

## C interfaces to GALAHAD BQP

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## **Chapter 1**

# GALAHAD C package bqp

## 1.1 Introduction

#### 1.1.1 Purpose

This package uses a preconditioned, projected-gradient method to solve the **convex bound-constrained quadratic programming problem** 

$$\text{minimize} \ \ q(x) = \frac{1}{2} x^T H x + g^T x + f$$

subject to the simple bound constraints

$$x_i^l \le x_j \le x_i^u, \quad j = 1, \dots, n,$$

where the n by n symmetric postive semi-definite matrix H, the vectors g,  $x^l$ ,  $x^u$  and the scalar f are given. Any of the constraint bounds  $x^l_j$  and  $x^u_j$  may be infinite. Full advantage is taken of any zero coefficients in the matrix H; the matrix need not be provided as there are options to obtain matrix-vector products involving H by reverse communication.

#### 1.1.2 Authors

N. I. M. Gould, STFC-Rutherford Appleton Laboratory, England.

C interface, additionally J. Fowkes, STFC-Rutherford Appleton Laboratory.

## 1.1.3 Originally released

November 2009, C interface February 2022.

## 1.1.4 Terminology

The required solution x necessarily satisfies the primal optimality conditions

$$x^l \le x \le x^u$$
,

the dual optimality conditions

$$Hx + g = z$$

where

$$z = z^{l} + z^{u}, z^{l} > 0$$
 and  $z^{u} < 0$ ,

and the complementary slackness conditions

$$(x-x^l)^T z^l = 0$$
 and  $(x-x^u)^T z^u = 0$ ,

where the vector z is known as the dual variables for the bounds, respectively, and where the vector inequalities hold component-wise.

#### 1.1.5 Method

The method is iterative. Each iteration proceeds in two stages. Firstly, the so-called generalized Cauchy point for the quadratic objective is found. (The purpose of this point is to ensure that the algorithm converges and that the set of bounds which are satisfied as equations at the solution is rapidly identified.) Thereafter an improvement to the objective is sought using either a direct-matrix or truncated conjugate-gradient algorithm.

#### 1.1.6 Reference

This is a specialised version of the method presented in

A. R. Conn, N. I. M. Gould and Ph. L. Toint (1988). Global convergence of a class of trust region algorithms for optimization with simple bounds. SIAM Journal on Numerical Analysis **25** 433-460,

#### 1.1.7 Call order

To solve a given problem, functions from the bqp package must be called in the following order:

- bqp\_initialize provide default control parameters and set up initial data structures
- bgp read specfile (optional) override control values by reading replacement values from a file
- · set up problem data structures and fixed values by caling one of
  - bqp\_import in the case that H is explicitly available
  - $bqp_import_without_h$  in the case that only the effect of applying H to a vector is possible
- bqp\_reset\_control (optional) possibly change control parameters if a sequence of problems are being solved
- · solve the problem by calling one of
  - bgp solve given h solve the problem using values of H
  - $bqp\_solve\_reverse\_h\_prod$  solve the problem by returning to the caller for products of H with specified vectors
- bgp information (optional) recover information about the solution and solution process
- bqp\_terminate deallocate data structures

See Section 4.1 for examples of use.

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### 1.1.8 Symmetric matrix storage formats

If it is explicitly available, the symmetric n by n objective Hessian matrix H may be presented and stored in a variety of formats. But crucially symmetry is exploited by only storing values from the lower triangular part (i.e, those entries that lie on or below the leading diagonal).

#### 1.1.8.1 Dense storage format

The matrix H is stored as a compact dense matrix by rows, that is, the values of the entries of each row in turn are stored in order within an appropriate real one-dimensional array. Since H is symmetric, only the lower triangular part (that is the part  $h_{ij}$  for  $0 \le j \le i \le n-1$ ) need be held. In this case the lower triangle should be stored by rows, that is component i\*i/2+j of the storage array H\_val will hold the value  $h_{ij}$  (and, by symmetry,  $h_{ji}$ ) for  $0 \le j \le i \le n-1$ .

#### 1.1.8.2 Sparse co-ordinate storage format

Only the nonzero entries of the matrices are stored. For the l-th entry,  $0 \le l \le ne-1$ , of H, its row index i, column index j and value  $h_{ij}$ ,  $0 \le j \le i \le n-1$ , are stored as the l-th components of the integer arrays H\_row and H\_col and real array H\_val, respectively, while the number of nonzeros is recorded as H\_ne = ne. Note that only the entries in the lower triangle should be stored.

#### 1.1.8.3 Sparse row-wise storage format

Again only the nonzero entries are stored, but this time they are ordered so that those in row i appear directly before those in row i+1. For the i-th row of H the i-th component of the integer array H\_ptr holds the position of the first entry in this row, while H\_ptr(n) holds the total number of entries plus one. The column indices j,  $0 \le j \le i$ , and values  $h_{ij}$  of the entries in the i-th row are stored in components I = H\_ptr(i), ..., H\_ptr(i+1)-1 of the integer array H\_col, and real array H\_val, respectively. Note that as before only the entries in the lower triangle should be stored. For sparse matrices, this scheme almost always requires less storage than its predecessor.

#### 1.1.8.4 Diagonal storage format

If H is diagonal (i.e.,  $H_{ij}=0$  for all  $0 \le i \ne j \le n-1$ ) only the diagonals entries  $H_{ii}$ ,  $0 \le i \le n-1$  need be stored, and the first n components of the array H\_val may be used for the purpose.

