

# C interfaces to GALAHAD RQS

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# **Chapter 1**

# GALAHAD C package rqs

#### 1.1 Introduction

# 1.1.1 Purpose

Given real n by n symmetric matrices H and M (with M diagonally dominant), another real m by n matrix A, a real n vector c and scalars  $\sigma>0$ , p>2 and f, this package finds an approximate minimizer of the regularised quadratic objective function  $\frac{1}{2}x^THx+c^Tx+f+\frac{1}{p}\sigma\|x\|_M^p$ , where the vector x may additionally be required to satisfy Ax=0, and where the M-norm of x is  $\|x\|_M=\sqrt{x^TMx}$ .

This problem commonly occurs as a subproblem in nonlinear optimization calculations. The matrix M need not be provided in the commonly-occurring  $\ell_2$ -regularisation case for which M=I, the n by n identity matrix.

Factorization of matrices of the form  $H + \lambda M$ —or

$$(1) \quad \left( \begin{array}{cc} H + \lambda M & A^T \\ A & 0 \end{array} \right)$$

in cases where Ax=0 is imposed—for a succession of scalars  $\lambda$  will be required, so this package is most suited for the case where such a factorization may be found efficiently. If this is not the case, the GALAHAD package GLRT may be preferred.

#### 1.1.2 Authors

N. I. M. Gould and H. S. Thorne, STFC-Rutherford Appleton Laboratory, England, and D. P. Robinson, Oxford University, England.

C interface, additionally J. Fowkes, STFC-Rutherford Appleton Laboratory.

#### 1.1.3 Originally released

November 2008, C interface December 2021.

#### 1.1.4 Method

The required solution  $x_*$  necessarily satisfies the optimality condition  $Hx_* + \lambda_* Mx_* + A^Ty_* + c = 0$  and  $Ax_* = 0$ , where  $\lambda_* = \sigma \|x_*\|^{p-2}$  is a Lagrange multiplier corresponding to the regularisation and  $y_*$  are Lagrange multipliers for the linear constraints Ax = 0, if any. In addition in all cases, the matrix  $H + \lambda_* M$  will be positive semi-definite on the null-space of A; in most instances it will actually be positive definite, but in special 'hard' cases singularity is a possibility.

The method is iterative, and proceeds in two phases. Firstly, lower and upper bounds,  $\lambda_L$  and  $\lambda_U$ , on  $\lambda_*$  are computed using Gershgorin's theorems and other eigenvalue bounds. The first phase of the computation proceeds by progressively shrinking the bound interval  $[\lambda_L, \lambda_U]$  until a value  $\lambda$  for which  $\|x(\lambda)\|_M \ge \sigma \|x(\lambda)\|_M^{p-2}$  is found. Here  $x(\lambda)$  and its companion  $y(\lambda)$  are defined to be a solution of

(2) 
$$(H + \lambda M)x(\lambda) + A^T y(\lambda) = -c$$
 and  $Ax(\lambda) = 0$ .

Once the terminating  $\lambda$  from the first phase has been discovered, the second phase consists of applying Newton or higher-order iterations to the nonlinear 'secular" equation  $\|x(\lambda)\|_M = \sigma \|x(\lambda)\|_M^{p-2}$  with the knowledge that such iterations are both globally and ultimately rapidly convergent. It is possible in the 'hard" case that the interval in the first-phase will shrink to the single point  $\lambda_*$ , and precautions are taken, using inverse iteration with Rayleigh-quotient acceleration to ensure that this too happens rapidly.

The dominant cost is the requirement that we solve a sequence of linear systems (2). In the absence of linear constraints, an efficient sparse Cholesky factorization with precautions to detect indefinite  $H + \lambda M$  is used. If Ax = 0 is required, a sparse symmetric, indefinite factorization of (1) is used rather than a Cholesky factorization.

#### 1.1.5 Reference

The method is described in detail in

H. S. Dollar, N. I. M. Gould and D. P. Robinson. On solving trust-region and other regularised subproblems in optimization. Mathematical Programming Computation **2(1)** (2010) 21–57.

#### 1.1.6 Call order

To solve a given problem, functions from the rqs package must be called in the following order:

- rqs\_initialize provide default control parameters and set up initial data structures
- rqs\_read\_specfile (optional) override control values by reading replacement values from a file
- rqs\_import set up problem data structures and fixed values
- rqs\_import\_m (optional) set up problem data structures and fixed values for the scaling matrix M, if any
- rqs\_import\_a (optional) set up problem data structures and fixed values for the constraint matrix A, if any
- rqs\_reset\_control (optional) possibly change control parameters if a sequence of problems are being solved
- rqs\_solve\_problem solve the regularised quadratic problem
- rqs\_information (optional) recover information about the solution and solution process
- rqs\_terminate deallocate data structures

See Section 4.1 for examples of use.

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#### 1.1.7 Unsymmetric matrix storage formats

The unsymmetric m by n constraint matrix A may be presented and stored in a variety of convenient input formats.

Both C-style (0 based) and fortran-style (1-based) indexing is allowed. Choose control.f\_indexing as false for C style and true for fortran style; the discussion below presumes C style, but add 1 to indices for the corresponding fortran version.

