Numerical Methods Runtime Table

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1 Equations

We have used the same 25 equations with each method and run each method (Bisection, False Position, and Hybrid) 500 times for each problem and then we have calculated the average time. We have also calculated the number of iterations each method have taken for each problem.

We have also used the same tolerance for each method which is 10^{-14}

These are the equations that we have used with each method:

1.1 Our Equations

In these equations we have tried to use different types of functions and intervals to test our methods.

Table 1: Our Equations Table

No	Equation	Equation Code	Interval
$P1$	$f(x) = x^3 + 4x^2 - 10 = 0$	`x**3 + 4*x**2 - 10`	`[0, 4]`
$P2$	$f(x) = x^2 - 4$	`x**2 - 4`	`[0, 4]`
$P3$	$f(x) = e^x - 2$	`math.exp(x) - 2`	`[0, 2]`
$P4$	$f(x) = \sin(x)$	`math.sin(x)`	`[2, 6]`
$P5$	$f(x) = x^3 - 6x^2 + 11x - 6$	`x**3 - 6*x**2 + 11*x - 6`	`[1, 2.5]`
$P6$	$f(x) = x^2 + 3x + 2$	`x**2 + 3*x + 2`	`[-2.5, -1.5]`
$P7$	$f(x) = \cos(x) - x$	`math.cos(x) - x`	`[0, 1]`
$P8$	$f(x) = 2^x - 8$	`2**x - 8`	`[2,4]`
$P9$	$f(x) = \tan(x)$	`math.tan(x)`	`[-1, 1]`
$P10$	$f(x) = x^4 - 8x^3 + 18x^2 - 9x + 1$	`x**4 - 8*x**3 + 18*x**2 - 9*x + 1`	`[2, 4]`

1.2 Equations From Paper

We got these equations from this paper and we have used the same intervals too.

Table 2: Equations From Paper Table

No	Equation	Equation Code	Interval	Reference
$P11$	$f(x) = x^2 - 3$	`x**2 - 3`	`[1,2]`	Harder [18]
$P12$	$f(x) = x^2 - 5$	`x**2 - 5`	`[2,7]`	Srivastava[9]
$P13$	$f(x) = x^2 - 10$	`x**2 - 10`	`[3,4]`	Harder [18]
$P14$	$f(x) = x^2 - x - 2$	`x**2 - x - 2`	`[1,4]`	Moazzam [10]
$P15$	$f(x) = x^2 + 2x - 7$	`x**2 + 2*x - 7`	`[1,3]`	Nayak[11]
$P16$	$f(x) = x^3 - 2$	`x**3 - 2`	`[0,2]`	Harder [18]
$P17$	$f(x) = xe^x - 7$	`x * math.exp(x) - 7`	`[0,2]`	Callhoun [19]
$P18$	$f(x) = x - \cos(x)$	`x - math.cos(x)`	`[0,1]`	Ehiwario [6]

No	Equation	Equation Code	Interval	Reference
P19	$f(x) = x\sin(x) - 1$	`x * math.sin(x) - 1`	[0,2]	Mathews [20]
P20	$f(x) = x\cos(x) + 1$	`x * math.cos(x) + 1`	[-2,4]	Esfandiari [21]
P21	$f(x) = x^{10} - 1$	`x**10 - 1`	[0,1.3]	Chapra [17]
P22	$f(x) = x^2 + e^{x/2} - 5$	`x**2 + (2.71828**(x/2)) - 5`	[1,2]	Esfandiari [21]
P23	$f(x) = \sin(x)\sinh(x) + 1$	`math.sin(x) * math.sinh(x) + 1`	[3,4]	Esfandiari [21]
P24	$f(x) = e^x - 3x - 2$	`(2.71828**x) - 3*x - 2`	[2,3]	Hoffman [22]
P25	$f(x) = \sin(x) - x^2$	`math.sin(x) - x**2`	[0.5,1]	Chapra[17]

2 Results

These are the results we got with each method. We have run each method 500 times on each equation and took the average time to get the highest accuracy possible.