# Chapter 2

# File Index

## 2.1 File List

| Here is a l | list of | all t | iles | wi | th t | orie | f de | esc | ript | ion | ıs: |  |  |  |  |  |  |  |  |  |  |  |   |
|-------------|---------|-------|------|----|------|------|------|-----|------|-----|-----|--|--|--|--|--|--|--|--|--|--|--|---|
| bqp.h       |         |       |      |    |      |      |      |     |      |     |     |  |  |  |  |  |  |  |  |  |  |  | 7 |

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## **Chapter 3**

## **File Documentation**

## 3.1 bqp.h File Reference

```
#include <stdbool.h>
#include "galahad_precision.h"
#include "sbls.h"
```

#### **Data Structures**

- · struct bap control type
- struct bqp time type
- struct bqp\_inform\_type

#### **Functions**

- void bqp\_initialize (void \*\*data, struct bqp\_control\_type \*control, int \*status)
- void bqp read specfile (struct bqp control type \*control, const char specfile[])
- void bqp\_import (struct bqp\_control\_type \*control, void \*\*data, int \*status, int n, const char H\_type[], int ne, const int H\_row[], const int H\_col[], const int H\_ptr[])
- void bqp\_import\_without\_h (struct bqp\_control\_type \*control, void \*\*data, int \*status, int n)
- void bqp\_reset\_control (struct bqp\_control\_type \*control, void \*\*data, int \*status)
- void bqp\_solve\_given\_h (void \*\*data, int \*status, int n, int h\_ne, const real\_wp\_ H\_val[], const real\_wp\_ g[], const real\_wp\_ f, const real\_wp\_ x\_l[], const real\_wp\_ x\_u[], real\_wp\_ y[], real\_wp\_ z[], int x\_stat[])
- void bqp\_solve\_reverse\_h\_prod (void \*\*data, int \*status, int n, const real\_wp\_ g[], const real\_wp\_ f, const real\_wp\_ x\_l[], const real\_wp\_ x\_u[], real\_wp\_ y[], real\_wp\_ z[], int x\_stat[], real\_wp\_ v[], const real\_wp\_ prod[], int nz\_v[], int \*nz\_v\_start, int \*nz\_v\_end, const int nz\_prod[], int nz\_prod\_end)
- void bqp\_information (void \*\*data, struct bqp\_inform\_type \*inform, int \*status)
- void bqp\_terminate (void \*\*data, struct bqp\_control\_type \*control, struct bqp\_inform\_type \*inform)

#### 3.1.1 Data Structure Documentation

## 3.1.1.1 struct bqp\_control\_type

control derived type as a C struct

**Examples** 

bapt.c, and baptf.c.

## **Data Fields**

| bool     | f_indexing             | use C or Fortran sparse matrix indexing  |
|----------|------------------------|--|
| int      | error                  | unit number for error and warning diagnostics  |
| int      | out                    | general output unit number   |
| int      | print_level            | the level of output required   |
| int      | start_print            | on which iteration to start printing   |
| int      | stop_print             | on which iteration to stop printing  |
| int      | print_gap              | how many iterations between printing   |
| int      | maxit                  | how many iterations to perform (-ve reverts to HUGE(1)-1)  |
| int      | cold_start             | cold_start should be set to 0 if a warm start is required  |
|          |                        | (with variable assigned according to B_stat, see below), and to any other value if the values given in prob.X suffice  |
| int      | ratio_cg_vs_sd         | the ratio of how many iterations use CG rather steepest descent  |
| int      | change_max             | the maximum number of per-iteration changes in the working set permitted when allowing CG rather than steepest descent   |
| int      | cg_maxit               | how many CG iterations to perform per BQP iteration (-ve reverts to n+1)   |
| int      | sif_file_device        | the unit number to write generated SIF file describing the current probl   |
| real_wp_ | infinity               | any bound larger than infinity in modulus will be regarded as infinite   |
| real_wp_ | stop_p                 | the required accuracy for the primal infeasibility   |
| real_wp_ | stop_d                 | the required accuracy for the dual infeasibility   |
| real_wp_ | stop_c                 | the required accuracy for the complementary slackness  |
| real_wp_ | identical_bounds_tol   | any pair of constraint bounds (x_l,x_u) that are closer than i dentical_bounds_tol will be reset to the average of their values  |
| real_wp_ | stop_cg_relative       | the CG iteration will be stopped as soon as the current norm of the preconditioned gradient is smaller than max( stop_cg_relative * initial preconditioned gradient, stop_cg_absolut     |
| real_wp_ | stop_cg_absolute       | see stop_cg_relative   |
| real_wp_ | zero_curvature         | threshold below which curvature is regarded as zero  |
| real_wp_ | cpu_time_limit         | the maximum CPU time allowed (-ve = no limit)  |
| bool     | exact_arcsearch        | exact_arcsearch is true if an exact arcsearch is required, and false if approximation suffices   |
| bool     | space_critical         | if space_critical is true, every effort will be made to use as little space as possible. This may result in longer computation times   |
| bool     | deallocate_error_fatal | if deallocate_error_fatal is true, any array/pointer deallocation error will terminate execution. Otherwise, computation will continue   |
| bool     | generate_sif_file      | if generate_sif_file is true, a SIF file describing the current problem will be generated  |
| char     | sif_file_name[31]      | name (max 30 characters) of generated SIF file containing input problem  |
| char     | prefix[31]             | all output lines will be prefixed by a string (max 30 characters) prefix(2:LEN(TRIM(.prefix))-1) where prefix contains the required string enclosed in quotes, e.g. "string" or 'string' |

## **Data Fields**

| struct sbls_control_type   sbls_control | control parameters for SBLS |
|---|-----------------------------|
|---|-----------------------------|

## 3.1.1.2 struct bqp\_time\_type

time derived type as a C struct

## **Data Fields**

| real_sp_ | total     | total time                         |
|----------|-----------|------------------------------------|
| real_sp_ | analyse   | time for the analysis phase        |
| real_sp_ | factorize | time for the factorization phase   |
| real_sp_ | solve     | time for the linear solution phase |

## 3.1.1.3 struct bqp\_inform\_type

inform derived type as a C struct

## Examples

bqpt.c, and bqptf.c.

