



Science and  
Technology  
Facilities Council



# GALAHAD

DPS

USER DOCUMENTATION

GALAHAD Optimization Library version 4.0

## 1 SUMMARY

Given a real  $n$  by  $n$  symmetric matrix  $\mathbf{H}$ , this package **construct a symmetric, positive definite matrix  $\mathbf{M}$  so that  $\mathbf{H}$  is diagonal in the norm  $\|\mathbf{v}\|_{\mathbf{M}} = \sqrt{\mathbf{v}^T \mathbf{M} \mathbf{v}}$  induced by  $\mathbf{M}$ .** Subsequently the package can be use to **solve the trust-region subproblem**

$$\text{minimize } q(\mathbf{x}) = \frac{1}{2} \mathbf{x}^T \mathbf{H} \mathbf{x} + \mathbf{c}^T \mathbf{x} + f \text{ subject to } \|\mathbf{x}\|_{\mathbf{M}} \leq \Delta \quad (1.1)$$

or the **regularized quadratic problem**

$$\text{minimize } q(\mathbf{x}) + \frac{1}{p} \sigma \|\mathbf{x}\|_{\mathbf{M}}^p \quad (1.2)$$

for a real  $n$  vector  $\mathbf{c}$  and scalars  $f, \Delta > 0, \sigma > 0$  and  $p \geq 2$ .

A factorization of the matrix  $\mathbf{H}$  will be required, so this package is most suited for the case where such a factorization, either dense or sparse, may be found efficiently.

**ATTRIBUTES — Versions:** GALAHAD\_DPS\_single, GALAHAD\_DPS\_double. **Uses:** GALAHAD\_CLOCK, GALAHAD\_SYMBOLS, GALAHAD\_SPACE, GALAHAD\_NORMS, GALAHAD\_SMT, GALAHAD\_SPECFILE, GALAHAD\_SLS, GALAHAD\_TRS, GALAHAD\_RQS **Date:** March 2018. **Origin:** N. I. M. Gould. **Language:** Fortran 2003. **Parallelism:** Some options may use OpenMP and its runtime library.

## 2 HOW TO USE THE PACKAGE

Access to the package requires a USE statement such as

*Single precision version*

```
USE GALAHAD_DPS_single
```

*Double precision version*

```
USE GALAHAD_DPS_double
```

If it is required to use both modules at the same time, the derived types SMT\_TYPE, DPS\_control\_type, DPS\_history\_type, DPS\_inform\_type, DPS\_data\_type, (Section 2.4) and the subroutines DPS\_initialize, DPS\_solve, DPS\_terminate (Section 2.5) and DPS\_read\_specfile (Section 2.7) must be renamed on one of the USE statements.

### 2.1 Matrix storage formats

The matrix  $\mathbf{H}$  may be stored in a variety of input formats.

#### 2.1.1 Dense storage format

The matrix  $\mathbf{H}$  is stored as a compact dense matrix by rows, that is, the values of the entries of each row in turn are stored in order within an appropriate real one-dimensional array. Since  $\mathbf{H}$  is symmetric, only the lower triangular part (that is the part  $h_{ij}$  for  $1 \leq j \leq i \leq n$ ) need be held. In this case the lower triangle should be stored by rows, that is component  $i * (i - 1) / 2 + j$  of the storage array H%val will hold the value  $h_{ij}$  (and, by symmetry,  $h_{ji}$ ) for  $1 \leq j \leq i \leq n$ .

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### 2.1.2 Sparse co-ordinate storage format

Only the nonzero entries of the matrices are stored. For the  $l$ -th entry of  $\mathbf{H}$ ,  $1 \leq j \leq i \leq n$ , its row index  $i$ , column index  $j$  and value  $h_{ij}$  are stored in the  $l$ -th components of the integer arrays `H%row`, `H%col` and real array `H%val`, respectively. Note that only the entries in the lower triangle should be stored.

### 2.1.3 Sparse row-wise storage format

Again only the nonzero entries are stored, but this time they are ordered so that those in row  $i$  appear directly before those in row  $i + 1$ . For the  $i$ -th row of  $\mathbf{H}$ , the  $i$ -th component of the integer array `A%ptr` holds the position of the first entry in this row, while `A%ptr(m + 1)` holds the total number of entries plus one. The column indices  $j$ ,  $1 \leq j \leq i$ , and values  $h_{ij}$  of the entries in the  $i$ -th row are stored in components  $l = \text{H\%ptr}(i), \dots, \text{H\%ptr}(i + 1) - 1$  of the integer array `H%col`, and real array `H%val`, respectively. Note that as before only the entries in the lower triangle should be stored. For sparse matrices, this scheme almost always requires less storage than its predecessor.

## 2.2 Integer kinds

We use the term long `INTEGER` to denote `INTEGER(kind=long)`, where `long = selected_int_kind(18)`.

## 2.3 OpenMP

OpenMP may be used by the `GALAHAD_DPS` package to provide parallelism for some solver options in shared memory environments. See the documentation for the `GALAHAD` package SLS for more details. To run in parallel, OpenMP must be enabled at compilation time by using the correct compiler flag (usually some variant of `-openmp`). The number of threads may be controlled at runtime by setting the environment variable `OMP_NUM_THREADS`.

The code may be compiled and run in serial mode.

## 2.4 The derived data types

Five derived data types are accessible from the package.

### 2.4.1 The derived data type for holding matrices

The derived data type `SMT_TYPE` is used to hold the matrices  $\mathbf{H}$  and perhaps  $\mathbf{M}$  and/or  $\mathbf{A}$ . The components of `SMT_TYPE` used here are:

`m` is a scalar component of type default `INTEGER`, that holds the number of rows in the matrix.

