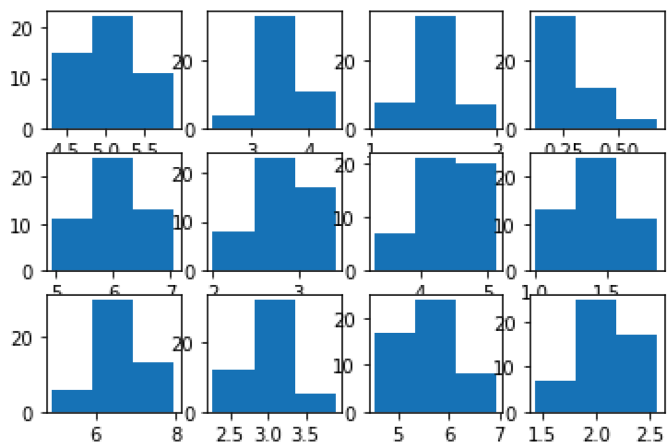
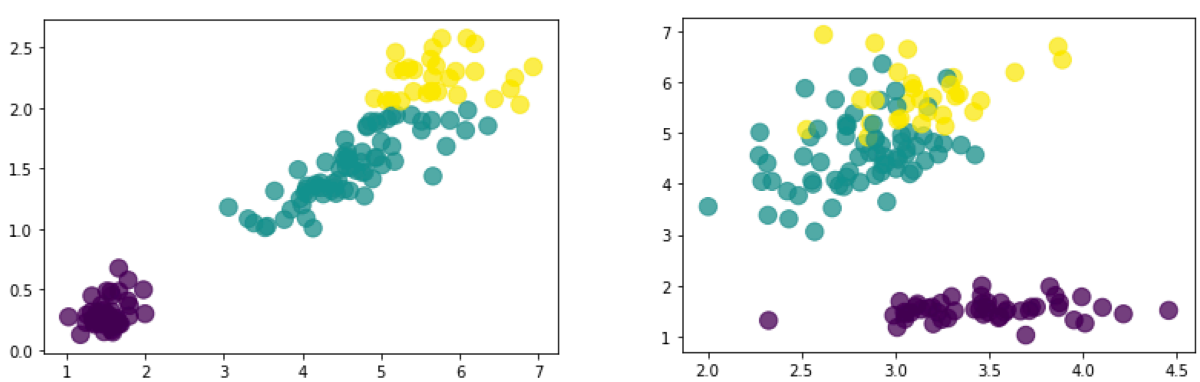
```
---PROBLEM 1---

-Part 1-
Features: 4
Data Points: 148

-Part 2-
```

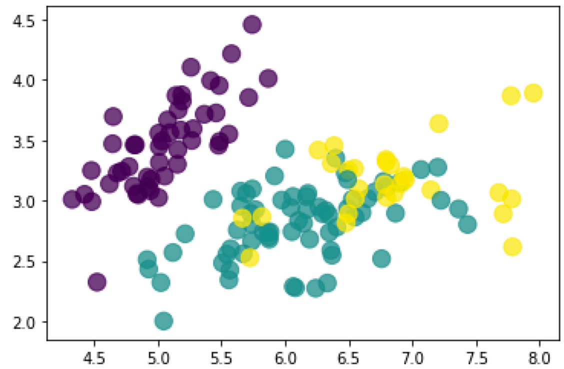
N



```
-Part 3-

Feature 1
Mean: 5.900103764189188
Variance: 0.6945590049046649
Standard Deviation: 0.833402066774894

Feature 2
Mean: 3.098930916891892
Variance: 0.19035056790635788
Standard Deviation: 0.43629183800107685

Feature 3
Mean: 3.8195548405405404
Variance: 3.076716342840002
Standard Deviation: 1.7540571093439352

Feature 4
Mean: 1.2525554845945945
Variance: 0.5757356415417657
Standard Deviation: 0.7587724570263247
```



