Quantum Walks and Monte Carlo Project

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1 Classical part

The Galton Box is a classical random generator and an example of a Monte Carlo device, which is used to model computational randomness. It works by allowing a ball to fall through multiple layers of pegs. The layers are arranged from top to bottom, and each layer contains a number of pegs equal to its position in the sequence.

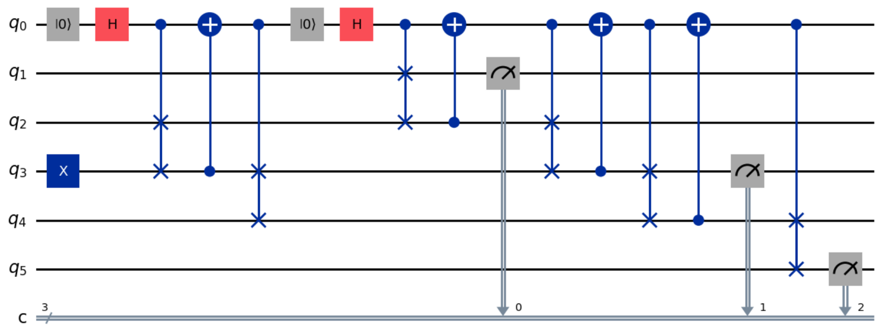
Ideally, each peg has a 50% chance of deflecting the ball to the right and a 50% chance of deflecting it to the left. However, the probabilities can vary. For example, if a peg is more likely to push the ball to the right, then the probability of the ball going left increases for the next peg, and vice versa.

When many balls are dropped, their final positions at the bottom of the Galton Box typically follow a Gaussian (normal) distribution.

2 Quantum part

A natural question arises: Why use quantum systems when classical systems already exist? My point of view that, for some reasons which are :

- Quantum systems (spins) aare naturally random when they are in superposition. We can predict mathematically what the next measurement value might be, but it is not certain until we actually measure it. Moreover, our mathematical prediction could be wrong. In other words, it is nearly impossible to be 100% sure about the prediction if there is superposition.
- On high numbers, at some limit Quantum systems become faster and cheaper than Classical systems. If the number of all possible states = k and the number of particles used = n, in Classical systems $k = n$ and in Quantum systems $k = 2^n$, which means Quantum can present more states with fewer particles above a certain point.



Explaining the the curcuit():

- The Hadamard gate is used to create superposition in the controlled qubit with equal probability
- The Reset gate removes any effect from previous operations on the controlled qubit, so it becomes ready for equal superposition again without any corruption.
- The X gate is used to show the active state as a ball.
- The first controlled-swap gate has a 50% chance to work because of the superposition of the controlled qubit.
- In the case that the first controlled-swap gate does not work, the CNOT gate forces the controlled qubit to turn the second controlled-swap gate on.

The probability in the pegs at each level is the same as in Pascal's triangle and also fits a Gaussian distribution.
If I want an unequal probability distribution (falling left is not the same as falling right), all I should do is apply an Rx(θ) gate after the Hadamard gate. This causes a shift in the curve.
From the previously given points, I could produce an exponential distribution by shifting the curve far right or far left so that the slope of the plotted data does not decrease before the curve ends. This effect happens because the Gaussian distribution has an exponential in it, and its beginning/ending slope is the same as that of an exponential distribution.

3 references

[1] `https://arxiv.org/pdf/2202.01735`