Wrappers will automatically convert between 0-based (C) and 1-based (fortran) array indexing, so may be used transparently from C. This conversion involves both time and memory overheads that may be avoided by supplying data that is already stored using 1-based indexing.

#### 1.1.7.1 Dense storage format

The matrix A is stored as a compact dense matrix by rows, that is, the values of the entries of each row in turn are stored in order within an appropriate real one-dimensional array. In this case, component n\*i+j of the storage array A\_val will hold the value  $A_{ij}$  for  $0 \le i \le m-1$ ,  $0 \le j \le n-1$ .

#### 1.1.7.2 Sparse co-ordinate storage format

Only the nonzero entries of the matrices are stored. For the l-th entry,  $0 \le l \le ne-1$ , of A, its row index i, column index j and value  $A_{ij}$ ,  $0 \le i \le m-1$ ,  $0 \le j \le n-1$ , are stored as the l-th components of the integer arrays A\_row and A\_col and real array A\_val, respectively, while the number of nonzeros is recorded as A\_ne = ne.

#### 1.1.7.3 Sparse row-wise storage format

Again only the nonzero entries are stored, but this time they are ordered so that those in row i appear directly before those in row i+1. For the i-th row of A the i-th component of the integer array A\_ptr holds the position of the first entry in this row, while A\_ptr(m) holds the total number of entries plus one. The column indices j,  $0 \le j \le n-1$ , and values  $A_{ij}$  of the nonzero entries in the i-th row are stored in components I = A\_ptr(i), ..., A\_ptr(i+1)-1,  $0 \le i \le m-1$ , of the integer array A\_col, and real array A\_val, respectively. For sparse matrices, this scheme almost always requires less storage than its predecessor.

# 1.1.8 Symmetric matrix storage formats

Likewise, the symmetric n by n objective Hessian matrix H and scaling matrix M may be presented and stored in a variety of formats. But crucially symmetry is exploited by only storing values from the lower triangular part (i.e, those entries that lie on or below the leading diagonal). In what follows, we refer to H but this applies equally to M.

#### 1.1.8.1 Dense storage format

The matrix H is stored as a compact dense matrix by rows, that is, the values of the entries of each row in turn are stored in order within an appropriate real one-dimensional array. Since H is symmetric, only the lower triangular part (that is the part  $h_{ij}$  for  $0 \le j \le i \le n-1$ ) need be held. In this case the lower triangle should be stored by rows, that is component i\*i/2+j of the storage array H\_val will hold the value  $h_{ij}$  (and, by symmetry,  $h_{ji}$ ) for  $0 \le j \le i \le n-1$ .

#### 1.1.8.2 Sparse co-ordinate storage format

Only the nonzero entries of the matrices are stored. For the l-th entry,  $0 \le l \le ne-1$ , of H, its row index i, column index j and value  $h_{ij}$ ,  $0 \le j \le i \le n-1$ , are stored as the l-th components of the integer arrays H\_row and H\_col and real array H\_val, respectively, while the number of nonzeros is recorded as H\_ne = ne. Note that only the entries in the lower triangle should be stored.

#### 1.1.8.3 Sparse row-wise storage format

Again only the nonzero entries are stored, but this time they are ordered so that those in row i appear directly before those in row i+1. For the i-th row of H the i-th component of the integer array H\_ptr holds the position of the first entry in this row, while H\_ptr(n) holds the total number of entries plus one. The column indices j,  $0 \le j \le i$ , and values  $h_{ij}$  of the entries in the i-th row are stored in components I = H\_ptr(i), ..., H\_ptr(i+1)-1 of the integer array H\_col, and real array H\_val, respectively. Note that as before only the entries in the lower triangle should be stored. For sparse matrices, this scheme almost always requires less storage than its predecessor.

#### 1.1.8.4 Diagonal storage format

If H is diagonal (i.e.,  $H_{ij}=0$  for all  $0 \le i \ne j \le n-1$ ) only the diagonals entries  $H_{ii}$ ,  $0 \le i \le n-1$  need be stored, and the first n components of the array H\_val may be used for the purpose.

# **Chapter 2**

# File Index

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Here is a list of all files with brief descriptions:		
rqs.h		7

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# **Chapter 3**

# **File Documentation**

# 3.1 rqs.h File Reference

```
#include <stdbool.h>
#include "galahad_precision.h"
#include "sls.h"
#include "ir.h"
```

#### **Data Structures**

- struct rqs\_control\_type
- struct rqs\_time\_type
- struct rqs\_history\_type
- struct rqs\_inform\_type

#### **Functions**

- void rqs\_initialize (void \*\*data, struct rqs\_control\_type \*control, int \*status)
- void rqs\_read\_specfile (struct rqs\_control\_type \*control, const char specfile[])
- void rqs\_import (struct rqs\_control\_type \*control, void \*\*data, int \*status, int n, const char H\_type[], int H\_ne, const int H\_row[], const int H\_col[], const int H\_ptr[])
- void rqs\_import\_m (void \*\*data, int \*status, int n, const char M\_type[], int M\_ne, const int M\_row[], const int M\_col[], const int M\_ptr[])
- void rqs\_import\_a (void \*\*data, int \*status, int m, const char A\_type[], int A\_ne, const int A\_row[], const int A\_col[], const int A\_ptr[])
- void rgs reset control (struct rgs control type \*control, void \*\*data, int \*status)
- void rqs\_solve\_problem (void \*\*data, int \*status, int n, const real\_wp\_ power, const real\_wp\_ weight, const real\_wp\_ f, const real\_wp\_ c[], int H\_ne, const real\_wp\_ H\_val[], real\_wp\_ x[], int M\_ne, const real\_wp\_ M\_val[], int m, int A\_ne, const real\_wp\_ A\_val[], real\_wp\_ y[])
- void rgs information (void \*\*data, struct rgs inform type \*inform, int \*status)
- void rqs\_terminate (void \*\*data, struct rqs\_control\_type \*control, struct rqs\_inform\_type \*inform)