2.1 False Position

These are the results we got with False Position method:

Table 3: False Position Table

Problem	Bisection Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
P1	80	0.000229008	1.3652300134140964	-7.11E-15	1.3652300134140964	4
P2	33	4.399728775024414e-05	1.9999999999999978	-8.88E-15	1.9999999999999978	4
P3	51	5.6000232696533204e-05	0.6931471805599422	-6.22E-15	0.6931471805599422	2
P4	8	6.000041961669922e-06	3.141592653589793	1.2246467991473532e-16	3.141592653589793	3.1415926535899232
P5	2	0	1	0	1	2.5
P6	31	4.800844192504883e-05	-2	-5.33E-15	-2.5	-2
P7	12	1.101541519165039e-05	0.7390851332151551	9.2148511043888e-15	0.7390851332151551	1
P8	30	4.401159286499024e-05	2.9999999999999987	-7.11E-15	2.9999999999999987	4
P9	2	1.991748809814453e-06	0	0	0	1
P10	13	4.0007591247558594e-05	3.1117486563092474	0	3.1117486563092474	3.1117486563092482
P11	14	1.7997264862060548e-05	1.732050807568876	-4.00E-15	1.732050807568876	2
P12	50	6.600427627563476e-05	2.2360679774997876	-9.77E-15	2.2360679774997876	7
P13	17	2.2464752197265626e-05	3.162277660168379	-1.78E-15	3.162277660168379	4
P14	38	5.301380157470703e-05	1.9999999999999971	-8.66E-15	1.9999999999999971	4
P15	21	3.1998634338378904e-05	1.8284271247461896	-2.66E-15	1.8284271247461896	3
P16	41	5.600643157958984e-05	1.2599210498948719	-6.22E-15	1.2599210498948719	2
P17	30	3.40123176574707e-05	1.5243452049841437	-7.99E-15	1.5243452049841437	2
P18	12	1.2005805969238282e-05	0.7390851332151551	-9.21E-15	0.7390851332151551	1
P19	7	7.99846649169922e-06	1.1141571408719306	8.881784197001252e-16	1.0997501702946164	1.1141571408719306
P20	13	1.1332988739013672e-05	2.0739328090912146	7.771561172376096e-16	2.0739328090912146	2.5157197710146586
P21	139	0.000183961	0.9999999999999991	-8.88E-15	0.9999999999999991	1.3
P22	16	3.3281803131103514e-05	1.6490135532979475	-1.78E-15	1.6490135532979475	2
P23	45	7.994651794433594e-05	3.2215883990939416	6.328271240363392e-15	3.2215883990939416	4
P24	45	6.818151473999023e-05	2.1253934262332246	-9.77E-15	2.1253934262332246	3
P25	17	2.703714370727539e-05	0.8767262153950554	7.882583474838611e-15	0.8767262153950554	1

2.2 Bisection Method

These are the results we got with Bisection method:

Table 4: Bisection Table

Problem	Bisection Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
$P1$	50	7.303380966186524e-05	1.3652300134140951	-2.84E-14	1.3652300134140916	1.3652300134140987
$P2$	1	0	2	0	0	4
$P3$	49	4.200363159179687e-05	0.6931471805599436	-3.33E-15	0.6931471805599401	0.6931471805599472
$P4$	50	3.406333923339844e-05	3.141592653589793	1.2246467991473532e-16	3.1415926535897896	3.1415926535897967
$P5$	48	7.496118545532227e-05	2.0000000000000018	0	1.9999999999999964	2.000000000000007
$P6$	1	1.9011497497558594e-06	-2	0	-2.5	-1.5
$P7$	48	3.201484680175781e-05	0.7390851332151591	2.55351295663786e-15	0.7390851332151556	0.7390851332151627
$P8$	1	1.9893646240234374e-06	3	0	2	4
$P9$	1	1.991748809814453e-06	0	0	-1	1
$P10$	49	9.199857711791992e-05	3.111748656309249	1.0658141036401503e-14	3.1117486563092456	3.1117486563092527
$P11$	48	4.000377655029297e-05	1.7320508075688785	4.440892098500626e-15	1.732050807568875	1.732050807568882
$P12$	50	3.901958465576172e-05	2.2360679774997854	-1.95E-14	2.236067977499781	2.236067977
$P13$	48	3.7988662719726566e-05	3.1622776601683817	1.5987211554602254e-14	3.162277660168378	3.1622776601683853
$P14$	50	4.400014877319336e-05	1.9999999999999991	-2.66E-15	1.9999999999999964	2.0000000000000018
$P15$	49	5.607509613037109e-05	1.828427124746188	-1.15E-14	1.8284271247461845	1.8284271247461916
$P16$	49	3.8086414337158205e-05	1.2599210498948743	5.329070518200751e-15	1.2599210498948707	1.2599210498948779
$P17$	49	3.905820846557617e-05	1.5243452049841473	3.375077994860476e-14	1.5243452049841437	1.5243452049841508
$P18$	48	2.9998779296875e-05	0.7390851332151591	-2.55E-15	0.7390851332151556	0.7390851332151627
$P19$	49	0.000136974	1.114157140871928	-3.00E-15	1.1141571408719244	1.1141571408719315
$P20$	51	5.606412887573242e-05	2.0739328090912155	-1.33E-15	2.073932809091213	2.073932809091218
$P21$	48	4.004716873168945e-05	1.000000000000001	1.1102230246251565e-14	0.9999999999999966	1.0000000000000058
$P22$	44	5.988311767578125e-05	1.649013553297948	0	1.6490135532978911	1.6490135532980048
$P23$	48	6.889772415161133e-05	3.2215883990939425	-5.55E-15	3.221588399093939	3.221588399093946
$P24$	48	4.5994281768798825e-05	2.1253934262332272	5.329070518200751e-15	2.1253934262332237	2.125393426233231
$P25$	47	6.799602508544923e-05	0.8767262153950632	-8.88E-16	0.8767262153950597	0.8767262153950668

2.3 Hybrid Method

These are the results we got with hybrid method:

Table 5: Hybrid Table

Problem	Bisection Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
$P1$	10	3.6006927490234375e-05	1.3652300134140964	-7.11E-15	1.365230013413779	1.3675001980274413
$P2$	1	1.9969940185546874e-06	2	0	0	4
$P3$	10	1.399993896484375e-05	0.6931471805599453	0	0.6931471805599334	0.695162706
$P4$	6	1.006174087524414e-05	3.141592653589793	1.2246467991473532e-16	3.1415903579556947	3.141592653604888
$P5$	1	3.940105438232422e-06	1	0	1	2.5
$P6$	1	1.9888877868652345e-06	-2	0	-2.5	-1.5
$P7$	8	1.1938095092773438e-05	0.7390851332151606	1.1102230246251565e-16	0.739085133	0.7422270732175922
$P8$	1	2.0036697387695312e-06	3	0	2	4
$P9$	1	2.0928382873535157e-06	0	0	-1	1
$P10$	8	2.4066925048828126e-05	3.1117486563092474	0	3.1085379927858856	3.1117486563092536
$P11$	8	1.7096519470214843e-05	1.7320508075688772	-4.44E-16	1.7320508075688001	1.7350578402209837
$P12$	10	1.4061450958251953e-05	2.236067977499789	-3.55E-15	2.236067977499364	2.243929153983615
$P13$	8	1.393747329711914e-05	3.1622776601683795	1.7763568394002505e-15	3.16227766	3.1672187190124017

