

# Numerical Methods Runtime Table

Mohamed Emary

November 2, 2023

## 1 Equations

We have used the same 25 equations with each method and run each method (Bisection, False Position, and Hybrid) 500 times for each problem and then we have calculated the average time. We have also calculated the number of iterations each method have taken for each problem.

We have also used the same tolerance for each method which is  $10^{-14}$

These are the equations that we have used with each method:

### 1.1 Our Equations

In these equations we have tried to use different types of functions and intervals to test our methods.

Table 1: Our Equations Table

No	Equation	Equation Code	Interval
<i>P1</i>	$f(x) = x^3 + 4x^2 - 10 = 0$	<code>x**3 + 4*x**2 - 10</code>	[0, 4]
<i>P2</i>	$f(x) = x^2 - 4$	<code>x**2 - 4</code>	[0, 4]
<i>P3</i>	$f(x) = e^x - 2$	<code>math.exp(x) - 2</code>	[0, 2]
<i>P4</i>	$f(x) = \sin(x)$	<code>math.sin(x)</code>	[2, 6]
<i>P5</i>	$f(x) = x^3 - 6x^2 + 11x - 6$	<code>x**3 - 6*x**2 + 11*x - 6</code>	[1, 2.5]
<i>P6</i>	$f(x) = x^2 + 3x + 2$	<code>x**2 + 3*x + 2</code>	[-2.5, -1.5]
<i>P7</i>	$f(x) = \cos(x) - x$	<code>math.cos(x) - x</code>	[0, 1]
<i>P8</i>	$f(x) = 2^x - 8$	<code>2**x - 8</code>	[2,4]
<i>P9</i>	$f(x) = \tan(x)$	<code>math.tan(x)</code>	[-1, 1]
<i>P10</i>	$f(x) = x^4 - 8x^3 + 18x^2 - 9x + 1$	<code>x**4 - 8*x**3 + 18*x**2 - 9*x + 1</code>	[2, 4]

### 1.2 Equations From Paper

We got these equations from [this paper](#) and we have used the same intervals too.

Table 2: Equations From Paper Table

No	Equation	Equation Code	Interval	Reference
P11	$f(x) = x^2 - 3$	<code>x**2 - 3</code>	[1,2]	Harder [18]
P12	$f(x) = x^2 - 5$	<code>x**2 - 5</code>	[2,7]	Srivastava[9]
P13	$f(x) = x^2 - 10$	<code>x**2 - 10</code>	[3,4]	Harder [18]
P14	$f(x) = x^2 - x - 2$	<code>x**2 - x - 2</code>	[1,4]	Moazzam [10]
P15	$f(x) = x^2 + 2x - 7$	<code>x**2 + 2*x - 7</code>	[1,3]	Nayak[11]
P16	$f(x) = x^3 - 2$	<code>x**3 - 2</code>	[0,2]	Harder [18]
P17	$f(x) = xe^x - 7$	<code>x * math.exp(x) - 7</code>	[0,2]	Callhoun [19]
P18	$f(x) = x - \cos(x)$	<code>x - math.cos(x)</code>	[0,1]	Ehiwario [6]
P19	$f(x) = x \sin(x) - 1$	<code>x * math.sin(x) - 1</code>	[0,2]	Mathews [20]
P20	$f(x) = x \cos(x) + 1$	<code>x * math.cos(x) + 1</code>	[-2,4]	Esfandiari [21]
P21	$f(x) = x^{10} - 1$	<code>x**10 - 1</code>	[0,1.3]	Chapra [17]
P22	$f(x) = x^2 + e^{x/2} - 5$	<code>x**2 + (2.71828**(x/2)) - 5</code>	[1,2]	Esfandiari [21]
P23	$f(x) = \sin(x) \sinh(x) + 1$	<code>math.sin(x) * math.sinh(x) + 1</code>	[3,4]	Esfandiari [21]
P24	$f(x) = e^x - 3x - 2$	<code>(2.71828**x) - 3*x - 2</code>	[2,3]	Hoffman [22]
P25	$f(x) = \sin(x) - x^2$	<code>math.sin(x) - x**2</code>	[0.5,1]	Chapra[17]

## 2 Results

These are the results we got with each method. We have run each method 500 times on each equation and took the average time to get the highest accuracy possible.

