



# GALAHAD

# CHECK

USER DOCUMENTATION

GALAHAD Optimization Library version 3.3

## 1 SUMMARY

This package uses finite difference approximations to **check the gradient of an objective function  $f(\mathbf{x})$ , the Jacobian matrix of a constraint function  $\mathbf{c}(\mathbf{x})$ , and the second derivative Hessian matrix of the Lagrangian function  $L(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}) - \mathbf{c}(\mathbf{x})^T \mathbf{y}$ .** These quantities are typically associated with a nonlinear optimization problem

$$\text{minimize } f(\mathbf{x})$$

subject to the general linear constraints

$$\mathbf{a}_i^l \leq \mathbf{a}_i^T \mathbf{x} \leq \mathbf{a}_i^u, \quad i = 1, \dots, m_a,$$

general nonlinear constraints

$$c_i^l \leq c_i(\mathbf{x}) \leq c_i^u, \quad i = 1, \dots, m,$$

and simple bound constraints

$$x_j^l \leq x_j \leq x_j^u, \quad j = 1, \dots, n,$$

where the vectors  $\mathbf{a}_i$ ,  $\mathbf{a}^l$ ,  $\mathbf{a}^u$ ,  $\mathbf{c}^l$ ,  $\mathbf{c}^u$ ,  $\mathbf{x}^l$ , and  $\mathbf{x}^u$  are given, and the vectors  $\mathbf{x} \in \mathbb{R}^n$  and  $\mathbf{y} \in \mathbb{R}^m$  are known as the primal and dual (Lagrange multiplier) vectors, respectively. The user may choose to perform a “cheap” verification of the requested derivatives, or a more detailed and “expensive” check. Function values can be supplied via internal subroutine evaluation or reverse communication.

**ATTRIBUTES — Versions:** GALAHAD\_CHECK\_single and GALAHAD\_CHECK\_double. **Uses:** GALAHAD\_SYMBOLS, GALAHAD\_SPECFILE, GALAHAD\_SPACE, GALAHAD\_MOP, GALAHAD\_SMT, and GALAHAD\_NLPT. **Date:** September 2010. **Origin:** D. P. Robinson, University of Oxford, UK, and N. I. M. Gould, Rutherford Appleton Laboratory. **Language:** Fortran 95 + TR 15581 or Fortran 2003.

## 2 HOW TO USE THE PACKAGE

Access to the package requires a USE statement such as

*Single precision version*

```
USE GALAHAD_CHECK_single
```

*Double precision version*

```
USE GALAHAD_CHECK_double
```

If it is required to use both modules at the same time, the derived types SMT\_type, NLPT\_problem\_type, CHECK\_control\_type, CHECK\_inform\_type, CHECK\_data\_type, CHECK\_reverse\_communication\_type and NLPT\_userdata\_type (Section 2.2), and the subroutines CHECK\_initialize, CHECK\_verify, CHECK\_terminate (Section 2.3), and CHECK\_read\_specfile (Section 2.7) must be renamed on one of the USE statements.

### 2.1 Matrix storage formats

The Jacobian matrix  $\mathbf{J} = \nabla_{\mathbf{x}} \mathbf{c}(\mathbf{x})$  and the Hessian matrix  $\mathbf{H} = \nabla_{\mathbf{xx}} L(\mathbf{x}, \mathbf{y})$  may be stored in a variety of input formats.

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### 2.1.1 Dense storage format

The matrix **J** is stored as a compact dense matrix by rows, that is, the values of the entries of each row in turn are stored in order within an appropriate real one-dimensional array. Component  $n * (i - 1) + j$  of the storage array `J%val` will hold the value  $j_{ij}$  for  $i = 1, \dots, m$  and  $j = 1, \dots, n$ . Since **H** is symmetric, only the lower triangular part (that is the part  $h_{ij}$  for  $1 \leq j \leq i \leq n$ ) should be stored. In this case the lower triangle will be stored by rows, that is component  $i * (i - 1) / 2 + j$  of the storage array `H%val` will hold the value  $h_{ij}$  (and, by symmetry,  $h_{ji}$ ) for  $1 \leq j \leq i \leq n$ .

### 2.1.2 Sparse co-ordinate storage format

Only the nonzero entries of the matrices are stored. For the  $l$ -th entry of **J**, its row index  $i$ , column index  $j$  and value  $j_{ij}$  are stored in the  $l$ -th components of the integer arrays `J%row`, `J%col` and real array `J%val`. The order is unimportant, but the total number of entries `J%ne` is also required. Since **H** is symmetric, the same scheme is applicable, except that only the entries in the lower triangle should be stored.

### 2.1.3 Sparse row-wise storage format

Again only the nonzero entries are stored, but this time they are ordered so that those in row  $i$  appear directly before those in row  $i + 1$ . For the  $i$ -th row of **J**, the  $i$ -th component of an integer array `J%ptr` holds the position of the first entry in this row, while `J%ptr(m + 1)` holds the total number of entries plus one. The column indices  $j$  and values  $j_{ij}$  of the entries in the  $i$ -th row are stored in components  $l = \text{J\%ptr}(i), \dots, \text{J\%ptr}(i + 1) - 1$  of the integer array `J%col`, and real array `J%val`, respectively. Since **H** is symmetric, the same scheme is applicable, except that only the entries in the lower triangle should be stored.

For sparse matrices, this scheme almost always requires less storage than its predecessor.

### 2.1.4 Sparse column-wise storage format

Again only the nonzero entries are stored, but this time they are ordered so that those in column  $j$  appear directly before those in column  $j + 1$ . For the  $j$ -th column of **J**, the  $j$ -th component of the integer array `J%ptr` holds the position of the first entry in this column, while `J%ptr(n + 1)` holds the total number of entries plus one. The row indices  $i$  and values  $j_{ij}$  of the entries in the  $j$ -th column are stored in components  $l = \text{J\%ptr}(j), \dots, \text{J\%ptr}(j + 1) - 1$  of the integer array `J%row`, and real array `J%val`, respectively. Since **H** is symmetric, the same scheme is applicable, except that only the entries in the lower triangle should be stored.

### 2.1.5 Diagonal storage format

If **J** is diagonal (i.e.,  $j_{ij} = 0$  for all  $1 \leq i \neq j \leq n$ ) only the diagonal entries  $j_{ii}$  for  $1 \leq i \leq n$  should be stored, and the first  $n$  components of the array `J%val` should be used for this purpose. The same holds for **H**.

## 2.2 The derived data types

Seven derived data types are accessible from the package.

### 2.2.1 The derived data type for holding matrices

The derived data type `SMT_TYPE` is used to hold the Jacobian matrix **J** and the Hessian matrix **H**. The components of `SMT_TYPE` used here are:

- `m` is a scalar component of type default `INTEGER` that holds the number of rows of the matrix.
- `n` is a scalar component of type default `INTEGER` that holds the number of columns of the matrix.

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- `ne` is a scalar variable of type default `INTEGER` that holds the number of matrix entries.
- `type` is a rank-one allocatable array of type default `CHARACTER` that is used to indicate the matrix storage scheme used. Its precise length and content depends on the type of matrix to be stored (see §2.2.2).
- `val` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) and dimension at least `ne`, that holds the values of the entries. Each pair of off-diagonal entries  $h_{ij} = h_{ji}$  for the *symmetric* matrix  $\mathbf{H}$  is represented as a single entry (see §2.1.1–2.1.3). Any duplicated entries that appear in the sparse co-ordinate, row-wise, or column-wise schemes will be summed.
- `row` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `ne`, that may hold the row indices of the entries (see §2.1.2).
- `col` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `ne`, that may hold the column indices of the entries (see §2.1.2–2.1.3).
- `ptr` is a rank-one allocatable array of type default `INTEGER`. If sparse row-wise storage is used (see §2.1.3), then it must have dimension at least `m + 1` and hold the pointers to the first entry in each row. If sparse column-wise storage is used (see §2.1.4), then it must have dimension at least `n + 1` and hold the pointers to the first entry in each column.