#### 3.1.1 Data Structure Documentation

#### 3.1.1.1 struct rqs\_control\_type

control derived type as a C struct

**Examples** 

rqst.c, and rqstf.c.

# Data Fields

bool	f_indexing	use C or Fortran sparse matrix indexing
int	error	unit for error messages
int	out	unit for monitor output
int	problem	unit to write problem data into file problem_file
int	print_level	controls level of diagnostic output
int	dense_factorization	should the problem be solved by dense factorization? Possible values are
		0 sparse factorization will be used
		1 dense factorization will be used
		other the choice is made automatically depending on the dimension and sparsity
int	new_h	how much of ${\cal H}$ has changed since the previous call. Possible values are
		0 unchanged
		1 values but not indices have changed
		2 values and indices have changed
int	new_m	how much of ${\cal M}$ has changed since the previous call. Possible values are
		0 unchanged
		1 values but not indices have changed
		2 values and indices have changed
int	new_a	how much of $A$ has changed since the previous call. Possible values are 0 unchanged 1 values but not
		indices have changed 2 values and indices have changed
int	max_factorizations	the maximum number of factorizations (=iterations) allowedve implies no limit
int	inverse_itmax	the number of inverse iterations performed in the "maybe hard" case
int	taylor_max_degree	maximum degree of Taylor approximant allowed
real_wp_	initial_multiplier	initial estimate of the Lagrange multipler
real_wp_	lower	lower and upper bounds on the multiplier, if known
real_wp_	upper	see lower
real_wp_	stop_normal	$\begin{split} & \text{stop when }  \ x\  - (multiplier/\sigma)^(1/(p-2))  \leq \\ & \text{stop\_normal } * \max \\ & (\ x\ , (multiplier/\sigma)^(1/(p-2))) \text{ REAL ( KIND =} \\ & \text{wp ) } :: \text{stop\_normal = epsmch } ** 0.75 \end{split}$
real_wp_	stop_hard	stop when bracket on optimal multiplier <= stop_hard * max( bracket ends ) REAL ( KIND = wp ) :: stop_hard = epsmch ** 0.75
real_wp_	start_invit_tol	start inverse iteration when bracket on optimal multiplier <= stop_start_invit_tol * max( bracket ends )

#### **Data Fields**

real_wp_	start_invitmax_tol	start full inverse iteration when bracket on multiplier <= stop_start_invitmax_tol * max( bracket ends)
bool	use_initial_multiplier	ignore initial_multiplier?
bool	initialize_approx_eigenvector	should a suitable initial eigenvector should be chosen or should a previous eigenvector may be used?
bool	space_critical	if space is critical, ensure allocated arrays are no bigger than needed
bool	deallocate_error_fatal	exit if any deallocation fails
char	problem_file[31]	name of file into which to write problem data
char	symmetric_linear_solver[31]	symmetric (indefinite) linear equation solver
char	definite_linear_solver[31]	definite linear equation solver
char	prefix[31]	all output lines will be prefixed by prefix(2:LEN(TRIM(.prefix))-1) where prefix contains the required string enclosed in quotes, e.g. "string" or 'string'
struct sls_control_type	sls_control	control parameters for the Cholesky factorization and solution (see sls_c documentation)
struct ir_control_type	ir_control	control parameters for iterative refinement (see ir_c documentation)

# 3.1.1.2 struct rqs\_time\_type

time derived type as a C struct

#### **Data Fields**

real_wp_	total	total CPU time spent in the package
real_wp_	assemble	CPU time spent building $H + \lambda M$ .
real_wp_	analyse	CPU time spent reordering $H + \lambda M$ prior to factorization.
real_wp_	factorize	CPU time spent factorizing $H + \lambda M$ .
real_wp_	solve	CPU time spent solving linear systems inolving $H + \lambda M$ .
real_wp_	clock_total	total clock time spent in the package
real_wp_	clock_assemble	clock time spent building $H+\lambda M$
real_wp_	clock_analyse	clock time spent reordering $H + \lambda M$ prior to factorization
real_wp_	clock_factorize	clock time spent factorizing $H + \lambda M$
real_wp_	clock_solve	clock time spent solving linear systems inolving $H + \lambda M$

# 3.1.1.3 struct rqs\_history\_type

history derived type as a C struct

### **Data Fields**

real_wp_	lambda	the value of $\lambda$
real_wp_	x_norm	the corresponding value of $\ x(\lambda)\ _M$

# 3.1.1.4 struct rqs\_inform\_type

inform derived type as a C struct

# Examples

rqst.c, and rqstf.c.