## Data Fields

| int                     | status               | reported return status:                          |
|-------------------------|----------------------|--|
|                         |                      | • 0 success                                      |
|                         |                      | -1 allocation error                              |
|                         |                      | • -2 deallocation error                          |
|                         |                      | • -3 matrix data faulty (.n $<$ 1, .ne $<$ 0)    |
|                         |                      | -20 alegedly +ve definite matrix is not          |
| int                     | alloc_status         | Fortran STAT value after allocate failure.       |
| int                     | factorization_status | status return from factorization                 |
| int                     | iter                 | number of iterations required                    |
| int                     | cg_iter              | number of CG iterations required                 |
| real_wp_                | obj                  | current value of the objective function          |
| real_wp_                | norm_pg              | current value of the projected gradient          |
| char                    | bad_alloc[81]        | name of array which provoked an allocate failure |
| struct bqp_time_type    | time                 | times for various stages                         |
| struct sbls_inform_type | sbls_inform          | inform values from SBLS                          |

## 3.1.2 Function Documentation

## 3.1.2.1 bqp\_initialize()

Set default control values and initialize private data

#### **Parameters**

| in,out | data    | holds private internal data   |
|--------|---------|---|
| out    | control | is a struct containing control information (see bqp_control_type)   |
| out    | status  | is a scalar variable of type int, that gives the exit status from the package. Possible values are (currently): |
|        |         | 0. The import was succesful.  |

#### **Examples**

bqpt.c, and bqptf.c.

### 3.1.2.2 bqp\_read\_specfile()

Read the content of a specification file, and assign values associated with given keywords to the corresponding control parameters

#### **Parameters**

| in,out | control  | is a struct containing control information (see <a href="bqp_control_type">bqp_control_type</a> ) |
|--------|----------|---|
| in     | specfile | is a character string containing the name of the specification file                               |

## 3.1.2.3 bqp\_import()

```
void ** data,
int * status,
int n,
const char H_type[],
int ne,
const int H_row[],
const int H_col[],
const int H_ptr[])
```

Import problem data into internal storage prior to solution.

## **Parameters**

| in     | control | is a struct whose members provide control paramters for the remaining prcedures (see bqp_control_type)  |
|--------|---------|---|
| in,out | data    | holds private internal data   |
| in,out | status  | is a scalar variable of type int, that gives the exit status from the package. Possible values are:   |
|        |         | 1. The import was succesful, and the package is ready for the solve phase   |
|        |         | <ul> <li>-1. An allocation error occurred. A message indicating the offending array is written on unit control.error, and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>                |
|        |         | <ul> <li>-2. A deallocation error occurred. A message indicating the offending array is<br/>written on unit control.error and the returned allocation status and a string<br/>containing the name of the offending array are held in inform.alloc_status and<br/>inform.bad_alloc respectively.</li> </ul>    |
|        |         | <ul> <li>-3. The restriction n &gt; 0 or requirement that type contains its relevant string<br/>'dense', 'coordinate', 'sparse_by_rows' or 'diagonal' has been violated.</li> </ul>   |
| in     | n       | is a scalar variable of type int, that holds the number of variables.   |
| in     | H_type  | is a one-dimensional array of type char that specifies the symmetric storage scheme used for the Hessian. It should be one of 'coordinate', 'sparse_by_rows', 'dense', 'diagonal' or 'absent', the latter if access to the Hessian is via matrix-vector products; lower or upper case variants are allowed.   |
| in     | ne      | is a scalar variable of type int, that holds the number of entries in the lower triangular part of H in the sparse co-ordinate storage scheme. It need not be set for any of the other three schemes.   |
| in     | H_row   | is a one-dimensional array of size ne and type int, that holds the row indices of the lower triangular part of H in the sparse co-ordinate storage scheme. It need not be set for any of the other three schemes, and in this case can be NULL  |
| in     | H_col   | is a one-dimensional array of size ne and type int, that holds the column indices of the lower triangular part of H in either the sparse co-ordinate, or the sparse row-wise storage scheme. It need not be set when the dense or diagonal storage schemes are used, and in this case can be NULL             |
| in     | H_ptr   | is a one-dimensional array of size n+1 and type int, that holds the starting position of each row of the lower triangular part of H, as well as the total number of entries plus one, in the sparse row-wise storage scheme. It need not be set when the other schemes are used, and in this case can be NULL |

## Examples

bqpt.c, and bqptf.c.

## 3.1.2.4 bqp\_import\_without\_h()

```
void bqp_import_without_h (
          struct bqp_control_type * control,
          void ** data,
          int * status,
          int n )
```

Import problem data into internal storage prior to solution.

#### **Parameters**

| in     | control | is a struct whose members provide control paramters for the remaining prcedures (see bqp_control_type)  |
|--------|---------|---|
| in,out | data    | holds private internal data   |
| in,out | status  | is a scalar variable of type int, that gives the exit status from the package. Possible values are:   |
|        |         | <ul> <li>1. The import was succesful, and the package is ready for the solve phase</li> <li>-1. An allocation error occurred. A message indicating the offending array is written on unit control.error, and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul> |
|        |         | <ul> <li>-2. A deallocation error occurred. A message indicating the offending array is<br/>written on unit control.error and the returned allocation status and a string<br/>containing the name of the offending array are held in inform.alloc_status and<br/>inform.bad_alloc respectively.</li> </ul>  |
|        |         | <ul> <li>-3. The restriction n &gt; 0 has been violated.</li> </ul>   |
| in     | n       | is a scalar variable of type int, that holds the number of variables.   |

## Examples

bqpt.c, and bqptf.c.

## 3.1.2.5 bqp\_reset\_control()

```
void bqp_reset_control (
          struct bqp_control_type * control,
          void ** data,
          int * status )
```

Reset control parameters after import if required.