`n` is a scalar component of type default `INTEGER`, that holds the number of columns in the matrix.

`ne` is a scalar variable of type default `INTEGER`, that holds the number of matrix entries.

`type` is a rank-one allocatable array of type default `CHARACTER`, that is used to indicate the matrix storage scheme used. Its precise length and content depends on the type of matrix to be stored.

`val` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_DPS_double`) and dimension at least `ne`, that holds the values of the entries. Each pair of off-diagonal entries  $h_{ij} = h_{ji}$  of the *symmetric* matrix  $\mathbf{H}$  is represented as a single entry (see §2.1.1–2.1.3). Any duplicated entries that appear in the sparse co-ordinate or row-wise schemes will be summed.

`row` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `ne`, that may hold the row indices of the entries. (see §2.1.2).

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`col` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `ne`, that may hold the column indices of the entries (see §2.1.2–2.1.3).

`ptr` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `n + 1`, that may holds the pointers to the first entry in each row (see §2.1.3).

## 2.4.2 The derived data type for holding control parameters

The derived data type `DPS_control_type` is used to hold controlling data. Default values may be obtained by calling `DPS_initialize` (see Section 2.5.1). The components of `DPS_control_type` are:

`error` is a scalar variable of type default `INTEGER`, that holds the stream number for error messages. Printing of error messages in `DPS_solve` and `DPS_terminate` is suppressed if `error`  $\leq 0$ . The default is `error` = 6.

`out` is a scalar variable of type default `INTEGER`, that holds the stream number for informational messages. Printing of informational messages in `DPS_solve` is suppressed if `out`  $< 0$ . The default is `out` = 6.

`print_level` is a scalar variable of type default `INTEGER`, that is used to control the amount of informational output which is required. No informational output will occur if `print_level`  $\leq 0$ . If `print_level` = 1 a single line of output will be produced for each iteration of the process. If `print_level`  $\geq 2$  this output will be increased to provide significant detail of each iteration. The default is `print_level` = 0.

`new_h` is a scalar variable of type default `INTEGER`, that is used to indicate how **H** has changed (if at all) since the previous call to `DPS_solve`. Possible values are:

- 0 **H** is unchanged.
- 1 the values in **H** have changed, but its nonzero structure is as before.
- 2 both the values and structure of **H** have changed.

The default is `new_h` = 2.

`taylor_max_degree` is a scalar variable of type default `INTEGER`, that specifies the maximum degree of Taylor approximant that will be used to approximate the secular function when trying to improve  $\lambda$ ; a first-degree approximant results in Newton's method. The higher the degree, the better in general the improvement, but the larger the cost. Thus there is a balance between many cheap low-degree approximants and a few more expensive higher-degree ones. Our experience favours higher-degree approximants. The default is `taylor_max_degree` = 3, which is the highest degree currently supported.

`eigen_min` is a scalar variables of type `REAL` (double precision in `GALAHAD_DPS_double`), that specifies the smallest allowable value of an eigenvalue of the block diagonal factor of **H**. Any eigenvalue smaller than `eigen_min` will be set to `eigen_min` when constructing **M**. See Section 4 for more details. The default is  $\sqrt{u}$ , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in `GALAHAD_DPS_double`).

`lower` is a scalar variables of type `REAL` (double precision in `GALAHAD_DPS_double`), that holds the value of any known lower bound on the required multiplier  $\lambda_*$ . A good lower bound may sometimes dramatically improve the performance of the package, but an incorrect value might cause the method to fail. Thus resetting `lower` from its default should be used with caution. The default is `lower` = - `HUGE(1.0)` (-`HUGE(1.0D0)` in `GALAHAD_DPS_double`).

`upper` is a scalar variables of type `REAL` (double precision in `GALAHAD_DPS_double`), that holds the value of any known upper bound on the required multiplier  $\lambda_*$ . A good upper bound may sometimes dramatically improve the performance of the package, but an incorrect value might cause the method to fail. Thus resetting `upper` from its default should be used with caution. The default is `upper` = `HUGE(1.0)` (`HUGE(1.0D0)` in `GALAHAD_DPS_double`).

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`stop_normal` and `stop_absolute_normal` are scalar variables of type REAL (double precision in GALAHAD\_DPS\_double), that hold values for the standard convergence tolerances of the method (see Section 4). In particular, the method is deemed to have converged for the trust-region subproblem when the computed solution  $\mathbf{x}$  and its multiplier  $\lambda$  satisfy either  $\lambda = 0$  and  $\|\mathbf{x}\|_{\mathbf{M}} < \Delta$  or  $\|\mathbf{x}\|_{\mathbf{M}} - \Delta \leq \max(\text{stop\_normal} * \Delta, \text{stop\_absolute\_normal})$ , while convergence in the regularization case happens when  $\|\mathbf{x}\|_{\mathbf{M}} - (\lambda/\sigma)^{1/(p-2)} \leq \text{stop\_normal} * \max(1, \|\mathbf{x}\|_{\mathbf{M}}, (\lambda/\sigma)^{1/(p-2)})$ . The defaults are `stop_normal = stop_absolute_normal =  $u^{0.75}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_DPS\_double).

`goldfarb` is a scalar variable of type default LOGICAL, that may be set `.TRUE.` if the user wishes the package to use Goldfarb's method to build  $\mathbf{M}$  from  $\mathbf{H}$ , and `.FALSE.` if the modified-absolute-value method is to be used instead. The default is `goldfarb = .FALSE.`.

`space_critical` is a scalar variable of type default LOGICAL, that may be set `.TRUE.` if the user wishes the package to allocate as little internal storage as possible, and `.FALSE.` otherwise. The package may be more efficient if `space_critical` is set `.FALSE.`. The default is `space_critical = .FALSE.`.

