag{3}$$

Taking into account that the maximal field strength in BI theory is $b = 10^{18}$ Volt/cm, such fields are not necessarily small. In the linear limit, obtained when $b \to \infty$, one recovers the Maxwell Lagrangian, $L = -F$.

2.2. QED Lagrangian

Exploring quantum mechanical corrections for electromagnetic processes Weisskopf (1936) [2], and later, in a covariant form, Schwinger (1951) [3] deduced that nonlinear effects take place due to the probability of electron-positron pair creation (vacuum polarization). The approach of QED is based on a gauge invariant effective action which takes into account modifications of Maxwell electrodynamics induced by several processes: quantum vacuum responds as if it were a classical medium. Similar effects appear inside a dielectric medium that responds non-linearly to an external stimulus. In fact such nonlinear electromagnetic effects as photon-photon scattering or vacuum polarization have been observed in laboratory at fields of the critical order of $E_{crit} \approx 10^{16}$ Volt/m [7].

The one-loop effective Lagrangian for vacuum polarization due to a constant external electromagnetic field depends on the invariants of the electromagnetic field in the same way as BI Lagrangian Eq. (3). It is given by

$$
\begin{aligned}
L_{eff} &= L_M + L^{(1)}, \\
L^{(1)} &= a(E^2 - B^2)^2 + a^\star (E \cdot B)^2 \cdots, \\
a &= \frac{2\alpha^2}{45 m_e^4}, \quad a^\star = \frac{7}{4} a,
\end{aligned}
$$

where α is the fine structure constant, m_e is the rest mass of the electron, $L_M = -F$, is the Maxwell Lagrangian and $L^{(1)}$ is the part of the radiative corrections to first order. The previous Lagrangian was derived for slowly varying fields, $\frac{\hbar}{m_e c}|\nabla F| << |F|$ and $\frac{\hbar}{m_e c^2}|\frac{\partial F}{\partial t}| << |F|$. Moreover, it is assumed that the production of real electron-positron pairs is negligible, i.e. the threshold for energies is $|E| < \frac{m_e^2}{|e|} = \frac{m_e^2 c^3}{|e|\hbar}$.

This result gives support to the explicit form of the Lagrangian of Born-Infeld among the wide variety of nonlinear electromagnetic Lagrangians that one can propose.

3. Coupling gravity with nonlinear electromagnetic fields

For situations where strong electromagnetic and gravitational fields occur, as early universes or close to stars with strong magnetic fields, it makes sense to couple gravitation to NLED. The field equations are derived from the action,

$$
S = \int d^4 x \sqrt{-g} \{ \frac{R}{16\pi} - L_{\mathrm{NLED}} \}, \tag{4}
$$

where R denotes the scalar curvature, $g := \det|g_{\mu\nu}|$ and L_{NLED} is the electromagnetic part, that depends in nonlinear way on the invariants of the electromagnetic field. Being the Born-Infeld Lagrangian a particular case, in what follows we consider the general case, $L = L(F, G)$. The corresponding stress-energy tensor and the scalar curvature are given by

$$
\begin{aligned}
4\pi T_{\mu\nu} &= -L_{,F} F_\mu^\alpha F_{\alpha\nu} + (G L_{,G} - L) g_{\mu\nu}, \\
R &= 8\pi (L - F L_{,F} - G L_{,G}) = -8\pi T.
\end{aligned} \tag{5}
$$

where $L_{,x} = dL/dx$.

3.1. Breakdown of conformal invariance

From Eq. (5) it is evident that conformal invariance of Maxwell equations is broken by the non-vanishing of the trace of the energy-momentum tensor,

$$
T_\alpha^\alpha = L_{,F} F + G L_{,G} - L, \tag{6}
$$

The Weyl (trace or conformal) anomaly arises also when a scalar field (inflaton, dilaton, pseudo-Goldstone bosons, axions) is included in the action. Also when adding extra dimensions or couplings to curvature terms. The interaction of photons with other fields breaks conformal invariance as well. Remind that only equations describing massless particles are conformally invariant.