### 2.1 False Position

These are the results we got with False Position method:

Table 3: False Position Table

Problem	False Position Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
<i>P1</i>	80	0.000229008	1.3652300134140964	-7.11E-15	1.3652300134140964	4
<i>P2</i>	33	4.399728775024414e-05	1.9999999999999978	-8.88E-15	1.9999999999999978	4
<i>P3</i>	51	5.6000232696533204e-05	0.6931471805599422	-6.22E-15	0.6931471805599422	2
<i>P4</i>	8	6.000041961669922e-06	3.141592653589793	1.2246467991473532e-16	3.141592653589793	3.1415926535899232
<i>P5</i>	2	0	1	0	1	2.5
<i>P6</i>	31	4.800844192504883e-05	-2	-5.33E-15	-2.5	-2
<i>P7</i>	12	1.101541519165039e-05	0.7390851332151551	9.2148511043888e-15	0.7390851332151551	1
<i>P8</i>	30	4.401159286499024e-05	2.9999999999999987	-7.11E-15	2.9999999999999987	4
<i>P9</i>	2	1.991748809814453e-06	0	0	0	1
<i>P10</i>	13	4.0007591247558594e-05	3.1117486563092474	0	3.1117486563092474	3.1117486563092482
<i>P11</i>	14	1.7997264862060548e-05	1.732050807568876	-4.00E-15	1.732050807568876	2
<i>P12</i>	50	6.600427627563476e-05	2.2360679774997876	-9.77E-15	2.2360679774997876	7
<i>P13</i>	17	2.2464752197265626e-05	3.162277660168379	-1.78E-15	3.162277660168379	4
<i>P14</i>	38	5.301380157470703e-05	1.9999999999999971	-8.66E-15	1.9999999999999971	4
<i>P15</i>	21	3.1998634338378904e-05	1.8284271247461896	-2.66E-15	1.8284271247461896	3
<i>P16</i>	41	5.600643157958984e-05	1.2599210498948719	-6.22E-15	1.2599210498948719	2
<i>P17</i>	30	3.40123176574707e-05	1.5243452049841437	-7.99E-15	1.5243452049841437	2
<i>P18</i>	12	1.2005805969238282e-05	0.7390851332151551	-9.21E-15	0.7390851332151551	1
<i>P19</i>	7	7.99846649169922e-06	1.1141571408719306	8.881784197001252e-16	1.0997501702946164	1.1141571408719306
<i>P20</i>	13	1.1332988739013672e-05	2.0739328090912146	7.771561172376096e-16	2.0739328090912146	2.5157197710146586
<i>P21</i>	139	0.000183961	0.9999999999999991	-8.88E-15	0.9999999999999991	1.3
<i>P22</i>	16	3.3281803131103514e-05	1.6490135532979475	-1.78E-15	1.6490135532979475	2
<i>P23</i>	45	7.994651794433594e-05	3.2215883990939416	6.328271240363392e-15	3.2215883990939416	4
<i>P24</i>	45	6.818151473999023e-05	2.1253934262332246	-9.77E-15	2.1253934262332246	3
<i>P25</i>	17	2.703714370727539e-05	0.8767262153950554	7.882583474838611e-15	0.8767262153950554	1

## 2.2 Bisection Method

These are the results we got with Bisection method:

Table 4: Bisection Table

Problem	Bisection Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
<i>P1</i>	50	7.303380966186524e-05	1.3652300134140951	-2.84E-14	1.3652300134140916	1.3652300134140987
<i>P2</i>	1	0	2	0	0	4
<i>P3</i>	49	4.200363159179687e-05	0.6931471805599436	-3.33E-15	0.6931471805599401	0.6931471805599472
<i>P4</i>	50	3.406333923339844e-05	3.141592653589793	1.2246467991473532e-16	3.1415926535897896	3.1415926535897967
<i>P5</i>	48	7.496118545532227e-05	2.0000000000000018	0	1.9999999999999964	2.0000000000000007
<i>P6</i>	1	1.9011497497558594e-06	-2	0	-2.5	-1.5
<i>P7</i>	48	3.201484680175781e-05	0.7390851332151591	2.55351295663786e-15	0.7390851332151556	0.7390851332151627
<i>P8</i>	1	1.9893646240234374e-06	3	0	2	4
<i>P9</i>	1	1.991748809814453e-06	0	0	-1	1
<i>P10</i>	49	9.199857711791992e-05	3.111748656309249	1.0658141036401503e-14	3.1117486563092456	3.1117486563092527
<i>P11</i>	48	4.000377655029297e-05	1.7320508075688785	4.440892098500626e-15	1.732050807568875	1.732050807568882
<i>P12</i>	50	3.901958465576172e-05	2.2360679774997854	-1.95E-14	2.236067977499781	2.236067977
<i>P13</i>	48	3.7988662719726566e-05	3.1622776601683817	1.5987211554602254e-14	3.162277660168378	3.1622776601683853
<i>P14</i>	50	4.400014877319336e-05	1.9999999999999991	-2.66E-15	1.9999999999999964	2.00000000000000018
<i>P15</i>	49	5.607509613037109e-05	1.828427124746188	-1.15E-14	1.8284271247461845	1.8284271247461916
<i>P16</i>	49	3.8086414337158205e-05	1.2599210498948743	5.329070518200751e-15	1.2599210498948707	1.2599210498948779
<i>P17</i>	49	3.905820846557617e-05	1.5243452049841473	3.375077994860476e-14	1.5243452049841437	1.5243452049841508
<i>P18</i>	48	2.9998779296875e-05	0.7390851332151591	-2.55E-15	0.7390851332151556	0.7390851332151627
<i>P19</i>	49	0.000136974	1.114157140871928	-3.00E-15	1.1141571408719244	1.1141571408719315
<i>P20</i>	51	5.606412887573242e-05	2.0739328090912155	-1.33E-15	2.073932809091213	2.073932809091218
<i>P21</i>	48	4.004716873168945e-05	1.0000000000000001	1.1102230246251565e-14	0.9999999999999966	1.0000000000000058
<i>P22</i>	44	5.988311767578125e-05	1.649013553297948	0	1.6490135532978911	1.6490135532980048
<i>P23</i>	48	6.889772415161133e-05	3.2215883990939425	-5.55E-15	3.221588399093939	3.221588399093946
<i>P24</i>	48	4.5994281768798825e-05	2.1253934262332272	5.329070518200751e-15	2.1253934262332237	2.125393426233231
<i>P25</i>	47	6.799602508544923e-05	0.8767262153950632	-8.88E-16	0.8767262153950597	0.8767262153950668