Problem	Bisection Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
$P14$	2	2.0089149475097657e-06	2	0	1.5	2.5
$P15$	5	8.056163787841797e-06	1.828427125	0	1.8284271247430004	1.8284271247493797
$P16$	9	1.2000083923339844e-05	1.2599210498948723	-4.00E-15	1.259921049893984	1.2611286403176987
$P17$	11	1.3935565948486329e-05	1.5243452049841444	0	1.5243452049841386	1.526033337108763
$P18$	8	1.0064601898193359e-05	0.7390851332151606	-1.11E-16	0.739085133	0.7422270732175922
$P19$	6	8.002758026123047e-06	1.1141571408719302	2.220446049250313e-16	1.1132427327642702	1.1141571408719768
$P20$	10	1.5938282012939452e-05	2.073932809091215	-2.22E-16	2.0739328090911866	2.078935003337393
$P21$	12	1.6058921813964842e-05	0.9999999999999999	-1.11E-15	0.9999999999999305	1.000343363282986
$P22$	8	2.393531799316406e-05	1.6490135532979473	-3.55E-15	1.6490135532974015	1.6531560376633945
$P23$	9	1.7997264862060548e-05	3.221588399093942	3.3306690738754696e-16	3.2215883990939242	3.2224168881395068
$P24$	9	1.2019157409667969e-05	2.125393426233225	-7.11E-15	2.1253934262325003	2.1275213330097245
$P25$	7	1.1998653411865234e-05	0.8767262153950581	4.773959005888173e-15	0.8767262153886713	0.8772684454348731

As we see from the table above the hybrid method tend to be faster and take much less iterations than both Bisection and False Position methods.
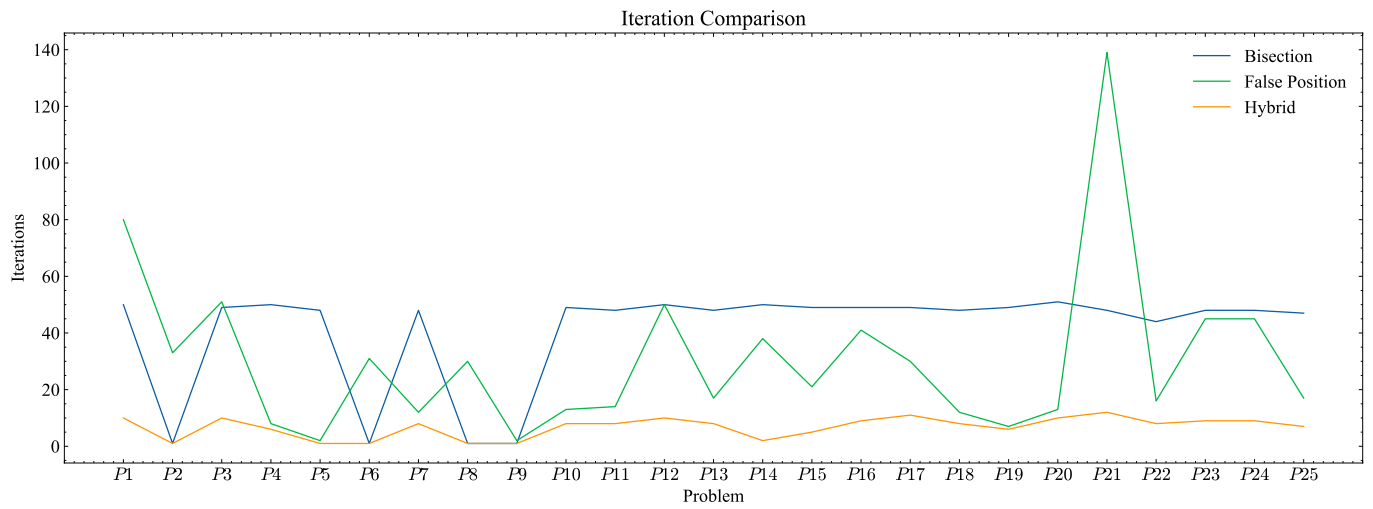
2.4 Final Results

