Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

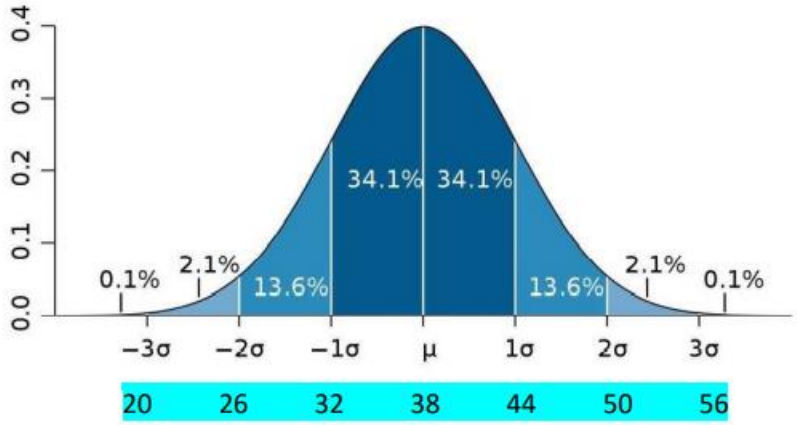
    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Ans: Let the prob of not meeting commitment be P(E).
We have to calculate the z-score first for the given scenario
Given : μ = 45 , σ = 8 , time = 60 – 10 = 50
Minutes Z-Score at 50 => (time – mean time)/std dev => (50-45)/8 = 0.625
Corresponding probability from Z-table = 0.7324
P(E) = 1 – 0.7324 = **0.2676 (Answer = Option B)**

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.

    Ans: False

    Explanation: Consider the following Normal Distribution Graph. The range between ages 38 and 44 is within one standard deviation from the mean. This means that it contains about 34.1% of 400 approx. = 136 people. Going beyond age 44 will result in about 16% approx. = 64 people which is less than the former. Therefore, the answer is False.



B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True Explanation: Finding the corresponding probability at age 30 gives approx. 36 people

Code:

```
import scipy.stats as ss
xcritical = 30
mean = 38
stdev = 6
#to get z-score and probability in python
p = ss.norm.cdf(x=xcritical,loc=mean,scale=stdev)
z_score = ss.norm.ppf(p)
print(z_score)
print(p)
print(p*400)


Output:
-1.3333333333333333
0.09121121972586788
36.484487890347154 <= approx. 36
```

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

   Ans: iid stands for independent, identically distributed random variables. A good example is a succession of throws of a fair coin. As per the question, consider X1 and X2 be the outcomes of two die rolls. They iid normal random variables. Then X1+X2 is the sum of the numbers on the two dice and 2X1 is twice the number on the first die. These don't have the same distribution - for example, X1+X2 can be odd, and 2X1 is always even.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

   Ans: In case of 0.99 symmetric prob, to get symmetry about mean => (1-0.99)/2 = 0.005
   z-score corresponding to the value is -2.57.
   To find the a,b values => 20x(-2.57) +/- 100
   would give approx. (48.6, 151.4)

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$

respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: According sum of normal random variables rules,
we can add up the profits. Annual_profit ~ N(5+7, 32 + 42) => N(12, 5 2)
Rupee Range = [99008103.48, 980991896.52]
Rupee Range ~ 99MillionRupees to 980MillionRupees **(Answer)**

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans: We already have the upper and lower range of the Annual_profit.
We can calculate the 5th percentile using python.
5 th percentile of profit = 143.1 Million Rupees **(Answer)**

Code:

```
import scipy.stats as ss
mean, std, p = 12, 5, 0.05
mean = mean*(10**6)*45
std = std*(10**6)*45
#to get z-score and rupee value
y = ss.scoreatpercentile([99008103.48, 980991896.52], 5)
print(y)
```

```
Output: 143107293.132 ~ approx. 143.1Million
```

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Division 1 will have larger probability for making a loss. **(Answer)**

Code:

```
import scipy.stats as ss
#prob of division1 to make profit less than 0
div1 = ss.norm.cdf(0,5,3)
print("P(div1 <0) = {:2f}".format(div1))
#prob of division2 to make profit less than 0
div2 = ss.norm.cdf(0,7,4)
print("P(div2 <0) = {:2f}".format(div2))
if div1 > div2:
    print("Division1 has larger prob for loss")
else: print("Division2 has larger prob for loss")
```

```
Output:
P(div1 <0) = 0.047790
P(div2 <0) = 0.040059
Division1 has larger prob for loss
```