`deallocate_error_fatal` is a scalar variable of type default LOGICAL, that may be set `.TRUE.` if the user wishes the package to return to the user in the unlikely event that an internal array deallocation fails, and `.FALSE.` if the package should be allowed to try to continue. The default is `deallocate_error_fatal = .FALSE.`.

`symmetric_linear_solver` is a scalar variable of type default CHARACTER and length 30, that specifies the external package to be used to solve any symmetric linear system that might arise. Current possible choices are `'sils'`, `'ma27'`, `'ma57'`, `'ma77'`, `'ma86'`, `'ma97'`, `ssids`, `'pardiso'` and `'wsmp'`, although only `'sils'` and, for OMP 4.0-compliant compilers, `'ssids'` are installed by default. See the documentation for the GALAHAD package SLS for further details. The default is `symmetric_linear_solver = 'sils'`.

`prefix` is a scalar variable of type default CHARACTER and length 30, that may be used to provide a user-selected character string to preface every line of printed output. Specifically, each line of output will be prefaced by the string `prefix(2:LEN(TRIM(prefix))-1)`, thus ignoring the first and last non-null components of the supplied string. If the user does not want to preface lines by such a string, they may use the default `prefix = ""`.

`SLS_control` is a scalar variable of type `SLS_control_type` that is used to control various aspects of the factorization package SLS. See the documentation for GALAHAD\_SLS for more details.

### 2.4.3 The derived data type for holding timing information

The derived data type `DPS_time_type` is used to hold elapsed CPU and system clock times for the various parts of the calculation. The components of `DPS_time_type` are:

`total` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the total CPU time spent in the package.

`analyse` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the CPU time spent analysing required matrices prior to factorization.

`factorize` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the CPU time spent factorizing the required matrices.

`solve` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the CPU time spent solving diagonal trust-region or regularization subproblems.

`clock_total` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the total elapsed system clock time spent in the package.

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`clock_analyse` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the elapsed system clock time spent analysing required matrices prior to factorization.

`clock_factorize` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the elapsed system clock time spent factorizing the required matrices.

`clock_solve` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that gives the elapsed time spent solving diagonal trust-region or regularization subproblems.

#### 2.4.4 The derived data type for holding informational parameters

The derived data type `DPS_inform_type` is used to hold parameters that give information about the progress and needs of the algorithm. The components of `DPS_inform_type` are:

`status` is a scalar variable of type default INTEGER, that gives the current status of the algorithm. See Section 2.6 for details.

`alloc_status` is a scalar variable of type default INTEGER, that gives the status of the last internal array allocation or deallocation. This will be 0 if `status` = 0.

`bad_alloc` is a scalar variable of type default CHARACTER and length 80, that gives the name of the last internal array for which there were allocation or deallocation errors. This will be the null string if `status` = 0.

`mod_1by1` is a scalar variable of type default INTEGER, that gives the number of eigenvalues from 1 by 1 blocks from the factorization of  $\mathbf{H}$  that were modified when constructing  $\mathbf{M}$ .

`mod_2by2` is a scalar variable of type default INTEGER, that gives the number of eigenvalues from 2 by 2 blocks from the factorization of  $\mathbf{H}$  that were modified when constructing  $\mathbf{M}$ .

`obj` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that holds the value of the objective function  $\frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{c}^T\mathbf{x} + f$ .

`obj_regularized` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that holds the value of the regularized objective function  $\frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{c}^T\mathbf{x} + f + \frac{1}{p}\boldsymbol{\sigma}\|\mathbf{x}\|_{\mathbf{M}}^p$ .

`multiplier` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that holds the value of the Lagrange multiplier  $\lambda$  associated with the constraint.

`x_norm` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that holds the value of  $\|\mathbf{x}\|_{\mathbf{M}}$ .

`pole` is a scalar variable of type default REAL (double precision in GALAHAD\_DPS\_double), that holds a lower bound on  $\max(0, -\lambda_1)$ , where  $\lambda_1$  is the left-most eigenvalue of the matrix pencil  $(H, M)$ .

`hard_case` is a scalar variable of type default LOGICAL, that will be `.TRUE.` if the “hard-case” has occurred (see Section 4) and `.FALSE.` otherwise.

`time` is a scalar variable of type `DPS_time_type` whose components are used to hold elapsed CPU and system clock times for the various parts of the calculation (see Section 2.4.3).

`SLS_inform` is a scalar variable of type `SLS_inform_type`, that holds informational parameters concerning the analysis, factorization and solution phases performed by the GALAHAD sparse matrix factorization package SLS. See the documentation for the package SLS for details of the derived type `SLS_inform_type`.

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### 2.4.5 The derived data type for holding problem data

The derived data type `DPS_data_type` is used to hold all the data for a particular problem between calls of DPS procedures. This data should be preserved, untouched, from the initial call to `DPS_initialize` to the final call to `DPS_terminate`.

## 2.5 Argument lists and calling sequences

There are three procedures for user calls (see Section 2.7 for further features):

1. The subroutine `DPS_initialize` is used to set default values and initialize private data.
2. The subroutine `DPS_solve` is called to compute the desired matrix  $\mathbf{M}$  and then solve the appropriate quadratic subproblem.
3. The subroutine `DPS_resolve` is used to resolve the problem when the only data that has changed since the last solution are the values of the radius  $\Delta$  or the regularization weight  $\sigma$  and, optionally, the linear and constant terms  $\mathbf{c}$  and  $f$ .
4. The subroutine `DPS_terminate` is provided to allow the user to automatically deallocate array components of the private data, allocated by `DPS_solve`, at the end of the solution process.