#### **Data Fields**

int	status	reported return status:
		0 the solution has been found
		<ul> <li>-1 an array allocation has failed</li> </ul>
		<ul> <li>-2 an array deallocation has failed</li> </ul>
		• -3 n and/or sigma is not positive and/or p $<$ = 2
		9 the analysis phase of the factorization of $H + \lambda M$ failed
		• -10 the factorization of $H + \lambda M$ failed
		15 $M$ does not appear to be strictly diagonally dominant
		-16 ill-conditioning has prevented furthr progress
int	alloc_status	STAT value after allocate failure.
int	factorizations	the number of factorizations performed
int	max_entries_factors	the maximum number of entries in the factors
int	len_history	the number of $(\ x\ _M,\lambda)$ pairs in the history
real_wp_	obj	the value of the quadratic function
real_wp_	obj_regularized	the value of the regularized quadratic function
real_wp_	x_norm	the $M$ -norm of $x$ , $\ x\ _M$
real_wp_	multiplier	the Lagrange multiplier corresponding to the regularization
real_wp_	pole	a lower bound max $(0,-\lambda_1)$ , where $\lambda_1$ is the left-most eigenvalue of $(H,M)$
bool	dense_factorization	was a dense factorization used?
bool	hard_case	has the hard case occurred?
char	bad_alloc[81]	name of array which provoked an allocate failure
struct rqs_time_type	time	time information
struct rqs_history_type	history[100]	history information
struct sls_inform_type	sls_inform	cholesky information (see sls_c documentation)
struct ir_inform_type	ir_inform	iterative_refinement information (see ir_c documentation)

# 3.1.2 Function Documentation

#### 3.1.2.1 rqs\_initialize()

Set default control values and initialize private data

#### **Parameters**

in,out	data	holds private internal data	
out	control	ntrol is a struct containing control information (see rqs_control_type)	
out	out status is a scalar variable of type int, that gives the exit status from the package. Possible values are (currently):		
		0. The import was succesful.	

#### **Examples**

rqst.c, and rqstf.c.

#### 3.1.2.2 rqs\_read\_specfile()

Read the content of a specification file, and assign values associated with given keywords to the corresponding control parameters

#### **Parameters**

in,out	control	is a struct containing control information (see rqs_control_type)
in	specfile	is a character string containing the name of the specification file

#### 3.1.2.3 rqs\_import()

```
const int H_row[],
const int H_col[],
const int H_ptr[])
```

Import problem data into internal storage prior to solution.

#### **Parameters**

		is a stance where an amplication of the stance of the stan	
in	control	is a struct whose members provide control paramters for the remaining proedures (see rqs_control_type)	
in,out	data	holds private internal data	
in,out	status	is a scalar variable of type int, that gives the exit status from the package. Possible values are:  • 0. The import was succesful	
		<ul> <li>-1. An allocation error occurred. A message indicating the offending array is written on unit control.error, and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>	
		<ul> <li>-2. A deallocation error occurred. A message indicating the offending array is written on unit control.error and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>	
		• -3. The restrictions $n>0$ and $m>0$ or requirement that a type contains its relevant string 'dense', 'coordinate', 'sparse_by_rows', diagonal' or 'identity' has been violated.	
in	n	is a scalar variable of type int, that holds the number of rows (and columns) of H.	
in	H_type	is a one-dimensional array of type char that specifies the symmetric storage scheme used for the Hessian, $H$ . It should be one of 'coordinate', 'sparse_by_rows', 'dense', or 'diagonal'; lower or upper case variants are allowed.	
in	H_ne	is a scalar variable of type int, that holds the number of entries in the lower triangular part of $H$ in the sparse co-ordinate storage scheme. It need not be set for any of the other schemes.	
in	H_row	is a one-dimensional array of size $H_ne$ and type int, that holds the row indices of the lower triangular part of $H$ in the sparse co-ordinate storage scheme. It need not be set for any of the other three schemes, and in this case can be NULL.	
in	H_col	is a one-dimensional array of size $H_ne$ and type int, that holds the column indices of the lower triangular part of $H$ in either the sparse co-ordinate, or the sparse row-wise storage scheme. It need not be set when the dense or diagonal storage schemes are used, and in this case can be NULL.	
in	H_ptr	is a one-dimensional array of size $n+1$ and type int, that holds the starting position of each row of the lower triangular part of $H$ , as well as the total number of entries plus one, in the sparse row-wise storage scheme. It need not be set when the other schemes are used, and in this case can be NULL.	

# Examples

rqst.c, and rqstf.c.

#### 3.1.2.4 rqs\_import\_m()

```
void rqs_import_m (
    void ** data,
    int * status,
    int n,
    const char M_type[],
    int M_ne,
    const int M_row[],
    const int M_col[],
    const int M_ptr[])
```

Import data for the scaling matrix M into internal storage prior to solution.