However, conformal anomaly can be advantageous in certain scenarios, for instance in inflationary epochs. If conformal invariance is broken during inflation, then inflation provides a mechanism to amplify perturbations, like primordial magnetic seed fields [8], [9], [10]. If we want that large scale correlated fields be produced during inflation, conformal invariance should be broken, otherwise they are not conveniently enlarged during inflation.

3.2. NLED energy conditions

Let us explore what kind of energy conditions arise from a stress-energy tensor derived from a nonlinear electromagnetic theory.

Consider a timelike vector, V^α, $V_\alpha V^\alpha < 1$. Assuming that the local energy density should be non-negative (weak energy condition, WEC),

$$T_{\mu\nu}V^\mu V^\nu \geq 0, \tag{7}$$

and the requirement that the local energy flow vector be non-spacelike (DEC) amounts to,

$$T_{\alpha\beta}T^\alpha_\gamma V^\beta V^\gamma \leq 0. \tag{8}$$

In terms of the Lagrangian $L(F,G)$ both conditions hold provided

$$L_{,F} > 0, \quad (L - FL_{,F} - GL_{,G}) \geq 0. \tag{9}$$

While for the strong energy condition (SEC),

$$R_{\mu\nu}V^\mu V^\nu \geq 0, \tag{10}$$

using the Einstein equations it can be settled as,

$$R_{\mu\nu}V^\mu V^\nu = 8\pi(T_{\mu\nu}V^\mu V^\nu + \frac{T}{2}) \geq 0. \tag{11}$$

From Eq. (11) note that NLED matter can violate SEC if the trace T is negatively enough. Moreover, this fact has consequences in the focussing of neighbouring geodesics as the analysis of the Raychaudhuri equation shows.

The rate of change of expansion of timelike congruences $\Theta(\lambda)$, is ruled out by the Raychaudhuri equation,

$$\frac{d\Theta(\lambda)}{d\lambda} = -R_{\mu\nu}V^\mu V^\nu + 2\omega^2 - 2\sigma^2 - \frac{\Theta^2}{3} + \dot{V}^\alpha_{;\alpha}. \tag{12}$$

Let us consider a geodesic congruence, $V^\alpha = 0$, without vorticity $\omega = 0$, then the expansion $\Theta(\lambda)$ will monotonically decrease along a geodesic, since $-R_{\mu\nu}V^\mu V^\nu < 0$ induces contraction of the geodesic lines. Then focussing of neighbouring geodesics is unavoidable if SEC is fulfilled, since the other terms on the r. h. s. of Eq. (12) are also negative. The situation is different if SEC is violated, $R_{\mu\nu}V^\mu V^\nu < 0$, provided

$$-R_{\mu\nu}V^\mu V^\nu > 2\sigma^2 + \frac{\Theta^2}{3}, \tag{13}$$

the possibility exists of avoiding singularities, i.e. stopping the focusing of geodesics (non existence of trapped surfaces) [11], [12]. This effect can also be obtained from the inclusion of exotic matter, phantom matter, a cosmological constant or fluids with negative pressure, all of them generate a repulsive effect. In here there is no exotic matter, the question is if the produced repulsive effect is enough to some purposes. For wormholes this aspect has been explored [13], as well as in cosmologies [14]. In Fig. 1 it is shown the behaviour of BI radiation in a Bianchi space [15].

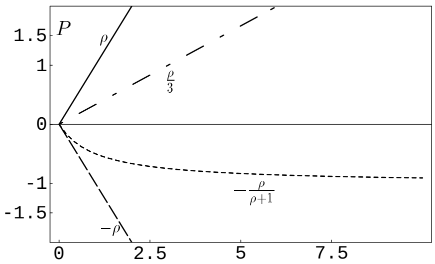

Figure 1. Behaviour of radiation in a Bianchi space [15]. It is shown the pressure P as function of the energy density ρ. Nonlinear electromagnetic radiation corresponds to the dashed curve $P = -\rho/(\rho+1)$ compared with vacuum-like state, $P = -\rho$ (broken curve) and normal radiation, $P = \rho/3$.

