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1 *Chapter One: Introduction*

1.1 Introduction

The ever-evolving landscape of cyber threats demands constant innovation in the field of cryptography. Existing encryption algorithms, while providing valuable protection, are often riddled with limitations. Computational complexity can hinder performance, and the rise of quantum computing casts a shadow on the future of established methods. This project presents a groundbreaking departure from tradition, introducing a novel encryption algorithm that leverages the potent combination of polynomials and root finding methods.

This paper delves into the intricate details of the algorithm, meticulously explaining each step of the encryption and decryption processes. We provide a comprehensive analysis of its performance, Comparing it with established methods such as AES, showcasing its significant speed advantage.

1.2 Problem & Project Aim

Encryption, regardless of its application, inevitably requires some processing time. This could be for file encryption on a disk or network encryption via a VPN. The extent of this slowdown is contingent on the encryption algorithms employed and, crucially, the proficiency of the programmer who crafted the encryption and decryption code.

Algorithms with high computational complexity can become performance bottlenecks, particularly for systems necessitating real-time data encryption and decryption. This can adversely affect real-time applications such as video conferencing, secure voice calls, or high-speed data transfers. The processing delays induced by encryption can result in lag, disruptions, and a bad user experience.

The aim of this project is to develop a high-speed encryption algorithm that can be used for real-time applications. The algorithm should be able to encrypt and decrypt data at a much faster rate than traditional encryption algorithms, then it will be used to encrypt and decrypt data in real-time messaging web application.

2 *Chapter Two: Numerical Methods*

2.1 Root Finding Methods

At the heart of our innovative encryption algorithm lies a powerful mathematical tool: root finding methods. These methods, while seemingly abstract, play a crucial role in ensuring the security and efficiency of our solution. But before we delve into their specific application, let's unpack what they are and why they hold such significance.

In essence, root finding methods aim to solve the equation $f(x) = 0$, where $f(x)$ is any function. They essentially seek the "roots" of the function, which are the values of x that make the function evaluate to zero. This seemingly simple task becomes incredibly powerful in cryptography.

In our algorithm, we leverage this power by strategically designing the function $f(x)$ to incorporate the encryption key as an unknown variable. Through carefully chosen root finding methods, we iteratively approach the function's roots, and in the decryption process, utilize these roots to recover the original data. The elegance of this approach lies in its inherent security: without knowledge of both the root finding method and how the key is embedded within the function, an attacker would face a near-impossible task of finding the correct roots, keeping your data safe.

However, the importance of root finding methods extends far beyond encryption. They have diverse applications across various fields. In numerical analysis, they are used for solving differential equations and optimization problems among other things. In engineering design, they are crucial for calculating parameters in fields like fluid dynamics and structural analysis. In computer graphics, they are essential for generating realistic images and animations.

Root finding algorithms also have a significant role in machine learning. They are utilized in optimization methods like gradient descent, which is a common technique for training models. The goal of these methods is to minimize a loss function, thereby improving the model's accuracy.

They also have a vital role in economics and finance. They are used to calculate internal rates of return, solve equilibrium equations in economic models, and find optimal investment strategies.

And finally, we will be using root finding methods to solve the polynomial equations that we will be using in our encryption algorithm.

2.2 Bisection Method

The Bisection Method is a straightforward and reliable numerical method used for solving equations in mathematics, particularly in the field of engineering. It solves equations by repeatedly bisecting an interval and then selecting a subinterval in which a root must lie for further processing.

2.2.1 How Does the Bisection Method Work?

If we have a function $f(x)$ that is continuous on the interval $[a, b]$ and $f(a) \cdot f(b) < 0$ (the signals of $f(x)$ at the ends a and b are different), then the function has at least one root in the interval $[a, b]$. The Bisection Method works by repeatedly bisecting the interval and then selecting a subinterval in which a root must lie for further processing. The value of x at the midpoint of the interval is equal to $\frac{a+b}{2}$, if $f(\frac{a+b}{2}) = 0$, then $\frac{a+b}{2}$ is the root of the equation. If $f(a) \cdot f(\frac{a+b}{2}) < 0$, then the root lies in the interval $[a, \frac{a+b}{2}]$, and if $f(\frac{a+b}{2}) \cdot f(b) < 0$, then the root lies in the interval $[\frac{a+b}{2}, b]$. This process is repeated until we reach the desired accuracy.

The number of iterations required to reach the desired accuracy can be calculated using the formula:

$$n = \lceil \log_2(\frac{b-a}{\epsilon}) \rceil \tag{1}$$

Where n is the number of iterations, a and b are the lower and upper bounds of the interval, and ϵ is the desired accuracy.

The accuracy of the Bisection Method can be calculated using the formula:

$$\epsilon = \frac{b-a}{2^n} \tag{2}$$

2.2.2 Bisection Method Advantages

There are several key advantages to the bisection method:

- Guaranteed convergence. The bracketing approach is known as the bisection method, and it is always convergent.
- Errors can be managed. Increasing the number of iterations in the bisection method always results in a more accurate root.
- Doesn't demand complicated calculations. There are no complicated calculations required when using the bisection method. To use the bisection method, we only need to take the average of two values.
- The bisection method is simple and straightforward to programme on a computer.
- In the case of several roots, the bisection procedure is quick.

2.2.3 Bisection Method Disadvantages

There are also some limitations to the bisection method:

- Although the Bisection method's convergence is guaranteed, it is often slow.
- Choosing a guess that is close to the root may necessitate numerous iterations to converge.
- Some equations' roots cannot be found. Because there are no bracketing values, like $f(x) = x^2$.
- Its rate of convergence is linear.
- It is incapable of determining complex roots.
- If the guess interval contains discontinuities, it cannot be used.
- It cannot be applied over an interval where the function returns values of the same sign.

2.3 False Position Method

In mathematics, the regula falsi, method of false position, or false position method is a very old method for solving an equation with one unknown; this method, in modified form, is still in use. In simple terms, the method is the trial and error technique of using test ("false") values for the variable and then adjusting the test value according to the outcome. This is sometimes also referred to as "guess and check". Versions of the method predate the advent of algebra and the use of equations.

2.3.1 How Does the False Position Method Work?

If we have a function $f(x)$ that is continuous on the interval $[a, b]$ and $f(a) \cdot f(b) < 0$ (the signals of $f(x)$ at the ends a and b are different), then the function has at least one root in the interval $[a, b]$.

The interval $[a, b]$ have different signs. The false position method uses two endpoints of the interval $[a, b]$ with initial values $(r_0 = a, r_1 = b)$. The connecting line between the two points $(r_0, f(r_0))$ and $(r_1, f(r_1))$ intersects the x-axis at the next estimate, r_2. Now, we can determine the successive estimates, r_n from the following relationship:

$$r_n = r_{n-1} - \frac{f(r_{n-1})(r_{n-1} - r_{n-2})}{f(r_{n-1}) - f(r_{n-2})} \tag{3}$$

2.3.2 False Position Method Advantages

There are several key advantages to the false position method:

- Convergence is guarenteed: this method is bracketing method and it is always convergent.
- Error can be controlled: increasing number of iteration always yields more accurate root.
- Does not require derivative: this method does not require derivative calculation.

2.3.3 False Position Method Disadvantages

There are also some limitations to the false position method:

- Slow Rate of Convergence: Although convergence of Regula Falsi method is guaranteed, it is generally slow.
- Can not find root of some equations. For example: $f(x) = x^2$ as there are no bracketing values.
- It has linear rate of convergence.
- It fails to determine complex roots.
- It can not be applied if there are discontinuities in the guess interval.
- It can not be applied over an interval where the function takes values of the same sign.

2.4 Secant Method

In numerical analysis, the secant method is a root-finding algorithm that uses a sequence of roots of secant lines to better approximate a root of a function f. Unlike the method of false position, which keeps one endpoint fixed, the secant method uses two moving points. The secant method can be thought of as a finite-difference approximation of Newton's method. However, the method was developed independently of Newton's method and predates it by many years.

2.4.1 How Does the Secant Method Work?

The secant method begins with two initial approximations x_0 and x_1 for the root. These points should ideally be close to the actual root. The method then uses the secant line through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ to obtain a new approximation x_2, which is the x-intercept of this line. The formula for the new approximation is:

$$x_n = x_{n-1} - f(x_{n-1})\frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})} \tag{4}$$

This process is repeated until the difference between successive approximations is less than a predetermined tolerance level.

2.4.2 Secant Method Advantages

The secant method has several advantages:

- **Efficiency**: It typically converges faster than the method of false position and bisection method.
- **Simplicity**: It does not require the function's derivative, unlike Newton's method.
- **Flexibility**: It can handle a wide range of functions.

2.4.3 Secant Method Disadvantages

However, the secant method also has its disadvantages:

- **Convergence is not guaranteed**: If the initial guesses are not close to the actual root, the method may fail to converge.
- **Sensitive to initial guesses**: The closer the initial guesses to the root, the faster the convergence.
- **Possibility of divergence**: If the function is not well-behaved, the secant method can diverge.
- **Multiple roots**: The method may have difficulty distinguishing between multiple roots that are close to each other.
- **Requires two initial values**: Unlike the bisection method, which only needs a single interval, the secant method requires two initial approximations.

2.5 HybridBF Algorithm

The HybridBF algorithm is a hybrid algorithm between the bisection method and false position method. The algorithm works as follows:

1. Take the polynomial and the interval that contains the root.
2. In each iteration, the algorithm will apply the bisection method and the false position method and get the root from each method.
3. The algorithm will choose the root that will give the smallest absolute value of the polynomial $|f(x)|$.
4. The algorithm will stop when the absolute value of the polynomial is less than a certain tolerance we define.

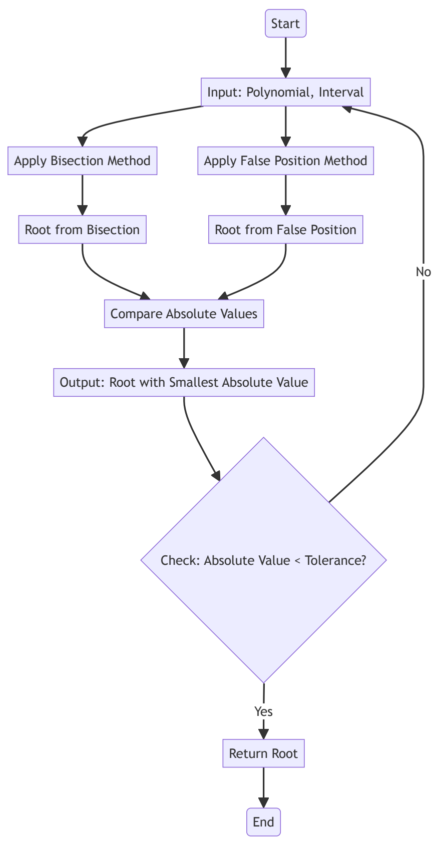
Start
Input: Polynomial, Interval
Apply Bisection Method
Apply False Position Method
Root from Bisection
Root from False Position
Compare Absolute Values
Output: Root with Smallest Absolute Value
Check: Absolute Value < Tolerance?
No
Yes
Return Root
End

Figure 1: HybridBF Steps Flowchart

2.6 HybridSF Algorithm

It has the same steps as HybridBF but it uses the secant method instead of the bisection method.

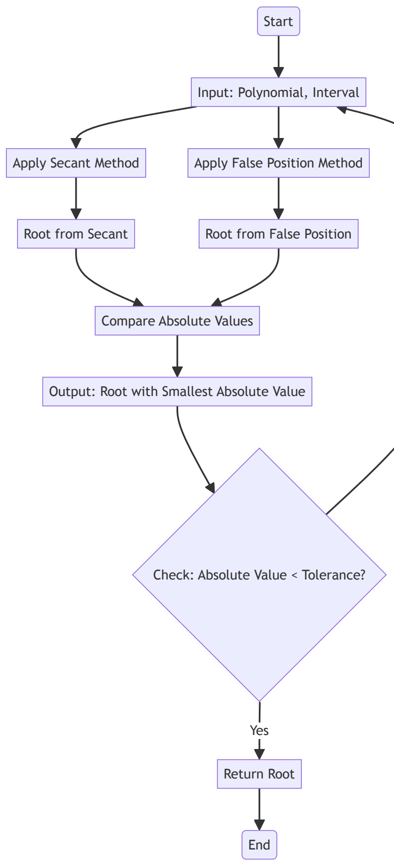
Start
Input: Polynomial, Interval
Apply Secant Method
Apply False Position Method
Root from Secant
Root from False Position
Compare Absolute Values
Output: Root with Smallest Absolute Value
Check: Absolute Value < Tolerance?
No
Yes
Return Root
End

Figure 2: HybridSF Steps Flowchart

To test the algorithm we have used the same 25 equations with each method and run each method (Bisection, False Position, HybridBF, Secant, and HybridSF) 500 times for each problem and then we have calculated the average time and the number of iterations each method have taken for each problem.

We have also used the same tolerance for each method which is $\epsilon = 10^{-14}$

These are the equations that we have used with each method:

2.7 Equations That Serve as Test Cases

In these equations we have tried to use different types of functions like polynomial, exponential, trigonometric, and logarithmic functions to ensure that the algorithm works with different types of functions. We have also used different intervals with each algorithm depending on where the roots of the equations are.

Table 1: Test Cases Equations

No	Equation	Interval
P_1	$f(x) = x^3 + 4x^2 - 10 = 0$	$[0, 4]$

No	Equation	Interval
P_2	$f(x) = x^2 - 4$	$[0, 4]$
P_3	$f(x) = e^x - 2$	$[0, 2]$
P_4	$f(x) = \sin(x)$	$[2, 6]$
P_5	$f(x) = x^3 - 6x^2 + 11x - 6$	$[1, 2.5]$
P_6	$f(x) = x^2 + 3x + 2$	$[-2.5, -1.5]$
P_7	$f(x) = \cos(x) - x$	$[0, 1]$
P_8	$f(x) = 2^x - 8$	$[2, 4]$
P_9	$f(x) = \tan(x)$	$[-1, 1]$
P_{10}	$f(x) = x^4 - 8x^3 + 18x^2 - 9x + 1$	$[2, 4]$

2.8 Extra Equations From Paper

We got these equations from this paper and we have used the same intervals too.

Table 2: Equations From Paper

No	Equation	Interval	Reference
P_{11}	$f(x) = x^2 - 3$	$[1, 2]$	Harder [18]
P_{12}	$f(x) = x^2 - 5$	$[2, 7]$	Srivastava[9]
P_{13}	$f(x) = x^2 - 10$	$[3, 4]$	Harder [18]
P_{14}	$f(x) = x^2 - x - 2$	$[1, 4]$	Moazzam [10]
P_{15}	$f(x) = x^2 + 2x - 7$	$[1, 3]$	Nayak[11]
P_{16}	$f(x) = x^3 - 2$	$[0, 2]$	Harder [18]
P_{17}	$f(x) = xe^x - 7$	$[0, 2]$	Callhoun [19]
P_{18}	$f(x) = x - \cos(x)$	$[0, 1]$	Ehiwario [6]
P_{19}	$f(x) = x\sin(x) - 1$	$[0, 2]$	Mathews [20]
P_{20}	$f(x) = x\cos(x) + 1$	$[-2, 4]$	Esfandiari [21]
P_{21}	$f(x) = x^{10} - 1$	$[0, 1.3]$	Chapra [17]
P_{22}	$f(x) = x^2 + e^{x/2} - 5$	$[1, 2]$	Esfandiari [21]
P_{23}	$f(x) = \sin(x)\sinh(x) + 1$	$[3, 4]$	Esfandiari [21]
P_{24}	$f(x) = e^x - 3x - 2$	$[2, 3]$	Hoffman [22]
P_{25}	$f(x) = \sin(x) - x^2$	$[0.5, 1]$	Chapra[17]

2.9 Root Finding Algorithms Performance Results

These are the results we got with each method. We have run each method 500 times on each equation and took the average time to get the highest accuracy possible.