#### **Parameters**

in,out	data	holds private internal data	
in,out	status	is a scalar variable of type int, that gives the exit status from the package. Possible values are:	
		0. The import was succesful	
		<ul> <li>-1. An allocation error occurred. A message indicating the offending array is written on unit control.error, and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>	
		<ul> <li>-2. A deallocation error occurred. A message indicating the offending array is written on unit control.error and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>	
		<ul> <li>-3. The restrictions n &gt; 0 and m &gt; 0 or requirement that a type contains its relevant string 'dense', 'coordinate', 'sparse_by_rows', diagonal' or 'identity' has been violated.</li> </ul>	
in	n	is a scalar variable of type int, that holds the number of rows (and columns) of M.	
in	M_type	is a one-dimensional array of type char that specifies the symmetric storage scheme used for the scaling matrix, $M$ . It should be one of 'coordinate', 'sparse_by_rows', 'dense', or 'diagonal'; lower or upper case variants are allowed.	
in	M_ne	is a scalar variable of type int, that holds the number of entries in the lower triangular part of $M$ in the sparse co-ordinate storage scheme. It need not be set for any of the other schemes.	
in	M_row	is a one-dimensional array of size M_ne and type int, that holds the row indices of the lower triangular part of $M$ in the sparse co-ordinate storage scheme. It need not be set for any of the other three schemes, and in this case can be NULL.	
in	M_col	is a one-dimensional array of size M_ne and type int, that holds the column indices of the lower triangular part of $M$ in either the sparse co-ordinate, or the sparse row-wise storage scheme. It need not be set when the dense, diagonal or identity storage schemes are used, and in this case can be NULL.	
in	M_ptr	is a one-dimensional array of size n+1 and type int, that holds the starting position of each row of the lower triangular part of $M$ , as well as the total number of entries plus one, in the sparse row-wise storage scheme. It need not be set when the other schemes are used, and in this case can be NULL.	

# Examples

rqst.c, and rqstf.c.

# 3.1.2.5 rqs\_import\_a()

Import data for the constraint matrix A into internal storage prior to solution.

#### **Parameters**

in,out	data	holds private internal data	
in,out	status	is a scalar variable of type int, that gives the exit status from the package. Possible values are:	
		0. The import was succesful	
		<ul> <li>-1. An allocation error occurred. A message indicating the offending array is written on unit control.error, and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>	
		<ul> <li>-2. A deallocation error occurred. A message indicating the offending array is written on unit control.error and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>	
		<ul> <li>-3. The restrictions n &gt; 0 and m &gt; 0 or requirement that a type contains its relevant string 'dense', 'coordinate' or 'sparse_by_rows' has been violated.</li> </ul>	
in	m	is a scalar variable of type int, that holds the number of general linear constraints, i.e., the number of rows of A, if any. m must be non-negative.	
in	A_type	is a one-dimensional array of type char that specifies the unsymmetric storage scheme used for the constraint Jacobian, $A$ if any. It should be one of 'coordinate', 'sparse_by_rows' or 'dense'; lower or upper case variants are allowed.	
in	A_ne	is a scalar variable of type int, that holds the number of entries in $A$ , if used, in the sparse co-ordinate storage scheme. It need not be set for any of the other schemes.	
in	A_row	is a one-dimensional array of size A_ne and type int, that holds the row indices of $A$ in the sparse co-ordinate storage scheme. It need not be set for any of the other schemes, and in this case can be NULL.	
in	A_col	is a one-dimensional array of size A_ne and type int, that holds the column indices of $A$ in either the sparse co-ordinate, or the sparse row-wise storage scheme. It need not be set when the dense or diagonal storage schemes are used, and in this case can be NULL.	
in	A_ptr	is a one-dimensional array of size $n+1$ and type int, that holds the starting position of each row of $A$ , as well as the total number of entries plus one, in the sparse row-wise storage scheme. It need not be set when the other schemes are used, and in this case can be NULL.	

#### Examples

```
rqst.c, and rqstf.c.
```

### 3.1.2.6 rqs\_reset\_control()

Reset control parameters after import if required.

#### **Parameters**

in	control	is a struct whose members provide control paramters for the remaining prcedures (see rqs_control_type)	
in,out	data	holds private internal data	
in,out	status	is a scalar variable of type int, that gives the exit status from the package. Possible values are:	
		0. The import was succesful.	

#### 3.1.2.7 rqs\_solve\_problem()

```
void rqs_solve_problem (
             void ** data,
             int * status,
             int n,
             const real_wp_ power,
             const real_wp_ weight,
             const real_wp_ f,
             const real_wp_ c[],
             int H_ne,
             const real_wp_ H_val[],
             real_wp_ x[],
             int M_ne,
             const real_wp_ M_val[],
             int m_{*}
             int A_ne,
             const real_wp_ A_val[],
             real_wp_ y[] )
```

Solve the regularised quadratic problem.