We use square brackets [ ] to indicate OPTIONAL arguments.

### 2.5.1 The initialization subroutine

Default values are provided as follows:

```
CALL DPS_initialize( data, control, inform )
```

`data` is a scalar `INTENT(INOUT)` argument of type `DPS_data_type` (see Section 2.4.5). It is used to hold data about the problem being solved.

`control` is a scalar `INTENT(OUT)` argument of type `DPS_control_type` (see Section 2.4.2). On exit, `control` contains default values for the components as described in Section 2.4.2. These values should only be changed after calling `DPS_initialize`.

`inform` is a scalar `INTENT(OUT)` argument of type `DPS_inform_type` (see Section 2.4.4). A successful call to `DPS_initialize` is indicated when the component `status` has the value 0. For other return values of `status`, see Section 2.6.

### 2.5.2 The optimization problem solution subroutine

The optimization problem solution algorithm is called as follows:

```
CALL DPS_solve( n, H, c, f, X, data, control, inform[, delta, sigma, p] )
```

`n` is a scalar `INTENT(IN)` argument of type default `INTEGER`, that must be set to the number of unknowns,  $n$ .  
**Restriction:**  $n > 0$ .

`H` is scalar `INTENT(IN)` argument of type `SMT_TYPE` that holds the Hessian matrix  $\mathbf{H}$ . The following components are used here:

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`H%type` is an allocatable array of rank one and type default `CHARACTER`, that is used to indicate the storage scheme used. If the dense storage scheme (see Section 2.1.1) is used, the first five components of `H%type` must contain the string `DENSE`. For the sparse co-ordinate scheme (see Section 2.1.2), the first ten components of `H%type` must contain the string `COORDINATE`, and for the sparse row-wise storage scheme (see Section 2.1.3), the first fourteen components of `H%type` must contain the string `SPARSE_BY_ROWS`.

For convenience, the procedure `SMT_put` may be used to allocate sufficient space and insert the required keyword into `H%type`. For example, if we wish to store **M** using the co-ordinate scheme, we may simply

```
CALL SMT_put( H%type, 'COORDINATE' )
```

See the documentation for the GALAHAD package `SMT` for further details on the use of `SMT_put`.

`H%ne` is a scalar variable of type default `INTEGER`, that holds the number of entries in the **lower triangular** part of **H** in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be set for any of the other three schemes.

`H%val` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_DPS_double`), that holds the values of the entries of the **lower triangular** part of the Hessian matrix **H** in any of the storage schemes discussed in Section 2.1.

`H%row` is a rank-one allocatable array of type default `INTEGER`, that holds the row indices of the **lower triangular** part of **H** in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be allocated for any of the other schemes.

`H%col` is a rank-one allocatable array variable of type default `INTEGER`, that holds the column indices of the **lower triangular** part of **H** in either the sparse co-ordinate (see Section 2.1.2), or the sparse row-wise (see Section 2.1.3) storage scheme. It need not be allocated when the dense storage scheme is used.

`H%ptr` is a rank-one allocatable array of dimension `n+1` and type default `INTEGER`, that holds the starting position of each row of the **lower triangular** part of **H**, as well as the total number of entries plus one, in the sparse row-wise storage scheme (see Section 2.1.3). It need not be allocated when the other schemes are used.

`C` is an array `INTENT(IN)` argument of dimension `n` and type default `REAL` (double precision in `GALAHAD_DPS_double`), whose *i*-th entry holds the component  $c_i$  of the vector **c** for the objective function.

`f` is a scalar `INTENT(IN)` variable of type default `REAL` (double precision in `GALAHAD_DPS_double`), that holds the scalar value *f* for the objective function.

`X` is an array `INTENT(OUT)` argument of dimension `n` and type default `REAL` (double precision in `GALAHAD_DPS_double`), that holds an estimate of the solution **x** of the problem (1.1) or (1.2) on exit.

`data` is a scalar `INTENT(INOUT)` argument of type `DPS_data_type` (see Section 2.4.5). It is used to hold data about the problem being solved. It must not have been altered **by the user** since the last call to `DPS_initialize`.

`control` is a scalar `INTENT(IN)` argument of type `DPS_control_type`. (see Section 2.4.2). Default values may be assigned by calling `DPS_initialize` prior to the first call to `DPS_solve`.

`inform` is a scalar `INTENT(INOUT)` argument of type `DPS_inform_type` (see Section 2.4.4) whose components need not be set on entry. A successful call to `DPS_solve` is indicated when the component `status` has the value 0. For other return values of `status`, see Section 2.6.

`delta` is an OPTIONAL scalar `INTENT(IN)` variable of type default `REAL` (double precision in `GALAHAD_DPS_double`), that must be set on initial entry to the value of the radius of the trust-region constraint,  $\Delta$  if the solution to the trust-region subproblem (1.1) is required. If `delta` is not `PRESENT`, the trust-region subproblem will not be solved, but when it is `PRESENT`, the regularization subproblem will not be solved regardless of the status of `sigma` (see below). **Restriction:** `delta` > 0.

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`sigma` is an OPTIONAL scalar INTENT(IN) variable of type default REAL (double precision in GALAHAD\_DPS\_double), that must be set on initial entry to the value of the regularization weight,  $\sigma$  if the solution to the regularization subproblem (1.2) is required. If `sigma` is not PRESENT, the regularization subproblem will not be solved.