2.9.1 False Position

These are the results we got with False Position method:

Table 3: False Position

Problem	False Position Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
P_1	80	0.000108872890472412	1.3652300134141	-7.105427357601E-15	1.3652300134141	4
P_2	33	2.54607200622559E-05	2	-8.88178419700125E-15	2	4
P_3	51	3.37719917297363E-05	0.693147180559942	-6.21724893790088E-15	0.693147180559942	2
P_4	8	4.61483001708984E-06	3.14159265358979	1.22464679914735E-16	3.14159265358979	3.14159265358992
P_5	2	2.02751159667969E-06	1	0	1	2.5
P_6	31	3.35359573364258E-05	-2	-5.32907051820075E-15	-2.5	-2
P_7	12	7.18259811401367E-06	0.739085133215155	9.2148511043888E-15	0.739085133215155	1
P_8	30	2.37202644348144E-05	3	-7.105427357601E-15	3	4
P_9	2	9.75608825683594E-07	0	0	0	1
P_{10}	13	2.66284942626953E-05	3.11174865630925	0	3.11174865630925	3.11174865630925
P_{11}	14	1.06868743896484E-05	1.73205080756888	-3.99680288865056E-15	1.73205080756888	2
P_{12}	50	3.87668609619141E-05	2.23606797749979	-9.76996261670138E-15	2.23606797749979	7
P_{13}	17	1.34563446044922E-05	3.16227766016838	-1.77635683940025E-15	3.16227766016838	4
P_{14}	38	3.21516990661621E-05	2	-8.65973959207622E-15	2	4
P_{15}	21	2.0256519317627E-05	1.82842712474619	-2.66453525910038E-15	1.82842712474619	3
P_{16}	41	3.12848091125488E-05	1.25992104989487	-6.21724893790088E-15	1.25992104989487	2
P_{17}	30	2.12483406066895E-05	1.52434520498414	-7.99360577730113E-15	1.52434520498414	2
P_{18}	12	7.22360610961914E-06	0.739085133215155	-9.2148511043888E-15	0.739085133215155	1
P_{19}	7	5.02967834472656E-06	1.11415714087193	8.88178419700125E-16	1.09975017029462	1.11415714087193
P_{20}	13	9.73367691040039E-06	2.07393280909121	7.7715611723761E-16	2.07393280909121	2.51571977101466
P_{21}	139	0.000106982231140137	0.999999999999999	-8.88178419700125E-15	0.999999999999999	1.3
P_{22}	16	1.86800956726074E-05	1.64901326830319	-8.88178419700125E-16	1.64901326830319	2
P_{23}	45	5.06892204284668E-05	3.22158839909394	6.43929354282591E-15	3.22158839909394	4
P_{24}	45	3.95450592041016E-05	2.12539119881113	-8.88178419700125E-15	2.12539119881113	3
P_{25}	17	1.45211219787598E-05	0.876726215395055	7.88258347483861E-15	0.876726215395055	1

2.9.2 Bisection Method

These are the results we got with Bisection method:

Table 4: Bisection

Problem	Bisection Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
P_1	50	3.78618240356445E-05	1.3652300134141	-2.8421709430404E-14	1.36523001341409	1.3652300134141
P_2	1	9.20772552490234E-07	2	0	0	4
P_3	49	2.00080871582031E-05	0.693147180559944	-3.33066907387547E-15	0.69314718055994	0.693147180559947
P_4	50	1.82290077209473E-05	3.14159265358979	1.22464679914735E-16	3.14159265358979	3.1415926535898
P_5	48	4.16674613952637E-05	2	0	2	2.00000000000001
P_6	1	1.10149383544922E-06	-2	0	-2.5	-1.5
P_7	48	1.79662704467773E-05	0.739085133215159	2.55351295663786E-15	0.739085133215156	0.739085133215163
P_8	1	9.37938690185547E-07	3	0	2	4
P_9	1	7.75814056396484E-07	0	0	-1	1
P_{10}	49	5.54814338684082E-05	3.11174865630925	1.06581410364015E-14	3.11174865630925	3.11174865630925
P_{11}	48	2.20985412597656E-05	1.73205080756888	4.44089209850063E-15	1.73205080756888	1.73205080756888

Problem	Bisection Algorithm					
	Iter	**Avg CPU Time**	**Approximate Root**	**Function Value**	**Lower Bound**	**Upper Bound**
P_{12}	50	2.28300094604492E-05	2.23606797749979	-1.95399252334028E-14	2.23606797749978	2.23606797749979
P_{13}	48	2.22716331481934E-05	3.16227766016838	1.59872115546023E-14	3.16227766016838	3.16227766016839
P_{14}	50	2.49624252319336E-05	2	-2.66453525910038E-15	2	2
P_{15}	49	2.77295112609863E-05	1.82842712474619	-1.15463194561016E-14	1.82842712474618	1.82842712474619
P_{16}	49	2.24394798278809E-05	1.25992104989487	5.32907051820075E-15	1.25992104989487	1.25992104989488
P_{17}	49	2.14519500732422E-05	1.52434520498415	3.37507799486048E-14	1.52434520498414	1.52434520498415
P_{18}	48	1.79176330566406E-05	0.739085133215159	-2.55351295663786E-15	0.739085133215156	0.739085133215163
P_{19}	49	2.24361419677734E-05	1.11415714087193	-2.99760216648792E-15	1.11415714087192	1.11415714087193
P_{20}	51	2.31366157531738E-05	2.07393280909122	-1.33226762955019E-15	2.07393280909121	2.07393280909122
P_{21}	48	2.20670700073242E-05	1	1.11022302462516E-14	0.999999999999996	1.00000000000001
P_{22}	48	3.11980247497559E-05	1.64901326830319	-3.5527136788005E-15	1.64901326830319	1.64901326830319
P_{23}	48	3.00378799438477E-05	3.22158839909394	-5.55111512312578E-15	3.22158839909394	3.22158839909395
P_{24}	46	2.39477157592773E-05	2.12539119881113	0	2.12539119881112	2.12539119881114
P_{25}	47	2.58064270019531E-05	0.876726215395063	-8.88178419700125E-16	0.87672621539506	0.876726215395067

2.9.3 Secant Method

These are the results we got with Secant method:

Table 5: Secant

Problem	Secant Algorithm					
	Iter	**Avg CPU Time**	**Approximate Root**	**Function Value**	**Lower Bound**	**Upper Bound**
P_1	12	6.93511962890625E-06	1.3652300134141	0	1.3652300134141	1.3652300134141
P_2	9	3.46088409423828E-06	2	0	2	2
P_3	9	3.33023071289062E-06	0.693147180559945	0	0.693147180559945	0.693147180559945
P_4	7	2.45857238769531E-06	6.28318530717959	-2.44929359829471E-16	6.28318530717959	6.28318530717959
P_5	2	1.84392929077148E-06	1	0	1	1
P_6	10	4.70542907714844E-06	-2	0	-2	-2
P_7	7	2.44808197021484E-06	0.739085133215161	0	0.739085133215161	0.739085133215161
P_8	8	3.28588485717773E-06	3	0	3	3
P_9	2	1.04379653930664E-06	0	0	0	0
P_{10}	17	1.45139694213867E-05	0.481389601149957	0	0.481389601149957	0.481389601149957
P_{11}	7	2.82144546508789E-06	1.73205080756888	4.44089209850063E-16	1.73205080756888	1.73205080756888
P_{12}	8	3.25679779052734E-06	2.23606797749979	8.88178419700125E-16	2.23606797749979	2.23606797749979
P_{13}	6	2.61688232421875E-06	3.16227766016838	-1.77635683940025E-15	3.16227766016838	3.16227766016838
P_{14}	9	3.77416610717773E-06	2	0	2	2
P_{15}	7	3.43656539916992E-06	1.82842712474619	8.88178419700125E-16	1.82842712474619	1.82842712474619
P_{16}	11	4.30393218994141E-06	1.25992104989487	0	1.25992104989487	1.25992104989487
P_{17}	10	3.88193130493164E-06	1.52434520498414	0	1.52434520498414	1.52434520498414
P_{18}	7	2.60496139526367E-06	0.739085133215161	0	0.739085133215161	0.739085133215161
P_{19}	6	2.66456604003906E-06	1.11415714087193	2.22044604925031E-16	1.11415714087193	1.11415714087193
P_{20}	9	3.63922119140625E-06	2.07393280909121	-2.22044604925031E-16	2.07393280909121	2.07393280909121
P_{21}				***No Solution Was Found***		
P_{22}	7	4.0740966796875E-06	1.64901326830319	0	1.64901326830319	1.64901326830319
P_{23}	8	4.34207916259766E-06	3.22158839909394	3.33066907387547E-16	3.22158839909394	3.22158839909394
P_{24}	8	3.73697280883789E-06	2.12539119881113	0	2.12539119881113	2.12539119881113
P_{25}	8	3.88669967651367E-06	0.876726215395062	0	0.876726215395062	0.876726215395062

2.9.4 HybridBF Method

These are the results we got with HybridBF method:

Table 6: HybridBF

Problem	HybridBF Algorithm					
	Iter	**Avg CPU Time**	**Approximate Root**	**Function Value**	**Lower Bound**	**Upper Bound**
P_1	10	1.51724815368652E-05	1.3652300134141	-7.105427357601E-15	1.36523001341378	1.36750019802744
P_2	1	9.86576080322266E-07	2	0	0	4
P_3	10	1.01175308227539E-05	0.693147180559945	0	0.693147180559933	0.695162706464415
P_4	6	5.41830062866211E-06	3.14159265358979	1.22464679914735E-16	3.14159035795569	3.14159265360489
P_5	1	1.57594680786133E-06	1	0	1	2.5
P_6	1	1.17969512939453E-06	-2	0	-2.5	-1.5
P_7	8	7.32851028442383E-06	0.739085133215161	1.11022302462516E-16	0.739085133215147	0.742227073217592
P_8	1	1.02519989013672E-06	3	0	2	4
P_9	1	8.06331634521484E-07	0	0	-1	1
P_{10}	8	1.61228179931641E-05	3.11174865630925	0	3.10853799278589	3.11174865630925
P_{11}	8	8.37850570678711E-06	1.73205080756888	-4.44089209850063E-16	1.7320508075688	1.73505784022098
P_{12}	10	1.055908203125E-05	2.23606797749979	-3.5527136788005E-15	2.23606797749936	2.24392915398361
P_{13}	8	8.51917266845703E-06	3.16227766016838	1.77635683940025E-15	3.16227766016837	3.1672187190124
P_{14}	2	2.20155715942383E-06	2	0	1.5	2.5
P_{15}	5	6.05535507202148E-06	1.82842712474619	0	1.828427124743	1.82842712474938
P_{16}	9	9.37175750732422E-06	1.25992104989487	-3.99680288865056E-15	1.25992104989398	1.2611286403177
P_{17}	11	1.14202499389648E-05	1.52434520498414	0	1.52434520498414	1.52603333710876
P_{18}	8	7.33852386474609E-06	0.739085133215161	-1.11022302462516E-16	0.739085133215147	0.742227073217592
P_{19}	6	6.33621215820313E-06	1.11415714087193	-2.22044604925031E-16	1.11324273276427	1.11415714087198
P_{20}	10	1.05037689208984E-05	2.07393280909121	-2.22044604925031E-16	2.07393280909119	2.07893500333739
P_{21}	12	1.24101638793945E-05	1	-1.11022302462516E-15	0.99999999999993	1.00034336328299
P_{22}	8	1.04570388793945E-05	1.64901326830319	-3.5527136788005E-15	1.64901326830264	1.65315575626948
P_{23}	9	1.19032859802246E-05	3.22158839909394	3.33066907387547E-16	3.22158839909392	3.22241688813951
P_{24}	9	1.03793144226074E-05	2.12539119881113	-6.21724893790088E-15	2.1253911988104	2.12751913344632
P_{25}	7	7.85541534423828E-06	0.876726215395058	4.77395900588817E-15	0.876726215388671	0.877268445434873

As we see from the table above the hybrid method tend to be faster and take much less iterations than both Bisection and False Position methods.

2.9.5 HybridSF Method

These are the results we got with HybridSF method:

Table 7: HybridSF

Problem	HybridSF Algorithm					
	Iter	**Avg CPU Time**	**Approximate Root**	**Function Value**	**Lower Bound**	**Upper Bound**
P_1	40	4.32667732238769E-05	1.3652300134141	-7.105427357601E-15	1.3652300134141	4
P_2	17	1.1502742767334E-05	2	-2.66453525910038E-15	2	4
P_3	26	1.56879425048828E-05	0.693147180559944	-3.10862446895044E-15	0.693147180559942	2
P_4	6	3.37457656860352E-06	3.14159265358979	1.22464679914735E-16	3.14159265358979	3.14159265363241
P_5	1	1.11484527587891E-06	1	0	1	2.5
P_6	16	1.37491226196289E-05	-2	-4.44089209850063E-15	-2.00000000000001	-1.75
P_7	6	3.53384017944336E-06	0.739085133215155	9.2148511043888E-15	0.73908513321505	1
P_8	15	1.04451179504395E-05	3	-7.105427357601E-15	3	4

Problem	Hybrid Algorithm					
	Iter	Avg CPU Time	Approximate Root	Function Value	Lower Bound	Upper Bound
P_9	1	6.39915466308594E-07	0	0	-1	1
P_{10}	7	1.10006332397461E-05	3.11174865630925	7.105427357601E-15	3.11174865630925	3.11174865630925
P_{11}	7	4.94670867919922E-06	1.73205080756888	-3.99680288865056E-15	1.73205080756886	2
P_{12}	25	1.68638229370117E-05	2.23606797749979	-9.76996261670138E-15	2.23606797749979	7
P_{13}	9	6.21271133422852E-06	3.16227766016838	1.77635683940025E-15	3.16227766016838	4
P_{14}	19	1.37357711791992E-05	2	-8.65973959207622E-15	1.99999999999999	4
P_{15}	11	9.09614562988281E-06	1.82842712474619	0	1.82842712474619	3
P_{16}	21	1.41358375549316E-05	1.25992104989487	-3.10862446895044E-15	1.25992104989487	2
P_{17}	15	9.67836380004883E-06	1.52434520498414	-7.99360577730113E-15	1.52434520498414	2
P_{18}	6	3.55148315429688E-06	0.739085133215155	-9.2148511043888E-15	0.73908513321505	1
P_{19}	5	3.5557746887207E-06	1.11415714087193	2.22044604925031E-16	1.11415714087193	1.11415714087196
P_{20}	7	0.0000048828125	2.07393280909121	1.77635683940025E-15	2.07393280909119	2.51571977101466
P_{21}	70	4.62303161621094E-05	1	-6.66133814775094E-15	0.999999999999999	1.3
P_{22}	8	7.51972198486328E-06	1.64901326830319	-8.88178419700125E-16	1.64901326830319	2
P_{23}	23	2.04296112060547E-05	3.22158839909394	3.33066907387547E-16	3.22158839909394	4
P_{24}	23	1.71117782592773E-05	2.12539119881113	-1.77635683940025E-15	2.12539119881113	3
P_{25}	9	6.75392150878906E-06	0.876726215395061	1.11022302462516E-15	0.876726215395055	1

2.9.6 Conclusion

These are our conclusions based on the tables and the plots:

2.9.6.1 Iterations

2.9.6.1.1 Bisection, False Position, HybridBF As you see in the plot below the HybridBF method demonstrates superior performance compared to both the bisection and false position methods in terms of the number of iterations required.

As we see here in P_{21} the false position method have much more number of iterations than both hybrid and bisection methods which will lead to more CPU time as we will see in the next section.

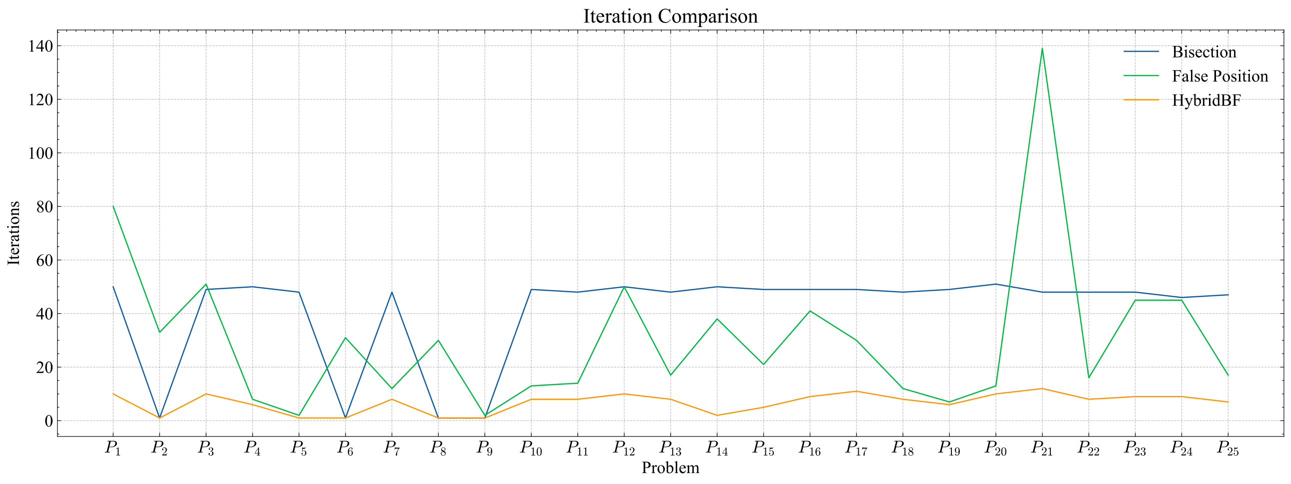

Figure 3: Iterations Comparison Bisection, False Position, HybridBF

2.9.6.1.2 Secant, False Position, HybridSF The HybridSF method here didn't have the same performance improvement as in the previous case. The secant method is the fastest in terms of iterations, followed by the hybrid method then false position, However There are 7 problems where the HybridSF method is faster than the secant method with 1 or 2 iterations, these are P_4, P_5, P_7, P_9, P_{18}, P_{19}, P_{20}.