## **Parameters**

| in          | control | is a struct whose members provide control paramters for the remaining proedures (see                      |
|-------------|---------|---|
|             |         | bqp_control_type)   |
| in,out      | data    | holds private internal data   |
| in,out      | status  | is a scalar variable of type int, that gives the exit status from the package. Possible values are:       |
| GALAHAD 4.0 |         | C interfaces to GALAHAD BQP     1. The import was succesful, and the package is ready for the solve phase |

## 3.1.2.6 bqp\_solve\_given\_h()

```
void bqp_solve_given_h (
    void ** data,
    int * status,
    int n,
    int h_ne,
    const real_wp_ H_val[],
    const real_wp_ g[],
    const real_wp_ f,
    const real_wp_ x_l[],
    const real_wp_ x_u[],
    real_wp_ y[],
    real_wp_ z[],
    int x_stat[])
```

Solve the bound-constrained quadratic program when the Hessian  ${\cal H}$  is available.

#### **Parameters**

| in,out | data | holds private internal data |  |
|--------|------|-----------------------------|--|
|--------|------|-----------------------------|--|

## **Parameters**

| in,out | status | is a scalar variable of type int, that gives the entry and exit status from the package. On initial entry, status must be set to 1. Possible exit are:   |
|--------|--------|--|
|        |        | 0. The run was succesful.  |
|        |        | <ul> <li>-1. An allocation error occurred. A message indicating the offending array is<br/>written on unit control.error, and the returned allocation status and a string<br/>containing the name of the offending array are held in inform.alloc_status and<br/>inform.bad_alloc respectively.</li> </ul> |
|        |        | <ul> <li>-2. A deallocation error occurred. A message indicating the offending array is<br/>written on unit control.error and the returned allocation status and a string<br/>containing the name of the offending array are held in inform.alloc_status and<br/>inform.bad_alloc respectively.</li> </ul> |
|        |        | <ul> <li>-3. The restriction n &gt; 0 or requirement that a type contains its relevant string<br/>'dense', 'coordinate', 'sparse_by_rows' or 'diagonal' has been violated.</li> </ul>  |
|        |        | -4. The simple-bound constraints are inconsistent.   |
|        |        | <ul> <li>-9. The analysis phase of the factorization failed; the return status from the<br/>factorization package is given in the component inform.factor_status</li> </ul>  |
|        |        | <ul> <li>-10. The factorization failed; the return status from the factorization package is<br/>given in the component inform.factor_status.</li> </ul>  |
|        |        | <ul> <li>-11. The solution of a set of linear equations using factors from the factorization<br/>package failed; the return status from the factorization package is given in the<br/>component inform.factor_status.</li> </ul>   |
|        |        | -16. The problem is so ill-conditioned that further progress is impossible.  |
|        |        | -17. The step is too small to make further impact.   |
|        |        | <ul> <li>-18. Too many iterations have been performed. This may happen if control.maxit<br/>is too small, but may also be symptomatic of a badly scaled problem.</li> </ul>  |
|        |        | <ul> <li>-19. The CPU time limit has been reached. This may happen if<br/>control.cpu_time_limit is too small, but may also be symptomatic of a badly scaled<br/>problem.</li> </ul>   |
|        |        | ullet -20. The Hessian matrix $H$ appears to be indefinite. specified.   |
|        |        | ullet -23. An entry from the strict upper triangle of $H$ has been   |
| in     | n      | is a scalar variable of type int, that holds the number of variables   |
| in     | h_ne   | is a scalar variable of type int, that holds the number of entries in the lower triangular part of the Hessian matrix ${\cal H}.$  |
| in     | H_val  | is a one-dimensional array of size h_ne and type double, that holds the values of the entries of the lower triangular part of the Hessian matrix $H$ in any of the available storage schemes.  |
| in     | g      | is a one-dimensional array of size n and type double, that holds the linear term $g$ of the objective function. The j-th component of g, j = 0,, n-1, contains $g_j$ .   |
| in     | f      | is a scalar of type double, that holds the constant term $f$ of the objective function.  |
| in     | x_I    | is a one-dimensional array of size n and type double, that holds the lower bounds $x^l$ on the variables $x$ . The j-th component of x_l, j = 0,, n-1, contains $x^l_j$ .  |
| in     | x_u    | is a one-dimensional array of size n and type double, that holds the upper bounds $x^l$ on the variables $x$ . The j-th component of x_u, j = 0,, n-1, contains $x_j^l$ .  |
|        |        | -  |

#### **Parameters**

| in,out | X      | is a one-dimensional array of size n and type double, that holds the values $x$ of the optimization variables. The j-th component of x, j = 0,, n-1, contains $x_j$ .   |
|--------|--------|---|
| in,out | Z      | is a one-dimensional array of size n and type double, that holds the values $z$ of the dual variables. The j-th component of z, j = 0,, n-1, contains $z_j$ .   |
| in,out | x_stat | is a one-dimensional array of size n and type int, that gives the optimal status of the problem variables. If $x\_stat(j)$ is negative, the variable $x_j$ most likely lies on its lower bound, if it is positive, it lies on its upper bound, and if it is zero, it lies between its bounds. |

## Examples

bqpt.c, and bqptf.c.

## 3.1.2.7 bqp\_solve\_reverse\_h\_prod()

```
void bqp_solve_reverse_h_prod (
             void ** data,
            int * status,
            int n,
             const real_wp_ g[],
            const real_wp_ f,
             const real_wp_ x_1[],
             const real_wp_ x_u[],
             real_wp_ y[],
             real_wp_ z[],
             int x_stat[],
             real_wp_ v[],
             const real_wp_ prod[],
             int nz_v[],
             int * nz_v_start,
             int * nz_v_end,
             const int nz_prod[],
             int nz_prod_end )
```

Solve the bound-constrained quadratic program when the products of the Hessian H with specified vectors may be computed by the calling program.

