Your name
CS 163/265, Winter 2020
Homework 7
Due Mar 6

List of collaborators:

**1.** In class we described the Gale–Shapley algorithm in terms of job applications and the positions they are applying for. Each vacant position sends out offer letters, and each job applicant tentatively accepts the best offer letter they have received so far.

**(a, CS 163 students only)** Find a system of preferences for three job applicants and three positions such that, using this algorithm, each applicant ends up matched to their least-preferred position.

**(b, CS 265 students only)** Find a system of preferences for four job applicants and four positions such that, using this algorithm, none of the positions ends up matched to its most-preferred applicant.

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**2.** **(a)** Find a rooted tree such that, if the greedy coloring algorithm colors the vertices of the tree in bottom-up order (postorder, every vertex colored after all of its children) it will use four different colors. (Note: you should not use a binary tree, because bottom-up greedy coloring of binary trees will always use at most three colors.)

**(b)** What is the Strahler number of your tree? (For non-binary trees, use the rule that the number at each node is either the maximum number of child, if only one child has the maximum, or the maximum plus one if more than one child has the maximum.) Show the numbers calculated in this way at each node of your tree.

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**3.** Find a set of intervals whose interval graph is `https://commons.wikimedia.org/wiki/File:Identity_graph2.svg`

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**4.** Suppose that a graph $G$ can be drawn with three colors, and that $G$ also contains a three-vertex complete subgraph. Does this imply that $G$ is perfect? If so, explain why; if not, find a counterexample (a graph that is 3-colorable and contains a 3-vertex complete subgraph but is not perfect).

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