You can also notice that the graph of the secant method is ***not continuous*** on P_{21} since secant method is not guaranteed to converge, so it wasn't able to find the root in this problem.

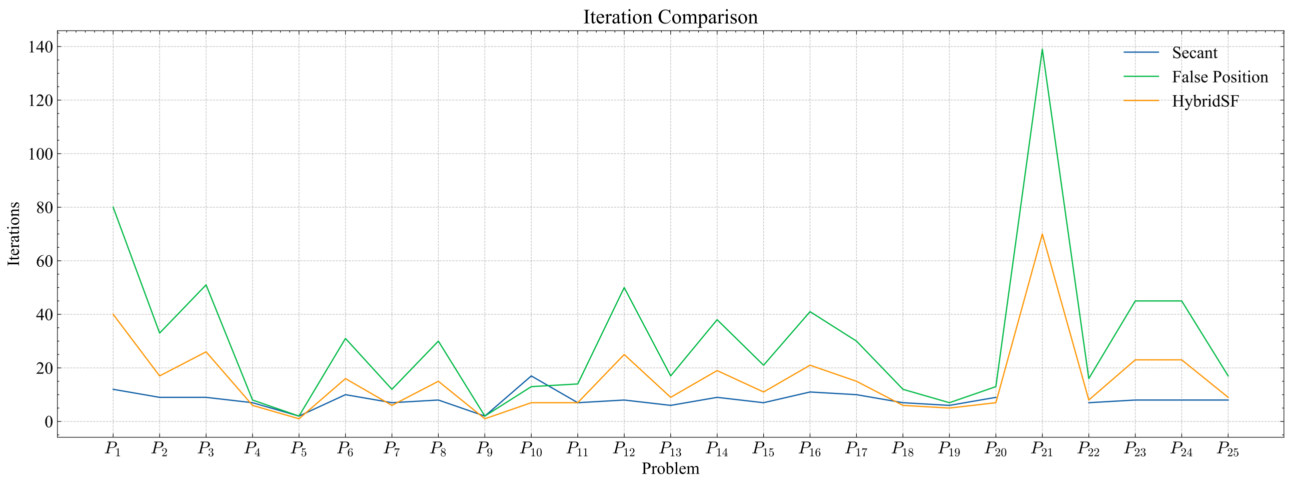

Figure 4: Iterations Comparison Secant, False Position, HybridSF

2.9.6.2 CPU Time

2.9.6.2.1 Bisection, False Position, HybridBF As a result of having less number of iterations the hybridBF method shows significant improvement over the bisection method in terms of CPU time, with a ratio of 21:4. This translates to approximately 84% for the hybrid method and 16% for the bisection method.

It also shows an improvement over the false position method in terms of CPU time, with a ratio of 19:6. This translates to approximately 76% for the hybrid method and 24% for the false position method.

As a general trend, the hybrid method is faster than both the bisection and false position methods when it comes to finding the approximate root.

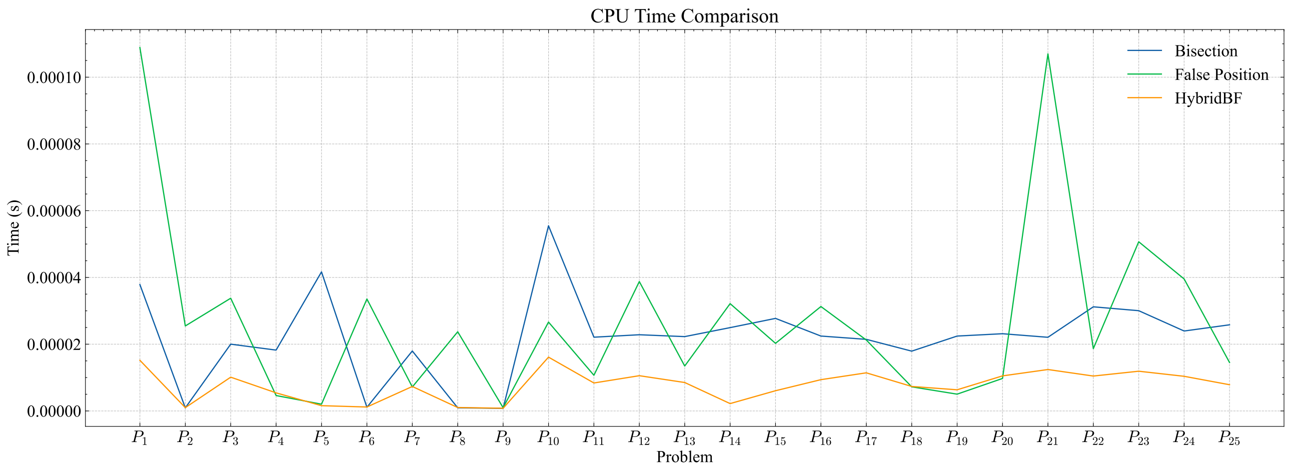

Figure 5: CPU Time Comparison Bisection, False Position, HybridBF

2.9.6.2.2 Secant, False Position, HybridSF As a result of secant method having less number of iterations than both HybridSF and false position and much more simple implementation than HybridSF, it has less CPU time. The secant method is the fastest in terms of CPU time, followed by the hybrid method then false position.

This happens in all problems except only two problems which are P_5, P_9 where the HybridSF method is *slightly* faster than the secant method.

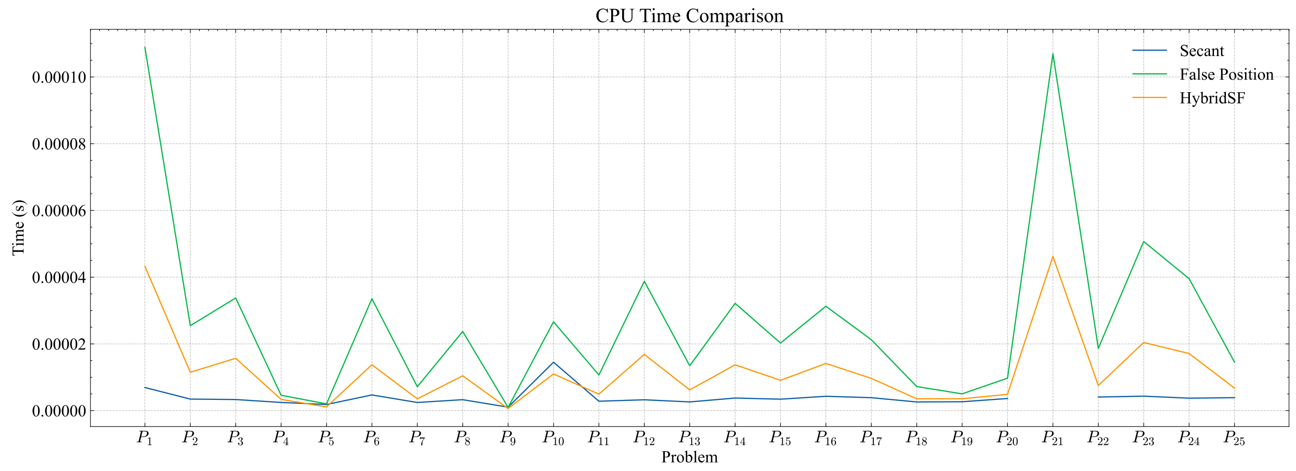

Figure 6: CPU Time Comparison Secant, False Position, HybridSF

2.9.6.3 Function Value

2.9.6.3.1 Bisection, False Position, HybridBF The hybrid method outperforms both the bisection and false position methods in terms of function value, with smaller absolute values that are closer to zero.

This happens in all problems except only one problem P_{24} in which the bisection method had the nearest value to zero.

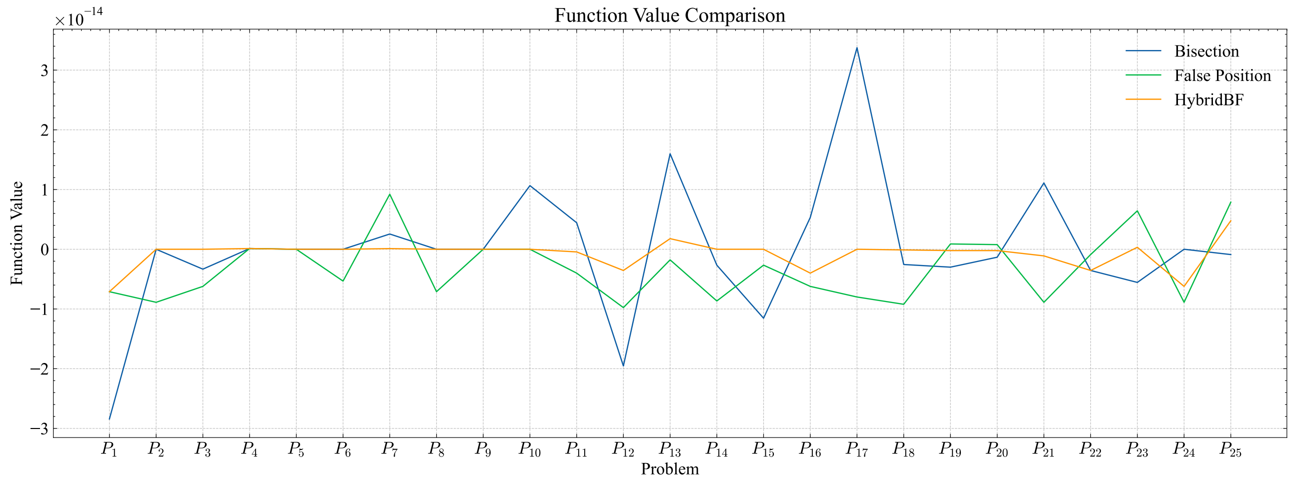

Figure 7: Function Value Comparison Bisection, False Position, HybridBF

2.9.6.3.2 Secant, False Position, HybridSF Again the secant method shows an improvement over both the false position and HybridSF methods in terms of function value, with smaller absolute values that are closer to zero.

This happens in all problems except two P_4 in which both HybridSF and false position methods had the nearest value to zero, and P_{15} in which the HybridSF had a value closer to zero.

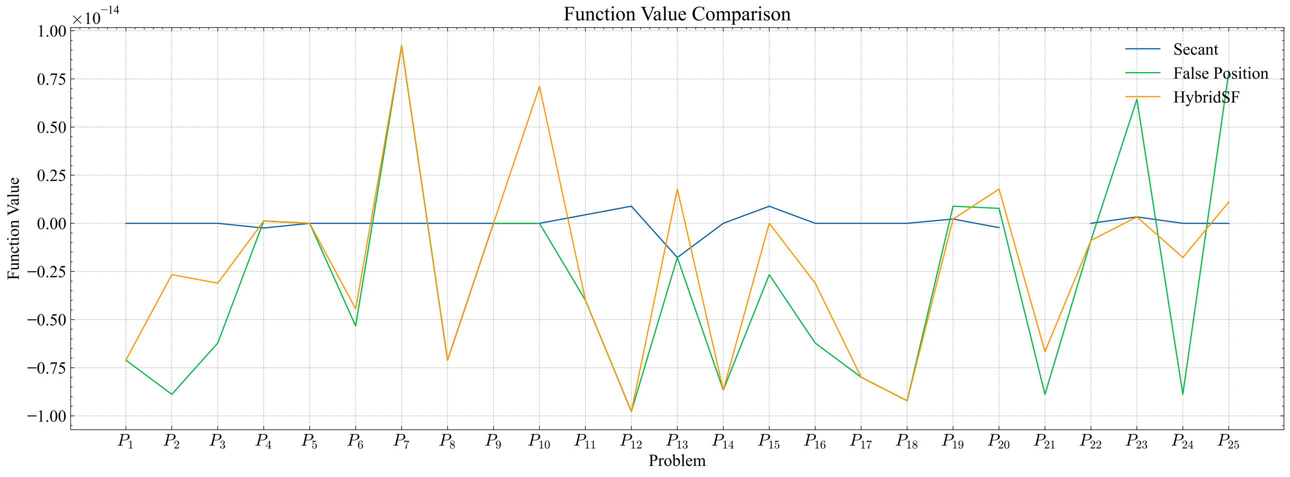

Figure 8: Function Value Comparison Secant, False Position, HybridSF

2.9.6.4 Secant vs HybridBF When comparing the results of the fastest two algorithms which are the secant and HybridBF methods, we found the following:

2.9.6.4.1 Iterations There is no winner here, the secant method is faster in some problems and the HybridBF method is faster in others but there is a slight advantage for the HybridSF method over the secant method.

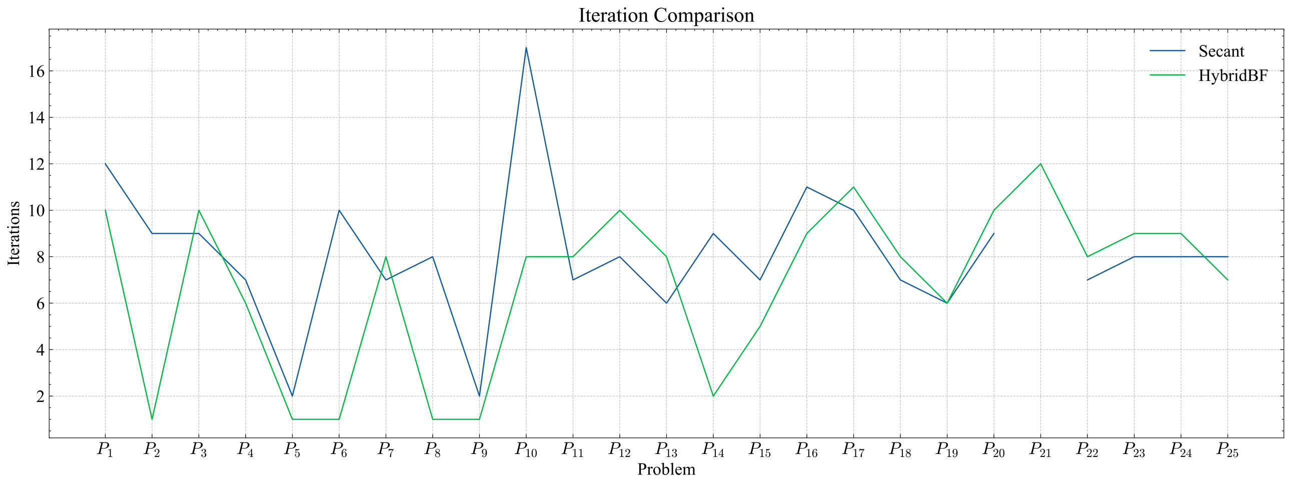

Figure 9: Iterations Comparison Secant, HybridBF

2.9.6.4.2 CPU Time When comparing the CPU time of both algorithms we found that the secant method is faster in all problems except for 6 problems which are P_2, P_5, P_6, P_8, P_9, P_{14}.

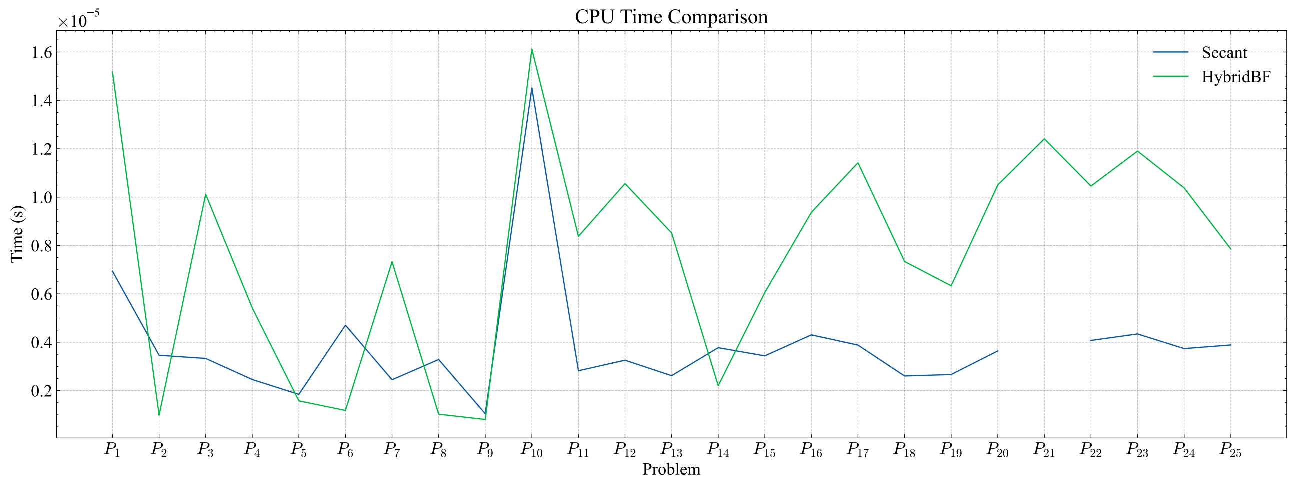

Figure 10: CPU Time Comparison Secant, HybridBF

2.9.6.4.3 Function Value When comparing function values with absolute values closer to zero, the results of secant method were better than the HybridBF method in nearly all problems except for P_{15}.

