Your name
CS 163/265, Winter 2020
Homework 5
Due Feb 21

List of collaborators:

1. Recall that the dynamic programming algorithm for TSP takes time $O(n^2 2^n)$ and space $O(n2^n)$. Suppose the constant factors in these running times are that it takes $n^2 2^n/10^{10}$ seconds to run an instance of size n, and that it uses $8n2^n$ bytes of memory. How big a problem can you solve in at most an hour on a computer with 4×10^9 bytes of memory? What is the limiting factor for this computation, time or memory?

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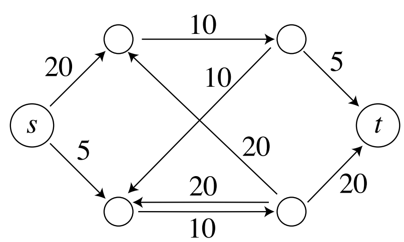
2. For the flow network shown with its edge capacities in the figure below, find the flow from s to t that you get, starting from a zero flow, by sending as much flow as possible along a single widest path in the flow network. Draw the residual network for your flow, and find an augmenting path in the residual network. (Make sure to simplify your residual graph. It should not have any edges into s or out of t, zero-capacity edges, or more than one edge between the same two vertices.)



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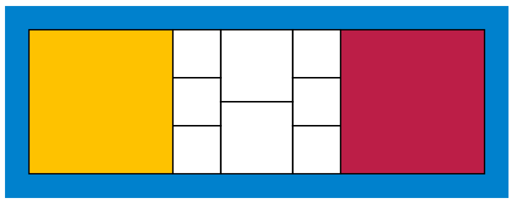
3. (a) (all students). The figure shows the map of an island, divided into ten squares. The people in the yellow country in the west want to build a wall separating them from the red country in the east, and they can choose to route it along any of the black line segments between two squares (only the line segments that are inland, not the line segments along the beaches). The cost for building a wall on each segment is not yet known (but can be assumed to be positive) and different line segments might have different costs. (Some go through swamps, some over mountains, etc.) The wall should go all the way across the island, so that it separates the island into two countries, one for each city. Draw a directed graph with a vertex for each square (with two vertices labeled s and t, but with no edge capacities) such that, once we find out the cost for building each segment of wall, we can use those costs as edge capacities and solve a minimum cut problem on the resulting flow network to find the cheapest wall.



(b) (265 students only). Describe a line segment between two squares as "one-way" if, for all possible optimal walls that use that line segment, the yellow country is always on the same side of that line segment (for example always on the north side, or always on the west side). Call it "two-way" if there are two different optimal walls (for two different sets of construction costs) that have the yellow country on opposite sides of that line segment. Which of the line segments of the map are one-way, and which are two-way?

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4. Suppose that Edna, Harpo, and Pat wish to share some food between them. They have 1500 calories of bread, 1000 calories of beef, and 500 calories of broccoli, and they each need 1000 calories of food. However, Edna does not eat broccoli, Harpo will not eat beef, and Pat cannot eat bread. There are many ways to meet these constraints and give them all enough food, and it is easy to find a solution. The goal of this problem is to model the space of all solutions as a maximum flow problem.
Draw an input to a flow problem (a directed graph with a capacity on each edge) that has a vertex for each person, a vertex for each type of food, a source vertex s, and a destination vertex t. Your network should have the property that, for any maximum flow, the flow amount from each food type x to each person y can be used as the amount of food of type x to give to person y in an assignment of food meeting the constraints above, and that every valid assignment of food can be modeled by flow amounts in this way.

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