C interfaces to GALAHAD GLTR

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Chapter 1

GALAHAD C package gltr

1.1 Introduction

1.1.1 Purpose

Given real n by n symmetric matrices H and M (with M positive definite), a real n vector c and scalars $\Delta > 0$ and f_0, this package finds an **approximate minimizer of the quadratic objective function $\frac{1}{2}x^T Hx + c^T x + f_0$, where the vector x is required to satisfy the constraint $\|x\|_M \leq \Delta$**, and where the M-norm of x is $\|x\|_M = \sqrt{x^T Mx}$. This problem commonly occurs as a trust-region subproblem in nonlinear optimization calculations. The method may be suitable for large n as no factorization of H is required. Reverse communication is used to obtain matrix-vector products of the form Hz and $M^{-1}z$.

The package may also be used to solve the related problem in which x is instead required to satisfy the **equality constraint $\|x\|_M = \Delta$**.

1.1.2 Authors

N. I. M. Gould, STFC-Rutherford Appleton Laboratory, England.

C interface, additionally J. Fowkes, STFC-Rutherford Appleton Laboratory.

1.1.3 Originally released

April 1997, C interface December 2021.

1.1.4 Terminology

1.1.5 Method

The required solution x necessarily satisfies the optimality condition $Hx + \lambda Mx + c = 0$, where $\lambda \geq 0$ is a Lagrange multiplier corresponding to the constraint $\|x\|_M \leq \Delta$. In addition, the matrix $H + \lambda M$ will be positive definite.

The method is iterative. Starting with the vector $M^{-1}c$, a matrix of Lanczos vectors is built one column at a time so that the k-th column is generated during iteration k. These columns span a so-called Krylov space. The resulting n by k matrix Q_k has the property that $Q_k^T H Q_k = T_k$, where T_k is tridiagonal. An approximation to the required solution may then be expressed formally as

$$x_{k+1} = Q_k y_k,$$

where y_k solves the `'tridiagonal'' subproblem of minimizing

$$(1) \quad \frac{1}{2} y^T T_k y + \|c\|_{M^{-1}} e_1^T y \text{ subject to the constraint } \|y\|_2 \leq \Delta,$$

and where e_1 is the first unit vector.

If the solution to (1) lies interior to the constraint, the required solution x_{k+1} may simply be found as the k-th (preconditioned) conjugate-gradient iterate. This solution can be obtained without the need to access the whole matrix Q_k. These conjugate-gradient iterates increase in M-norm, and thus once one of them exceeds Δ in M-norm, the solution must occur on the constraint boundary. Thereafter, the solution to (1) is less easy to obtain, but an efficient inner iteration to solve (1) is nonetheless achievable because T_k is tridiagonal. It is possible to observe the optimality measure $\|Hx + \lambda Mx + c\|_{M^{-1}}$ without computing x_{k+1}, and thus without needing Q_k. Once this measure is sufficiently small, a second pass is required to obtain the estimate x_{k+1} from y_k. As this second pass is an additional expense, a record is kept of the optimal objective function values for each value of k, and the second pass is only performed so far as to ensure a given fraction of the final optimal objective value. Large savings may be made in the second pass by choosing the required fraction to be significantly smaller than one.

A cheaper alternative is to use the Steihuag-Toint strategy, which is simply to stop at the first boundary point encountered along the piecewise linear path generated by the conjugate-gradient iterates. Note that if H is significantly indefinite, this strategy often produces a far from optimal point, but is effective when H is positive definite or almost

1.1.6 Reference

The method is described in detail in

N. I. M. Gould, S. Lucidi, M. Roma and Ph. L. Toint, Solving the trust-region subproblem using the Lanczos method. SIAM Journal on Optimization **9:2** (1999), 504-525.