### 2.2.2 The derived data type for holding the problem

The derived data type `NLPT_problem_type` holds the problem. The relevant components of `NLPT_problem_type` are:

- `m` is a scalar variable of type default `INTEGER` that holds the number of nonlinear constraints  $m$ .
- `n` is a scalar variable of type default `INTEGER` that holds the number of optimization variables  $n$ .
- `H` is scalar variable of type `SMT_TYPE` that holds the Hessian matrix  $\mathbf{H} = \nabla_{xx}f(\mathbf{x})$ . The following components are used here:

`H%type` is an allocatable array of rank one and type default `CHARACTER`, that is used to indicate the storage scheme used. If the dense storage scheme (see Section 2.1.1) is used, the first five components of `H%type` must contain the string `DENSE`. For the sparse co-ordinate scheme (see Section 2.1.2), the first ten components of `H%type` must contain the string `COORDINATE`, for the sparse row-wise storage scheme (see Section 2.1.3), the first fourteen components of `H%type` must contain the string `SPARSE_BY_ROWS`, for the sparse column-wise storage scheme (see Section 2.1.4), the first seventeen components of `H%type` must contain the string `SPARSE_BY_COLUMNS`, and for the diagonal storage scheme (see Section 2.1.5), the first eight components of `H%type` must contain the string `DIAGONAL`.

For convenience, the procedure `SMT_put` may be used to allocate sufficient space and insert the required keyword into `H%type`. For example, if `nlp` is of derived type `CHECK_problem_type` and involves a Hessian we wish to store using the co-ordinate scheme, we may simply

```
CALL SMT_put( nlp%H%type, 'COORDINATE' )
```

See the documentation for the GALAHAD package `SMT` for further details on the use of `SMT_put`.

`H%ne` is a scalar variable of type default `INTEGER`, that holds the number of entries in the **lower triangular** part of  $\mathbf{H}$  in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be set for any of the other four schemes.

`H%val` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`), that holds the values of the entries of the **lower triangular** part of the Hessian matrix  $\mathbf{H}$  in any of the storage schemes discussed in Section 2.1.

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- `H%row` is a rank-one allocatable array of type default `INTEGER`, that holds the row indices of the **lower triangular** part of  $\mathbf{H}$  in the sparse co-ordinate storage scheme (see Section 2.1.2) and the sparse column-wise scheme (see Section 2.1.4). It need not be allocated for any of the other three schemes.
- `H%col` is a rank-one allocatable array variable of type default `INTEGER`, that holds the column indices of the **lower triangular** part of  $\mathbf{H}$  in either the sparse co-ordinate (see Section 2.1.2), or the sparse row-wise (see Section 2.1.3) storage scheme. It need not be allocated for any of the other three schemes.
- `H%ptr` is a rank-one allocatable array of dimension  $n+1$  and type default `INTEGER` that holds the starting position of each row of the **lower triangular** part of  $\mathbf{H}$ , as well as the total number of entries plus one, in the sparse row-wise storage scheme (see Section 2.1.3), or the starting position of each column of the **lower triangular** part of  $\mathbf{H}$ , as well as the total number of entries plus one, in the sparse column-wise storage scheme (see Section 2.1.4). It need not be allocated when the other schemes are used.
- `J` is scalar variable of type `SMT_TYPE` that holds the Jacobian matrix  $\mathbf{J} = \nabla_x \mathbf{c}(\mathbf{x})$ . The following components are used here:
- `J%type` is an allocatable array of rank one and type default `CHARACTER`, that is used to indicate the storage scheme used. If the dense storage scheme (see Section 2.1.1) is used, the first five components of `J%type` must contain the string `DENSE`. For the sparse co-ordinate scheme (see Section 2.1.2), the first ten components of `J%type` must contain the string `COORDINATE`, for the sparse row-wise storage scheme (see Section 2.1.3), the first fourteen components of `J%type` must contain the string `SPARSE_BY_ROWS`, for the sparse column-wise storage scheme (see Section 2.1.4), the first seventeen components of `J%type` must contain the string `SPARSE_BY_COLUMNS`, and for the diagonal storage scheme (see Section 2.1.5), the first eight components of `J%type` must contain the string `DIAGONAL`.
- For convenience, the procedure `SMT_put` may be used to allocate sufficient space and insert the required keyword into `J%type`. For example, if `nlp` is of derived type `CHECK_problem_type` and involves a Jacobian we wish to store using the co-ordinate scheme, we may simply
- ```
CALL SMT_put( nlp%J%type, 'COORDINATE' )
```
- See the documentation for the GALAHAD package `SMT` for further details on the use of `SMT_put`.
- `J%ne` is a scalar variable of type default `INTEGER` that holds the number of entries in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be set for any of the other schemes.
- `J%val` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the values of the entries of the Jacobian matrix  $\mathbf{J}$  in any of the storage schemes discussed in Section 2.1.
- `J%row` is a rank-one allocatable array of type default `INTEGER` that holds the row indices of  $\mathbf{J}$  in the sparse co-ordinate storage scheme discussed in Section 2.1.2 and the sparse column-wise storage scheme discussed in Section 2.1.4. It need not be allocated for any of the other three schemes.
- `J%col` is a rank-one allocatable array variable of type default `INTEGER` that holds the column indices of  $\mathbf{J}$  in either the sparse co-ordinate scheme discussed in Section 2.1.2 or the sparse row-wise scheme discussed in Section 2.1.3. It need not be allocated for any of the other three schemes.
- `J%ptr` for the sparse row-wise storage scheme discussed in Section 2.1.3, it is a rank-one allocatable array of dimension  $m+1$  and type default `INTEGER` that holds the starting position of each row of  $\mathbf{J}$  as well as the total number of entries plus one. For the sparse column-wise storage scheme discussed in Section 2.1.4, it is a rank-one allocatable array of dimension  $n+1$  and type default `INTEGER` that holds the starting position of each column of  $\mathbf{J}$  as well as the total number of entries plus one. It need not be allocated for any of the other schemes.
- `G` is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_CHECK_double`), that holds the gradient  $\mathbf{g}$  of the objective function. The  $j$ -th component of  $\mathbf{G}$ ,  $j = 1, \dots, n$ , contains  $\mathbf{g}_j$ .

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- C** is a rank-one allocatable array of dimension  $m$  and type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the value of the constraint function. The  $j$ -th component of **C**,  $j = 1, \dots, m$ , contains  $c_j$ .
- f** is a scalar variable of type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the value of the objective function.
- X** is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_CHECK_double`), that holds the values  $\mathbf{x}$  of the optimization variables. The  $j$ -th component of **X**,  $j = 1, \dots, n$ , contains  $x_j$ .
- X\_l** is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the lower bounds on the optimization variables  $\mathbf{x}$ .
- X\_u** is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds upper bounds on the optimization variables  $\mathbf{x}$ .
- Y** is a rank-one allocatable array of dimension  $m$  and type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the value  $\mathbf{y}$  of the Lagrange multiplier estimate. The  $j$ -th component of **Y**,  $j = 1, \dots, m$ , contains  $y_j$ .

### 2.2.3 The derived data type for holding control parameters

The derived data type `CHECK_control_type` is used to hold controlling data. Default values may be obtained by calling `CHECK_initialize` (see Section 2.3.1), while components may also be changed by calling `GALAHAD_CHECK_read_spec` (see Section 2.7.1). The components of `CHECK_control_type` are:

- error** is a scalar variable of type default `INTEGER`, that holds the stream number for error messages. Printing of error messages in `CHECK_verify` and `CHECK_terminate` is suppressed if  $\text{error} \leq 0$ . The default is  $\text{error} = 6$ .
- out** is a scalar variable of type default `INTEGER`, that holds the stream number for informational messages. Printing of informational messages in `CHECK_verify` is suppressed if  $\text{out} \leq 0$ . The default is  $\text{out} = 6$ .
- print\_level** is a scalar variable of type default `INTEGER`, that is used to control the amount of informational output which is required. No informational output will occur if  $\text{print\_level} \leq 0$ . If  $\text{print\_level} = 1$ , a brief summary of the derivative verification is provided. If  $\text{print\_level} \geq 2$ , this output will be increased to provide significant detail of each iteration (see Section 2.8 for more details). The default is  $\text{print\_level} = 0$ .
- verify\_level** is a scalar variable of type default `INTEGER` that determines the detail of verification performed. A “cheap” check will be performed if  $\text{verify\_level} = 1$ . If  $\text{verify\_level} \geq 2$ , an “expensive”—but more detailed—verification of the derivatives is done. No checking is performed if  $\text{verify\_level} \leq 0$ . The default is  $\text{verify\_level} = 2$ .
- f\_availability** is a scalar variable of type default `INTEGER` that controls how the user is expected to supply objective function values, when required. The user should set  $\text{f\_availability} = 1$  if an appropriate evaluation routine is supplied (see Section 2.4.1), and  $\text{f\_availability} = 2$  if reverse communication will be used (see Section 2.5).
- c\_availability** is a scalar variable of type default `INTEGER` that controls how the user is expected to supply constraint function values, when required. The user should set  $\text{c\_availability} = 1$  if an appropriate evaluation routine is supplied (see Section 2.4.2), and  $\text{c\_availability} = 2$  if reverse communication will be used (see Section 2.5). Any other value will result in an error message.
- g\_availability** is a scalar variable of type default `INTEGER` that controls how the user is expected to supply the gradient of the objective function, when required. The user should set  $\text{g\_availability} = 1$  if an appropriate evaluation routine is supplied (see Section 2.4.3), and  $\text{g\_availability} = 2$  if reverse communication will be used (see Section 2.5). Any other value will result in an error message.