## 2.3 Hybrid Method

These are the results we got with hybrid method:

Table 5: Hybrid Table

Problem	Hybrid Algorithm (Bisection & False Position)					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
P1	10	3.6006927490234375e-05	1.3652300134140964	-7.11E-15	1.365230013413779	1.3675001980274413
P2	1	1.9969940185546874e-06	2	0	0	4
P3	10	1.399993896484375e-05	0.6931471805599453	0	0.6931471805599334	0.695162706
P4	6	1.006174087524414e-05	3.141592653589793	1.2246467991473532e-16	3.1415903579556947	3.141592653604888
P5	1	3.940105438232422e-06	1	0	1	2.5
P6	1	1.9888877868652345e-06	-2	0	-2.5	-1.5
P7	8	1.1938095092773438e-05	0.7390851332151606	1.1102230246251565e-16	0.739085133	0.7422270732175922
P8	1	2.0036697387695312e-06	3	0	2	4
P9	1	2.0928382873535157e-06	0	0	-1	1
P10	8	2.4066925048828126e-05	3.1117486563092474	0	3.1085379927858856	3.1117486563092536
P11	8	1.7096519470214843e-05	1.7320508075688772	-4.44E-16	1.7320508075688001	1.7350578402209837
P12	10	1.4061450958251953e-05	2.236067977499789	-3.55E-15	2.236067977499364	2.243929153983615
P13	8	1.393747329711914e-05	3.1622776601683795	1.7763568394002505e-15	3.16227766	3.1672187190124017
P14	2	2.0089149475097657e-06	2	0	1.5	2.5
P15	5	8.056163787841797e-06	1.828427125	0	1.8284271247430004	1.8284271247493797
P16	9	1.2000083923339844e-05	1.2599210498948723	-4.00E-15	1.259921049893984	1.2611286403176987
P17	11	1.3935565948486329e-05	1.5243452049841444	0	1.5243452049841386	1.526033337108763
P18	8	1.0064601898193359e-05	0.7390851332151606	-1.11E-16	0.739085133	0.7422270732175922
P19	6	8.002758026123047e-06	1.1141571408719302	2.220446049250313e-16	1.1132427327642702	1.1141571408719768
P20	10	1.5938282012939452e-05	2.073932809091215	-2.22E-16	2.0739328090911866	2.078935003337393
P21	12	1.6058921813964842e-05	0.9999999999999999	-1.11E-15	0.9999999999999305	1.000343363282986
P22	8	2.393531799316406e-05	1.6490135532979473	-3.55E-15	1.6490135532974015	1.6531560376633945
P23	9	1.7997264862060548e-05	3.221588399093942	3.3306690738754696e-16	3.2215883990939242	3.2224168881395068
P24	9	1.2019157409667969e-05	2.125393426233225	-7.11E-15	2.1253934262325003	2.1275213330097245
P25	7	1.1998653411865234e-05	0.8767262153950581	4.773959005888173e-15	0.8767262153886713	0.8772684454348731

As we see from the table above the hybrid method tend to be faster and take much less iterations than both Bisection and False Position methods.