#### **Parameters**

|         |      | Line en e |
|---------|------|--|
| in, out | data | holds private internal data                |

## **Parameters**

| in,out | status | is a scalar variable of type int, that gives the entry and exit status from the package.  Possible exit are:   |
|--------|--------|--|
|        |        | 0. The run was succesful.  |
|        |        | <ul> <li>-1. An allocation error occurred. A message indicating the offending array<br/>is written on unit control.error, and the returned allocation status and a<br/>string containing the name of the offending array are held in<br/>inform.alloc_status and inform.bad_alloc respectively.</li> </ul> |
|        |        | <ul> <li>-2. A deallocation error occurred. A message indicating the offending array<br/>is written on unit control.error and the returned allocation status and a<br/>string containing the name of the offending array are held in<br/>inform.alloc_status and inform.bad_alloc respectively.</li> </ul> |
|        |        | <ul> <li>-3. The restriction n &gt; 0 or requirement that a type contains its relevant<br/>string 'dense', 'coordinate', 'sparse_by_rows' or 'diagonal' has been<br/>violated.</li> </ul>  |
|        |        | -4. The simple-bound constraints are inconsistent.   |
|        |        | <ul> <li>-9. The analysis phase of the factorization failed; the return status from the<br/>factorization package is given in the component inform.factor_status</li> </ul>  |
|        |        | <ul> <li>-10. The factorization failed; the return status from the factorization<br/>package is given in the component inform.factor_status.</li> </ul>  |
|        |        | <ul> <li>-11. The solution of a set of linear equations using factors from the<br/>factorization package failed; the return status from the factorization<br/>package is given in the component inform.factor_status.</li> </ul>   |
|        |        | -16. The problem is so ill-conditioned that further progress is impossible.  |
|        |        | -17. The step is too small to make further impact.   |
|        |        | <ul> <li>-18. Too many iterations have been performed. This may happen if<br/>control.maxit is too small, but may also be symptomatic of a badly scaled<br/>problem.</li> </ul>  |
|        |        | <ul> <li>-19. The CPU time limit has been reached. This may happen if<br/>control.cpu_time_limit is too small, but may also be symptomatic of a badly<br/>scaled problem.</li> </ul>   |
|        |        | ullet -20. The Hessian matrix $H$ appears to be indefinite. specified.   |
|        |        | ullet -23. An entry from the strict upper triangle of $H$ has been specified.  |

## **Parameters**

|        | status      | (continued)  |
|--------|-------------|--|
|        |             | • 2. The product $Hv$ of the Hessian $H$ with a given output vector $v$ is required from the user. The vector $v$ will be stored in $v$ and the product $Hv$ must be returned in prod, and bqp_solve_reverse_h_prod re-entered with all other arguments unchanged.   |
|        |             | • 3. The product $Hv$ of the Hessian H with a given output vector $v$ is required from the user. Only components $nz_v[nz_v_{start-1}:nz_v_{end-1}]$ of the vector $v$ stored in v are nonzero. The resulting product $Hv$ must be placed in prod, and $pz_v_{end}$ and $pz_v_{end}$ reverse_h_prod re-entered with all other arguments unchanged.   |
|        |             | • 4. The product $Hv$ of the Hessian H with a given output vector $v$ is required from the user. Only components $nz_v[nz_v_{start-1}:nz_v_{end-1}]$ of the vector $v$ stored in v are nonzero. The resulting <b>nonzeros</b> in the product $Hv$ must be placed in their appropriate components of prod, while a list of indices of the nonzeros placed in $nz_prod[0:nz_prod_{end-1}]$ . $pq_solve_reverse_h_prod_{end-1}$ by v will be very sparse (i.e., $pz_p_{end-1}$ by $pz_$ |
| in     | n           | is a scalar variable of type int, that holds the number of variables   |
| in     | g           | is a one-dimensional array of size n and type double, that holds the linear term $g$ of the objective function. The j-th component of g, j = 0,, n-1, contains $g_j$ .   |
| in     | f           | is a scalar of type double, that holds the constant term $f$ of the objective function.  |
| in     | x_I         | is a one-dimensional array of size n and type double, that holds the lower bounds $x^l$ on the variables $x$ . The j-th component of x_l, j = 0,, n-1, contains $x_j^l$ .  |
| in     | x_u         | is a one-dimensional array of size n and type double, that holds the upper bounds $x^l$ on the variables $x$ . The j-th component of x_u, j = 0,, n-1, contains $x^l_j$ .  |
| in,out | х           | is a one-dimensional array of size n and type double, that holds the values $x$ of the optimization variables. The j-th component of x, j = 0,, n-1, contains $x_j$ .  |
| in,out | Z           | is a one-dimensional array of size n and type double, that holds the values $z$ of the dual variables. The j-th component of z, j = 0,, n-1, contains $z_j$ .  |
| in,out | x_stat      | is a one-dimensional array of size n and type int, that gives the optimal status of the problem variables. If $x_{stat}(j)$ is negative, the variable $x_{j}$ most likely lies on its lower bound, if it is positive, it lies on its upper bound, and if it is zero, it lies between its bounds.   |
| out    | V           | is a one-dimensional array of size n and type double, that is used for reverse communication (see status=2-4 above for details)  |
| in     | prod        | is a one-dimensional array of size n and type double, that is used for reverse communication (see status=2-4 above for details)  |
| out    | nz_v        | is a one-dimensional array of size n and type int, that is used for reverse communication (see status=3-4 above for details)   |
| out    | nz_v_start  | is a scalar of type int, that is used for reverse communication (see status=3-4 above for details)   |
| out    | nz_v_end    | is a scalar of type int, that is used for reverse communication (see status=3-4 above for details)   |
| in     | nz_prod     | is a one-dimensional array of size n and type int, that is used for reverse communication (see status=4 above for details)   |
| in     | nz_prod_end | is a scalar of type int, that is used for reverse communication (see status=4 above for details)   |

## Examples

bqpt.c, and bqptf.c.