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`J_availability` is a scalar variable of type default `INTEGER` that controls how the user is expected to supply the Jacobian of the constraint function, when required. The user should set `J_availability = 1` if an appropriate evaluation routine is supplied (see Section 2.4.4), `J_availability = 2` if reverse communication will be used to obtain Jacobian values (see Section 2.5), `J_availability = 3` if an appropriate Jacobian-vector product routine is supplied (see Section 2.4.6), and `J_availability = 4` if reverse communication will be used to get Jacobian-vector products (see Section 2.5). Any other value will result in an error message.

`H_availability` is a scalar variable of type default `INTEGER` that controls how the user is expected to supply the Hessian of the Lagrangian function, when required. The user should set `H_availability = 1` if an appropriate evaluation routine is supplied (see Section 2.4.5), `H_availability = 2` if reverse communication will be used to obtain Hessian values (see Section 2.5), `H_availability = 3` if an appropriate Hessian-vector product routine is supplied (see Section 2.4.7), and `H_availability = 4` if reverse communication will be used to get Hessian-vector products (see Section 2.5). Any other value will result in an error message.

`checkG` is a scalar variable of type default `LOGICAL` that should be set `.TRUE.` if the gradient of the objective function should be checked. Otherwise, it should be set `.FALSE.`. The default is `checkG=.TRUE.`.

`checkJ` is a scalar variable of type default `LOGICAL` that should be set `.TRUE.` if the Jacobian of the constraint function should be checked. Otherwise, it should be set `.FALSE.`. The default is `checkJ=.TRUE.`.

`checkH` is a scalar variable of type default `LOGICAL` that should be set `.TRUE.` if the Hessian of the Lagrangian function should be checked. Otherwise, it should be set `.FALSE.`. The default is `checkH=.TRUE.`.

`deallocate_error_fatal` is a scalar variable of type default `LOGICAL`, that must be set `.TRUE.` if the user wishes to terminate execution if a deallocation fails, and `.FALSE.` if an attempt to continue will be made. The default is `deallocate_error_fatal=.FALSE.`

## 2.2.4 The derived data type for holding informational parameters

The derived data type `CHECK_inform_type` is used to hold parameters that give information about the progress and needs of the algorithm. The components of `CHECK_inform_type` are:

`status` is a scalar variable of type default `INTEGER` that gives the exit status of the algorithm. See Sections 2.5 and 2.6 for further details.

`alloc_status` is a scalar variable of type default `INTEGER` that gives the status of the last attempted array allocation or deallocation. This will be 0 if `status = 0`.

`bad_alloc` is a scalar variable of type default `CHARACTER` and length 80 that gives the name of the last internal array for which there was an allocation or deallocation error. This will be the null string if `status = 0`.

`numG_wrong` is a scalar variable of type default `INTEGER` that gives the number of components of the gradient of the objective function that appear to be wrong.

`numJ_wrong` is a scalar variable of type default `INTEGER` that gives the number of entries of the Jacobian of the constraint function that appear to be wrong.

`numH_wrong` is a scalar variable of type default `INTEGER` that gives the number of entries of the Hessian of the Lagrangian function that appear to be wrong.

`derivative_ok` is a scalar variable of type default `LOGICAL` that is `.TRUE.` if all derivatives appear to be correct, and set `.FALSE.` otherwise.

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### 2.2.5 The derived data type for holding problem data

The derived data type `CHECK_data_type` is used to hold all the data for a particular problem, or sequences of problems with the same structure, between calls of `CHECK` procedures. The only data that should be changed by the user from the initial call to `CHECK_initialize` to the final call to `CHECK_terminate` is the component `RC` of type `CHECK_reverse_communication_type` (see Section 2.2.6), and this should be done only as directed from `GALAHAD_CHECK_verify` with positive values of `inform%status` as described in Section 2.5.

### 2.2.6 The derived data type for holding reverse communication data

The derived data type `CHECK_reverse_communication_type` is used to hold data for reverse communication, when needed. The components of `CHECK_reverse_communication_type` are:

- `X` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the values of the optimization variables at which the user must perform function computation.
- `Y` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the values of the Lagrange multipliers that the user must use when evaluating the Hessian of the Lagrangian.
- `F` is a scalar variable of type default `REAL` (double precision in `GALAHAD_CHECK_double`) in which the user places the value of the objective function evaluated at `X`, when required (see Section 2.5).
- `C` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) in which the user places the value of the constraint function evaluated at `X`, when required (see Section 2.5).
- `G` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) in which the user places the gradient of the the objective function evaluated at `X`, when required (see Section 2.5).
- `V` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) that holds the vector for which a matrix-vector product is required (see Section 2.5).
- `U` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) in which the user places the result of any required matrix-vector product with the vector `V` from above (see Section 2.5).
- `Jval` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) in which the user places the entries of the Jacobian matrix evaluated at `X`, when required (see Section 2.5).
- `Hval` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`) in which the user places the **lower triangular** entries of the Hessian matrix of the Lagrangian evaluated at `X` and `Y`, when required (see Section 2.5).

### 2.2.7 The derived data type for holding user data

The derived data type `NLPT_userdata_type` is available to allow the user to pass data to and from user-supplied sub-routines for function and derivative calculations (see Section 2.4). Components of variables of type `NLPT_userdata_type` may be allocated as necessary. The following components are available:

- `integer` is a rank-one allocatable array of type default `INTEGER`.
- `real` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_CHECK_double`)
- `complex` is a rank-one allocatable array of type default `COMPLEX` (double precision complex in `GALAHAD_CHECK_double`).
- `character` is a rank-one allocatable array of type default `CHARACTER`.

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`logical` is a rank-one allocatable array of type default LOGICAL.

`integer_pointer` is a rank-one pointer array of type default INTEGER.

`real_pointer` is a rank-one pointer array of type default REAL (double precision in GALAHAD\_CHECK\_double)

`complex_pointer` is a rank-one pointer array of type default COMPLEX (double precision complex in GALAHAD\_CHECK\_double).

`character_pointer` is a rank-one pointer array of type default CHARACTER.

`logical_pointer` is a rank-one pointer array of type default LOGICAL.

### 2.3 Argument lists and calling sequences

There are three procedures for user calls (see Section 2.7 for further features):

1. The subroutine `CHECK_initialize` is used to set default values, and initialize private data, before verifying the derivatives of one or more problems with the same sparsity and bound structure.
2. The subroutine `CHECK_verify` is called to check the derivatives of the given problem.
3. The subroutine `CHECK_terminate` is provided to allow the user to automatically deallocate array components of the private data, allocated by `CHECK_verify`, at the end of the verification process. It is important to do this if the data object is re-used for another problem **with a different structure** since `CHECK_initialize` cannot test for this situation, and any existing associated targets will subsequently become unreachable.

We use square brackets [ ] to indicate OPTIONAL arguments.

#### 2.3.1 The initialization subroutine

Default values are provided as follows:

```
CALL CHECK_initialize( control )
```

`control` is a scalar INTENT(OUT) argument of type `CHECK_control_type` (see Section 2.2.3). On exit, `control` contains default values for the components as described in Section 2.2.3. These values should only be changed after calling `CHECK_initialize`.

