

Due date: Monday, October 21, 11:59 PM.

You will need to submit this as a PDF via GradeScope.

Please review the syllabus and course reference for the expectations of assignments in this class. Remember that problem sets are not online treasure hunts. You are welcome to discuss matters with classmates, but remember the Kenny Loggins rule.

Unless stated otherwise, all logarithms in CompSci 161 are base-2.

As with Problem Set 1, the option of implementing problems 2 and 3 is available to you. Your implementation must have the running time indicated in the problem statement. If you do programming for one or both, indicate that on the problem set you submit and briefly explain why your code complies with the running time requirement.

1. Give a closed form ( $\Theta$ -notation) solution to the following recurrence relations. You are encouraged to use the Master Theorem (see Thursday lecture of week 2). You are not required to show your work for this problem.

(a)  $T(n) = 9T(\frac{n}{3}) + n$

(b)  $T(n) = 8T(\frac{n}{4}) + n^{1.5}$

(c)  $T(n) = 9T(\frac{n}{3}) + n^3$

2. Consider the problem of finding a *local minimum* in an array. The value  $A[i]$  is a local minimum if - and only if -  $A[i-1] \geq A[i] \leq A[i+1]$ . Give an efficient **divide and conquer** algorithm to find a local minimum in an array  $A$  in which  $A[1] = A[n] = \infty$ . State the runtime of your algorithm, both as a recurrence relation and in asymptotic notation and briefly (1-2 sentences is sufficient) explain the correctness of your algorithm.

For example, if your input array  $A$  is:

$\infty$	10	23	17	20	1	2	3	4	-3	5	$\infty$
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then any of the grayed cells could be returned as a valid answer (you need only to find *one* local minimum, not all of them).

A solution with running time  $\Omega(n)$  will receive no credit for this problem.

3. Suppose you have two data sets, each of which contain  $n$  comparable elements. As a basic operation, you may ask one set to tell you the  $k$ th largest element of that set, for a value  $k$  you choose. Give an algorithm that, with  $\mathcal{O}(\log n)$  queries, determines the median value of the *union* of the two sets.

## Not Collected Questions

These questions will not be collected. Please do not submit your solutions for them. However, these are meant to help you to study and understand the course material better. You are encouraged to solve these as if they were normal homework problems.

This homework (approximately) covers §8.1, 8.2, 9.2 and Chapter 11. One of the strengths of this book is that it has a good variety and quality of practice problems.

If you need help deciding which problems to do, consider trying R-8.4, R-8.5, R-8.6, C-8.2, C-8.5, C-8.12, A-8.1, A-8.2, A-8.3, A-8.4, A-8.8, R-9.1, C-9.8, C-9.11, C-9.12, A-9.4, A-9.7, R-11.6, C-11.3, A-11.2, A-11.6.

It is also recommended that you be proficient with the Master Theorem.