

ASSIGNMENT 2

SET 2

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Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
 - A. 0.3875
 - B. 0.2676
 - C. 0.5
 - D. 0.6987

Ans: Let the prob of not meeting commitment be $P(E)$.

We have to calculate the z-score first for the given scenario

Given : $\mu = 45$, $\sigma = 8$, time = $60 - 10 = 50$

Minutes Z-Score at 50 \Rightarrow (time – mean time)/std dev \Rightarrow $(50-45)/8 = 0.625$

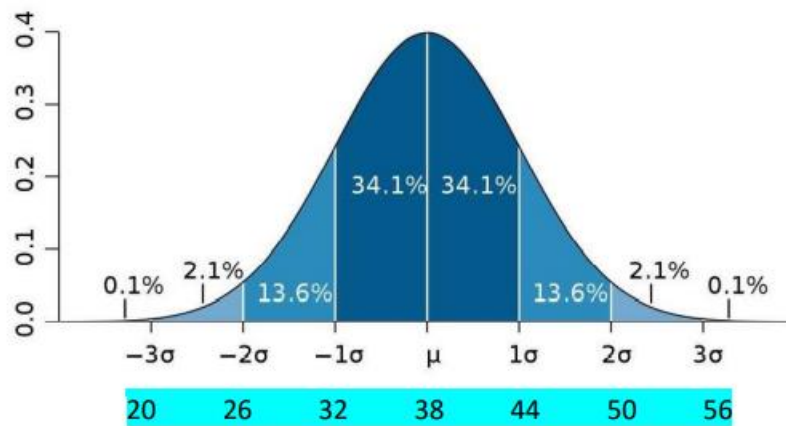
Corresponding probability from Z-table = 0.7324

$P(E) = 1 - 0.7324 = \mathbf{0.2676}$ (Answer = Option B)

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.
 - A. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False

Explanation: Consider the following Normal Distribution Graph. The range between ages 38 and 44 is within one standard deviation from the mean. This means that it contains about 34.1% of 400 approx. = 136 people. Going beyond age 44 will result in about 16% approx. = 64 people which is less than the former. Therefore, the answer is False.



- B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True Explanation: Finding the corresponding probability at age 30 gives approx. 36 people

Code:

```
import scipy.stats as ss
xcritical = 30
mean = 38
stdev = 6
#to get z-score and probability in python
p = ss.norm.cdf(x=xcritical,loc=mean,scale=stdev)
z_score = ss.norm.ppf(p)
print(z_score)
print(p)
print(p*400)
```

Output:

```
-1.3333333333333333
0.09121121972586788
36.484487890347154 <= approx. 36
```

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are iid normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans: iid stands for independent, identically distributed random variables. A good example is a succession of throws of a fair coin. As per the question, consider X_1 and X_2 be the outcomes of two die rolls. They iid normal random variables. Then $X_1 + X_2$ is the sum of the numbers on the two dice and $2X_1$ is twice the number on the first die. These don't have the same distribution - for example, $X_1 + X_2$ can be odd, and $2X_1$ is always even.

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- A. 90.5, 105.9
- B. 80.2, 119.8
- C. 22, 78
- D. 48.5, 151.5
- E. 90.1, 109.9

Ans: In case of 0.99 symmetric prob, to get symmetry about mean $\Rightarrow (1-0.99)/2 = 0.005$
z-score corresponding to the value is -2.57.

To find the a,b values $\Rightarrow 20x(-2.57) \pm 100$
would give approx. (48.6, 151.4)

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: According sum of normal random variables rules,
we can add up the profits. $\text{Annual_profit} \sim N(5+7, 3^2 + 4^2) \Rightarrow N(12, 5^2)$
Rupee Range = [99008103.48, 980991896.52]
Rupee Range \sim 99MillionRupees to 980MillionRupees **(Answer)**

- B. Specify the 5th percentile of profit (in Rupees) for the company

Ans: We already have the upper and lower range of the Annual_profit.
We can calculate the 5th percentile using python.
5 th percentile of profit = 143.1 Million Rupees **(Answer)**

Code:

```
import scipy.stats as ss
mean, std, p = 12, 5, 0.05
mean = mean*(10**6)*45
std = std*(10**6)*45
#to get z-score and rupee value
y = ss.scoreatpercentile([99008103.48, 980991896.52], 5)
print(y)

Output: 143107293.132 ~ approx. 143.1Million
```

- C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Division 1 will have larger probability for making a loss. **(Answer)**

Code:

```
import scipy.stats as ss
#prob of division1 to make profit less than 0
div1 = ss.norm.cdf(0,5,3)
print("P(div1 <0) = {:.2f}".format(div1))
#prob of division2 to make profit less than 0
div2 = ss.norm.cdf(0,7,4)
print("P(div2 <0) = {:.2f}".format(div2))
if div1 > div2:
    print("Division1 has larger prob for loss")
else: print("Division2 has larger prob for loss")
```

Output:

```
P(div1 <0) = 0.047790
P(div2 <0) = 0.040059
Division1 has larger prob for loss
```