Game Theory – Practice Test 2

Mohamed Abdelhamid
AAI-01,
Innopolis University
Innopolis, Russia
m.abdelhamid@innopolis.university

***Abstract*—In this report, I will attempt to find the best strategy for the snowball game by evaluating multiple strategies against each other in a tournament fashion, and comparing their average score. Finally, a winning solution is presented.**

Keywords—game theory, snowball fight, cooperation game

I. Introduction

This is a 2 players game and the environment is divided into 3 territories called A, B, and C fields. Fields A and B are the players' positions respectively, and field C is a hot field where players can discard their snowballs. The players cannot change their position during the game. The initial number of snowballs that each player starts with is N=100. Fields A and B have a Snowball Generating Machine (SGM) which increases the number of snowballs that the player has by 1 every minute. Each player has a Snowball Cannon (SC) that can be used to shoot balls at the other players. This SC does not make any damage to the balls so the number of snowballs for the other player will increase. The players can use SCs at most once per minute to the opponent's field and at most once per minute to the hot field (totally at most twice, it is like a sequential shot, where a shot to the opponent's field happens first. Each shot can contain 1 snowball or more. Not shooting is also allowed. If no shooting happened to the opponent's and hot fields, then it is assumed that during this minute shooting didn't happen. If SC was used once or twice per minute, then it is assumed as the presence of shooting during this minute. However, the shooting history affects the maximum number of snowballs shoot for the next minutes. The maximum number of snowballs shot by cannon per minute (together for both shots) is defined by the equation:

$$f(x) = \left\lfloor \frac{15 \times e^x}{15 + e^x} \right\rfloor \qquad (1)$$

where x is the number of minutes passed after the previous shot (presence of shooting). This game will be played for 60 rounds (minutes). The goal of the game is to minimize the number of snowballs left in your field after the game is over to maximize the payoff.

II. Observations about the game

A. When is a good time to shoot?

Whether the player is planning to shoot the other player or to the hot field, knowing when is the perfect time to shoot will make the player able to dispatch the largest number of balls during the game. To determine this time, I draw the SC function (1). Notice from figure 1, the value of the function (1) starts to plateau when $x > 6$, so there is no use in taking into consideration values of x greater than 6. In figure 2, you can see that when $x = 4$, the player can dispatch more than 160 balls which is enough to reach the optimal solution of 0 snowballs if both players cooperating.

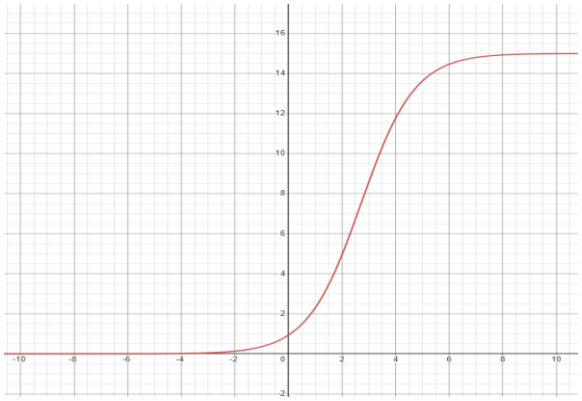

Figure 1:sigmoid SC function

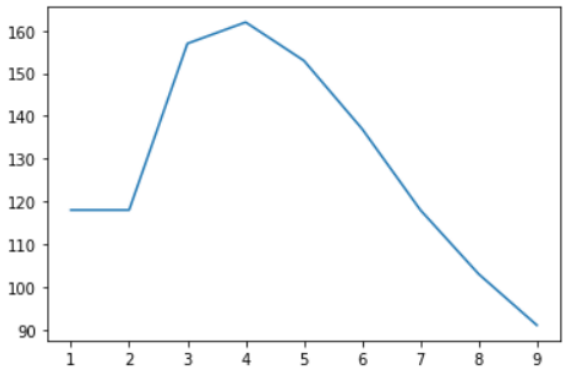

Figure 2: the max number of balls you can send for the given time

B. Cooperation is good

The definition of cooperation here is to send the balls to the hot field. This game has an interesting case where both players can finish with the lowest number of balls possible (which is zero). This can happen when both of them cooperate and send the balls to hot field every 4 minutes. This seems like a reasonable thing to do for both players. However, it has one problem which is trust. If all participants in this game trusted each other and cooperated, we will end up having the optimal solution of 0 balls, but can we? I wouldn't, because some people will try to sabotage the game for others in order for them to have higher rank in the tournament. In the following section, a solution that tries to solve this issue of trust will be presented.