## 3.1.2.8 bqp\_information()

#### Provides output information

#### **Parameters**

| in,out | data   | holds private internal data   |
|--------|--------|---|
| out    | inform | is a struct containing output information (see <a href="bags">bqp_inform_type</a> )                             |
| out    | status | is a scalar variable of type int, that gives the exit status from the package. Possible values are (currently): |
|        |        | 0. The values were recorded succesfully   |

## Examples

bqpt.c, and bqptf.c.

## 3.1.2.9 bqp\_terminate()

## Deallocate all internal private storage

### **Parameters**

| in,out | data    | holds private internal data   |
|--------|---------|---|
| out    | control | is a struct containing control information (see <a href="bqp_control_type">bqp_control_type</a> )     |
| out    | inform  | is a struct containing output information (see <a href="mailto:bqp_inform_type">bqp_inform_type</a> ) |

#### **Examples**

bqpt.c, and bqptf.c.

## **Chapter 4**

## **Example Documentation**

## 4.1 bqpt.c

This is an example of how to use the package to solve a quadratic program. A variety of supported Hessian and constraint matrix storage formats are shown.

Notice that C-style indexing is used, and that this is flaggeed by setting control.f\_indexing to false.

```
#include <stdio.h>
#include <math.h>
#include "bqp.h"
int main (void) {
             // Derived types
            void *data;
             struct bqp_control_type control;
              struct bqp_inform_type inform;
              // Set problem data
            int n=10; // dimension int H_n=2 \times n-1; // Hesssian elements, NB lower triangle int H_n=10 int H_n=10
              int H_col[H_ne]; // column indices int H_ptr[n+1]; // row pointers double H_val[H_ne]; // values
            double H_val[H_ne]; // values
double H_dense[H_dense_ne]; // dense values
double H_diag[n]; // diagonal values
double g[n]; // linear term in the objective
double f = 1.0; // constant term in the objective
double x_l[n]; // variable lower bound
double x_u[n]; // variable upper bound
double x[n]; // variables
double z[n]; // dual variables
              // Set output storage
              int x_stat[n]; // variable status
              int i, 1, status;
              printf("H_dense_ne = %i\n", H_dense_ne);
              for( int i = 1; i < n; i++) g[i] = 0.0;
              for( int i = 1; i < n; i++) x_1[i] = - INFINITY;
              x_u[0] = 1.0;
x_u[1] = INFINITY;
             for( int i = 2; i < n; i++) x_u[i] = 2.0;
// H = tridiag(2,1), H_dense = diag(2)
              H_{ptr[0]} = 1;
              H_row[1] = 0; H_col[1] = 0; H_val[1] = 2.0;
              for( int i = 1; i < n; i++)</pre>
                    1 = 1 + 1;
                    H_ptr[i] = 1;
                    H_row[1] = i; H_col[1] = i - 1; H_val[1] = 1.0;
```

```
1 = 1 + 1;
  H_row[1] = i; H_col[1] = i; H_val[1] = 2.0;
H_ptr[n] = 1 + 1;
1 = -1;
for(int i = 0; i < n; i++)
  H_{diag[i]} = 2.0;
  for( int j = 0; j <= i; j++)</pre>
    1 = 1 + 1;
    if ( j < i - 1 ) {
      H_{dense[1]} = 0.0;
    else if ( j == i - 1 ) {
  H_dense[l] = 1.0;
    else {
      H_dense[1] = 2.0;
    }
 }
printf(" C sparse matrix indexing\n\n");
printf(" basic tests of bqp storage formats\n\n");
for( int d=1; d <= 4; d++){</pre>
    // Initialize BQP
    bqp_initialize( &data, &control, &status );
    // Set user-defined control options
    control.f_indexing = false; // C sparse matrix indexing
    // Start from 0
    for( int i = 0; i < n; i++) x[i] = 0.0;
for( int i = 0; i < n; i++) z[i] = 0.0;
    switch(d){
        case 1: // sparse co-ordinate storage
    st = 'C';
             bqp_import( &control, &data, &status, n,
             "coordinate", H_ne, H_row, H_col, NULL ); bqp_solve_given_h( &data, &status, n, H_ne, H_val, g, f,
                                  x_1, x_u, x, z, x_stat );
        break;
printf(" case %1i break\n",d);
case 2: // sparse by rows
    st = 'R';
             bqp_solve_given_h( &data, &status, n, H_ne, H_val, g, f,
                                  x_1, x_u, x, z, x_stat );
        break;
case 3: // dense
st = 'D';
             bqp_import( &control, &data, &status, n,
                            "dense", H_dense_ne, NULL, NULL, NULL);
             bqp_solve_given_h( &data, &status, n, H_dense_ne, H_dense,
                                  g, f, x_1, x_u, x, z, x_stat );
             break;
        case 4: // diagonal
st = 'L';
             bqp_import( &control, &data, &status, n,
                            "diagonal", H_ne, NULL, NULL, NULL);
             bqp\_solve\_given\_h( \&data, \&status, n, n, H\_diag, g, f,
                                  x_1, x_u, x, z, x_stat );
             break;
    bqp_information( &data, &inform, &status );
    if(inform.status == 0){
        printf("%c:%6i iterations. Optimal objective value = %5.2f status = %1i\n",
                st, inform.iter, inform.obj, inform.status);
    }else{
        printf("%c: BQP_solve exit status = %1i\n", st, inform.status);
     //printf("x: ");
    //for( int i = 0; i < n; i++) printf("%f ", x[i]);
//printf("\n");
    //printf("gradient: ");
    //printf("\n");
//printf("\n");
     // Delete internal workspace
    bqp_terminate( &data, &control, &inform );
printf("\n tests reverse-communication options\n\n");
// reverse-communication input/output
int nz_v_start, nz_v_end, nz_prod_end;
int nz_v[n], nz_prod[n], mask[n];
double v[n], prod[n];
nz\_prod\_end = 0;
// Initialize BOP
bop initialize ( &data, &control, &status );
```