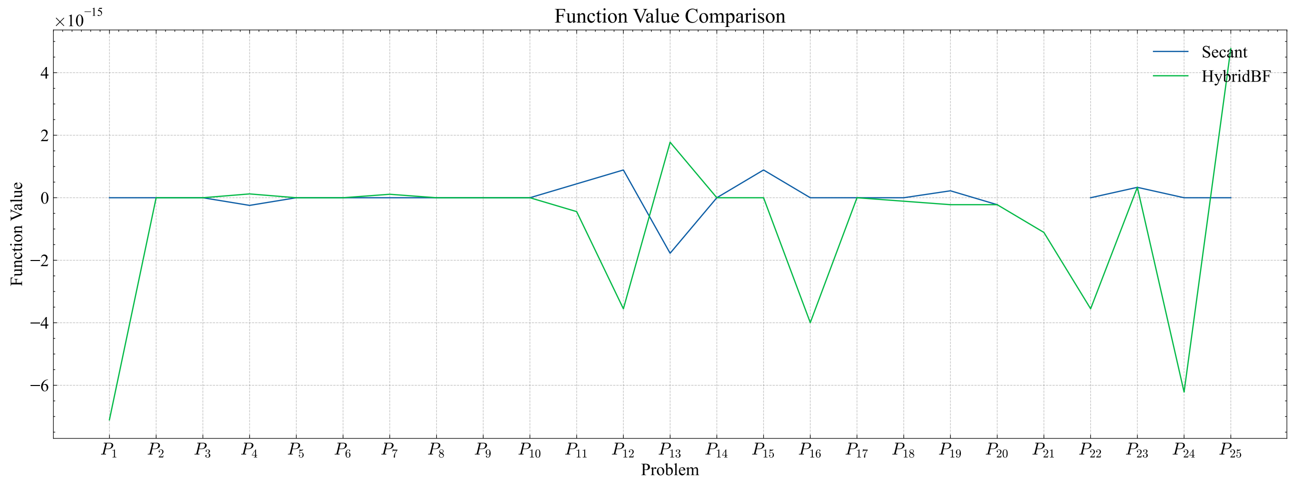

Figure 11: Function Value Comparison Secant, HybridBF

Development Environment:

- Operating System: Linux Mint 21.3 Cinnamon
- Python Version: 3.11.6
- Editor: Visual Studio Code 1.89.1
- Processor: Intel© Core™ i5-8300H CPU @ 2.30GHz × 4
- Memory: 16GB

3 *Chapter Three: IBGA Algorithm*

3.1 Introduction

This chapter provides an overview of the proposed polynomial roots based encryption algorithm IBGA. The chapter contains the encryption and decryption processes, the algorithm's steps, and the results of the algorithm's performance compared to AES.

Note:

Unlike BiNew algorithm which is mentioned in the paper, this algorithm doesn't have coefficients rotation step.

3.2 IBGA Algorithm Breakdown

3.2.1 Encryption

The algorithm encrypts plaintext message using a polynomial and root finding method to generate a ciphertext. The encryption process works as follows:

1. Take the plaintext message and convert each 10 characters to an integer value using their ASCII values.
2. Take the key from the user which will be used to generate the polynomial. The encryption key consists of a set of 5 integer values x_1, x_2, y, s, and r.
    1. x_1 and x_2 define x interval for the polynomial.
    2. y defines the start of y interval for the polynomial and the end will be the negative of y to ensure that the polynomial crosses the x-axis and has a root.
    3. s defines the number of sections that we want to divide the interval into which will affect the degree of the polynomial.
    4. r is used as a random state for the random number generator. The random values will always be the same for the same r value.
3. Use the encryption key to generate points that will be used to generate the polynomial. The points are generated using the following steps:
    1. Divide the interval $[x_1, x_2]$ into s equal sections.
    2. Divide the interval $[y, -y]$ into s equal sections.
    3. Generate points from the two intervals that will be used to generate the polynomial.
    4. Use the random numbers we got r to add some noise to the points.
    5. Apply Newton Forward Difference interpolation to the points to generate the polynomial Since the points are equally spaced.
4. Now we have the polynomial and the plaintext integer representation so we will normalize the plaintext integer representation to be between -1, 1 via this formula:

$$T' = \frac{T - min}{max - min}$$

$$\text{Where: } \begin{cases} T & \text{is the original plaintext integer representation} \\ T' & \text{is the normalized plaintext integer representation} \\ max & \text{is the maximum value of the plaintext integer representation} \\ min & \text{is the minimum value of the plaintext integer representation} \end{cases} \quad (5)$$

5. We get the root of the polynomial which will be the ciphertext using any of the root finding algorithm mentioned before.

6. Then we subtract the normalized plaintext integer value from the polynomial representation.

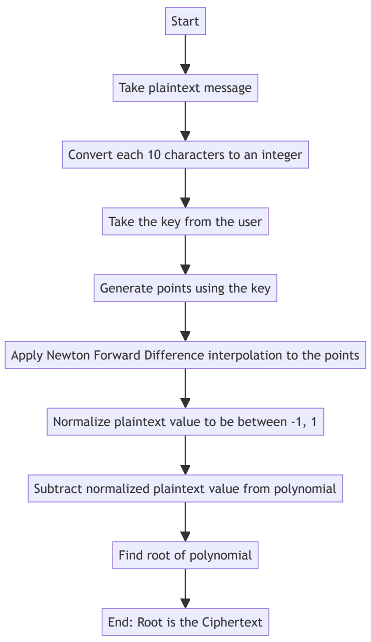
Start
Take plaintext message
Convert each 10 characters to an integer
Take the key from the user
Generate points using the key
Apply Newton Forward Difference interpolation to the points
Normalize plaintext value to be between -1, 1
Subtract normalized plaintext value from polynomial
Find root of polynomial
End: Root is the Ciphertext

Figure 12: Encryption Steps Flowchart

3.2.2 Decryption

The algorithm decrypts the ciphertext message using the polynomial. The decryption process works as follows:

1. Take the ciphertext and the key from the user which will be used to generate the polynomial again.
2. Use the key to generate the polynomial using the same steps as the encryption process.
3. Now we have the polynomial and the ciphertext so we will substitute the ciphertext in the polynomial to get the normalized plaintext integer representation.
4. Get the original plaintext integer representation from the normalized plaintext integer representation using the following formula:

$$T = (T^{'} \times (max - min)) + min \tag{6}$$

5. Convert the integer representation to the plaintext message by converting each integer to 10 characters using their ASCII values.

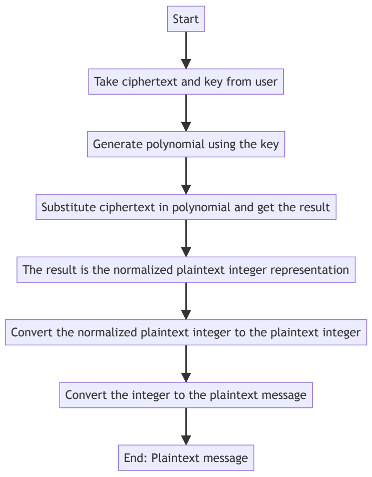
Start
Take ciphertext and key from user
Generate polynomial using the key
Substitute ciphertext in polynomial and get the result
The result is the normalized plaintext integer representation
Convert the normalized plaintext integer to the plaintext integer
Convert the integer to the plaintext message
End: Plaintext message

Figure 13: Decryption Steps Flowchart

3.3 Algorithm Pseudocode

3.3.1 Encryption

These are the encryption steps of the IBGA algorithm. The encryption process involves converting each chunk in the plaintext into an integer, generating points using the key, interpolating a polynomial using the points, normalizing the integer plaintext chunk, and finding the ciphertext chunk which is the root of the polynomial we got after subtracting the normalized plaintext chunk from the polynomial. Finally, we append the ciphertext chunk to the ciphertext array.

Algorithm 1 IBGA Algorithm Encryption Steps

```
1: **Input:** $plaintext, key$
2: **Output:** $ciphertext$
3: **procedure** ENCRYPTION($plaintext, key$)
4:     $x_1, x_2, y, s, r \leftarrow key$
5:     Initialize $ciphertext$ as an empty array
6:     **for** each $chunk$ in $plaintext$ **do**                ▷ Each $chunk$ is 10 characters
7:         $integer\_plaintext \leftarrow$ CONVERTTOINTEGER($chunk$)
8:         $points \leftarrow$ GENERATEPOINTS($x_1, x_2, y, s, r$)
9:         $polynomial \leftarrow$ NEWTONINTERPOLATION($points$)
10:        $normalized\_plaintext \leftarrow$ NORMALIZE($integer\_plaintext$)
11:        $new\_polynomial \leftarrow polynomial - normalized\_plaintext$
12:        $ciphertext\_chunk \leftarrow$ FINDROOT($new\_polynomial$)
13:        Append $ciphertext\_chunk$ to $ciphertext$ array
14:    **end for**
15:    **return** $ciphertext$ array
16: **end procedure**
```

3.3.2 Decryption

These are the decryption steps of the IBGA algorithm. The decryption process is the reverse of the encryption process. We first generate the points using the key, then interpolate the polynomial using the points. We then substitute the ciphertext chunk into the polynomial to get the normalized plaintext chunk. We denormalize the normalized plaintext chunk to get the integer plaintext chunk. Finally, we convert the integer plaintext chunk to a string and append it to the plaintext string.

Algorithm 2 IBGA Algorithm Decryption Steps

```
1:  procedure DECRYPTION($ciphertext, key$)
2:      Input: $ciphertext, key$
3:      Output: $plaintext$
4:      $x_1, x_2, y, s, r \leftarrow key$
5:      Initialize $plaintext$ as an empty string
6:      for each $ciphertext\_chunk$ in $ciphertext$ array do
7:                  ▷ $ciphertext\_chunk$ is the number we got from encrypting each text chunk
8:          $points \leftarrow$ GENERATEPOINTS($x_1, x_2, y, s, r$)
9:          $polynomial \leftarrow$ NEWTONINTERPOLATION($points$)
10:         $normalized\_plaintext \leftarrow polynomial(ciphertext\_chunk)$
11:         $integer\_plaintext \leftarrow$ DENORMALIZE($normalized\_plaintext$)
12:         $plaintext\_chunk \leftarrow$ CONVERTTOCHARACTERS($integer\_plaintext$)
13:         Append $plaintext\_chunk$ to $plaintext$ string
14:     end for
15:     return $plaintext$
16: end procedure
```

3.4 Results

The algorithm was tested using 1000 different plaintext messages and keys and was compared against AES encryption algorithm which is a symmetric encryption algorithm. The results showed that the algorithm is much faster than AES.

3.4.1 Encode Time Comparison

The algorithm showed a significant improvement in the encoding time compared to AES. The encoding time was measured using the time library in python and the results are shown in this figure:

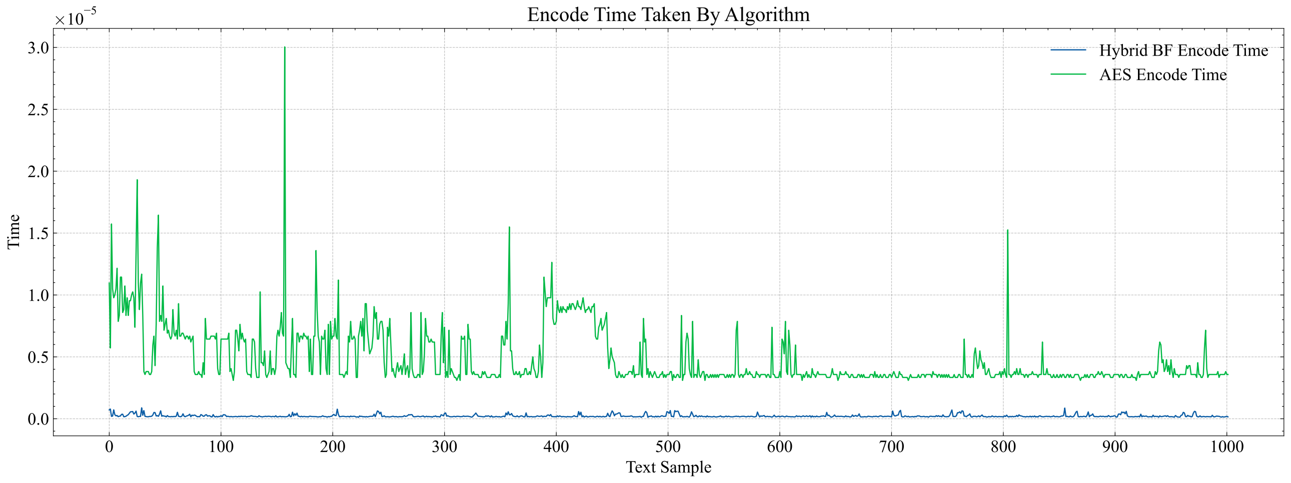

Figure 14: Encoding Time Comparison

And when we sum the encoding time for all the 1000 messages we get the following results:

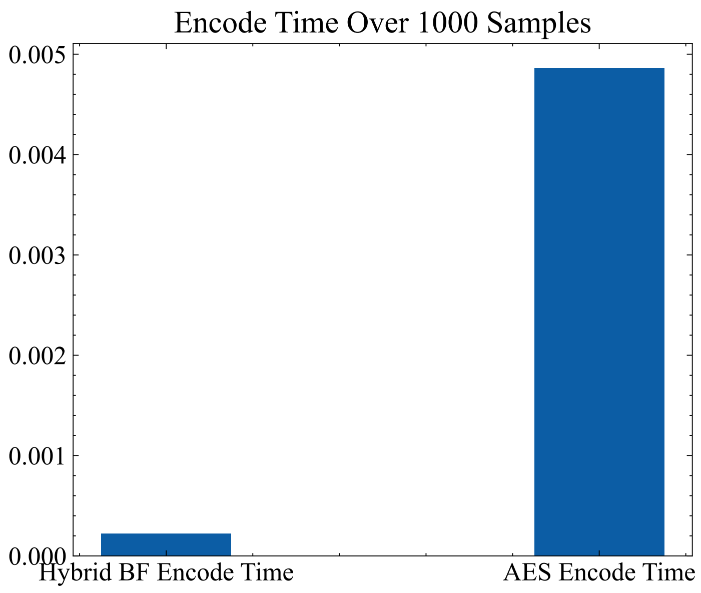

Figure 15: Total Encoding Time Comparison

3.4.2 Decode Time Comparison

The algorithm have also showed a significant improvement in the decoding time compared to AES. The results are shown in this figure:

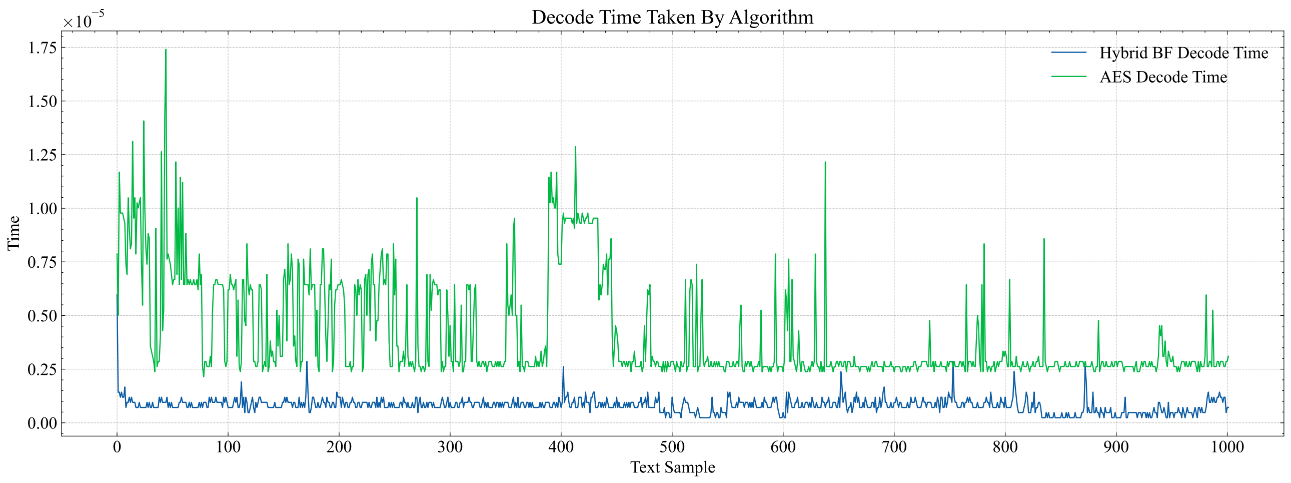

Figure 16: Decoding Time Comparison

And when we sum the decoding time for all the 1000 messages we get the following results:

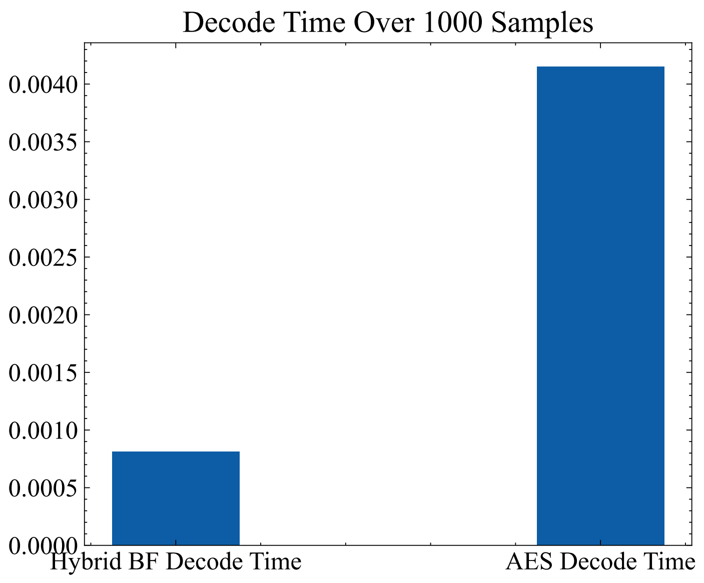

Figure 17: Total Decoding Time Comparison

3.4.3 Total Time Comparison

The total time for both algorithms was also measured and compared. The results are shown in this figure:

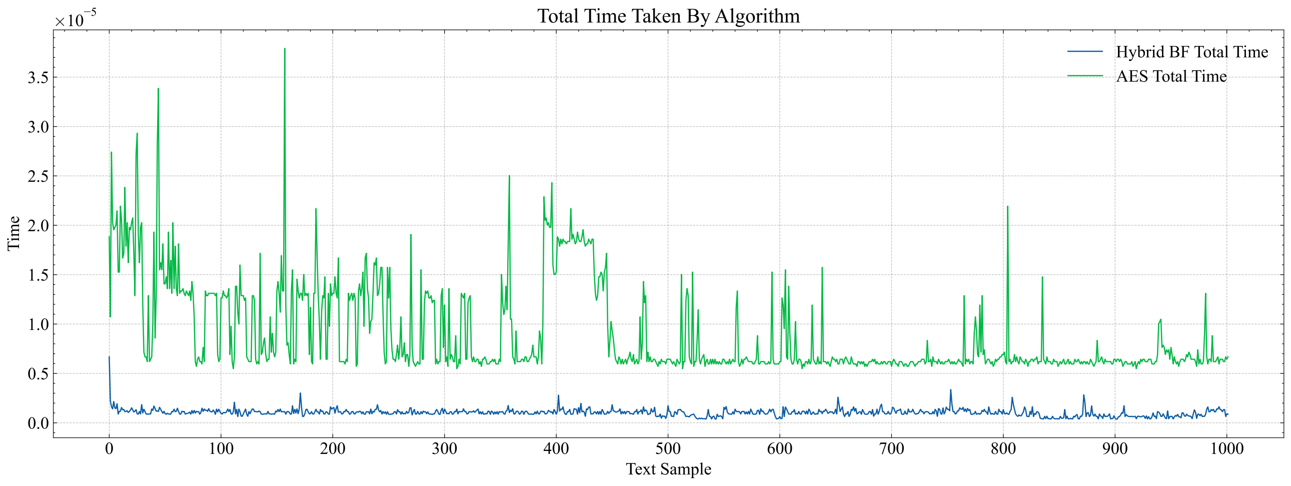

Figure 18: Total Time Comparison

And when we sum the total time for all the 1000 messages we get the following results:

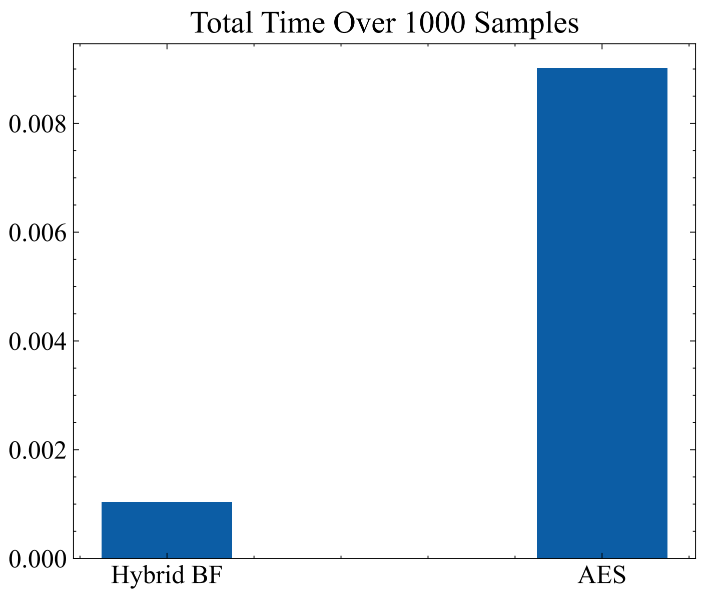

Figure 19: Total Time Comparison

4 *Chapter Four: Design and Analysis*

4.1 Introduction

The analysis and design section provides an overview of the proposed system architecture and data flow of the secure messaging web application. This section contains several software engineering diagrams depicting the structural and behavioral models of the system.

To enable secure communication, the application utilizes the cryptographic algorithm based on polynomial roots.

4.2 Functional & Non-Functional Requirements Table

Table: Functional & NonFunctional Requirements

Name	Functional	NonFunctional	Description	Priority	Actor
Registration	✓		Functionality to create account.	High	User & Admin
Login	✓		Functionality to get access.	High	User & Admin
Logout	✓		Functionality to delete session.	High	User & Admin
Add Chat	✓		Functionality for user to chat with other.	Medium	User
Remove Chat	✓		Functionality for user to remove chat.	Medium	User
Find Chat	✓		Functionality for user to search for a chat.	High	User
Block Account	✓		Functionality for user to block user account.	Medium	User
Send Message	✓		Functionality that send message.	High	User
Delete Message	✓		Functionality to delete message.	Medium	User & Admin
Read Message	✓		Functionality for user to read message	High	User
Privacy		✓	Users privacy.	High	Admin
Robustness		✓	to make able to deal with errors.	High	Admin
Performance		✓	Application performance must be better.	High	Admin
Usability		✓	To make able to use even for newbie.	High	Admin
Reliability		✓	To be trustworthy to users.	High	Admin
Supportability		✓	To be capable of being supportive.	High	Admin

Name	Functional	NonFunctional	Description	Priority	Actor
Portability		✓	Application must be able to run in many different system.	High	Admin

4.3 Diagrams

4.3.1 Use Case Diagram

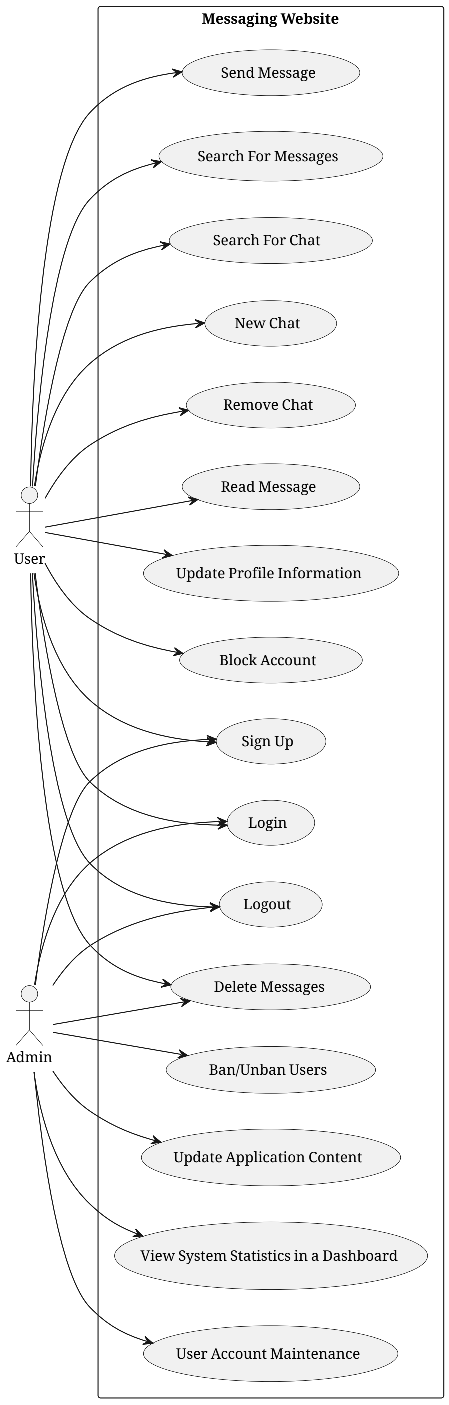

Figure 20: Use Case Diagram

4.3.2 Activity Diagram

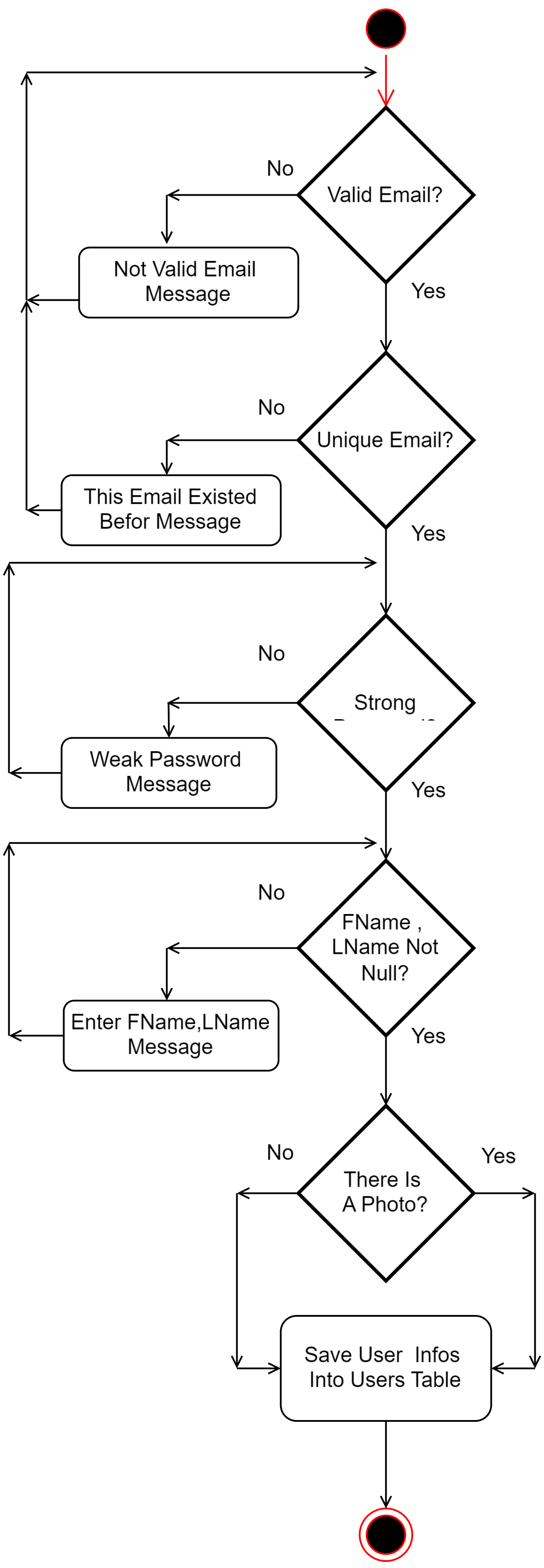

Figure 21: Sign up Activity Diagram

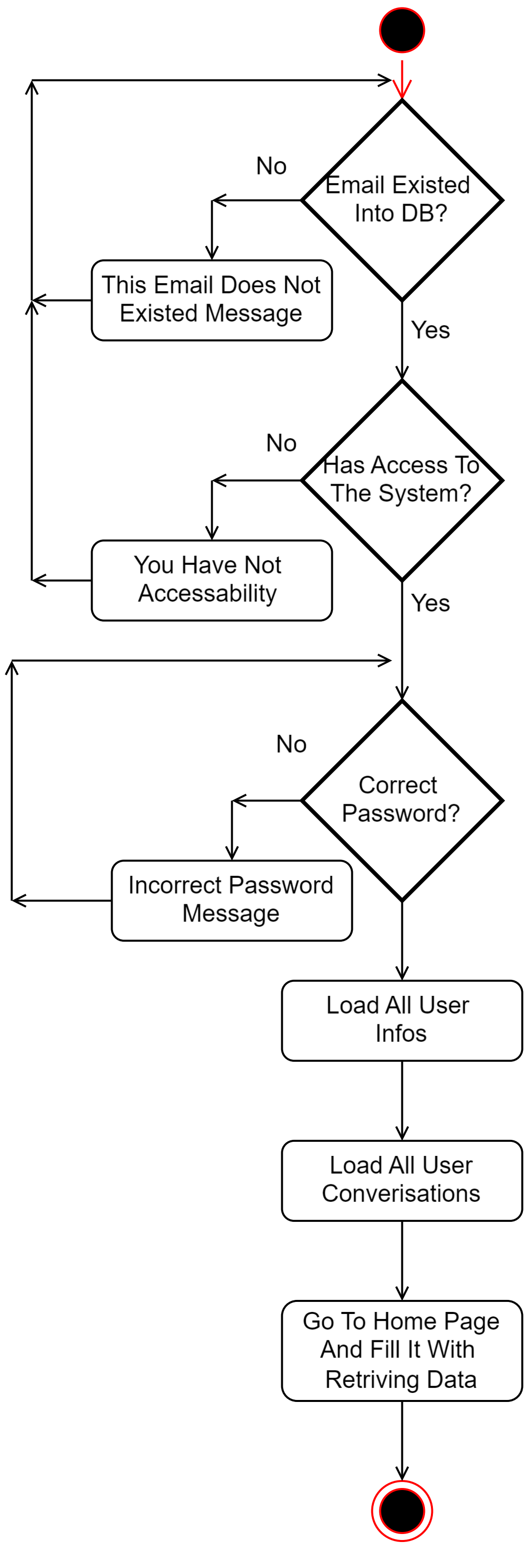
Email Existed Into DB?
No
This Email Does Not Existed Message
Yes
Has Access To The System?
No
You Have Not Accessability
Yes
Correct Password?
No
Incorrect Password Message
Load All User Infos
Load All User Converisations
Go To Home Page And Fill It With Retriving Data