-Part 4-



2) 1. A matrix is invertible iff its determinant is 0

2.
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{bmatrix}$$

$$\begin{aligned} Det(A) &= ((1\cdot -1\cdot 2) + (2\cdot 1\cdot 1) + (2\cdot 2\cdot 3)) \\ &\quad - ((1\cdot -1\cdot 2) + (3\cdot 1\cdot 1) + (2\cdot 2\cdot 2) \\ &= (-2 + 2 + 12) - (-2 + 3 + 8)) \\ &= 12 - 9 \\ &\boxed{= 3} \end{aligned}$$

$$\begin{aligned} Det(B) &= ((0\cdot -4\cdot 1) + (-3\cdot -2\cdot -3) + (-2\cdot 1\cdot 4)) \\ &\quad - ((-3\cdot -4\cdot -2) + (4\cdot -2\cdot 0) + (1\cdot 1\cdot -3)) \\ &= (0 - 18 - 8) - (-24 + 0 - 3) \\ &= -26 + 27 \\ &\boxed{= 1} \end{aligned}$$

3.
$$\begin{bmatrix} (-1)(2)-(3)(1) & (2)(2)-(1)(1) & (2)(3)-(1)(-1) \\ (2)(2)-(3)(2) & (1)(2)-(1)(2) & (1)(3)-(1)(-2) \\ (2)(1)-(-1)(2) & (1)(1)-(2)(2) & (1)(-1)-(2)(2) \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 3 & 7 \\ -2 & 0 & 1 \\ 4 & -3 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5 & -3 & 7 \\ 2 & 0 & -1 \\ 4 & 3 & -5 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & 2 & 4 \\ -3 & 0 & 3 \\ 7 & -1 & -5 \end{bmatrix}$$

$$Inv(A) = \begin{bmatrix} -5/3 & 2/3 & 4/3 \\ -1 & 0 & 1 \\ 7/3 & -1/3 & -5/3 \end{bmatrix}$$

$(-3)(-2)-(-4)(-2)$

$$\begin{bmatrix} (-4)(1)-(4)(-2) & (1)(1)-(-3)(-2) & (1)(4)-(-3)(-4) \\ (-3)(1)-(4)(-2) & (0)(1)-(-3)(-2) & (0)(4)-(-3)(-3) \\ (-3)(-2)-(-4)(-2) & (0)(-2)-(1)(-2) & (0)(-4)-(1)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -5 & -8 \\ 5 & -6 & -9 \\ -2 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 5 & -8 \\ -5 & -6 & 9 \\ 2 & -2 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & -5 & 2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{bmatrix}$$

$$Inv(B) = \begin{bmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{bmatrix}$$

4.
$$Inv(A^T) = \begin{bmatrix} -5/3 & -1 & 7/3 \\ 2/3 & 0 & -1/3 \\ 4/3 & 1 & -5/3 \end{bmatrix}$$

$$Inv(B^T) = \begin{bmatrix} 4 & 5 & -8 \\ -5 & -6 & 9 \\ -2 & -2 & 3 \end{bmatrix}$$

5. $Inv(AB) = Inv(A)\, Inv(B)$

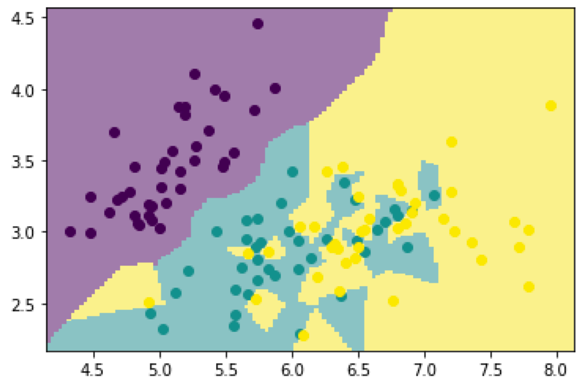
$$= \begin{bmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{bmatrix} \times \begin{bmatrix} -5/3 & 2/3 & 4/3 \\ -1 & 0 & 1 \\ 7/3 & -1/3 & -5/3 \end{bmatrix}$$