**Restriction:** `sigma` > 0.

`p` is an OPTIONAL scalar INTENT(IN) variable of type default REAL (double precision in GALAHAD\_DPS\_double), that must be set on initial entry to the value of the regularization order,  $p$  if the solution to the regularization subproblem (1.2) is required. If `p` is not PRESENT, the regularization order is taken to be 3.0. **Restriction:** `p` ≥ 2.0.

### 2.5.3 The optimization problem re-solution subroutine

The optimization problem solution algorithm may be recalled with modified problem data as follows:

```
CALL DPS_resolve( n, X, data, control, inform[, C, f, delta, sigma, p] )
```

`n` is a scalar INTENT(IN) argument of type default INTEGER, that must be set to the number of unknowns,  $n$ . This should not have been changed since the last call to `DPS_solve`.

`X` is an array INTENT(OUT) argument of dimension `n` and type default REAL (double precision in GALAHAD\_DPS\_double), that holds an estimate of the solution  $\mathbf{x}$  of the problem (1.1) or (1.2) on exit.

`data` is a scalar INTENT(INOUT) argument of type `DPS_data_type` (see Section 2.4.5). It is used to hold data about the problem being solved. It must not have been altered **by the user** since the last call to `DPS_solve`.

`control` is a scalar INTENT(IN) argument of type `DPS_control_type`. (see Section 2.4.2). Default values may be assigned by calling `DPS_initialize` prior to the first call to `DPS_solve`.

`inform` is a scalar INTENT(INOUT) argument of type `DPS_inform_type` (see Section 2.4.4) whose components need not be set on entry. A successful call to `DPS_solve` is indicated when the component `status` has the value 0. For other return values of `status`, see Section 2.6.

`C` is an OPTIONAL array INTENT(IN) argument of dimension `n` and type default REAL (double precision in GALAHAD\_DPS\_double), that if PRESENT whose  $i$ -th entry holds the component  $c_i$  of a new vector  $\mathbf{c}$  for the objective function.

`f` is an OPTIONAL scalar INTENT(IN) variable of type default REAL (double precision in GALAHAD\_DPS\_double), that if PRESENT holds the a new value of the scalar  $f$  for the objective function.

`delta` is an OPTIONAL scalar INTENT(IN) variable of type default REAL (double precision in GALAHAD\_DPS\_double), that must be set on entry to the value of the radius of the trust-region constraint,  $\Delta$  if the solution to the trust-region subproblem (1.1) is required. If `delta` is not PRESENT, the trust-region subproblem will not be solved, but when it is PRESENT, the regularization subproblem will not be solved regardless of the status of `sigma` (see below). **Restriction:** `delta` > 0.

`sigma` is an OPTIONAL scalar INTENT(IN) variable of type default REAL (double precision in GALAHAD\_DPS\_double), that must be set on entry to the value of the regularization weight,  $\sigma$  if the solution to the regularization subproblem (1.2) is required. If `sigma` is not PRESENT, the regularization subproblem will not be solved. **Restriction:** `sigma` > 0.

`p` is an OPTIONAL scalar INTENT(IN) variable of type default REAL (double precision in GALAHAD\_DPS\_double), that must be set on entry to the value of the regularization order,  $p$  if the solution to the regularization subproblem (1.2) is required. If `p` is not PRESENT, the regularization order is taken to be 3.0. **Restriction:** `p` ≥ 2.0.

---

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#### 2.5.4 The termination subroutine

All previously allocated arrays are deallocated as follows:

```
CALL DPS_terminate( data, control, inform )
```

`data` is a scalar INTENT (INOUT) argument of type `DPS_data_type` exactly as for `DPS_solve` that must not have been altered by the user since the last call to `DPS_initialize`. On exit, array components will have been deallocated.

`control` is a scalar INTENT (IN) argument of type `DPS_control_type` exactly as for `DPS_solve`.

`inform` is a scalar INTENT (OUT) argument of type `DPS_inform_type` exactly as for `DPS_solve`. Only the component status will be set on exit, and a successful call to `DPS_terminate` is indicated when this component status has the value 0. For other return values of `status`, see Section 2.6.

#### 2.6 Warning and error messages

A negative value of `inform%status` on exit from `DPS_solve` or `DPS_terminate` indicates that an error might have occurred. No further calls should be made until the error has been corrected. Possible values are:

- 1. An allocation error occurred. A message indicating the offending array is written on unit `control%error`, and the returned allocation status and a string containing the name of the offending array are held in `inform%alloc_status` and `inform%bad_alloc`, respectively.
- 2. A deallocation error occurred. A message indicating the offending array is written on unit `control%error` and the returned allocation status and a string containing the name of the offending array are held in `inform%alloc_status` and `inform%bad_alloc`, respectively.
- 3. (`DPS_solve` and `DPS_resolve` only) One of the restrictions  $n > 0$ ,  $radius > 0$ ,  $sigma > 0$  or  $p \geq 2$  has been violated.
- 9. (`DPS_solve` only) The analysis phase of the factorization of the matrix **H** failed.
- 10. (`DPS_solve` only) The factorization of the matrix **H** failed.
- 16. (`DPS_solve` and `DPS_resolve` only) The problem is so ill-conditioned that further progress is impossible.
- 40. (`DPS_solve` only) An error occurred when building the preconditioner.

#### 2.7 Further features

In this section, we describe an alternative means of setting control parameters, that is components of the variable `control` of type `DPS_control_type` (see Section 2.4.2), by reading an appropriate data specification file using the subroutine `DPS_read_specfile`. This facility is useful as it allows a user to change DPS control parameters without editing and recompiling programs that call DPS.