Figure 22: Sign in Activity Diagram

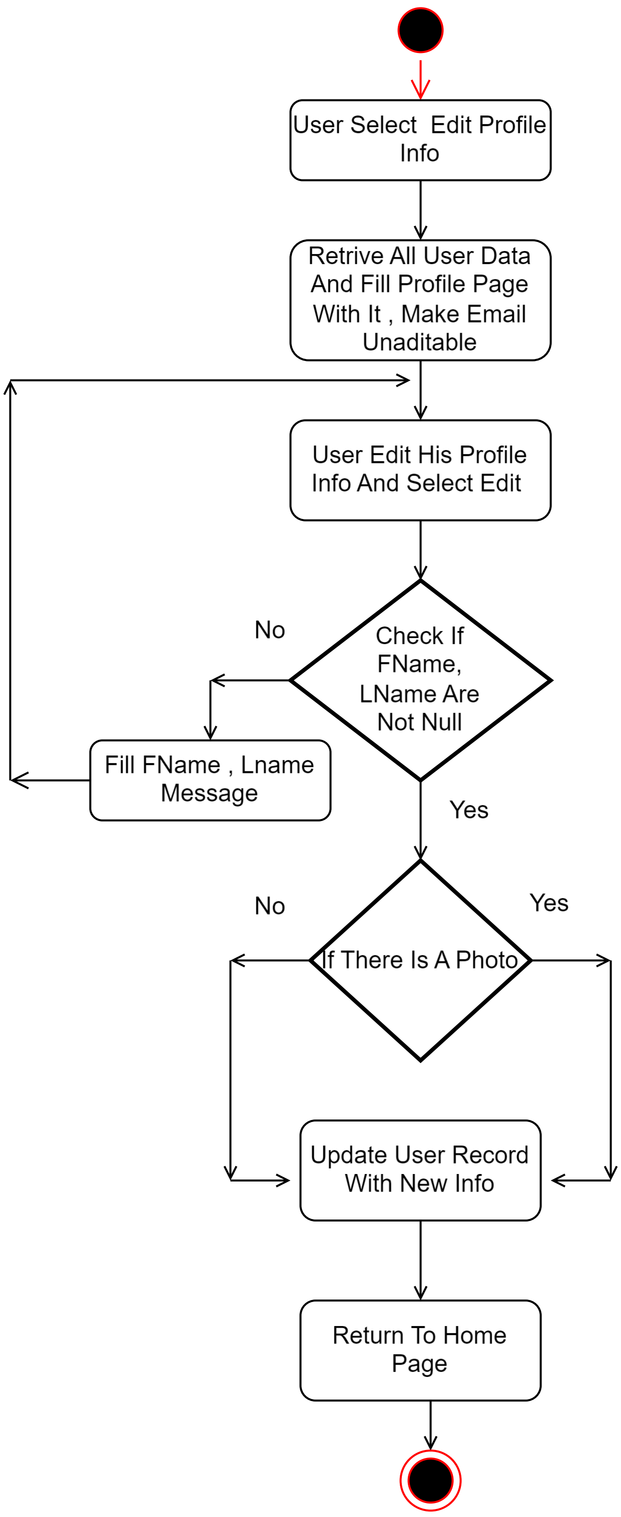
User Select Edit Profile Info
Retrive All User Data And Fill Profile Page With It , Make Email Unaditable
User Edit His Profile Info And Select Edit
Check If FName, LName Are Not Null
No
Fill FName , Lname Message
Yes
If There Is A Photo
No
Yes
Update User Record With New Info
Return To Home Page

Figure 23: Edit Profile Activity Diagram

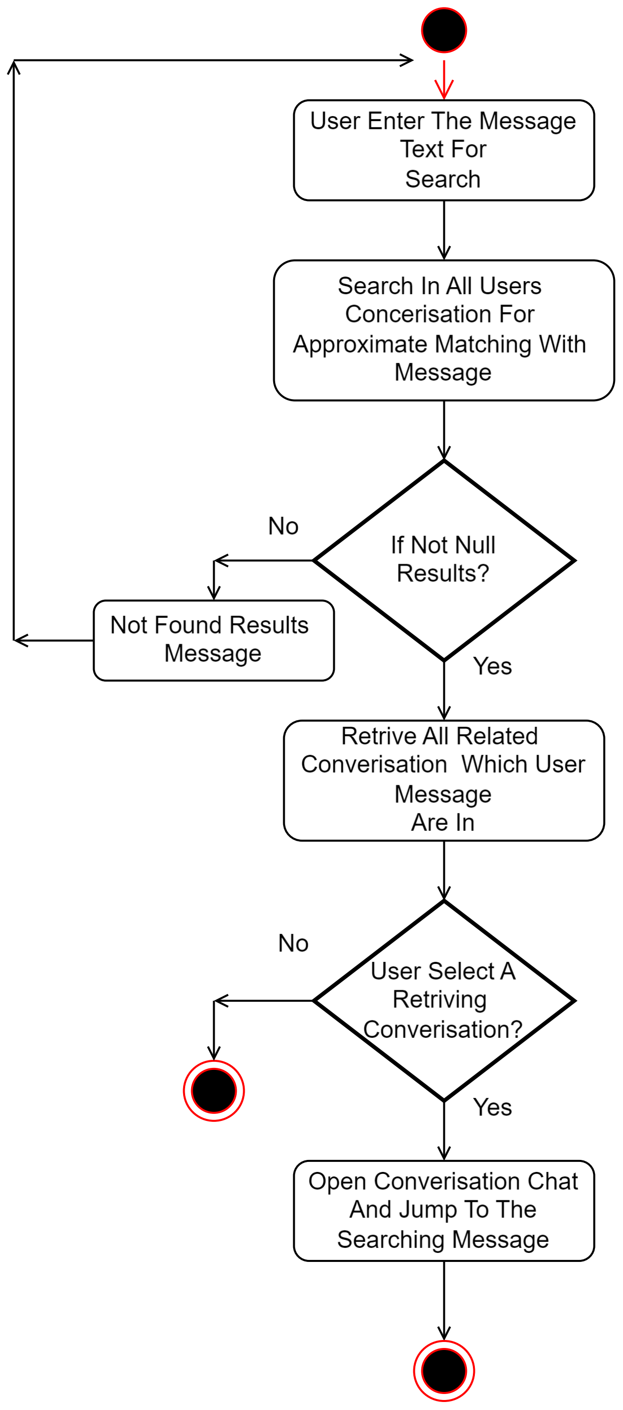
User Enter The Message Text For Search
Search In All Users Concerisation For Approximate Matching With Message
If Not Null Results?
No
Not Found Results Message
Yes
Retrive All Related Converisation Which User Message Are In
User Select A Retriving Converisation?
No
Yes
Open Converisation Chat And Jump To The Searching Message

Figure 24: Search Message Activity Diagram

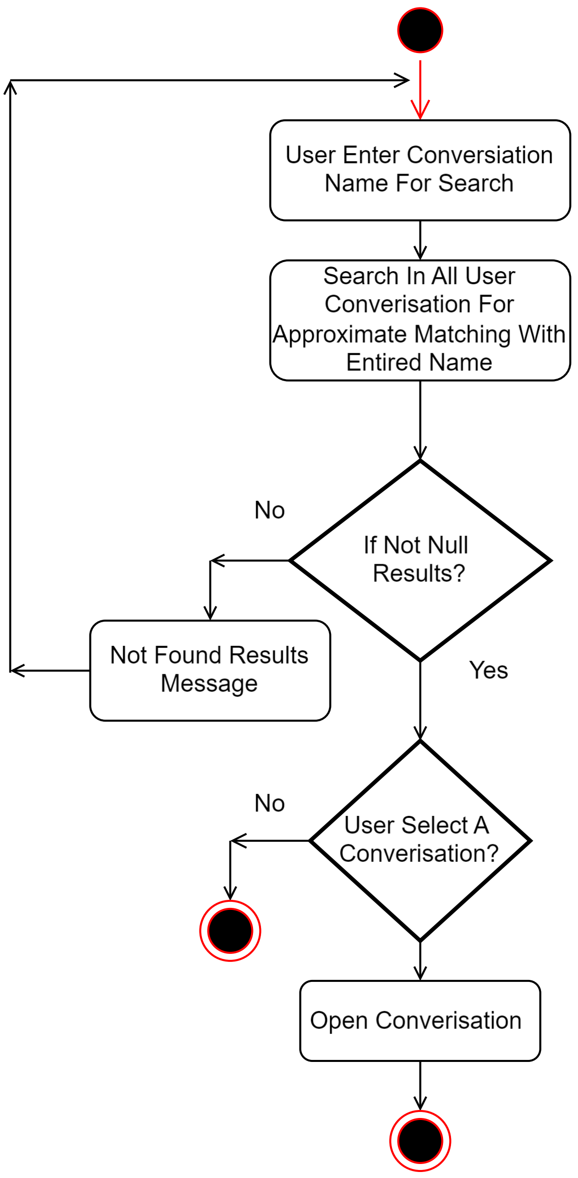
User Enter Converisation Name For Search
Search In All User Converisation For Approximate Matching With Entired Name
If Not Null Results?
No
Yes
Not Found Results Message
User Select A Converisation?
No
Open Converisation

Figure 25: Search For Conversation Activity Diagram

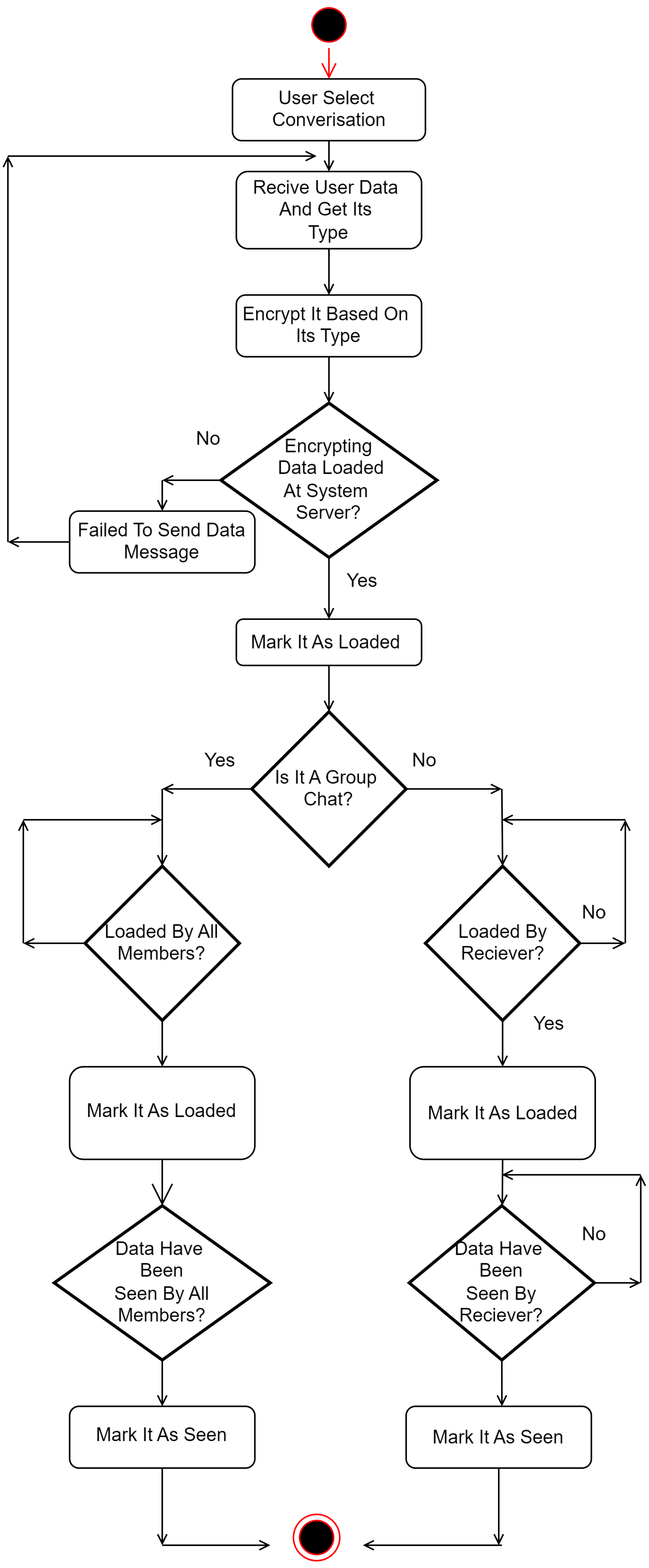
User Select Converisation
Recive User Data And Get Its Type
Encrypt It Based On Its Type
Encrypting Data Loaded At System Server?
No
Failed To Send Data Message
Yes
Mark It As Loaded
Is It A Group Chat?
Yes
No
Loaded By All Members?
Loaded By Reciever?
No
Yes
Mark It As Loaded
Mark It As Loaded
Data Have Been Seen By All Members?
Data Have Been Seen By Reciever?
No
Mark It As Seen
Mark It As Seen

Figure 26: Search/Receive Data Activity Diagram

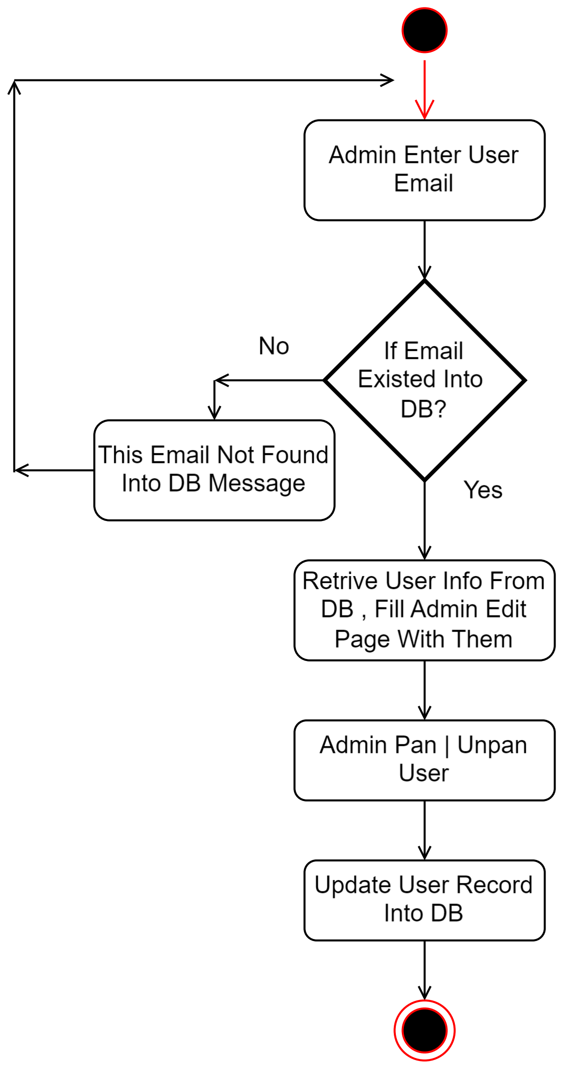
Admin Enter User Email
If Email Existed Into DB?
No
This Email Not Found Into DB Message
Yes
Retrive User Info From DB , Fill Admin Edit Page With Them
Admin Pan | Unpan User
Update User Record Into DB