#### **Parameters**

in,out	data	holds private internal data
--------	------	-----------------------------

# **Parameters**

in,out	status	is a scalar variable of type int, that gives the entry and exit status from the package.  On initial entry, status must be set to 1.  Possible exit are:
		0. The run was succesful.
		<ul> <li>-1. An allocation error occurred. A message indicating the offending array is written on unit control.error, and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>
		<ul> <li>-2. A deallocation error occurred. A message indicating the offending array is written on unit control.error and the returned allocation status and a string containing the name of the offending array are held in inform.alloc_status and inform.bad_alloc respectively.</li> </ul>
		<ul> <li>-3. The restrictions n &gt; 0, power &gt; 2, weight &gt; 0 and m &gt; 0 or requirement that a type contains its relevant string 'dense', 'coordinate', 'sparse_by_rows', 'diagonal' or 'identity' has been violated.</li> </ul>
		<ul> <li>-9. The analysis phase of the factorization of the matrix (1) failed.</li> </ul>
		-10. The factorization of the matrix (1) failed.
		-15. The matrix M appears not to be diagonally dominant.
		-16. The problem is so ill-conditioned that further progress is impossible.
		<ul> <li>-18. Too many factorizations have been required. This may happen if control.max factorizations is too small, but may also be symptomatic of a badly scaled problem.</li> </ul>
in	n	is a scalar variable of type int, that holds the number of variables.
in	power	is a scalar of type double, that holds the order of regularisation, $p$ , used. power must be no smaller than 2.
in	weight	is a scalar of type double, that holds the regularisation weight, $\sigma$ , used. weight must be strictly positive.
in	С	is a one-dimensional array of size n and type double, that holds the linear term $c$ of the objective function. The j-th component of c, j = 0,, n-1, contains $c_j$ .
in	f	is a scalar of type double, that holds the constant term $f$ of the objective function.
in	H_ne	is a scalar variable of type int, that holds the number of entries in the lower triangular part of the Hessian matrix ${\cal H}.$
in	H_val	is a one-dimensional array of size h_ne and type double, that holds the values of the entries of the lower triangular part of the Hessian matrix ${\cal H}$ in any of the available storage schemes.
out	X	is a one-dimensional array of size n and type double, that holds the values $x$ of the optimization variables. The j-th component of x, j = 0,, n-1, contains $x_j$ .
in	M_ne	is a scalar variable of type int, that holds the number of entries in the scaling matrix $M$ if it not the iedntity matrix.
in	M_val	is a one-dimensional array of size M_ne and type double, that holds the values of the entries of the scaling matrix $M$ , if it is not the identity matrix, in any of the available storage schemes. If M_val is NULL, M will be taken to be the identity matrix.
in	m	is a scalar variable of type int, that holds the number of general linear constraints, if any. m must be non-negative.
in	A_ne	is a scalar variable of type int, that holds the number of entries in the constraint Jacobian matrix $A$ if used. A_ne must be non-negative.

#### **Parameters**

in	A_val	is a one-dimensional array of size $A$ _ne and type double, that holds the values of the entries of the constraint Jacobian matrix $A$ , if used, in any of the available storage schemes. If $A$ val is $A$ 0 values of the entries of t	
out	у	is a one-dimensional array of size n and type double, that holds the values $y$ of the Lagrange multipliers for the equality constraints $Ax=0$ if used. The i-th component of y, i = 0,, m-1, contains $y_i$ .	

#### Examples

rqst.c, and rqstf.c.

# 3.1.2.8 rqs\_information()

#### Provides output information

#### **Parameters**

in,out	data	holds private internal data	
out	inform	is a struct containing output information (see rqs_inform_type)	
out	status	is a scalar variable of type int, that gives the exit status from the package. Possible values are (currently):	
		0. The values were recorded succesfully	

# Examples

rqst.c, and rqstf.c.

# 3.1.2.9 rqs\_terminate()

Deallocate all internal private storage

#### **Parameters**

in,	out	data	holds private internal data
out		control	is a struct containing control information (see rqs_control_type)
out		inform	is a struct containing output information (see rqs_inform_type)

# Examples

rqst.c, and rqstf.c.

# **Chapter 4**

# **Example Documentation**

# 4.1 rqst.c

This is an example of how to use the package to solve a regularised quadratic problem. A variety of supported Hessian, scaling and constraint matrix storage formats are shown.