$$= \begin{bmatrix} (4)(-5/3)+(-5)(-1)+(-2)(7/3) & (4)(2/3)+(-5)(0)+(-2)(-1/3) & (4)(4/3)+(-5)(1)+(-2)(-5/3) \\ (5)(-5/3)+(-6)(-1)+(-2)(7/3) & (5)(2/3)+(-6)(0)+(-2)(-1/3) & (5)(4/3)+(-6)(1)+(-2)(-5/3) \\ (-8)(-5/3)+(9)(-1)+(3)(7/3) & (-8)(2/3)+(9)(0)+(3)(-1/3) & (-8)(4/3)+(9)(1)+(3)(-5/3) \end{bmatrix}$$

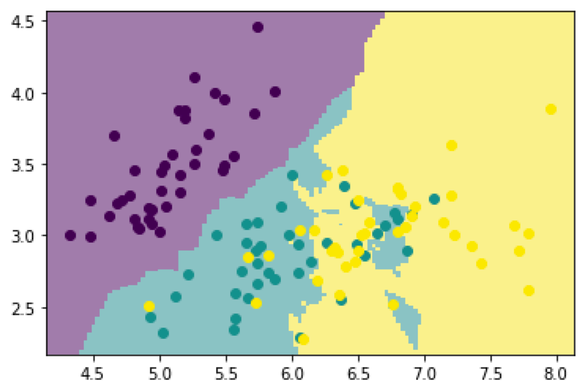
$$= \begin{bmatrix} -19/3 & 10/3 & 11/3 \\ -7 & 4 & 4 \\ 34/3 & -19/3 & -20/3 \end{bmatrix}$$

---PROBLEM 3---

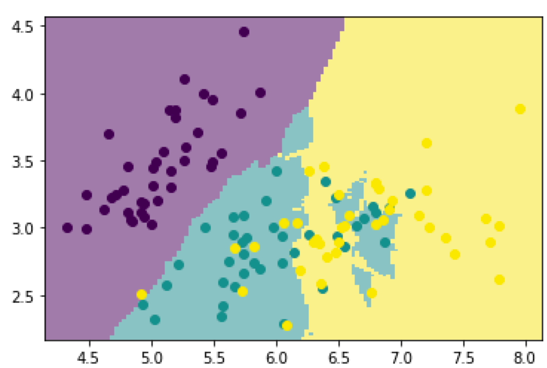
-Part 1-
k = 1



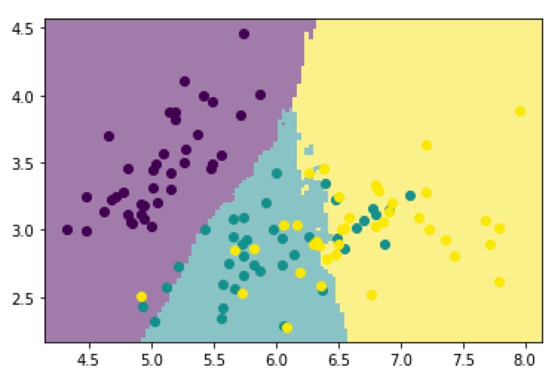
k = 5



k = 10



k = 50



Increasing the k values creates greater distinctions between the zones in the graph used to categorize outcomes. Having a k-value that is too high will however remove any islands from the graph. This could result in wrong predictions for some cases. Having the K too low does not do well because the AI doesn't learn much, the AI instead will just repeat the information it was given when trying to predict new info.

-Part 2-
[0.7333333333333333, 0.7333333333333333, 0.7666666666666667, 0.8, 0.8333333333333334, 0.8, 0.3]
I recommend a k-value of 10

-Part 3-
[0.9666666666666667, 0.9333333333333333, 0.9666666666666667, 1.0, 0.9, 0.6666666666666666, 0.26666666666666666]
The plots are not very different. The graph with more features has lower error, however that can be attributed to more information being present. I wouldn't change my k-value reccomendation.