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```
// control.print_level = 1;
// Set user-defined control options
 control.f_indexing = false; // C sparse matrix indexing
 // Start from 0
 for( int i = 0; i < n; i++) x[i] = 0.0;
for( int i = 0; i < n; i++) z[i] = 0.0;</pre>
 for( int i = 0; i < n; i++) mask[i] = 0;
 bqp_import_without_h( &control, &data, &status, n );
 while(true) { // reverse-communication loop
     bqp_solve_reverse_h_prod( &data, &status, n, g, f, x_l, x_u,
                                     x, z, x_stat, v, prod, nz_v, &nz_v_start, &nz_v_end,
                                     nz_prod, nz_prod_end );
     if(status == 0) { // successful termination
          break;
      }else if(status < 0){ // error exit.</pre>
         break;
      }else if(status == 2) { // evaluate Hv
       prod[0] = 2.0 * v[0] + v[1];
     fod(0] - 2.0 * v[0] + v[1];
for( int i = 1; i < n-1; i++) prod[i] = 2.0 * v[i] + v[i-1] + v[i+1];
prod[n-1] = 2.0 * v[n-1] + v[n-2];
}else if(status == 3) { // evaluate Hv for sparse v
for( int i = 0; i < n; i++) prod[i] = 0.0;
for( int l = nz_v_start - 1; l < nz_v_end; l++) {</pre>
          i = nz_v[1];
            if (i > 0) prod[i-1] = prod[i-1] + v[i];
           prod[i] = prod[i] + 2.0 * v[i];
if (i < n-1) prod[i+1] = prod[i+1] + v[i];</pre>
      }else if(status == 4){ // evaluate sarse Hv for sparse v
        nz_prod_end = 0;
        for( int 1 = nz_v_start - 1; 1 < nz_v_end; 1++) {</pre>
            i = nz_v[1];
            if (i > 0) {
              if (mask[i-1] == 0){
                mask[i-1] = 1;
                nz_prod[nz_prod_end] = i - 1;
                nz_prod_end = nz_prod_end + 1;
                prod[i-1] = v[i];
              }else{
                prod[i-1] = prod[i-1] + v[i];
              }
            if (mask[i] == 0) {
              mask[i] = 1;
              nz_prod[nz_prod_end] = i;
              nz_prod_end = nz_prod_end + 1;
prod[i] = 2.0 * v[i];
            }else{
             prod[i] = prod[i] + 2.0 * v[i];
            if (i < n-1) {
              if (mask[i+1] == 0){
                mask[i+1] = 1;
                nz prod[nz prod end] = i + 1;
                nz_prod_end = nz_prod_end + 1;
                prod[i+1] = prod[i+1] + v[i];
              }else{
               prod[i+1] = prod[i+1] + v[i];
             }
        for( int 1 = 0; 1 < nz_prod_end; 1++) mask[nz_prod[1]] = 0;</pre>
          printf(" the value %1i of status should not occur\n", status);
          break;
     }
 // Record solution information
 bqp_information( &data, &inform, &status );
 // Print solution details
 if(inform.status == 0){
     printf("%c:%6i iterations. Optimal objective value = %5.2f status = %1i\n",
              st, inform.iter, inform.obj, inform.status);
     printf("%c: BQP_solve exit status = %li\n", st, inform.status);
 //printf("x: ");
//for( int i = 0; i < n; i++) printf("%f ", x[i]);
//printf("\n");
 //printf("gradient: ");
 //for( int i = 0; i < n; i++) printf("%f ", g[i]);
//printf("\n");
 // Delete internal workspace
bqp_terminate( &data, &control, &inform );
```

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}

## 4.2 bqptf.c

This is the same example, but now fortran-style indexing is used.