Figure 27: Ban Unban Users Activity Diagram

4.3.3 Class Diagram

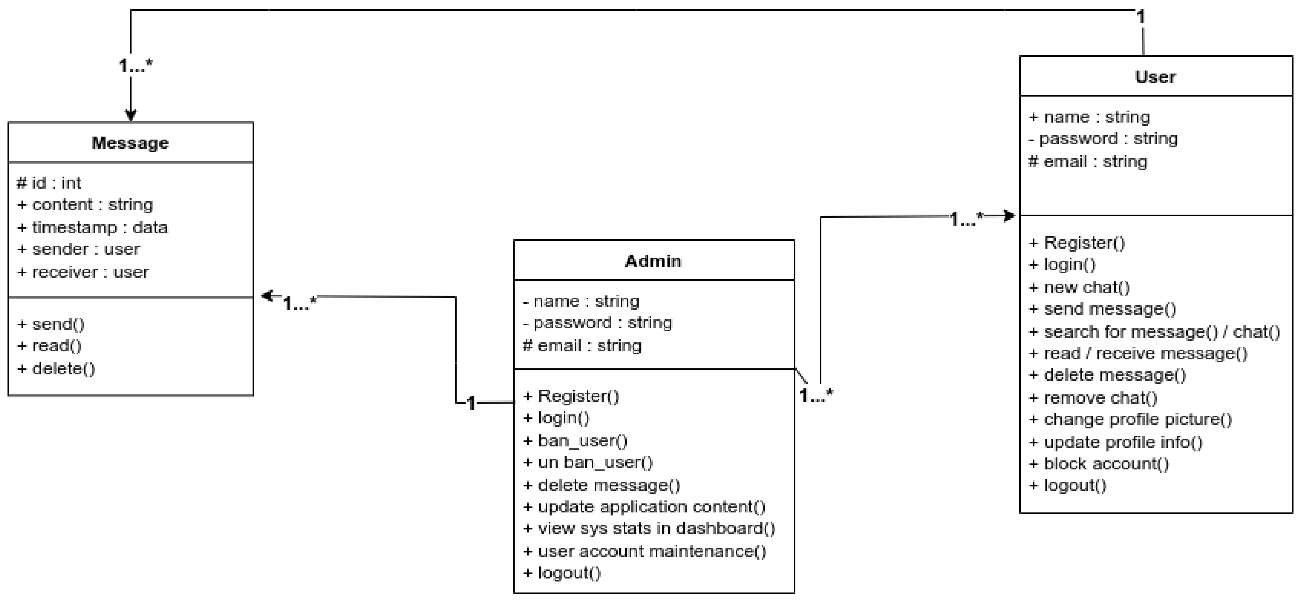

Figure 28: Class Diagram

4.3.4 Sequence Diagram

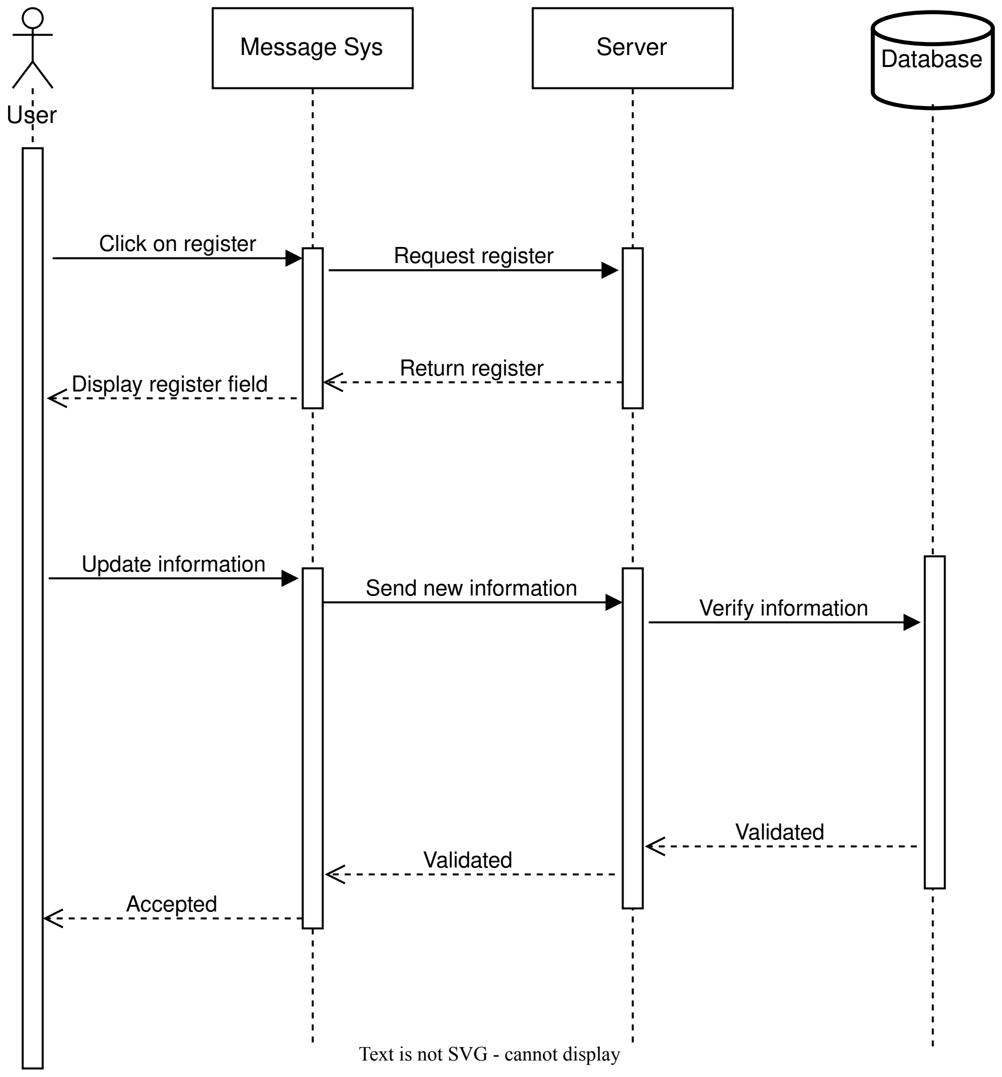

Figure 29: Registeration Sequence Diagram

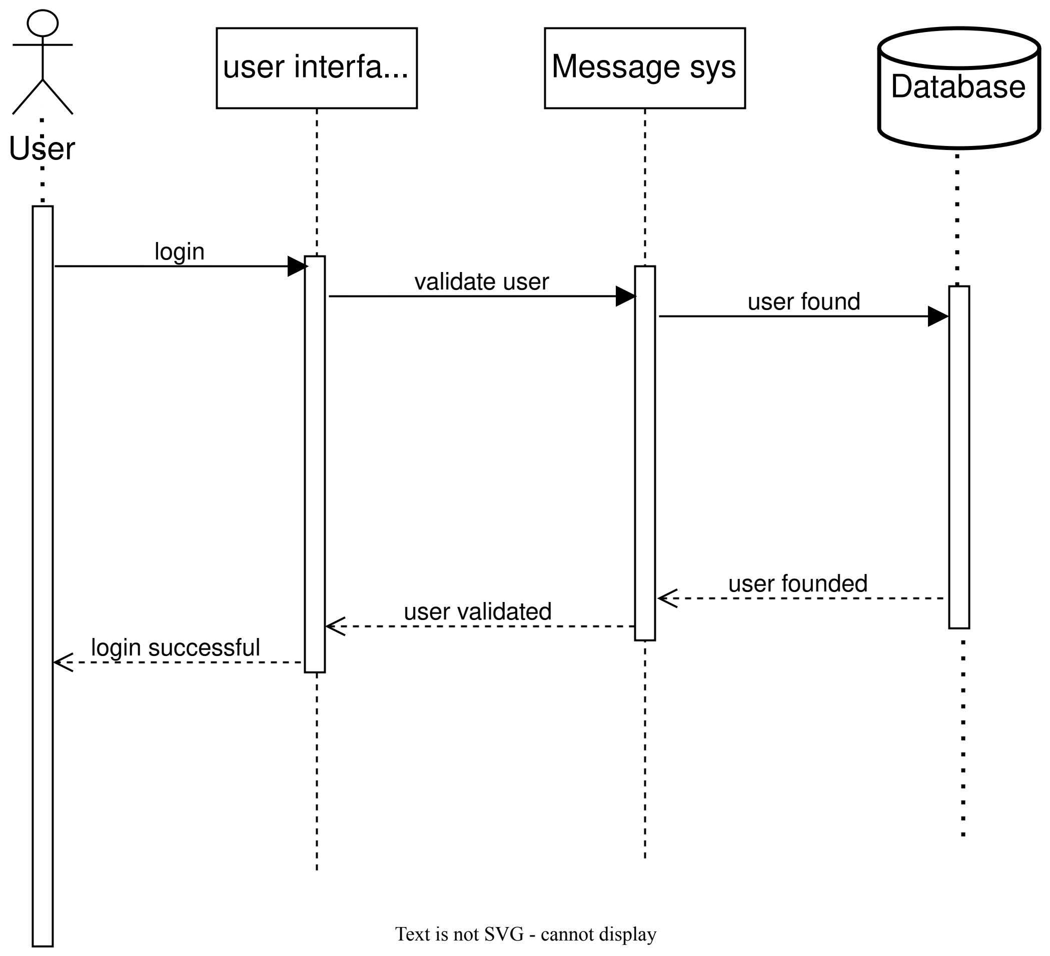

Figure 30: Login Sequence Diagram

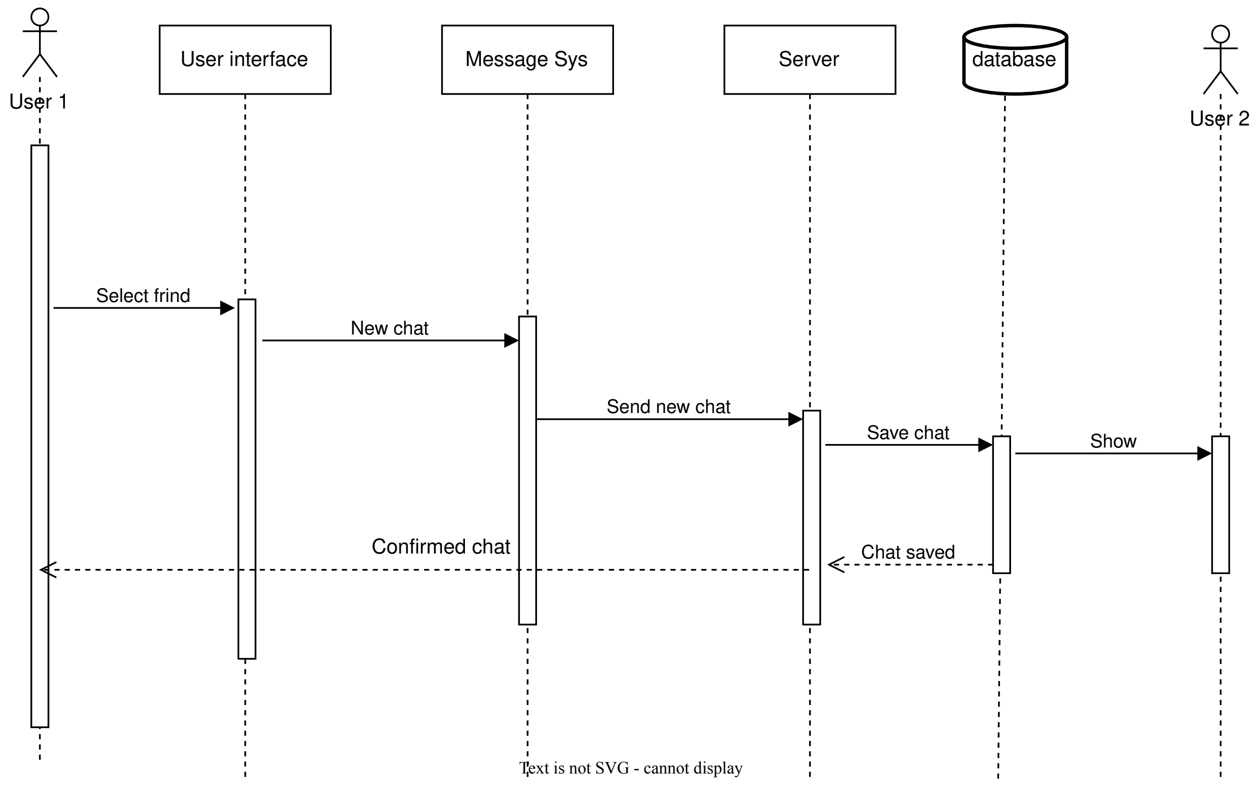

Figure 31: New Chat Sequence Diagram

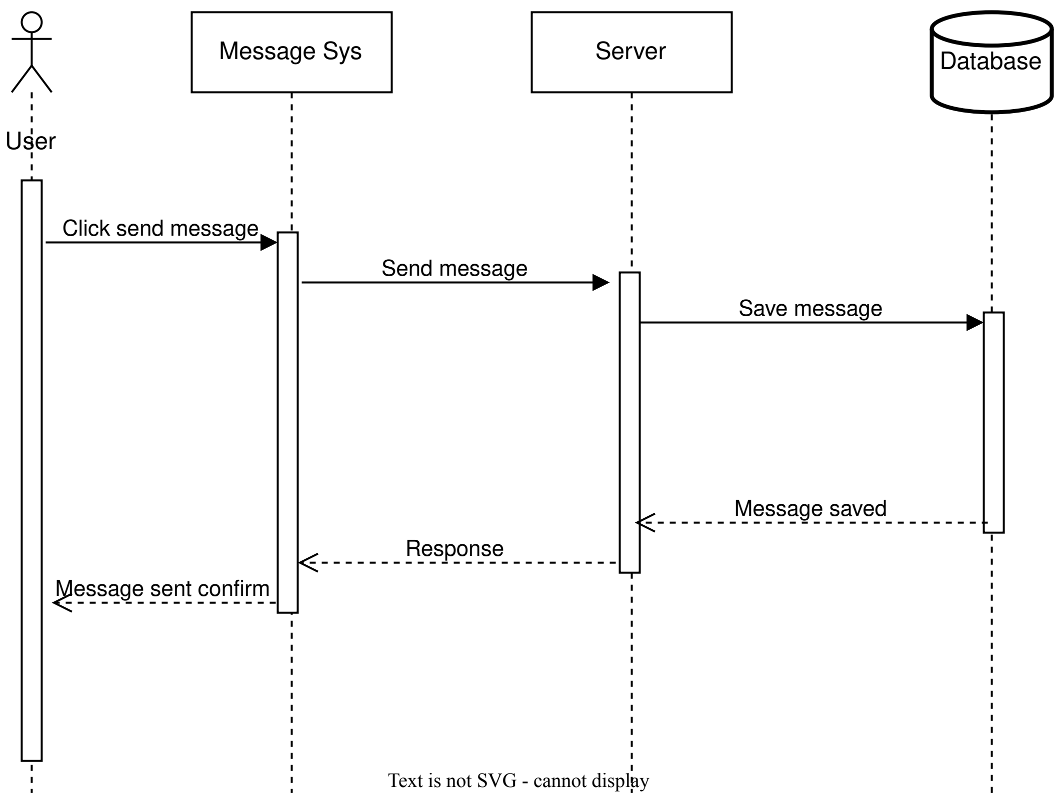

Figure 32: Send Message Sequence Diagram

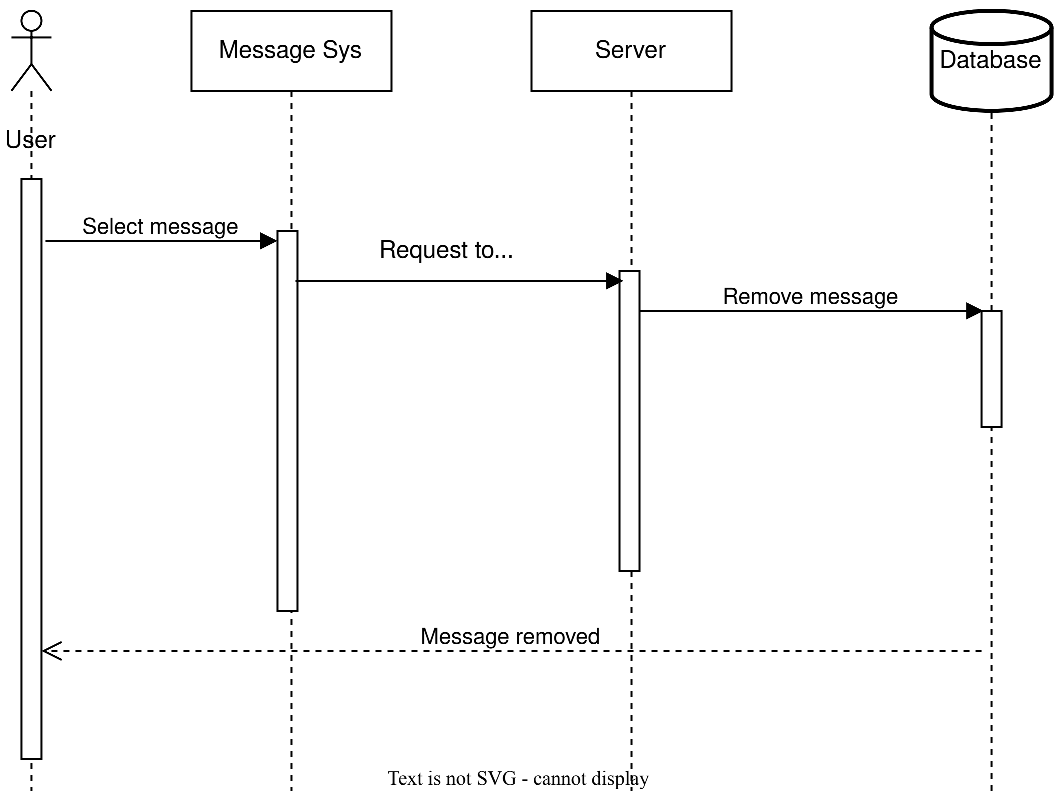

Figure 33: Remove Message Sequence Diagram

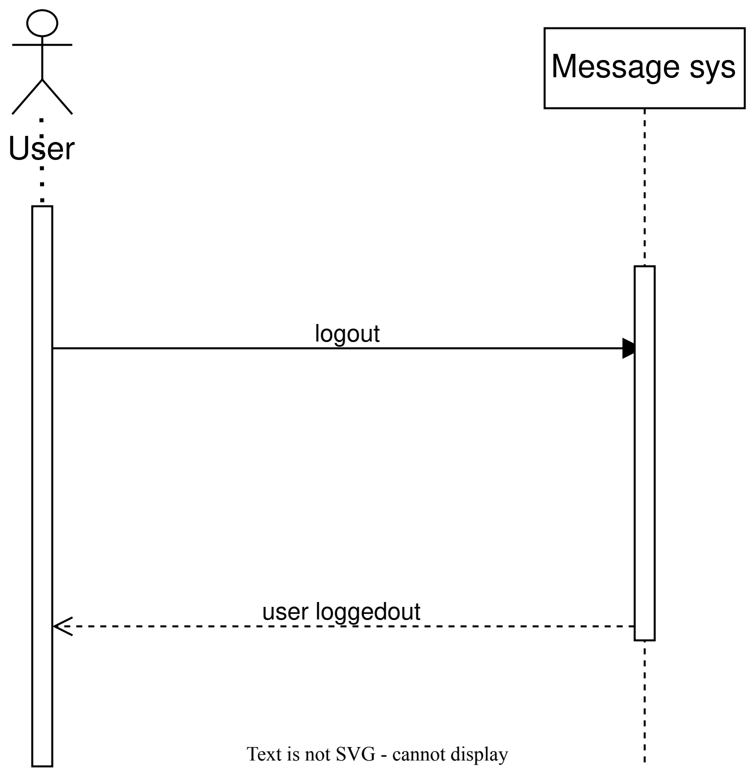

Figure 34: Logout Sequence Diagram

4.3.5 State Diagram

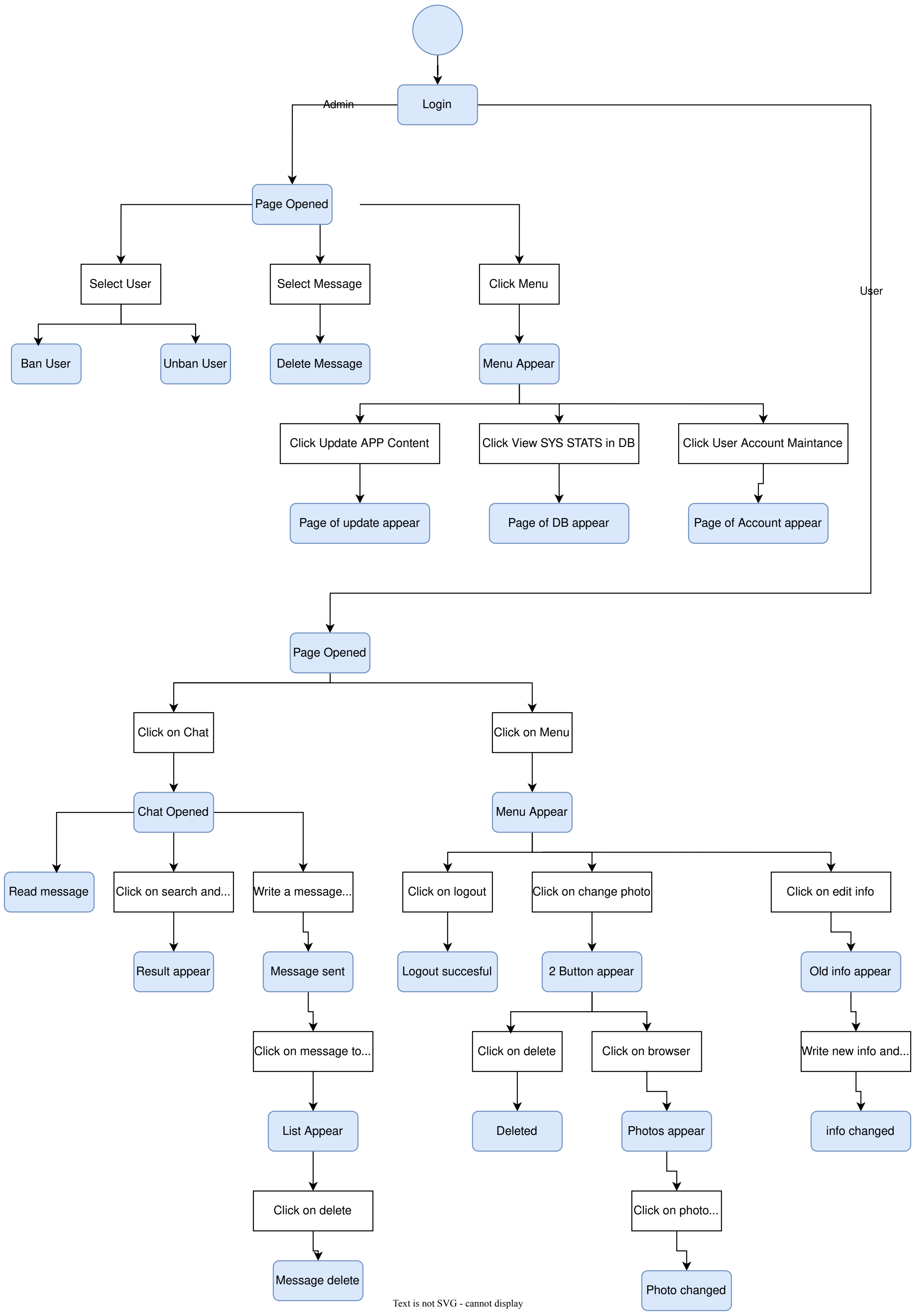

Figure 35: State Diagram

4.3.6 Context Diagram

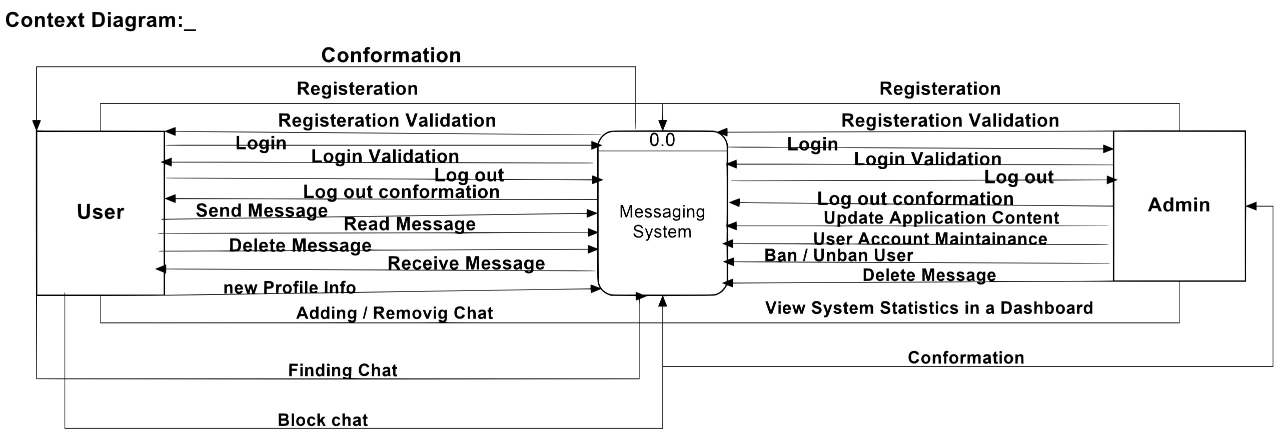

Figure 36: Context Diagram

4.3.7 Data Flow Diagram

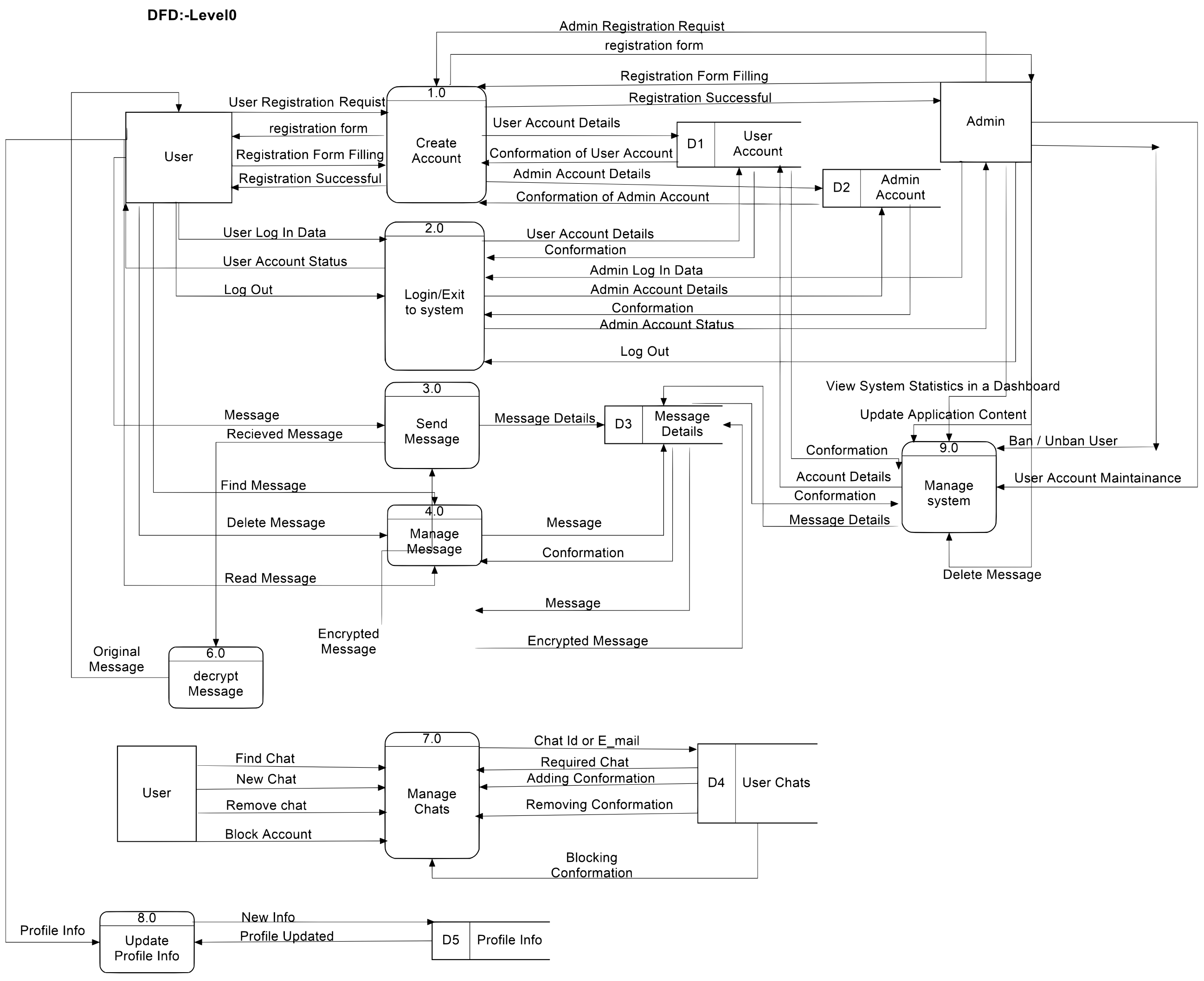

Figure 37: Data Flow Diagram

4.3.8 Entity Relationship Diagram

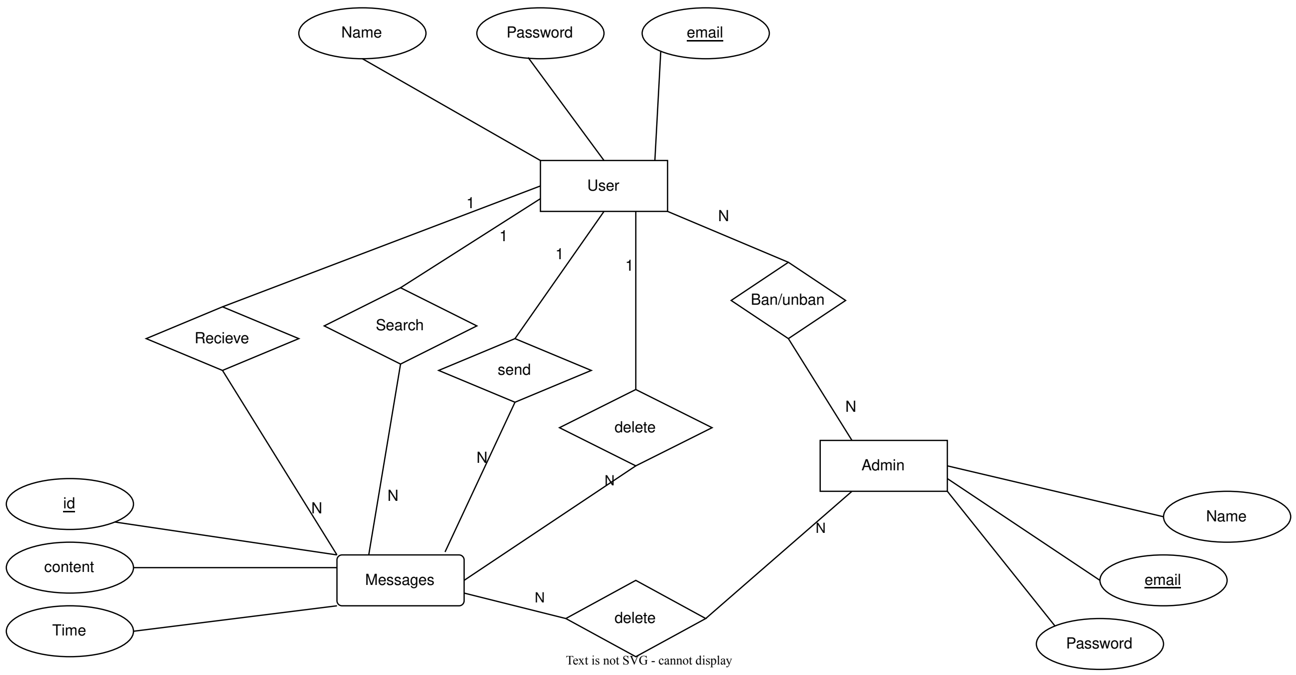

Figure 38: Entity Relationship Diagram

4.4 Tools & Libraries

4.4.1 Visual Studio Code

Visual Studio Code was used as the primary Integrated Development Environment (IDE) for the project. It's a source-code editor developed by Microsoft for Windows, Linux, and macOS. It includes support for debugging, embedded Git control, syntax highlighting, intelligent code completion, snippets, and code refactoring.

4.4.2 Python

The algorithm was written in Python which is a high-level, general-purpose programming language. It is known for its simplicity and easy-to-read syntax. It is widely used in scientific and numeric computing, web development, and artificial intelligence.

The following libraries were used:

4.4.2.1 `pandas` Pandas is a software library for the Python programming language that provides data manipulation and analysis capabilities.

We used it to read and manipulate the datasets from the CSV files.

4.4.2.2 `matplotlib` Matplotlib is a plotting library for the Python programming language. It provides an object-oriented API for embedding plots into applications using general-purpose GUI toolkits like Tkinter, wxPython, Qt, or GTK.

We used it to create the plots for the results of the algorithm.

4.4.2.3 `numpy` NumPy is a library for the Python programming language, adding support for large, multi-dimensional arrays and matrices, along with a large collection of high-level mathematical functions to operate on these arrays.

We used it to generate the points for most of the mathematical function and to generate the random numbers as it's much faster than the built-in python solutions.

4.4.2.4 `scienceplots` SciencePlots is a matplotlib style library that provides style sheets for plots to look like they would fit into a scientific publication.

It was used to make publication-ready plots.

4.4.2.5 `scipy` SciPy is a free and open-source Python library used for scientific computing and technical computing. It contains modules for optimization, linear algebra, integration, interpolation, special functions, FFT, signal and image processing, ODE solvers and other tasks common in science and engineering.