#### 2.3.2 The verification subroutine

The verification algorithm is called as follows:

```
CALL CHECK_verify( nlp, data, control, inform, userdata[, eval_F, eval_C,      &
                  eval_G, eval_J, eval_H, eval_Jv, eval_Hv] )
```

`nlp` is a scalar INTENT(INOUT) argument of type `NLPT_problem_type` (see Section 2.2.2). It is used to hold data about the problem whose derivatives are being verified. For a new problem, the user must allocate all the array components, and set values for `nlp%m`, `nlp%n`, and the required integer components of `nlp%J` and `nlp%H` that is determined by the values of `checkJ` and `checkH` as described in Section 2.2.3. Users are free to choose whichever of the matrix formats described in Section 2.1 is appropriate for **J** and **H** for their application.

The components `nlp%X` and `nlp%Y` must be set to initial values **x** and **y** of the primal and dual variables for the optimization problem. Prior to verification of the derivatives, the point `nlp%X` is modified internally to ensure feasibility with respect to the bound constraints  $\mathbf{x}^l$  and  $\mathbf{x}^u$ ; no modification of **y** is performed. The requested derivatives are then checked at the point `nlp%X` and `nlp%Y`.

**Restrictions:** `nlp%n` > 0 and `nlp%m` ≥ 0.

---

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`data` is a scalar `INTENT(INOUT)` argument of type `CHECK_data_type` (see Section 2.2.5). It is used to hold data about the problem derivatives being verified. With the possible exception of the component `RC` (see Sections 2.2.6 and 2.5), it must not have been altered **by the user** since the last call to `CHECK_initialize`.

`control` is a scalar `INTENT(IN)` argument of type `CHECK_control_type` (see Section 2.2.3). Default values may be assigned by calling `CHECK_initialize` prior to the first call to `CHECK_solve`.

`inform` is a scalar `INTENT(INOUT)` argument of type `CHECK_inform_type` (see Section 2.2.4). **On initial entry, the component status must be set to the value 1.** Other entries need not be set. A successful call to `CHECK_verify` is indicated when the component status has the value 0. For other return values of status, see Sections 2.5 and 2.6.

`userdata` is a scalar `INTENT(INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the OPTIONAL subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).

`eval_F` is an OPTIONAL user-supplied subroutine whose purpose is to evaluate the objective function  $f(\mathbf{x})$  at a given vector  $\mathbf{x}$ . See Section 2.4.1 for details. If `f_availability = 1` (see Section 2.2.3), then `eval_F` must be present and declared `EXTERNAL` in the calling program. If `f_availability = 2`, then `GALAHAD_CHECK_verify` will use reverse communication to obtain objective function values (see Section 2.5).

`eval_C` is an OPTIONAL user-supplied subroutine whose purpose is to evaluate the constraint function  $\mathbf{c}(\mathbf{x})$  at a given vector  $\mathbf{x}$ . See Section 2.4.2 for details. If `c_availability = 1` (see Section 2.2.3), then `eval_C` must be present and declared `EXTERNAL` in the calling program. If `c_availability = 2`, then `GALAHAD_CHECK_verify` will use reverse communication to obtain constraint function values (see Section 2.5).

`eval_G` is an OPTIONAL user-supplied subroutine whose purpose is to evaluate the gradient of the objective function  $\nabla_{\mathbf{x}}f(\mathbf{x})$  at a given vector  $\mathbf{x}$ . See Section 2.4.3 for details. If `g_availability = 1` (see Section 2.2.3), then `eval_G` must be present and declared `EXTERNAL` in the calling program. If `g_availability = 2`, then `GALAHAD_CHECK_verify` will use reverse communication to obtain gradient values (see Section 2.5).

`eval_J` is an OPTIONAL user-supplied subroutine whose purpose is to evaluate the Jacobian of the objective function  $\nabla_{\mathbf{xx}}f(\mathbf{x})$  at a given vector  $\mathbf{x}$ . See Section 2.4.5 for details. If `J_availability = 1` (see Section 2.2.3), then `eval_J` must be present and declared `EXTERNAL` in the calling program. Otherwise, `eval_J` need not be supplied.

`eval_H` is an OPTIONAL user-supplied subroutine whose purpose is to evaluate the Hessian of the Lagrangian  $\nabla_{\mathbf{xx}}L(\mathbf{x}, \mathbf{y})$  at a given point  $(\mathbf{x}, \mathbf{y})$ . See Section 2.4.5 for details. If `H_availability = 1` (see Section 2.2.3), then `eval_H` must be present and declared `EXTERNAL` in the calling program. Otherwise, `eval_H` need not be supplied.

`eval_Jv` is an OPTIONAL user-supplied subroutine whose purpose is to evaluate the value of the product of the Jacobian of the constraint function  $\nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x})$  with a given vector  $\mathbf{v}$ . See Section 2.4.7 for details. If `J_availability = 3` (see Section 2.2.3), then `eval_Jv` must be present and declared `EXTERNAL` in the calling program. Otherwise, `eval_Jv` need not be supplied.

`eval_Hv` is an OPTIONAL user-supplied subroutine whose purpose is to evaluate the value of the product of the Hessian of the Lagrangian function  $\nabla_{\mathbf{xx}}L(\mathbf{x}, \mathbf{y})$  with a given vector  $\mathbf{v}$ . See Section 2.4.7 for details. If `H_availability = 3` (see Section 2.2.3), then `eval_Hv` must be present and declared `EXTERNAL` in the calling program. Otherwise, `eval_Hv` need not be supplied.

### 2.3.3 The termination subroutine

All previously allocated arrays are deallocated as follows:

```
CALL CHECK_terminate( data, control, inform )
```

---

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`data` is a scalar `INTENT(INOUT)` argument of type `CHECK_data_type` exactly as for `CHECK_verify`, which must not have been altered **by the user** since the last call to `CHECK_initialize` (except possibly `data%RC` as described in Section 2.5). On exit, array components will have been deallocated.

`control` is a scalar `INTENT(IN)` argument of type `CHECK_control_type` exactly as for `CHECK_verify`.

`inform` is a scalar `INTENT(OUT)` argument of type `CHECK_inform_type` exactly as for `CHECK_verify`. Only the component `status` will be set on exit, and a successful call to `CHECK_terminate` is indicated when this component `status` has the value 0. For other return values of `status` see Section 2.6.

## 2.4 Function and derivative values

### 2.4.1 The objective function value via internal evaluation

If the control parameter `f_availability = 1` (see Section 2.2.3), then the argument `eval_F` must be present when calling `GALAHAD_CHECK_verify` and the user must provide a subroutine of that name to evaluate the value of the objective function  $f(\mathbf{x})$ . The routine must be specified as

```
SUBROUTINE eval_F( status, X, userdata, F )
```

whose arguments are as follows:

`status` is a scalar `INTENT(OUT)` argument of type default `INTEGER`, that should be set to 0 if the routine has been able to evaluate the objective function and to a non-zero value if the evaluation has not been possible.

`X` is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{x}$ .

`userdata` is a scalar `INTENT(INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).

`F` is a scalar `INTENT(OUT)` argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) that should be set to the value of the objective function  $f(\mathbf{x})$  evaluated at the vector  $\mathbf{x}$  input in `X`.

### 2.4.2 The constraint function value via internal evaluation

If the control parameter `c_availability = 1` (see Section 2.2.3), then the argument `eval_C` must be present when calling `GALAHAD_CHECK_verify` and the user must provide a subroutine of that name to evaluate the value of the constraint function  $\mathbf{c}(\mathbf{x})$ . The routine must be specified as

```
SUBROUTINE eval_C( status, X, userdata, C )
```

whose arguments are as follows:

`status` is a scalar `INTENT(OUT)` argument of type default `INTEGER`, that should be set to 0 if the routine has been able to evaluate the constraint function and to a non-zero value if the evaluation has not been possible.

`X` is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{x}$ .

`userdata` is a scalar `INTENT(INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).

`C` is a rank-one `INTENT(OUT)` argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) that should be set to the value of the constraint function  $\mathbf{c}(\mathbf{x})$  evaluated at the vector  $\mathbf{x}$  input in `X`.

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### 2.4.3 Gradient values via internal evaluation

If the control parameter `g_availability = 1` (see Section 2.2.3), then the argument `eval_G` must be present when calling `GALAHAD_CHECK_verify` and the user must provide a subroutine of that name to evaluate the value of the gradient the objective function  $\nabla_x f(\mathbf{x})$ . The routine must be specified as

```
SUBROUTINE eval_G( status, X, userdata, G )
```

whose arguments are as follows:

`status` is a scalar `INTENT (OUT)` argument of type default `INTEGER` that should be set to 0 if the routine has been able to evaluate the gradient of the objective function and to a non-zero value if the evaluation has not been possible.