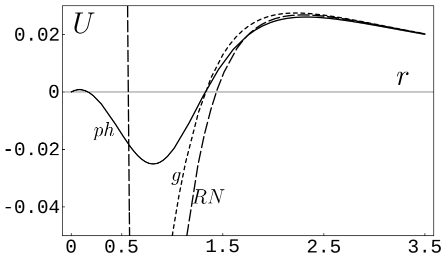

Figure 2. The potential $U(r)$ corresponding to null geodesics in Reissner-Nordstrom (RN) and Born-Infeld black holes [18] is shown. For the Born-Infeld black hole the potentials for null geodesics (g, dashed curve) and the light trajectories (ph, full curve) are not the same.

3.3. Photons obey an effective geometry

Another effect produced in spacetime by a nonlinear electromagnetic field is the modification of the geometry as *seen* by travelling photons; indeed, the velocity of light is different respect to the propagation in vacuum [16], [17]. From BI equations it is evident since they resemble Maxwell's equations in a material medium.

To determine the effective metric for light, we need to analyse the characteristic surfaces, these are surfaces that propagate the field discontinuities, i.e. they are like the wavefronts. For a curved spacetime the equation for the characteristic surfaces S is

$$g^{\mu\nu} S_{,\mu} S_{,\nu} = 0. \tag{14}$$

Locally these surfaces are normal to the light rays. In NLED the Eq. (14) is modified, in such a way that photons propagate in an effective metric, $\gamma_{\mu\nu}$. For the particular case of BI electrodynamics characteristic surfaces are given by,

$$(g^{\mu\nu} + \frac{4\pi}{b^2} T^{\mu\nu}) S_{,\mu} S_{,\nu} = \gamma^{\mu\nu} S_{,\mu} S_{,\nu} = 0. \tag{15}$$

Clearly the responsible of that deviation is the stress-energy tensor, $T^{\mu\nu}$. Hence, as far as b is finite, the propagation of gravitational and electromagnetic discontinuities do not occur at the same velocity. For a black hole with a Born-Infeld electromagnetic field, actually both trajectories converge at the horizon [18]; in Fig. 2 are shown the potentials for massless particles in such a BI black hole, compared with the potential for the Reissner-Nordstrom (RN) back hole, its linear counterpart.

The corresponding equation for the light propagation vectors k^{μ} in a spacetime with an arbitrary NLED, $L(F)$, is given by

$$(L_F g^{\mu\nu} - 4 L_{FF} F^{\mu\alpha} F^{\nu}_{\alpha})\, k_{,\mu} k_{,\nu} = \gamma^{\mu\nu} k_{,\mu} k_{,\nu} = 0. \tag{16}$$

In ordinary optics ($L = -F, g_{\mu\nu} = \eta_{\mu\nu}$) Eq. (16), this is known as the *eikonal* equation. In the case that both invariants $F \neq 0, G \neq 0$, $L(F,G)$, the photon trajectories follow the effective metric $\gamma^{\mu\nu}_{\pm}$ with

$$\gamma_{\pm}^{\mu\nu} = L_{,F} g^{\mu\nu} - 4[(L_{,FF} + \Omega_{\pm} L_{,FG}) F^{\mu}_{\lambda} F^{\lambda\nu} + (L_{,FG} + \Omega_{\pm} L_{,GG}) F^{\mu}_{\lambda} \hat{F}^{\lambda\nu}], \tag{17}$$

the function $\Omega_{\pm}$ satisfies a quadratic equation [6] thus two possibilities are allowed, γ_+ and γ_- then there are two possible trajectories for light and birefringence arises. The only nonlinear electrodynamics in which the speed of light does not depend on the polarization is the Born-Infeld theory [16].

4. Conclusions

Nowadays the motivations to study nonlinear generalizations of Maxwell electrodynamics is different from the one that inspired Born and Infeld. The interest is now related to cosmological and astrophysical observations, to understand the intense magnetic fields in neutron stars, the origin of the observed large-scale correlated magnetic fields, to favour inflation or as a mechanism to avoid singularities, these effects have been roughly explained in this contribution.

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