Notice that C-style indexing is used, and that this is flaggeed by setting control.f\_indexing to false.

```
/* rqst.c */
/* Full test for the RQS C interface using C sparse matrix indexing \star/
#include <stdio.h>
#include <string.h>
#include <math.h>
#include "rqs.h"
int main(void) {
     // Derived types
      void *data;
      struct rqs_control_type control;
      struct rqs_inform_type inform;
      // Set problem data
     int n = 3; // dimension of H int m = 1; // dimension of A
      int H_ne = 4; // number of elements of H
int M_ne = 3; // number of elements of M
      int A_ne = 3; // number of elements of A
      int H_dense_ne = 6; // number of elements of H
int M_dense_ne = 6; // number of elements of M
      int H_row[] = {0, 1, 2, 2}; // row indices, NB lower triangle int H_col[] = {0, 1, 2, 0};
      int H_ptr[] = {0, 1, 2, 4};
int M_row[] = {0, 1, 2}; // row indices, NB lower triangle
      int M_{col[]} = \{0, 1, 2\};
      int M_ptr[] = {0, 1, 2, 3};
int A_row[] = {0, 0, 0};
      int A_col[] = {0, 1, 2};
      int A_ptr[] = \{0, 3\};
      double H_val[] = {1.0, 2.0, 3.0, 4.0};
double M_val[] = {1.0, 2.0, 1.0};
     double A_val[] = {1.0, 1.0, 1.0};
double A_val[] = {1.0, 1.0, 1.0};
double H_dense[] = {1.0, 0.0, 2.0, 4.0, 0.0, 3.0};
double M_dense[] = {1.0, 0.0, 2.0, 0.0, 0.0, 1.0};
     double H_diag[] = {1.0, 0.0, 2.0};
double M_diag[] = {1.0, 2.0, 1.0};
      double f = 0.96;
     double power = 3.0;
double weight = 1.0;
double c[] = {0.0, 2.0, 0.0};
      char st;
      int status;
      double x[n];
     char ma[3];
printf(" C sparse matrix indexing\n\n");
     printf(" basic tests of storage formats\n\n");
for( int a_is=0; a_is <= 1; a_is++){ // add a linear constraint?</pre>
        for( int m_is=0; m_is <= 1; m_is++) { // include a scaling matrix?</pre>
```

```
if (a_is == 1 && m_is == 1 ) {
 strcpy(ma, "MA");
else if (a_is == 1) {
   strcpy(ma, "A ");
else if (m_is == 1) {
 strcpy(ma, "M ");
else {
 strcpy(ma, " ");
for( int storage_type=1; storage_type <= 4; storage_type++) {</pre>
  // Initialize RQS
  rqs_initialize( &data, &control, &status );
 // Set user-defined control options
control.f_indexing = false; // C sparse matrix indexing
  switch(storage_type){
     case 1: // sparse co-ordinate storage
    st = 'C';
          if (m_is == 1) {
           rqs_import_m( &data, &status, n,
                         "coordinate", M_ne, M_row, M_col, NULL );
          if (a_is == 1) {
           rqs_import_a( &data, &status, m, "coordinate", A_ne, A_row, A_col, NULL );
          // solve the problem
          if (a_is == 1 && m_is == 1 ) {
           rqs_solve_problem( &data, &status, n,
                             power, weight, f, c, H_ne, H_val, x, M_ne, M_val, m, A_ne, A_val, NULL);
         else if (a_is == 1) {
           rqs_solve_problem( &data, &status, n,
                             power, weight, f, c, H_ne, H_val, x,
                              0, NULL, m, A_ne, A_val, NULL);
         else if (m_is == 1) {
  rqs_solve_problem( &data, &status, n,
                              power, weight, f, c, H_ne, H_val, x,
                              M_ne, M_val, 0, 0, NULL, NULL );
         else {
           rqs_solve_problem( &data, &status, n,
                              power, weight, f, c, H_ne, H_val, x, 0, NULL, 0, 0, NULL, NULL);
         break;
     printf(" case %li break\n", storage_type );
case 2: // sparse by rows
st = 'R';
          // import the control parameters and structural data
         if (m_is == 1) {
           if (a_is == 1) {
            rqs_import_a( &data, &status, m,
                        "sparse_by_rows", A_ne, NULL, A_col, A_ptr );
          // solve the problem
          if (a_is == 1 && m_is == 1 ) {
           rqs_solve_problem( &data, &status, n,
                            power, weight, f, c, H_ne, H_val, x,
                              M_ne, M_val, m, A_ne, A_val, NULL );
          else if (a_is == 1) {
           rqs_solve_problem( &data, &status, n,
power, weight, f, c, H_ne, H_val, x,
                              0, NULL, m, A_ne, A_val, NULL);
          else if (m_is == 1) {
           else {
```

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```
break;
           case 3: // dense
   st = 'D';
   // import the control parameters and structural data
              if (m_is == 1) {
               if (a_is == 1) {
                // solve the problem
              if (a_is == 1 && m_is == 1 ) {
                rqs_solve_problem( &data, &status, n, power, weight, f, c, H_dense_ne, H_dense, x,
                                 M_dense_ne, M_dense, m, A_ne, A_val,
              else if (a_is == 1) {
                rqs_solve_problem( &data, &status, n, power, weight, f, c, H_dense_ne, H_dense, x, 0, NULL, m, A_ne, A_val, NULL);
              else if (m_is == 1) {
                rqs_solve_problem( &data, &status, n, power, weight,
                                 f, c, H_dense_ne, H_dense, x,
0, NULL, 0, 0, NULL, NULL);
              }
              break;
           case 4: // diagonal
st = 'L';
              \ensuremath{//} import the control parameters and structural data
              rqs_import( &control, &data, &status, n,
    "diagonal", H_ne, NULL, NULL, NULL);
              if (m_is == 1) {
               if (a_is == 1) {
                // solve the problem
               if (a_is == 1 && m_is == 1 ) {
                else if (a_is == 1) {
                else if (m_is == 1) {
                rqs_solve_problem( &data, &status, n, power, weight, f, c, n, H_diag, x,
                                 n, M_diag, 0, 0, NULL, NULL);
              else {
               break;
       rqs_information( &data, &inform, &status );
       printf("format %c%s: RQS_solve_problem exit status = %1i, f = %.2f\n",
       st, ma, inform.status, inform.obj_regularized);
//printf("x: ");
//for( int i = 0; i < n+m; i++) printf("%f ", x[i]);
// Delete internal workspace
}
       rqs_terminate( &data, &control, &inform );
```

# 4.2 rqstf.c

This is the same example, but now fortran-style indexing is used.