A specification file, or `specfile`, is a data file containing a number of "specification commands". Each command occurs on a separate line, and comprises a "keyword", which is a string (in a close-to-natural language) used to identify a control parameter, and an (optional) "value", which defines the value to be assigned to the given control parameter. All keywords and values are case insensitive, keywords may be preceded by one or more blanks but values must not contain blanks, and each value must be separated from its keyword by at least one blank. Values must not contain more than 30 characters, and each line of the `specfile` is limited to 80 characters, including the blanks separating keyword and value.

The portion of the specification file used by `DPS_read_specfile` must start with a "BEGIN DPS" command and end with an "END" command. The syntax of the `specfile` is thus defined as follows:

---

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```
( .. lines ignored by DPS_read_specfile .. )
BEGIN DPS
  keyword      value
  .....      .....
  keyword      value
END
( .. lines ignored by DPS_read_specfile .. )
```

where keyword and value are two strings separated by (at least) one blank. The “BEGIN DPS” and “END” delimiter command lines may contain additional (trailing) strings so long as such strings are separated by one or more blanks, so that lines such as

```
BEGIN DPS SPECIFICATION
```

and

```
END DPS SPECIFICATION
```

are acceptable. Furthermore, between the “BEGIN DPS” and “END” delimiters, specification commands may occur in any order. Blank lines and lines whose first non-blank character is ! or \* are ignored. The content of a line after a ! or \* character is also ignored (as is the ! or \* character itself). This provides an easy manner to “comment out” some specification commands, or to comment specific values of certain control parameters.

The value of a control parameters may be of three different types, namely integer, logical or real. Integer and real values may be expressed in any relevant Fortran integer and floating-point formats (respectively). Permitted values for logical parameters are “ON”, “TRUE”, “.TRUE.”, “T”, “YES”, “Y”, or “OFF”, “NO”, “N”, “FALSE”, “.FALSE.” and “F”. Empty values are also allowed for logical control parameters, and are interpreted as “TRUE”.

The specification file must be open for input when `DPS_read_specfile` is called, and the associated device number passed to the routine in `device` (see below). Note that the corresponding file is `REWINDed`, which makes it possible to combine the specifications for more than one program/routine. For the same reason, the file is not closed by `DPS_read_specfile`.

Control parameters corresponding to the components `SLS_control` and `IR_control` may be changed by including additional sections enclosed by “BEGIN SLS” and “END SLS”, and “BEGIN IR” and “END IR”, respectively. See the specification sheets for the packages `GALAHAD_SLS` and `GALAHAD_IR` for further details.

### 2.7.1 To read control parameters from a specification file

Control parameters may be read from a file as follows:

```
CALL DPS_read_specfile( control, device )
```

`control` is a scalar `INTENT(INOUT)` argument of type `DPS_control_type` (see Section 2.4.2). Default values should have already been set, perhaps by calling `DPS_initialize`. On exit, individual components of `control` may have been changed according to the commands found in the specfile. Specfile commands and the component (see Section 2.4.2) of `control` that each affects are given in Table 2.1.

`device` is a scalar `INTENT(IN)` argument of type `default_INTEGER`, that must be set to the unit number on which the specfile has been opened. If `device` is not open, `control` will not be altered and execution will continue, but an error message will be printed on unit `control%error`.

---

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command	component of control	value type
error-printout-device	%error	integer
printout-device	%out	integer
print-level	%print_level	integer
use-dense-factorization	%dense_factorization	integer
has-h-changed	%new_h	integer
max-degree-taylor-approximant	%taylor_max_degree	integer
smallest-eigenvalue-allowed	%eigen_min	real
lower-bound-on-multiplier	%lower	real
upper-bound-on-multiplier	%upper	real
stop-normal-case	%stop_normal	real
stop-absolute-normal-case	%stop_absolute_normal	real
build-goldfarb-preconditioner	%goldfarb	logical
space-critical	%space_critical	logical
deallocate-error-fatal	%deallocate_error_fatal	logical
symmetric-linear-equation-solver	%symmetric_linear_solver	character
output-line-prefix	%prefix	character

Table 2.1: Specfile commands and associated components of control.

## 2.8 Information printed

If `control%print_level` is positive, information about the progress of the algorithm will be printed on unit `control%out`. If `control%print_level = 1`, a single line of output will be produced for each iteration of the process. In the first phase of the algorithm, this will include the current estimate of the multiplier and known brackets on its optimal value. In the second phase, the residual  $\|\mathbf{x}\|_{\mathbf{M}} - \Delta$  (in the trust-region case), the current estimate of the multiplier and the size of the correction will be printed. If `control%print_level  $\geq 2$` , this output will be increased to provide significant detail of each iteration. This extra output includes times for various phases.

## 3 GENERAL INFORMATION

**Use of common:** None.

**Workspace:** Provided automatically by the module.

**Other routines called directly:** None.

**Other modules used directly:** `DPS_solve` and `DPS_resolve` call the GALAHAD packages `GALAHAD_CLOCK`, `GALAHAD_SYMBOLS`, `GALAHAD_SPACE`, `GALAHAD_NORMS`, `GALAHAD_SMT`, `GALAHAD_SPECFILE`, `GALAHAD_SLS`, `GALAHAD_TRS` and `GALAHAD_RQS`.

**Input/output:** Output is under control of the arguments `control%error`, `control%out` and `control%print_level`.

**Restrictions:**  $n > 0$ ,  $\Delta > 0$ ,  $\sigma > 0$ ,  $p \geq 2$ .

**Portability:** ISO Fortran 2003. The package is thread-safe.

## 4 METHOD

The required solution  $\mathbf{x}_*$  necessarily satisfies the optimality condition  $\mathbf{H}\mathbf{x}_* + \lambda_*\mathbf{M}\mathbf{x}_* + \mathbf{c} = \mathbf{0}$ , where  $\lambda_* \geq 0$  is a Lagrange multiplier that corresponds to the constraint  $\|\mathbf{x}\|_{\mathbf{M}} \leq \Delta$  in the trust-region case (1.1), and is given by

---

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$\lambda_* = \sigma \|\mathbf{x}_*\|^{p-2}$  for the regularization problem (1.2). In addition  $\mathbf{H} + \lambda_* \mathbf{M}$  will be positive semi-definite; in most instances it will actually be positive definite, but in special “hard” cases singularity is a possibility.

The matrix  $\mathbf{H}$  is decomposed as

$$\mathbf{H} = \mathbf{P} \mathbf{L} \mathbf{D} \mathbf{L}^T \mathbf{P}^T$$

by calling the GALAHAD package SLS. Here  $\mathbf{P}$  is a permutation matrix,  $\mathbf{L}$  is unit lower triangular and  $\mathbf{D}$  is block diagonal, with blocks of dimension at most two. The spectral decomposition of each diagonal block of  $\mathbf{D}$  is computed, and each eigenvalue  $\theta$  is replaced by  $\max(|\theta|, \theta_{\min})$ , where  $\theta_{\min}$  is a positive user-supplied value. The resulting block diagonal matrix is  $\mathbf{B}$ , from which we define the “modified-absolute-value”

$$\mathbf{M} = \mathbf{P} \mathbf{L} \mathbf{B} \mathbf{L}^T \mathbf{P}^T;$$

an alternative due to Goldfarb uses instead the simpler

$$\mathbf{M} = \mathbf{P} \mathbf{L} \mathbf{L}^T \mathbf{P}^T.$$

Given the factors of  $\mathbf{H}$  (and  $\mathbf{M}$ ), the required solution is found by making the change of variables  $\mathbf{y} = \mathbf{B}^{\frac{1}{2}} \mathbf{L}^T \mathbf{P}^T \mathbf{x}$  (or  $\mathbf{y} = \mathbf{L}^T \mathbf{P}^T \mathbf{x}$  in the Goldfarb case) which results in “diagonal” trust-region and regularization subproblems, whose solution may be easily obtained using a Newton or higher-order iteration of a resulting “secular” equation. If subsequent problems, for which  $\mathbf{H}$  and  $\mathbf{c}$  are unchanged, are to be attempted, the existing factorization and solution may easily be exploited.

The dominant cost is that for the factorization of the symmetric, but potentially indefinite, matrix  $\mathbf{H}$  using the GALAHAD package SLS.

## Reference:

The method is described in detail for the trust-region case in

N. I. M. Gould and J. Nocedal (1998). The modified absolute-value factorization for trust-region minimization. In “High Performance Algorithms and Software in Nonlinear Optimization” (R. De Leone, A. Murli, P. M. Pardalos and G. Toraldo, eds.), Kluwer Academic Publishers, pp. 225–241,

while the adaptation for the regularization case is obvious. The method used to solve the diagonal trust-region and regularization subproblems are as given by

H. S. Dollar, N. I. M. Gould and D. P. Robinson (2010). On solving trust-region and other regularised subproblems in optimization. *Mathematical Programming Computation* **2(1)** 21–57

with simplifications due to the diagonal Hessian.

## 5 EXAMPLE OF USE

Suppose we wish to solve the trust-region subproblem (1.1) in 10 unknowns, whose data is

$$\mathbf{H} = \begin{pmatrix} -2 & 1 & & & & \\ 1 & -2 & & & & \\ & & \cdot & \cdot & & \\ & & & \cdot & -2 & 1 \\ & & & & 1 & -2 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 1 \\ 1 \\ \cdot \\ 1 \\ 1 \end{pmatrix} \quad \text{and } f = 0,$$

with a radius  $\Delta = 1$  and then to change the first component of  $\mathbf{c}$  to 2, and to resolve with radii  $\Delta = 1$  and  $\Delta = 10$ . Then we may use the following code:

```
PROGRAM GALAHAD_DPS_EXAMPLE ! GALAHAD 3.0 - 23/03/2018 AT 07:30 GMT.
USE GALAHAD_DPS_DOUBLE ! double precision version
IMPLICIT NONE
```

---

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```

INTEGER, PARAMETER :: wp = KIND( 1.0D+0 )      ! set precision
INTEGER, PARAMETER :: n = 10                    ! problem dimension
INTEGER :: i
REAL ( KIND = wp ) :: f, delta
TYPE ( SMT_type ) :: H
REAL ( KIND = wp ), DIMENSION( n ) :: C, X
TYPE ( DPS_data_type ) :: data
TYPE ( DPS_control_type ) :: control
TYPE ( DPS_inform_type ) :: inform
INTEGER :: s
H%ne = 2 * n - 1                                ! set up problem
CALL SMT_put( H%type, 'COORDINATE', s )         ! specify co-ordinate for H
ALLOCATE( H%row( H%ne ), H%col( H%ne ), H%val( H%ne ), STAT = i )
DO i = 1, n - 1
    H%row( i ) = i ; H%col( i ) = i ; H%val( i ) = - 2.0_wp
    H%row( n + i ) = i + 1; H%col( n + i ) = i ; H%val( n + i ) = 1.0_wp
END DO
H%row( n ) = n ; H%col( n ) = n ; H%val( n ) = -2.0_wp
C = 1.0_wp ; f = 0.0_wp ; delta = 1.0_wp
CALL DPS_initialize( data, control, inform ) ! initialize control parameters
CALL DPS_solve( n, H, C, f, X, data, control, inform, delta = delta )
WRITE( 6, "( / A, ES12.4, A, / ( 5ES12.4 ) )" ) &
    ' optimal f =', inform%obj, ', optimal x = ', X
C( 1 ) = 2.0_wp                                ! change the first component of C to 2
CALL DPS_resolve( n, X, data, control, inform, C = C, delta = delta )
WRITE( 6, "( / A, /, A, ES12.4, A, / ( 5ES12.4 ) )" ) &
    ' change C:', ' optimal f =', inform%obj, ', optimal x = ', X
delta = 10.0_wp                                ! increase the radius
CALL DPS_resolve( n, X, data, control, inform, delta = delta )
WRITE( 6, "( / A, /, A, ES12.4, A, / ( 5ES12.4 ) )" ) &
    ' increase delta:', ' optimal f =', inform%obj, ', optimal x = ', X
CALL DPS_terminate( data, control, inform ) ! Deallocate arrays
END PROGRAM GALAHAD_DPS_EXAMPLE