2.4.1 Iterations

Figure 1: Iterations Comparison

2.4.2 CPU Time

Figure 2: CPU Time Comparison

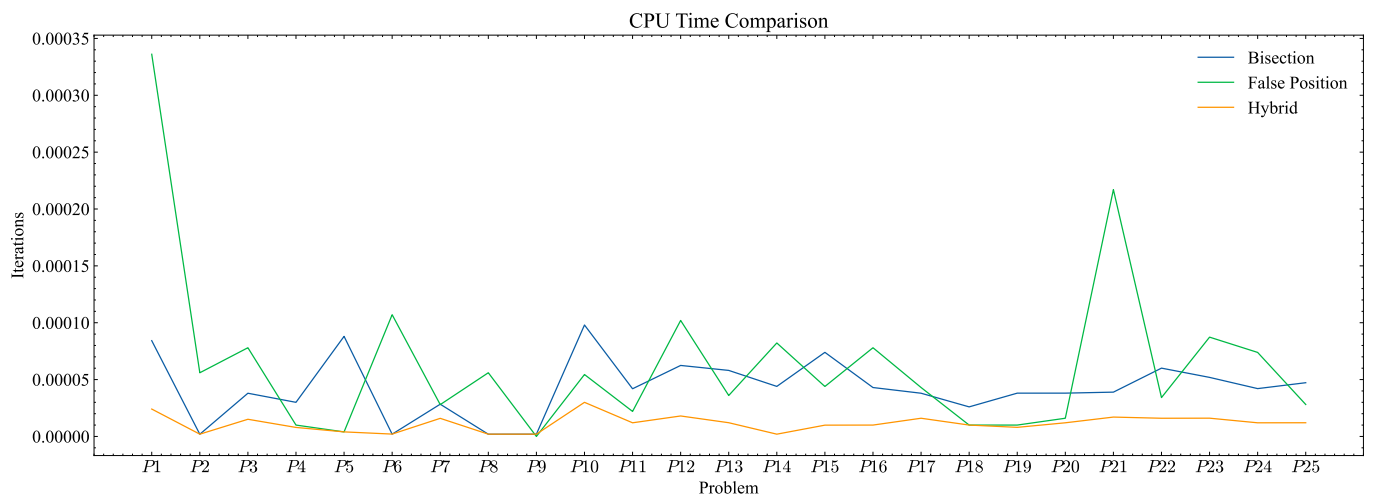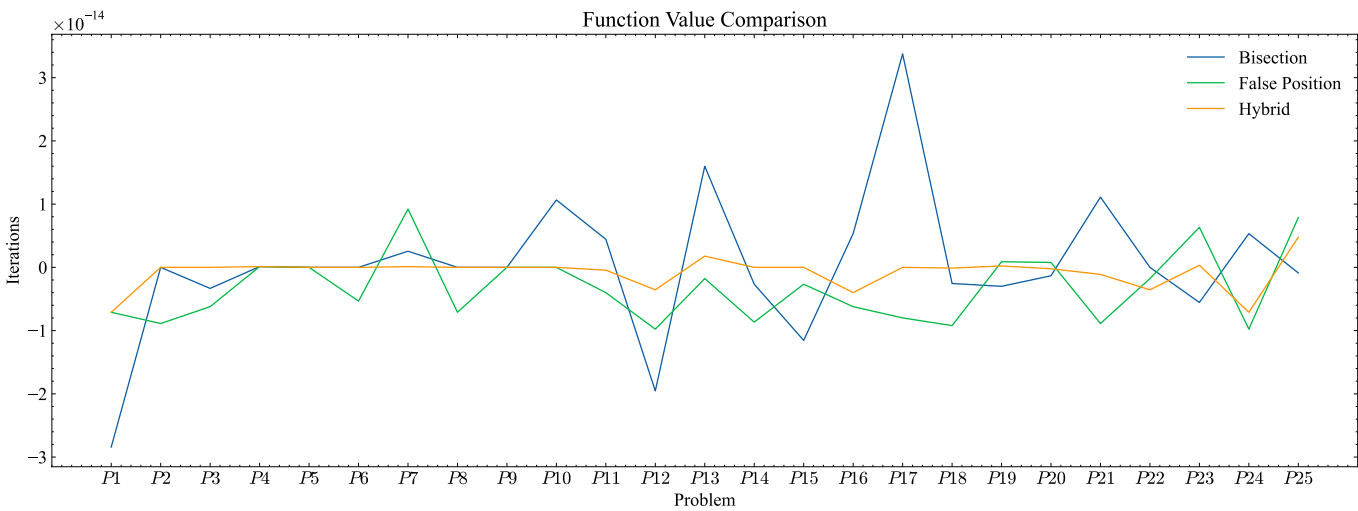### 2.4.3 Function Value

Figure 3: Function Value Comparison