It has optimized built-in functions for the bisection and false position methods, and newton forward difference interpolation.

4.4.2.6 `pycryptodome` PyCryptodome is a self-contained Python library for cryptography operations, such as symmetric encryption, asymmetric encryption, hashes, and digital signatures. It is a fork of the original PyCrypto library and aims to provide a more secure and updated alternative.

We used it to compare the results of the algorithm with AES the well-known encryption algorithm.

4.4.2.7 `time` The time module in Python provides various time-related functions. It is a part of Python's standard library and is used for handling time-related tasks like getting the current time, converting timestamps to readable formats, delaying the execution of functions, and more.

We used it to measure the execution time of each algorithm.

4.4.2.8 `string` The string module in Python includes functions to process standard Python strings. It contains constants for the printable ASCII characters, for various string operations, and for creating custom string transformations.

We used it to convert the plaintext message to an integer representation.

4.5 Future Work

After the successful implementation of the algorithm, we are planning to work on:

- Create a messaging application that uses the algorithm to secure the messages. The application will be a web application that will allow users to send and receive messages securely.
- The messaging application will feature end-to-end encryption, ensuring that only the intended recipient can read the messages. This is achieved by implementing our unique high-speed encryption algorithm.
- Users will be able to create an account, start conversations, and send send messages. Each message sent through the application is encrypted before it leaves the sender's device and can only be decrypted by the intended recipient. This ensures the privacy and security of the communication, even if the data is intercepted during transmission.

- The application will also include features such as group messaging, file sharing, and message notifications. The user interface will be intuitive and user-friendly, making it easy for users to navigate and use the application.

In our application, we'll employ a combination of symmetric and asymmetric encryption techniques to secure our communications. Our high-speed symmetric encryption algorithm will be used alongside the well-established RSA asymmetric encryption.

The role of the asymmetric encryption, in this case, is to facilitate a secure exchange of the symmetric encryption keys between the communicating parties. This guarantees that the symmetric keys are transmitted securely, allowing only the intended recipient to decrypt the messages.

The symmetric encryption algorithm, on the other hand, will be responsible for the actual message encryption. This ensures the privacy and security of the messages. Furthermore, our algorithm is designed for speed and efficiency, making it an ideal choice for real-time messaging applications.

5 References

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