`X` is a rank-one `INTENT (IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{x}$ .

`userdata` is a scalar `INTENT (INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).

`G` is a rank-one `INTENT (OUT)` argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`), whose components should be set to the values of the gradient of the objective function  $\nabla_x f(\mathbf{x})$  evaluated at the vector  $\mathbf{x}$  input in `X`.

### 2.4.4 Jacobian values via internal evaluation

If the control parameter `J_availability = 1` (see Section 2.2.3), then the argument `eval_J` must be present when calling `GALAHAD_CHECK_verify` and the user must provide a subroutine of that name to evaluate the values of the Jacobian of the constraint function  $\nabla_x \mathbf{c}(\mathbf{x})$ . The routine must be specified as

```
SUBROUTINE eval_J( status, X, userdata, Jval )
```

whose arguments are as follows:

`status` is a scalar `INTENT (OUT)` argument of type default `INTEGER` that should be set to 0 if the routine has been able to evaluate the Jacobian of the constraint function and to a non-zero value if the evaluation has not been possible.

`X` is a rank-one `INTENT (IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{x}$ .

`userdata` is a scalar `INTENT (INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).

`Jval` is a scalar `INTENT (OUT)` argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`), whose components should be set to the values of the Jacobian of the constraint function  $\nabla_x \mathbf{c}(\mathbf{x})$  evaluated at the vector  $\mathbf{x}$  input in `X`. The values should be input in the same order as that in which the array indices were given in `nlp%J`.

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### 2.4.5 Hessian values via internal evaluation

If the control parameter `H_availability = 1` (see Section 2.2.3), then the argument `eval_H` must be present when calling `GALAHAD_CHECK_verify` and the user must provide a subroutine of that name to evaluate the values of the Hessian of the Lagrangian  $\nabla_{xx}L(\mathbf{x}, \mathbf{y})$ . The routine must be specified as

```
SUBROUTINE eval_H( status, X, Y, userdata, Hval )
```

whose arguments are as follows:

`status` is a scalar `INTENT(OUT)` argument of type default `INTEGER`, that should be set to 0 if the routine has been able to evaluate the Hessian of the Lagrangian and to a non-zero value if the evaluation has not been possible.

`X` is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{x}$ .

`Y` is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the dual vector  $\mathbf{y}$ .

`userdata` is a scalar `INTENT(INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).

`Hval` is a scalar `INTENT(OUT)` argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`), whose components should be set to the values of the Hessian of the Lagrangian  $\nabla_{xx}L(\mathbf{x}, \mathbf{y})$  evaluated at the vector  $(\mathbf{x}, \mathbf{y})$  input in `X` and `Y`. The values should be input in the same order as that in which the array indices were given in `nlp%H`.

### 2.4.6 Jacobian-vector products via internal evaluation

If the control parameter `J_availability = 3` (see Section 2.2.3), then the argument `eval_Jv` must be present when calling `GALAHAD_CHECK_verify` and the user must provide a subroutine of that name to compute products of the Jacobian of the constraint function (and its transpose) of the form  $\mathbf{u} + \nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x})\mathbf{v}$  and  $\mathbf{u} + \nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x})^T\mathbf{v}$ . The routine must be specified as

```
SUBROUTINE eval_Jv( status, X, userdata, transpose, U, V )
```

whose arguments are as follows:

`status` is a scalar `INTENT(OUT)` argument of type default `INTEGER` that should be set to 0 if the routine has been able to perform the required calculation (see `transpose` below) and to a non-zero value if the computation has not been possible.

`X` is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{x}$ .

`userdata` is a scalar `INTENT(INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).

`transpose` is a scalar `INTENT(IN)` argument of type default `LOGICAL`. If `transpose = .FALSE.`, then the user should evaluate the sum  $\mathbf{u} + \nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x})\mathbf{v}$ . If `transpose = .TRUE.`, then the user should evaluate the sum  $\mathbf{u} + \nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x})^T\mathbf{v}$ .

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- U is a rank-one `INTENT(INOUT)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components on input contain the vector  $\mathbf{u}$  and on output contains either the sum  $\mathbf{u} + \nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x})\mathbf{v}$  or  $\mathbf{u} + \nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x})^T\mathbf{v}$  depending on the value of `transpose` given above.
- V is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{v}$ .

### 2.4.7 Hessian-vector products via internal evaluation

If the control parameter `H_availability = 3` (see Section 2.2.3), then the argument `eval_Hv` must be present when calling `GALAHAD_CHECK_verify` and the user must provide a subroutine of that name to evaluate the sum  $\mathbf{u} + \nabla_{\mathbf{xx}}L(\mathbf{x},\mathbf{y})\mathbf{v}$  involving the product of the Hessian of the Lagrangian  $\nabla_{\mathbf{xx}}L(\mathbf{x},\mathbf{y})$ . The routine must be specified as

```
SUBROUTINE eval_Hv( status, X, Y, userdata, U, V )
```

whose arguments are as follows:

- `status` is a scalar `INTENT(OUT)` argument of type default `INTEGER` that should be set to 0 if the routine has been able to perform the required calculation and to a non-zero value if the computation has not been possible.
- X is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{x}$ .
- Y is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{y}$ .
- `userdata` is a scalar `INTENT(INOUT)` argument of type `NLPT_userdata_type` whose components may be used to communicate user-supplied data to and from the subroutines `eval_F`, `eval_C`, `eval_G`, `eval_J`, `eval_H`, `eval_Jv`, and `eval_Hv` (see Section 2.2.7).
- U is a rank-one `INTENT(INOUT)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components on input contain the vector  $\mathbf{u}$  and on output the sum  $\mathbf{u} + \nabla_{\mathbf{xx}}L(\mathbf{x},\mathbf{y})\mathbf{v}$ .
- V is a rank-one `INTENT(IN)` array argument of type default `REAL` (double precision in `GALAHAD_CHECK_double`) whose components contain the vector  $\mathbf{v}$ .

## 2.5 Reverse Communication Information

A positive value of `inform%status` on exit from `CHECK_verify` indicates that `GALAHAD_CHECK_verify` is seeking further information—this will happen if the user has chosen not to evaluate function or derivative values internally (see Sections 2.2.3 and 2.4). The user should compute the required information and re-enter `GALAHAD_CHECK_verify` with all arguments (except those specifically mentioned below) unchanged.

Possible values of `inform%status` and the information required are

2. The user should compute the objective function value  $f(\mathbf{x})$  at the point  $\mathbf{x}$  indicated in `data%RC%X`. The required value should be set in `data%RC%F`. If the user is unable to evaluate  $f(\mathbf{x})$ —for instance, if the function is undefined at  $\mathbf{x}$ —the user need not set `data%RC%F`, but should then set `inform%status` to any negative value. Otherwise, the value of `inform%status` should remain unchanged.
3. The user should compute the constraint function value  $\mathbf{c}(\mathbf{x})$  at the point  $\mathbf{x}$  indicated in `data%RC%X`. The required value should be set in `data%RC%C`. If the user is unable to evaluate  $\mathbf{c}(\mathbf{x})$ —for instance, if the function is undefined at  $\mathbf{x}$ —the user need not set `data%RC%C`, but should then set `inform%status` to any negative value. Otherwise, the value of `inform%status` should remain unchanged.

