# Quiz 4

1. (3 points) Let $\mathbf{F}(x,y,z) = (z^3+2xy,\ x^2,\ 3xz^2)$ and $C$ be the curve consists of the straight line segments going from $(1,1,0)$ to $(0,1,0)$ then to $(0,0,1)$. Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{s}$.

**Note:** you did encounter this vector field in the homework.

1) $f(x,y,z) = (z^3+2xy,\ x^2,\ 3x^2)$

$(1,1,0) \to (0,1,0)$ & $(0,1,0) \to (0,0,1)$

① $r_1(t) = (1-t, 1, 0)$

$r_1'(t) = (-1, 0, 0)$

$x = 1-t,\ y = 1,\ z = 0$

② $r_2(s) = (0, 1-s, s)$

$r_2'(s) = (0, -1, 1)$

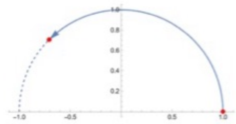
$f(x,y,z) = 2(1-t), (1-t)^2, 3(1-t)^2) \cdot (-1,0,0) = -2(1-t)$

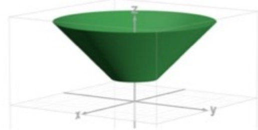
$-2\int_0^1 1-t\,dt = -2\left[t - \frac{1}{2}t^2 \Big|_0^1\right] = -1$

② $x=0, y=1-s, z=s \Rightarrow (s^3, 0, 0)\cdot(0,-1,1) = 0$

$\int_C f\cdot ds = -1$

2. (4 points) Let $\mathbf{G} = (-2xy,\ x^2 - 3y^2,\ xz)$ be a vector field in $\mathbb{R}^3$. Let $C$ be an arc on the unit circle on the $xy$-plane of $\mathbb{R}^3$ going counter-clockwise from $(1, 0, 0)$ to $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ (see picture below, on the left). Evaluate the line integral $\int_C \mathbf{G} \cdot d\mathbf{s}$. Note that $\mathbf{G}$ is **not** a gradient field and you may find the following fact useful: $\cos^2 t - \sin^2 t = 1 - 2\sin^2 t$.





$-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

$\downarrow$

2) $z = 0$, $x^2 + y^2 = 1$

$r(t) = (\cos t, \sin t, 0)$ $\quad 0 \le t \le 3\pi/4$

$\frac{dr}{dt} = (-\sin t, \cos t, 0)$

$G(\cos t, \sin t, 0) = (-2\cos t \sin t, \cos^2 t - 3\sin^2 t, 0)$

$G \cdot r' = 2\cos t \sin^2 t + \cos t(\cos^2 t - \sin^2 t)$

$= \cos t \cdot \cos(2t)$

$\int G \cdot ds = \int_0^{3\pi/4} \cos t \cos(2t)\, dt$

$\int_0^{3\pi/4} \frac{1}{2}[\cos t + \cos 3t]\, dt = \frac{\sqrt{2}}{3}$

3. (3 points) Let $S$ be the surface of the cone $z = \sqrt{x^2+y^2}$ between the planes $z = 1$ and $z = 5$ (see picture above, on the right).

Obtain a parametrization $\Phi$ for this surface then use it to find the area of $S$. Make sure to indicate the domain $D$ of $\Phi$ (i.e. the bounds for the variables of $\Phi$) and show all steps of your calculation process.

3) $z = x^2 + y^2$ $\quad 1 \leq z \leq \alpha$

$0 \leq \theta \leq 2\pi$

$\Phi(r,\theta) = (r\cos\theta, r\sin\theta, r)$

$$\frac{\partial \Phi}{\partial r} = (\cos\theta, \sin\theta, 1), \quad \frac{d\Phi}{d\theta} = (-r\sin\theta, r\cos\theta, 0)$$

$$\begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 1 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = (-r\cos\theta, r\sin\theta; r)$$

$$\sqrt{(-r\cos\theta)^2 + (r\sin\theta)^2 + r^2} = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + r^2} = \sqrt{r^2(\sin^2\theta + \cos^2\theta) + r^2}$$

$$= \sqrt{2r^2} = r\sqrt{2}$$

$$\sqrt{2}\int_0^{2\pi}\int_1^{\alpha} r\,dr\,d\theta = 2\pi\sqrt{2}\cdot\frac{\alpha^2-1}{2} =$$

$$2\pi\sqrt{2}(\alpha^2-1)$$