III. Strategy evaluation

In this section, I will present and evaluate a set of strategies.

A. Strategies:

- **Always attack (λ) (AA):** in this strategy the player will always attack the other players every λ period of time, and it will never cooperate.
- **Always cooperate (AC):** in this strategy the player will always cooperate even if the other player is attacking him.

- **Copycat (CC):** the player will start with cooperation then do the same thing that the other player did in the last time
- **Random (R):** the player will choose to attack or cooperate randomly at random instance of time.
- **Grouch (G):** the player will start with cooperation and will keep cooperating until the other player attacks him once, then the grouch player will keep attacking until the end of the game. Shoot every 4 minutes

B. *Notes about the strategies:*

During implementing **Copycat**, the player cannot know if the other player is discarding its ball or its waiting until the right moment to shoot at it, so to overcome this in my implementation I divided the game into episodes of 4 minutes, if the other player attacked me during those 4 minutes, I will attack him at the end of the 4 minutes. I chose 4 because I can send the maximum number of balls if I played every 4 minutes.

C. *Tournament*

In this tournament, I will make the strategies explained earlier play against each other for 10 matches and present the average result for both of them.

P1\ P2	AA (4)	AA (5)	AC	CC	R	G
AA(4)	160:160	149:171	6:160	149:160	92:193	149:160
AA(5)	171:149	160:160	17:149	138:149	100:183	160:149
AC	160:6	149:17	6:6	6:6	92:40	6:6
CC	160:149	149:138	6:6	6:6	87:185	6:6
R	193:92	183:100	40:92	191:90	121:126	191:95
G	160:149	149:160	6:6	6:6	95:191	6:6
Total	705	746	419	417	918	422

Table 1: Evaluating the preformance of strategies against each other

In table 1, the total row denotes how many snowballs are left with the column strategy. As we can see, the copycat that shoots every 4 minutes had the best score of 417 balls, coming in second place always cooperating, then the grouch strategy. At the first glance, copycat is the best strategy. However, if we look deeply at the scores of copycats, we will see that it scored 149:138 with always attack (5). Although always attack (5) shots every 5 minutes, which is not optimal, and copycat shots every 4 minutes which is optimal and allows copycat to shoot more balls than always attack. This happens because the way copycat work is by looking at the last 4 minutes, if the other player attacked yet, it assumes that it is cooperating and it will send its snowballs to the hot field. So, while the copycat was assuming that the other player was cooperating, in reality, it was not and it was just waiting until the 5th minute to shoot the copycat. The solution to this problem will be presented in the next section along with my strategy.

IV. GROUCH 2.0 (MY STRATEGY)

A. *Strategy explaination*

After analyzing the performance of different strategies and their performance, I noticed that if we made a mix between copycat, grouch and always attack, we can achieve a very decent solution that can beat all the presented strategies. It works as following:

1. Start with cooperating
2. If the other person attacked, then keep attacking him until the end of the game (grouch, and copycat)
3. If the other person cooperates, then keep cooperating (copycat) and shoot him in the last minute (always attack)

As we agreed in section II.B, cooperation is good, so I try to cooperate first. If the other person attacked at any time during the game, I will hold a grouch against him and I will keep attacking until the end of the game. This will solve the problem that copycat had earlier against always attack (5). If the other person cooperates, then there is no point in attacking him as we both are benefiting from cooperation except at the last step where I have to shoot him because if he was naïve and thought that I will still be cooperating until the end, then I will end up with score 1:11 where 1 is for me because he cannot return what I have shot at him in the last step as the game is over. And if he thought the same way, we will end up with a score of 6:6 because we will shoot at each other.

B. *Preformace*

Strategies vs Grouch 2.0	Grouch 2.0
AA (4)	157:152
AA (5)	168:141
AC	11:1
CC	11:1
R	201:85
G	11:1
Grouch 2.0	6:6
Total	387

Note: Grouch 2.0 is a nickname for my strategy here, but the class that I implemented in MohamedAbdelhamidCode.java is called MohamedAbdelhamidCode.

C. *Observations*

The reason why my strategy won against always attack (4) is that at the last minute, the minutesPassedAfterYourLastShot for both the strategies was 3, so my strategy decided to shote, while always attack (4) preferred to wait until it is 4. In addition, Grouch 2.0 was able to win against all the other strategies (as expected) which proves that it is the best strategy.

V. CONCULUSION

To conclude, the snowball game is about cooperation and trust between 2 players. If we know when to cooperate and attack, we will win. Following a greedy always attack approach to win in the tournament will lead to a tie and loss for both players. However, a good mix of cooperation and attacking will give the best results.