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4. The user should compute the gradient of the objective function  $\nabla_x f(\mathbf{x})$  at the point  $\mathbf{x}$  indicated in `data%RC%X`. The value of the  $i$ -th component of the gradient should be set in `data%RC%G(i)` for  $i = 1, \dots, n$ . If the user is unable to evaluate a component of  $\nabla_x f(\mathbf{x})$ —for instance, if a component of the gradient is undefined at  $\mathbf{x}$ —the user need not set `data%RC%G`, but should then set `inform%status` to a negative value. Otherwise, the value of `inform%status` should remain unchanged.
5. The user should compute the Jacobian of the constraint function  $\nabla_x \mathbf{c}(\mathbf{x})$  at the point  $\mathbf{x}$  indicated in `data%RC%X`. The  $l$ -th component of the Jacobian stored according to the scheme used to input `nlp%J` (see Section 2.2.2) should be set in `data%RC%Jval(l)` for  $l = 1, \dots, \text{nlp}\%J\%ne$ . If the user is unable to evaluate a component of  $\nabla_x \mathbf{c}(\mathbf{x})$ —for instance, if a component of the Jacobian is undefined at  $\mathbf{x}$ —the user need not set `data%RC%Jval`, but should then set `inform%status` to a negative number. Otherwise, the value of `inform%status` should remain unchanged.
6. The user should compute the value  $\mathbf{u} + \nabla_x \mathbf{c}(\mathbf{x})\mathbf{v}$ , which requires a product of the Jacobian of the constraint function  $\nabla_x \mathbf{c}(\mathbf{x})$  at the point  $\mathbf{x}$  with the vector  $\mathbf{v}$ ; the vectors  $\mathbf{x}$ ,  $\mathbf{u}$ , and  $\mathbf{v}$  are contained in `data%RC%X`, `data%RC%U`, and `data%RC%V`, respectively. On exit, the resulting vector  $\mathbf{u} + \nabla_x \mathbf{c}(\mathbf{x})\mathbf{v}$  should be stored in `data%RC%U`. If the user is unable to evaluate the product—for instance, if a component of the Jacobian is undefined at  $\mathbf{x}$ —the user need not set `data%RC%U`, but should then set `inform%status` to a negative value. Otherwise, the value of `inform%status` should remain unchanged.
7. The user should compute the value  $\mathbf{u} + \nabla_x \mathbf{c}(\mathbf{x})^T \mathbf{v}$ , which requires a product of the transpose of the Jacobian of the constraint function  $\nabla_x \mathbf{c}(\mathbf{x})$  at the point  $\mathbf{x}$  with the vector  $\mathbf{v}$ ; the vectors  $\mathbf{x}$ ,  $\mathbf{u}$ , and  $\mathbf{v}$  are contained in `data%RC%X`, `data%RC%U`, and `data%RC%V`, respectively. On exit, the resulting vector  $\mathbf{u} + \nabla_x \mathbf{c}(\mathbf{x})^T \mathbf{v}$  should be stored in `data%RC%U`. If the user is unable to evaluate the product—for instance, if a component of the Jacobian is undefined at  $\mathbf{x}$ —the user need not set `data%RC%U`, but should then set `inform%status` to a negative value. Otherwise, the value of `inform%status` should remain unchanged.
8. The user should compute the Hessian of the Lagrangian  $\nabla_{xx} L(\mathbf{x}, \mathbf{y})$  at the point  $(\mathbf{x}, \mathbf{y})$  indicated in `data%RC%X` and `data%RC%Y`. The  $l$ -th component of the Hessian stored according to the scheme used to input `nlp%H` (see Section 2.2.2) should be set in `data%RC%Hval(l)` for  $l = 1, \dots, \text{nlp}\%H\%ne$ . If the user is unable to evaluate a component of  $\nabla_{xx} L(\mathbf{x}, \mathbf{y})$ —for instance, if a component of the Hessian is undefined at  $(\mathbf{x}, \mathbf{y})$ —the user need not set `data%RC%Hval`, but should then set `inform%status` to a negative value. Otherwise, the value of `inform%status` should remain unchanged.
9. The user should compute the value  $\mathbf{u} + \nabla_{xx} L(\mathbf{x}, \mathbf{y})\mathbf{v}$ , which requires a product of the Hessian of the Lagrangian  $\nabla_{xx} L(\mathbf{x}, \mathbf{y})$  at the point  $(\mathbf{x}, \mathbf{y})$  with the vector  $\mathbf{v}$ ; the vectors  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $\mathbf{u}$ , and  $\mathbf{v}$  are contained in `data%RC%X`, `data%RC%Y`, `data%RC%U`, and `data%RC%V`, respectively. On exit, the resulting vector  $\mathbf{u} + \nabla_{xx} L(\mathbf{x}, \mathbf{y})\mathbf{v}$  should be stored in `data%RC%U`. If the user is unable to evaluate the product—for instance, if a component of the Hessian is undefined at  $(\mathbf{x}, \mathbf{y})$ —the user need not set `data%RC%U`, but should then set `inform%status` to a negative value. Otherwise, the value of `inform%status` should remain unchanged.

## 2.6 Warning and error messages

A negative value of `inform%status` on exit from `CHECK_verify` or `CHECK_terminate` indicates that an error has occurred. No further calls should be made until the error has been corrected. Possible values are:

- 1. An allocation error occurred. A message indicating the offending array is written on unit `control%error`, and the returned allocation status and a string containing the name of the offending array are held in `inform%alloc-status` and `inform%bad_alloc`, respectively.
- 2. A deallocation error occurred. A message indicating the offending array is written on unit `control%error` and the returned allocation status and a string containing the name of the offending array are held in `inform%alloc-status` and `inform%bad_alloc`, respectively.

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- 3. Either one of the restrictions  $nlp\%n > 0$  or  $nlp\%m \geq 0$  is violated, or the requirement that  $nlp\%J\_type$  and  $nlp\%H\_type$  contain a relevant string 'DENSE', 'COORDINATE', 'SPARSE\_BY\_ROWS', 'SPARSE\_BY\_COLUMNS', or 'DIAGONAL' is not satisfied.
- 50. The user has called `CHECK_verify` with  $inform\%status < 0$ , which indicates that the user was not able to perform a requested computation needed during reverse communication.
- 51. The user has called `CHECK_verify` with  $inform\%status = 0$ , which should never happen. The user should only set  $inform\%status$  in two situations:  $inform\%status = 1$  prior to the initial call to `CHECK_verify`, and  $inform\%status < 0$  when reverse communication is being used and the user is unable to perform the required computation as indicated by the value of  $inform\%status$  on return from `CHECK_verify` (see Section 2.5). The user should not change  $inform\%status$  for any other reason.
- 55. The user has input an invalid value for at least one of the control parameters  $f\_availability$ ,  $c\_availability$ ,  $g\_availability$ ,  $J\_availability$ , or  $H\_availability$  as described in Section 2.2.3.
- 56. Based on the values of the control parameters  $f\_availability$ ,  $c\_availability$ ,  $g\_availability$ ,  $J\_availability$ , and  $H\_availability$  (see Section 2.2.3), at least one optional dummy subroutine is missing in the call to `CHECK_verify`.
- 57. At least one component of  $nlp\%X\_l$  or  $nlp\%X\_u$  is inappropriate (see Section 2.2.2).
- 58. A user supplied function (see Sections 2.4.1–2.4.7) returned  $inform\%status \neq 0$ , implying that the function computation could not be performed at the required point.

## 2.7 Further features

In this section, we describe an alternative means of setting control parameters—that is components of the variable `control` of type `CHECK_control_type` (see Section 2.2.3)—by reading an appropriate data specification file using the subroutine `CHECK_read_specfile`. This facility is useful as it allows a user to change `CHECK` control parameters without editing and recompiling programs that call `CHECK`.

A specification file, or `specfile`, is a data file containing a number of "specification commands". Each command occurs on a separate line, and comprises a "keyword", which is a string (in a close-to-natural language) used to identify a control parameter, and an (optional) "value", which defines the value to be assigned to the given control parameter. All keywords and values are case insensitive, keywords may be preceded by one or more blanks but values must not contain blanks, and each value must be separated from its keyword by at least one blank. Values must not contain more than 30 characters, and each line of the `specfile` is limited to 80 characters, including the blanks separating keyword and value.

