

Topics: Confidence Intervals

1. For each of the following statements, indicate whether it is True/False. If false, explain why.

I. The sample size of the survey should at least be a fixed percentage of the population size in order to produce representative results.

Ans: True: The representation of the survey results should have a sample size. The sample size must be a fixed percentage of the total population size of the survey.

II. The sampling frame is a list of every item that appears in a survey sample, including those that did not respond to questions.

Ans: False: The sampling frame refers to a list of an item which responds to the question and not the ones which do not respond to the questions.

III. Larger surveys convey a more accurate impression of the population than smaller surveys.

Ans: True: The larger conveys a more accurate impression of the population as larger surveys involve large sample size which reduces the chances of error.

2. *PC Magazine* asked all of its readers to participate in a survey of their satisfaction with different brands of electronics. In the 2004 survey, which was included in an issue of the magazine that year, more than 9000 readers rated the products on a scale from 1 to 10. The magazine reported that the average rating assigned by 225 readers to a Kodak compact digital camera was 7.5. For this product, identify the following:

A. The population

Ans: All the readers of the PC magazine

B. The parameter of interest

Ans: The population mean that rated the digital camera

C. The sampling frame

Ans: 9000

D. The sample size

Ans: 225

E. The sampling design

Ans: Sampling Design = n/N

Where n – number of units to be samples

N – number of units in total population

Sampling Design = $225/9000 = 0.025$ (Answer)

F. Any potential sources of bias or other problems with the survey or sample

Ans: Selection of the readers, Selection of the issue which will contain the survey

3. For each of the following statements, indicate whether it is True/False. If false, explain why.

- I. If the 95% confidence interval for the average purchase of customers at a department store is \$50 to \$110, then \$100 is a plausible value for the population mean at this level of confidence.

Ans: True

Reason - The 95% confidence interval for the average purchase of customers at a department store is \$50 to \$110. Which means that there is a 95% chance that the population mean will fall between \$50 and \$110. Hence, as \$100 falls between \$50 and \$110, it is a plausible value for the population mean at this confidence level

- II. If the 95% confidence interval for the number of moviegoers who purchase concessions is 30% to 45%, this means that fewer than half of all moviegoers purchase concessions.

Ans: True

Reason - The 95% confidence interval for the number of moviegoers who purchase concessions is 30% to 45%, this means that there is a 95% chance that only 30% to 45 % of moviegoers purchase concessions, which is less than 50%. Hence, we can infer that fewer than half of all the moviegoers purchase concessions.

- III. The 95% Confidence-Interval for μ only applies if the sample data are nearly normally distributed.

Ans: False Reason – Most data we have are not normally distributed. But we can use central limit theorem to make a normal approximation and obtain an asymptotical confidence interval.

4. What are the chances that $\bar{X} > \mu$?

- A. $\frac{1}{4}$
- B. $\frac{1}{2}$
- C. $\frac{3}{4}$
- D. 1

Ans: D. 1

Reason: Due to the Central Limit Theory, the distribution of sample means will be normally distributed around the true mean.

5. In January 2005, a company that monitors Internet traffic (WebSideStory) reported that its sampling revealed that the Mozilla Firefox browser launched in 2004 had grabbed a 4.6% share of the market.

- I. If the sample were based on 2,000 users, could Microsoft conclude that Mozilla has a less than 5% share of the market?

Ans: No. It does not clearly mention the type of users the sample was based on

- II. WebSideStory claims that its sample includes all the daily Internet users. If that's the case, then can Microsoft conclude that Mozilla has a less than 5% share of the market?

Ans: Yes

6. A book publisher monitors the size of shipments of its textbooks to university bookstores. For a sample of texts used at various schools, the 95% confidence interval for the size of the shipment was 250 ± 45 books. Which, if any, of the following interpretations of this interval are correct?

- A. All shipments are between 205 and 295 books.

Ans: False

- B. 95% of shipments are between 205 and 295 books.

Ans: True

- C. The procedure that produced this interval generates ranges that hold the population mean for 95% of samples.

Ans: True

- D. If we get another sample, then we can be 95% sure that the mean of this second sample is between 205 and 295.

Ans: True

- E. We can be 95% confident that the range 160 to 340 holds the population mean.

Ans: False

7. Which is shorter: a 95% z-interval or a 95% t-interval for μ if we know that $\sigma = s$?

- A. The z-interval is shorter

- B. The t-interval is shorter

- C. Both are equal

- D. We cannot say

Ans: D. We cannot say.

It depends on estimate of standard deviation. A 95% t-interval for might be longer and a 95% z-interval for shorter due to z-critical value less than t-critical value and a 95% t-interval for might be shorter interval. The outcome depends on standard deviation instead of the sample because the standard deviation value is effect the length of an interval.

Questions 8 and 9 are based on the following: To prepare a report on the economy, analysts need to estimate the percentage of businesses that plan to hire additional employees in the next 60 days.

8. How many randomly selected employers (minimum number) must we contact in order to guarantee a margin of error of no more than 4% (at 95% confidence)?

- A. 600
- B. 400
- C. 550
- D. 1000

Ans: A. 600 We are required to find the value of n in order to create an estimate where we are 95% confident with a margin or error 4%.

Margin of error is given by

$$ME = Z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \text{ -----(1)}$$

Let n represent the size of the sample.

Let p^{\wedge} be the sample proportion and $q^{\wedge} = 1 - p^{\wedge}$

Since value of sample proportion p^{\wedge} has not been given and then we can take $p^{\wedge} = 0.5$ (which implies $q^{\wedge} = 1 - 0.5 = 0.5$ and $p^{\wedge} \times (1 - p^{\wedge}) = 0.25$), because this will result in the largest possible sample size and this will largest possible sample size will be appropriate for all sample proportions.

z-score corresponding to 95% = 1.96

Margin of Error, M.E = 0.04

Calculating n using eq(1)

$$n = 0.25 / ((M.E/z)^2) = 600.25 \sim \text{approx. } 600 \text{ (Ans } 0.25 / ((M.E/z)^2) \text{ wer)}$$

9. Suppose we want the above margin of error to be based on a 98% confidence level. What sample size (minimum) must we now use?

- A. 1000
- B. 757
- C. 848
- D. 543

Ans: C.848

Using above method, z-score corresponding to 98% = 2.33 , M.E = 0.04

$$n = 0.25 / ((M.E/z)^2) = 848.265 \sim \text{approx. } 848 \text{ (Answer)}$$