```

This produces the following output:

```

optimal f = -1.0988E+01, optimal x =
-4.7673E-01 -8.5812E-01 -1.1442E+00 -1.3348E+00 -1.4302E+00
-1.4302E+00 -1.3348E+00 -1.1442E+00 -8.5812E-01 -4.7673E-01

```

```

change C:
optimal f = -1.1496E+01, optimal x =
-5.3739E-01 -8.9290E-01 -1.1575E+00 -1.3311E+00 -1.4138E+00
-1.4055E+00 -1.3063E+00 -1.1161E+00 -8.3502E-01 -4.6298E-01

```

```

increase delta:
optimal f = -1.5996E+02, optimal x =
-5.3739E+00 -8.9290E+00 -1.1575E+01 -1.3311E+01 -1.4138E+01
-1.4055E+01 -1.3063E+01 -1.1161E+01 -8.3502E+00 -4.6298E+00

```

If, instead, we wish to solve the cubic ( $p = 3$ ) regularization subproblem (1.2) using the same problem data and initial weight  $\sigma = 1$ , and then to change the first component of  $c$  to 2, and to resolve with weights  $\sigma = 1$  and  $\sigma = 0.1$ , the following code is suitable:

```

PROGRAM GALAHAD_DPS_EXAMPLE2 ! GALAHAD 3.0 - 23/03/2018 AT 07:30 GMT.
USE GALAHAD_DPS_DOUBLE      ! double precision version

```

---

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```

IMPLICIT NONE
INTEGER, PARAMETER :: wp = KIND( 1.0D+0 )      ! set precision
INTEGER, PARAMETER :: n = 10                    ! problem dimension
INTEGER :: i
REAL ( KIND = wp ) :: f, sigma, p
TYPE ( SMT_type ) :: H
REAL ( KIND = wp ), DIMENSION( n ) :: C, X
TYPE ( DPS_data_type ) :: data
TYPE ( DPS_control_type ) :: control
TYPE ( DPS_inform_type ) :: inform
INTEGER :: s
H%ne = 2 * n - 1                                ! set up problem
CALL SMT_put( H%type, 'COORDINATE', s )         ! specify co-ordinate for H
ALLOCATE( H%row( H%ne ), H%col( H%ne ), H%val( H%ne ), STAT = i )
DO i = 1, n - 1
    H%row( i ) = i ; H%col( i ) = i ; H%val( i ) = - 2.0_wp
    H%row( n + i ) = i + 1; H%col( n + i ) = i ; H%val( n + i ) = 1.0_wp
END DO
H%row( n ) = n ; H%col( n ) = n ; H%val( n ) = -2.0_wp
C = 1.0_wp ; f = 0.0_wp ; sigma = 1.0_wp ; p = 3.0_wp
CALL DPS_initialize( data, control, inform ) ! initialize control parameters
CALL DPS_solve( n, H, C, f, X, data, control, inform, sigma = sigma, p = p )
WRITE( 6, "( / A, ES12.4, A, / ( 5ES12.4 ) )" ) &
    ' optimal f =', inform%obj, ', optimal x = ', X
C( 1 ) = 2.0_wp                                ! change the first component of C to 2
CALL DPS_resolve( n, X, data, control, inform, C = C, sigma = sigma )
WRITE( 6, "( / A, /, A, ES12.4, A, / ( 5ES12.4 ) )" ) &
    ' change C:', ' optimal f =', inform%obj, ', optimal x = ', X
sigma = 0.1_wp                                  ! decrease the weight
CALL DPS_resolve( n, X, data, control, inform, sigma = sigma )
WRITE( 6, "( / A, /, A, ES12.4, A, / ( 5ES12.4 ) )" ) &
    ' decrease sigma:', ' optimal f =', inform%obj, ', optimal x = ', X
CALL DPS_terminate( data, control, inform ) ! Deallocate arrays
END PROGRAM GALAHAD_DPS_EXAMPLE2

```

This produces the following output:

```

optimal f = -4.6745E+01, optimal x =
-1.8006E+00 -3.2410E+00 -4.3214E+00 -5.0416E+00 -5.4017E+00
-5.4017E+00 -5.0416E+00 -4.3214E+00 -3.2410E+00 -1.8006E+00

```

```

change C:
optimal f = -4.9797E+01, optimal x =
-2.0708E+00 -3.4408E+00 -4.4603E+00 -5.1293E+00 -5.4479E+00
-5.4160E+00 -5.0337E+00 -4.3010E+00 -3.2178E+00 -1.7841E+00

```

```

decrease sigma:
optimal f = -3.2079E+02, optimal x =
-8.9299E+00 -1.4837E+01 -1.9234E+01 -2.2119E+01 -2.3493E+01
-2.3355E+01 -2.1707E+01 -1.8547E+01 -1.3876E+01 -7.6935E+00

```