The portion of the specification file used by `CHECK_read_specfile` must start with a "BEGIN CHECK" command and end with an "END" command. The syntax of the `specfile` is thus defined as follows:

```
( .. lines ignored by CHECK_read_specfile .. )
BEGIN CHECK
  keyword      value
  .....
  keyword      value
END
( .. lines ignored by CHECK_read_specfile .. )
```

where `keyword` and `value` are two strings separated by (at least) one blank. The "BEGIN CHECK" and "END" delimiter command lines may contain additional (trailing) strings so long as such strings are separated by one or more blanks, so that lines such as

---

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```
BEGIN CHECK SPECIFICATION
```

and

```
END CHECK SPECIFICATION
```

are acceptable. Furthermore, between the “BEGIN CHECK” and “END” delimiters, specification commands may occur in any order. Blank lines and lines whose first non-blank character is ! or \* are ignored. The content of a line after a ! or \* character is also ignored (as is the ! or \* character itself). This provides an easy manner to “comment out” some specification commands, or to comment specific values of certain control parameters.

The value of a control parameters may be of three different types, namely integer, logical, or real. Integer and real values may be expressed in any relevant Fortran integer and floating-point formats (respectively). Permitted values for logical parameters are “ON”, “TRUE”, “.TRUE.”, “T”, “YES”, “Y”, or “OFF”, “NO”, “N”, “FALSE”, “.FALSE.” and “F”. Empty values are also allowed for logical control parameters, and are interpreted as “TRUE”.

The specification file must be open for input when `CHECK_read_specfile` is called, and the associated device number passed to the routine in `device` (see below). Note that the corresponding file is `REWINDed`, which makes it possible to combine the specifications for more than one program/routine. For the same reason, the file is not closed by `CHECK_read_specfile`.

### 2.7.1 To read control parameters from a specification file

Control parameters may be read from a file as follows:

```
CALL CHECK_read_specfile( control, device )
```

`control` is a scalar `INTENT(INOUT)` argument of type `CHECK_control_type` (see Section 2.2.3). Default values should have already been set, perhaps by calling `CHECK_initialize`. On exit, individual components of `control` may have been changed according to the commands found in the specfile. Specfile commands and the components (see Section 2.2.3) of `control` that they affect are given in Table 2.1.

| command                | component of control    | value type |
|------------------------|-------------------------|------------|
| error-printout-device  | %error                  | integer    |
| printout-device        | %out                    | integer    |
| print-level            | %print_level            | integer    |
| verification-level     | %verify_level           | integer    |
| f-availability         | %f_availability         | integer    |
| c-availability         | %c_availability         | integer    |
| g-availability         | %G_availability         | integer    |
| J-availability         | %J_availability         | integer    |
| H-availability         | %H_availability         | integer    |
| check-gradient         | %checkG                 | logical    |
| check-Jacobian         | %checkJ                 | logical    |
| check-Hessian          | %checkH                 | logical    |
| deallocate-error-fatal | %deallocate_error_fatal | logical    |

Table 2.1: Specfile commands and associated components of `control`.

`device` is a scalar `INTENT(IN)` argument of type `default_INTEGER`, that must be set to the unit number on which the specfile has been opened. If `device` is not open, `control` will not be altered and execution will continue, but an error message will be printed on unit `control%error`.

---

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## 2.8 Information printed

If `control%print_level` is positive, information about the derivative verification will be printed on unit `control%out`, provided `control%out > 0`. If `control%print_level = 1`, a basic summary of the derivative checking is produced. If `control%print_level = 2`, then in addition to the above there is detailed output of the derivative verification, control parameters are printed, and basic matrix data is produced. If `control%print_level = 3`, then in addition to the above, full matrix data is printed. Finally, `control%print_level ≥ 4` is used for debugging and in addition to the above also prints private data used during the verification process.

## 3 GENERAL INFORMATION

**Use of common:** None.

**Workspace:** Provided automatically by the module.

**Other routines called directly:** None.

**Other modules used directly:** `CHECK_verify` and `CHECK_terminate` call the GALAHAD packages `GALAHAD_MOP`, and `GALAHAD_SPACE`.

**Input/output:** Output is under control of the arguments `control%error`, `control%out` and `control%print_level`.

**Restrictions:** `nlp%n > 0` and `nlp%m ≥ 0`.

**Portability:** ISO Fortran 95 + TR 15581 or Fortran 2003. The package is thread-safe.

## 4 METHOD

Finite difference approximations are used to numerically “verify” requested derivatives. If `verify_level = 1`, we perform a “cheap” check of the gradient of the objective function by comparing  $(f(\mathbf{x}) - f(\mathbf{x} + \alpha \mathbf{s}))/\alpha$  with  $\nabla_x f(\mathbf{x})^T \mathbf{s}$  for some appropriately chosen vector  $\mathbf{s}$  and scalar  $0 < \alpha \ll 1$ . Similarly, for the Jacobian of the constraints and the Hessian of the Lagrangian, we compare  $(\mathbf{c}(\mathbf{x}) - \mathbf{c}(\mathbf{x} + \alpha \mathbf{s}))/\alpha$  to  $\nabla_x \mathbf{c}(\mathbf{x}) \mathbf{s}$  and  $(\nabla_x L(\mathbf{x}, \mathbf{y}) - \nabla_x L(\mathbf{x} + \alpha \mathbf{s}, \mathbf{y}))/\alpha$  to  $\nabla_{xx} L(\mathbf{x}, \mathbf{y}) \mathbf{s}$ , respectively. If `verify_level = 2`, we perform an “expensive” verification of the gradient of the objective function by comparing  $(f(\mathbf{x}) - f(\mathbf{x} + \alpha \mathbf{e}_i))/\alpha$  with  $[\nabla_x f(\mathbf{x})]_i$  for  $i = 1, \dots, n$ , where  $\mathbf{e}_i$  is the  $i$ th coordinate vector. Similarly, for the Jacobian of the constraints and the Hessian of the Lagrangian, we compare  $[(\mathbf{c}(\mathbf{x}) - \mathbf{c}(\mathbf{x} + \alpha \mathbf{e}_j))]_i/\alpha$  to  $[\nabla_x \mathbf{c}(\mathbf{x})]_{ij}$  and  $[(\nabla_x L(\mathbf{x}, \mathbf{y}) - \nabla_x L(\mathbf{x} + \alpha \mathbf{e}_j, \mathbf{y}))]_i/\alpha$  to  $[\nabla_{xx} L(\mathbf{x}, \mathbf{y})]_{ij}$ , respectively.

## 5 EXAMPLES OF USE

Suppose we wish to perform an “expensive” check of the derivatives associated with the objective function  $f(\mathbf{x}) = x_1 + x_2^3/3$  and the constraint function  $\mathbf{c}(\mathbf{x}) = (x_1 + x_2^2 + x_3^3 + x_3 x_2^2, -x_2^4)$  at the point  $\mathbf{x} = (4, 3, 2)$  and  $\mathbf{y} = (2, 3)$ , with bounds  $\mathbf{x}' = (-5, -5, -5)$  and  $\mathbf{x}'' = (5, 5, 5)$ . We may use the following code:

```
! THIS VERSION: GALAHAD 3.3 - 04/05/2021 AT 14:45 GMT.
PROGRAM GALAHAD_check_example
  USE GALAHAD_SMT_double    ! double precision version
  USE GALAHAD_USERDATA_double ! double precision version
  USE GALAHAD_NLPT_double   ! double precision version
  USE GALAHAD_MOP_double    ! double precision version
  USE GALAHAD_CHECK_double  ! double precision version
  IMPLICIT NONE
  integer, parameter :: wp = KIND( 1.0D+0 ) ! Define the working precision
```

---

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```

type( NLPT_problem_type ) :: nlp
type( GALAHAD_userdata_type ) :: userdata
type( CHECK_data_type ) :: data
type( CHECK_control_type ) :: control
type( CHECK_inform_type ) :: inform
integer :: stat, Jne, Hne, m, n
real (kind = wp), parameter :: one = 1.0_wp, two = 2.0_wp, three = 3.0_wp
real (kind = wp), parameter :: four = 4.0_wp, five = 5.0_wp
external funF, funC, funG, funJ, funH
nlp%m = 2 ; nlp%n = 3 ; m = nlp%m ; n = nlp%n
nlp%Jm = 2 ; nlp%Jn = 3 ; nlp%Jne = 4 ; Jne = nlp%Jne
nlp%Hm = 3 ; nlp%Hn = 3 ; nlp%Hne = 3 ; Hne = nlp%Hne
call SMT_put( nlp%Jid, 'Toy 2x3 matrix', stat )
call SMT_put( nlp%Jtype, 'COORDINATE', stat )
call SMT_put( nlp%Hid, 'Toy 3x3 hessian matrix', stat )
call SMT_put( nlp%Htype, 'COORDINATE', stat )
allocate( nlp%G(n), nlp%C(m), nlp%X(n), nlp%X_l(n), nlp%X_u(n), nlp%Y(m) )
allocate( nlp%Jrow(Jne), nlp%Jcol(Jne), nlp%Jval(Jne) )
allocate( nlp%Hrow(Hne), nlp%Hcol(Hne), nlp%Hval(Hne) )
nlp%Jrow = (/ 1, 1, 1, 2 /) ; nlp%Jcol = (/ 1, 2, 3, 2 /)
nlp%Hrow = (/ 2, 3, 3 /) ; nlp%Hcol = (/ 2, 2, 3 /)
nlp%X = (/ four, three, two /) ; nlp%X_l = -five ; nlp%X_u = five ; nlp%Y = (/ two, three /)
call CHECK_initialize( control ) ; control%print_level = 3
inform%status = 1
call CHECK_verify( nlp, data, control, inform, userdata, funF, funC, funG, funJ, funH )
call CHECK_terminate( data, control, inform )
END PROGRAM GALAHAD_check_example

SUBROUTINE funF( status, X, userdata, F )
  USE GALAHAD_USERDATA_double
  INTEGER, PARAMETER :: wp = KIND( 1.0D+0 )
  INTEGER, INTENT( OUT ) :: status
  REAL ( kind = wp ), INTENT( IN ), DIMENSION( : ) :: X
  REAL ( kind = wp ), INTENT( OUT ) :: F
  TYPE ( GALAHAD_userdata_type ), INTENT( INOUT ) :: userdata
  F = X(1) + X(2)**3 / 3.0_wp
  status = 0
  RETURN
END SUBROUTINE funF

SUBROUTINE funC(status, X, userdata, C)
  USE GALAHAD_USERDATA_double
  INTEGER, PARAMETER :: wp = KIND( 1.0D+0 )
  INTEGER, INTENT( OUT ) :: status
  REAL ( kind = wp ), INTENT( IN ), DIMENSION( : ) :: X
  REAL ( kind = wp ), DIMENSION( : ), INTENT( OUT ) :: C
  TYPE ( GALAHAD_userdata_type ), INTENT( INOUT ) :: userdata
  C(1) = X(1) + X(2)**2 + X(3)**3 + X(3)*X(2)**2
  C(2) = -X(2)**4
  status = 0
  RETURN
END SUBROUTINE funC

SUBROUTINE funG(status, X, userdata, G)
  USE GALAHAD_USERDATA_double
  INTEGER, PARAMETER :: wp = KIND( 1.0D+0 )
  INTEGER, INTENT( OUT ) :: status