```
/* Full test for the RQS C interface using Fortran sparse matrix indexing */
#include <stdio.h>
#include <string.h>
#include <math.h>
#include "rqs.h"
int main(void) {
     // Derived types
    void *data:
    struct rqs_control_type control;
struct rqs_inform_type inform;
     // Set problem data
     int n = 3; // dimension of H int m = 1; // dimension of A
     int H_ne = 4; // number of elements of H int M_ne = 3; // number of elements of M int A_ne = 3; // number of elements of A
     int H_dense_ne = 6; // number of elements of H
     int M_dense_ne = 6; // number of elements of M
     int H_row[] = \{1, 2, 3, 3\}; // row indices, NB lower triangle int <math>H_col[] = \{1, 2, 3, 1\};
     int H_ptr[] = {1, 2, 3, 5};
int M_row[] = {1, 2, 3}; // row indices, NB lower triangle
int M_col[] = {1, 2, 3};
     int M_ptr[] = \{1, 2, 3, 4\};
     int A_{row[]} = \{1, 1, 1\};
    int A_row[] = {1, 1, 1};
int A_col[] = {1, 2, 3};
int A_ptr[] = {1, 4};
double H_val[] = {1.0, 2.0, 3.0, 4.0};
double M_val[] = {1.0, 2.0, 1.0};
double A_val[] = {1.0, 1.0, 1.0};
     double H_dense[] = {1.0, 0.0, 2.0, 4.0, 0.0, 3.0};
double M_dense[] = {1.0, 0.0, 2.0, 0.0, 0.0, 1.0};
    double H_diag[] = {1.0, 0.0, 2.0};
double M_diag[] = {1.0, 2.0, 1.0};
double f = 0.96;
     double power = 3.0;
     double weight = 1.0;
     double c[] = \{0.0, 2.0, 0.0\};
     char st;
     int status:
     double x[n];
     char ma[3];
    else if (a_is == 1) {
   strcpy(ma, "A ");
          else if (m_is == 1) {
   strcpy(ma, "M ");
          else {
            strcpy(ma, " ");
           for( int storage type=1; storage type <= 4; storage type++) {</pre>
            // Initialize RQS
             rqs_initialize( &data, &control, &status );
             // Set user-defined control options
control.f_indexing = true; // fortran sparse matrix indexing
             switch(storage_type){
                  case 1: // sparse co-ordinate storage
    st = 'C';
                        // import the control parameters and structural data rqs_import( &control, &data, &status, n,
                                        "coordinate", H_ne, H_row, H_col, NULL );
                        if (m_is == 1) {
                          rqs_import_m( &data, &status, n, "coordinate", M_ne, M_row, M_col, NULL );
                        if (a_is == 1) {
                           rqs_import_a( &data, &status, m,
                                              "coordinate", A_ne, A_row, A_col, NULL);
                        // solve the problem
                        if (a_is == 1 && m_is == 1 ) {
                           rqs_solve_problem( &data, &status, n,
```

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```
power, weight, f, c, H_ne, H_val, x,
                    M_ne, M_val, m, A_ne, A_val, NULL );
   else if (a_is == 1) {
    else if (m_is == 1) {
    else {
    rqs_solve_problem( &data, &status, n,
                   power, weight, f, c, H_ne, H_val, x,
0, NULL, 0, 0, NULL, NULL);
printf(" case %li break\n", storage_type );
case 2: // sparse by rows
   st = 'R';
// import the control parameters and structural data
   if (m_is == 1) {
    rqs_import_m( &data, &status, n,
               "sparse_by_rows", M_ne, NULL, M_col, M_ptr );
   if (a_is == 1) {
    // solve the problem
   if (a_is == 1 && m_is == 1 ) {
    else if (a_is == 1) {
    else if (m_is == 1) {
    rqs_solve_problem( &data, &status, n,
                   power, weight, f, c, H_ne, H_val, x, M_ne, M_val, 0, 0, NULL, NULL);
   else {
    break;
case 3: // dense
st = 'D';
   // import the control parameters and structural data
   rqs_import( &control, &data, &status, n,
    "dense", H_ne, NULL, NULL, NULL);
   if (m_is == 1) {
    if (a_is == 1) {
    // solve the problem
   if (a_is == 1 && m_is == 1 ) {
    M_dense_ne, M_dense, m, A_ne, A_val,
                    NULL );
   else if (a_is == 1) {
    rqs_solve_problem( &data, &status, n, power, weight,
                    f, c, H_dense_ne, H_dense, x,
0, NULL, m, A_ne, A_val, NULL);
   else if (m_is == 1) {
    rqs_solve_problem( &data, &status, n, power, weight, f, c, H_dense_ne, H_dense, x, M_dense_ne, M_dense, 0, 0, NULL, NULL );
   else (
     rgs_solve_problem( &data, &status, n, power, weight,
```

```
f, c, H_dense_ne, H_dense, x,
                   0, NULL, 0, 0, NULL, NULL);
      if (a_is == 1) {
         // solve the problem
if (a_is == 1 && m_is == 1 ) {
         else if (a_is == 1) {
  rqs_solve_problem( &data, &status, n,
                  power, weight, f, c, n, H_diag, x,
                   0, NULL, m, A_ne, A_val, NULL);
        else if (m_is == 1) {
         else {
         break;
    // Delete internal workspace
}
    rqs_terminate( &data, &control, &inform );
```

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