```
/* Full test for the BQP C interface using fortran sparse matrix indexing */
#include <stdio.h>
#include <math.h>
#include "bqp.h'
int main(void) {
     // Derived types
     void *data;
     struct bqp_control_type control;
     struct bqp_inform_type inform;
     // Set problem data
int n = 10; // dimension
     int H_ne = 2 * n - 1; // Hesssian elements, NB lower triangle
     int H_dense_ne = n * (n + 1) / 2; // dense Hessian elements
int H_row[H_ne]; // row indices,
int H_col[H_ne]; // column indices
int H_ptr[n+1]; // row pointers
     double H_val[H_ne]; // values
     double H_dense[H_dense_ne]; // dense values
    double H_dense[H_dense_ne]; // dense values
double H_diag[n]; // diagonal values
double g[n]; // linear term in the objective
double f = 1.0; // constant term in the objective
double x_l[n]; // variable lower bound
double x_u[n]; // variable upper bound
double x[n]; // variables
double z[n]; // dual variables
     // Set output storage
     int x_stat[n]; // variable status
     char st;
     int i, 1, status;
printf("H_dense_ne = %i\n", H_dense_ne);
     g[0] = 2.0;
for( int i = 1; i < n; i++) g[i] = 0.0;
     x_1[0] = -1.0;
     for( int i = 1; i < n; i++) x_1[i] = - INFINITY;</pre>
     x_u[0] = 1.0;
x_u[1] = INFINITY;
     for ( int i = 2; i < n; i++) x_u[i] = 2.0;
     // H = tridiag(2,1), H_dense = diag(2)
     1 = 0:
     H_ptr[0] = 1 + 1;
H_row[1] = 1; H_col[1] = 1; H_val[1] = 2.0;
for( int i = 1; i < n; i++)</pre>
        H_ptr[i] = 1 + 1;
H_row[1] = i + 1; H_col[1] = i; H_val[1] = 1.0;
        1 = 1 + 1;
        H_row[1] = i + 1; H_col[1] = i + 1; H_val[1] = 2.0;
     H_ptr[n] = 1 + 2;
     for ( int i = 0; i < n; i++)
        H_{diag[i]} = 2.0;
        for( int j = 0; j <= i; j++)</pre>
          1 = 1 + 1;
           if ( j < i - 1 )
             H_{dense[1]} = 0.0;
           else if ( j == i - 1 ) {
            H_dense[1] = 1.0;
           else {
             H_{dense[1]} = 2.0;
           }
     printf(" fortran sparse matrix indexing\n\n"); printf(" basic tests of bqp storage formats\n');
     for( int d=1; d <= 4; d++) {
    // Initialize BQP
    bqp_initialize( &data, &control, &status );</pre>
           // Set user-defined control options
           control.f_indexing = true; // fortran sparse matrix indexing
           // Start from 0
           for( int i = 0; i < n; i++) x[i] = 0.0;
for( int i = 0; i < n; i++) z[i] = 0.0;
                 case 1: // sparse co-ordinate storage
```

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```
st = 'C';
              bqp_import( &control, &data, &status, n,
                            "coordinate", H_ne, H_row, H_col, NULL );
              bqp_solve_given_h( &data, &status, n, H_ne, H_val, g, f,
                                   x_1, x_u, x, z, x_stat );
              break;
         printf(" case %li break\n",d);
          case 2: // sparse by rows
st = 'R';
              break;
case 3: // dense
              st = 'D';
              g, f, x_1, x_u, x, z, x_stat );
          break;
case 4: // diagonal
st = 'L';
              x_1, x_u, x, z, x_stat );
     bqp_information( &data, &inform, &status );
     if(inform.status == 0){
         printf("%c:%6i iterations. Optimal objective value = %5.2f"
                  " status = %1i\n",
                 st, inform.iter, inform.obj, inform.status);
     }else{
         printf("%c: BQP_solve exit status = %1i\n", st, inform.status);
     //printf("x: ");
     //for( int i = 0; i < n; i++) printf("%f ", x[i]);
//printf("\n");
     //printf("gradient: ");
     //for( int i = 0; i < n; i++) printf("%f ", g[i]);
//printf("\n");
     // Delete internal workspace
     bqp_terminate( &data, &control, &inform );
 printf("\n tests reverse-communication options\n\n");
 // reverse-communication input/output
 int nz_v_start, nz_v_end, nz_prod_end;
int nz_v[n], nz_prod[n], mask[n];
 double v[n], prod[n];
nz_prod_end = 0;
 // Initialize BQP
bqp_initialize( &data, &control, &status );
// control.print_level = 1;
// Set user-defined control options
control.f_indexing = true; // fortran sparse matrix indexing
 // Start from 0
 for( int i = 0; i < n; i++) x[i] = 0.0;
for( int i = 0; i < n; i++) z[i] = 0.0;
st = 'I';
 for ( int i = 0; i < n; i++) mask[i] = 0;
 bqp_import_without_h( &control, &data, &status, n );
 while(true){ // reverse-communication loop
     bqp_solve_reverse_h_prod( &data, &status, n, g, f, x_l, x_u,
                                  x, z, x_stat, v, prod,
                                  nz_v, &nz_v_start, &nz_v_end,
                                  nz prod, nz prod end );
     if(status == 0){ // successful termination
         break;
     }else if(status < 0){ // error exit</pre>
         break;
     }else if(status == 2){ // evaluate Hv
       prod[0] = 2.0 * v[0] + v[1];
for( int i = 1; i < n-1; i++) prod[i] = 2.0 * v[i] + v[i-1] + v[i+1];
prod[n-1] = 2.0 * v[n-1] + v[n-2];</pre>
     }else if(status == 3) { // evaluate Hv for sparse v
       for( int i = 0; i < n; i++) prod[i] = 0.0;
for( int l = nz_v_start - 1; l < nz_v_end; l++){</pre>
          i = nz_v[1] - 1;
          if (i > 0) prod[i-1] = prod[i-1] + v[i];
prod[i] = prod[i] + 2.0 * v[i];
if (i < n-1) prod[i+1] = prod[i+1] + v[i];</pre>
        }
     }else if(status == 4){ // evaluate sarse Hv for sparse v
   nz_prod_end = 0;
       for( int 1 = nz_v_start - 1; 1 < nz_v_end; 1++) {</pre>
```

```
i = nz_v[1]-1;
          if (i > 0) {
  if (mask[i-1] == 0) {
               mask[i-1] = 1;
               nz_prod[nz_prod_end] = i - 1;
nz_prod_end = nz_prod_end + 1;
               prod[i-1] = v[i];
            prod[i-1] = prod[i-1] + v[i];
}
          if (mask[i] == 0) {
  mask[i] = 1;
             nz_prod[nz_prod_end] = i;
             nz_prod_end = nz_prod_end + 1;
prod[i] = 2.0 * v[i];
           lelset
            prod[i] = prod[i] + 2.0 * v[i];
           if (i < n-1) {
             if (mask[i+1] == 0){
               mask[i+1] = 1;
               nz_prod[nz_prod_end] = i + 1;
nz_prod_end = nz_prod_end + 1;
prod[i+1] = prod[i+1] + v[i];
            prod[i+1] = prod[i+1] + v[i];
}
       for( int 1 = 0; 1 < nz_prod_end; 1++) mask[nz_prod[1]] = 0;</pre>
     }else{
         printf(" the value %li of status should not occur\n", status);
    }
// Record solution information
bqp_information( &data, &inform, &status );
// Print solution details
if(inform.status == 0){
    }else{
    printf("%c: BQP_solve exit status = %li\n", st, inform.status);
//printf("x: ");
//for( int i = 0; i < n; i++) printf("%f ", x[i]);
//printf("\n");
//printf("(n");
//printf("gradient: ");
//for( int i = 0; i < n; i++) printf("%f ", g[i]);
//printf("\n");
// Delete internal workspace</pre>
bqp_terminate( &data, &control, &inform );
```

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