```

---

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```

REAL ( KIND = wp ), DIMENSION( : ), INTENT( IN ) :: X
REAL ( KIND = wp ), DIMENSION( : ), INTENT( OUT ) :: G
TYPE ( GALAHAD_userdata_type ), INTENT( INOUT ) :: userdata
G(1) = 1.0_wp
G(2) = X(2)**2
G(3) = 0.0_wp
status = 0
RETURN
END SUBROUTINE funG
SUBROUTINE funJ(status, X, userdata, Jval)
  USE GALAHAD_USERDATA_double
  INTEGER, PARAMETER :: wp = KIND( 1.0D+0 )
  INTEGER, INTENT( OUT ) :: status
  REAL ( KIND = wp ), DIMENSION( : ), INTENT( IN ) :: X
  REAL ( KIND = wp ), DIMENSION( : ), INTENT( OUT ) :: Jval
  TYPE ( GALAHAD_userdata_type ), INTENT( INOUT ) :: userdata
  Jval(1) = 1.0_wp
  Jval(2) = 2.0_wp * X(2) * ( 1.0_wp + X(3) )
  Jval(3) = 3.0_wp * X(3)**2 + X(2)**2
  Jval(4) = -4.0_wp * X(2)**3
  status = 0
  RETURN
END SUBROUTINE funJ
SUBROUTINE funH(status, X, Y, userdata, Hval)
  USE GALAHAD_USERDATA_double
  INTEGER, PARAMETER :: wp = KIND( 1.0D+0 )
  INTEGER, INTENT( OUT ) :: status
  REAL ( kind = wp ), DIMENSION( : ), INTENT( IN ) :: X
  REAL ( kind = wp ), DIMENSION( : ), INTENT( IN ) :: Y
  REAL ( kind = wp ), DIMENSION( : ), INTENT( OUT ) :: Hval
  TYPE ( GALAHAD_userdata_type ), INTENT( INOUT ) :: userdata
  Hval(1) = 2.0_wp * ( X(2) - Y(1) - Y(1)*X(3) + 6.0_wp*Y(2)*X(2)**2 )
  Hval(2) = -2.0_wp * Y(1) * X(2)
  Hval(3) = -6.0_wp * Y(1) * X(3)
  status = 0
  RETURN
END SUBROUTINE funH

```

The code produces the following output:

```

-----
-----BEGIN: CHECK_verify-----
-----
EXPENSIVE VERIFICATION OF THE GRADIENT G(X)

Component      Ok      Difference      Value      Error
-----
G(      1)      OK      9.999999891E-01      1.000000000E+00      5.437063107E-09
G(      2)      OK      9.000000276E+00      9.000000000E+00      2.755274155E-08
G(      3)      OK      0.000000000E+00      0.000000000E+00      0.000000000E+00

EXPENSIVE VERIFICATION OF THE JACOBIAN C(X)

Component      Ok      Difference      Value      Error
-----

```

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```

-----
J(  1,  1)  OK  9.999999891E-01  1.000000000E+00  5.437063107E-09
J(  2,  1)  OK  0.000000000E+00  0.000000000E+00  0.000000000E+00
J(  1,  2)  OK  1.800000025E+01  1.800000000E+01  1.328101027E-08
J(  2,  2)  OK -1.080000049E+02 -1.080000000E+02  4.540609994E-08
J(  1,  3)  OK  2.100000052E+01  2.100000000E+01  2.356511149E-08
J(  2,  3)  OK  0.000000000E+00  0.000000000E+00  0.000000000E+00

```

## EXPENSIVE VERIFICATION OF THE HESSIAN H(X,Y)

| Component  | Ok | Difference       | Value            | Error           |
|------------|----|------------------|------------------|-----------------|
| H(  1,  1) | OK | 0.000000000E+00  | 0.000000000E+00  | 0.000000000E+00 |
| H(  2,  1) | OK | 0.000000000E+00  | 0.000000000E+00  | 0.000000000E+00 |
| H(  3,  1) | OK | 0.000000000E+00  | 0.000000000E+00  | 0.000000000E+00 |
| H(  1,  2) | OK | 0.000000000E+00  | 0.000000000E+00  | 0.000000000E+00 |
| H(  2,  2) | OK | 3.180000098E+02  | 3.180000000E+02  | 3.083032558E-08 |
| H(  3,  2) | OK | -1.200000017E+01 | -1.200000000E+01 | 1.294047150E-08 |
| H(  1,  3) | OK | 0.000000000E+00  | 0.000000000E+00  | 0.000000000E+00 |
| H(  2,  3) | OK | -1.200000017E+01 | -1.200000000E+01 | 1.294047150E-08 |
| H(  3,  3) | OK | -2.400000049E+01 | -2.400000000E+01 | 1.943240311E-08 |

```

-----
|                          SUMMARY                          |
( Verify : Expensive )

```

THE GRADIENT OF THE OBJECTIVE FUNCTION IS ---- [OK]

THE JACOBIAN OF THE CONSTRAINT FUNCTION IS --- [OK]

THE HESSIAN OF THE LAGRANGIAN FUNCTION IS ---- [OK]

```

-----
CONTROL PARAMETERS

```

```

checkG = T   f_available = 1   deall_error_fatal = F
checkJ = T   c_available = 1   print_level      = 3
checkH = T   g_available = 1   verify_level     = 2
error   = 6   J_available = 1   out              = 6
           H_available = 1

```

```

-----
MATRIX DATA

```

```

J%type --- COORDINATE
J%id    --- Toy 2x3 matrix
H%type --- COORDINATE
H%id    --- Toy 3x3 hessian matrix

```

```

m =      2
n =      3

```



| J%row | J%col | J%val             |
|-------|-------|-------------------|
| ----  | ----  | -----             |
| 1     | 1     | 1.0000000000E+00  |
| 1     | 2     | 1.8000000571E+01  |
| 1     | 3     | 2.1000001142E+01  |
| 2     | 2     | -1.0800000000E+02 |

| H%row | H%col | H%val             |
|-------|-------|-------------------|
| ----  | ----  | -----             |
| 2     | 2     | 3.1800000000E+02  |
| 3     | 2     | -1.2000000000E+01 |
| 3     | 3     | -2.4000000000E+01 |

EXIT STATUS : 0

-----  
-----  
END: CHECK\_verify  
-----  
-----