1.1.7 Call order

To solve a given problem, functions from the gltr package must be called in the following order:

- gltr_initialize - provide default control parameters and set up initial data structures
- gltr_read_specfile (optional) - override control values by reading replacement values from a file
- gltr_import_control - import control parameters prior to solution
- gltr_solve_problem - solve the problem by reverse communication, a sequence of calls are made under control of a status parameter, each exit either asks the user to provide additional informaton and to re-enter, or reports that either the solution has been found or that an error has occurred
- gltr_information (optional) - recover information about the solution and solution process
- gltr_terminate - deallocate data structures

See Section 4.1 for an example of use.

Chapter 2

File Index

2.1 File List

Here is a list of all files with brief descriptions:

Chapter 3

File Documentation

3.1 gltr.h File Reference

```
#include <stdbool.h>
#include "galahad_precision.h"
```

Data Structures

- struct gltr_control_type
- struct gltr_inform_type

Functions

- void gltr_initialize (void **data, struct gltr_control_type *control, int *status)
- void gltr_read_specfile (struct gltr_control_type *control, const char specfile[ ])
- void gltr_import_control (struct gltr_control_type *control, void **data, int *status)
- void gltr_solve_problem (void **data, int *status, int n, const real_wp_ radius, real_wp_ x[ ], real_wp_ r[ ], real_wp_ vector[ ])
- void gltr_information (void **data, struct gltr_inform_type *inform, int *status)
- void gltr_terminate (void **data, struct gltr_control_type *control, struct gltr_inform_type *inform)

3.1.1 Data Structure Documentation

3.1.1.1 struct gltr_control_type

control derived type as a C struct

Examples

gltrt.c.

Data Fields

bool	f_indexing	use C or Fortran sparse matrix indexing
int	error	error and warning diagnostics occur on stream error
int	out	general output occurs on stream out
int	print_level	the level of output required is specified by print_level
int	itmax	the maximum number of iterations allowed (-ve = no bound)
int	Lanczos_itmax	the maximum number of iterations allowed once the boundary has been encountered (-ve = no bound)
int	extra_vectors	the number of extra work vectors of length n used
int	ritz_printout_device	the unit number for writing debug Ritz values
real_wp_	stop_relative	the iteration stops successfully when the gradient in the M(inverse) nor is smaller than max(stop_relative $*$ initial M(inverse) gradient norm, stop_absolute)
real_wp_	stop_absolute	see stop_relative
real_wp_	fraction_opt	an estimate of the solution that gives at least .fraction_opt times the optimal objective value will be found
real_wp_	f_min	the iteration stops if the objective-function value is lower than f_min
real_wp_	rminvr_zero	the smallest value that the square of the M norm of the gradient of the the objective may be before it is considered to be zero
real_wp_	f_0	the constant term, f_0, in the objective function
bool	unitm	is M the identity matrix ?
bool	steihaug_toint	should the iteration stop when the Trust-region is first encountered ?
bool	boundary	is the solution thought to lie on the constraint boundary ?
bool	equality_problem	is the solution required to lie on the constraint boundary ?
bool	space_critical	if .space_critical true, every effort will be made to use as little space as possible. This may result in longer computation time
bool	deallocate_error_fatal	if .deallocate_error_fatal is true, any array/pointer deallocation error will terminate execution. Otherwise, computation will continue
bool	print_ritz_values	should the Ritz values be written to the debug stream?
char	ritz_file_name[31]	name of debug file containing the Ritz values
char	prefix[31]	all output lines will be prefixed by .prefix(2:LEN(TRIM(.prefix))-1) where .prefix contains the required string enclosed in quotes, e.g. "string" or 'string'

3.1.1.2 struct gltr_inform_type

inform derived type as a C struct

Examples

gltrt.c.

Data Fields

int	status	return status. See gltr_solve_problem for details
int	alloc_status	the status of the last attempted allocation/deallocation
char	bad_alloc[81]	the name of the array for which an allocation/deallocation error ocurred
int	iter	the total number of iterations required

Data Fields

int	iter_pass2	the total number of pass-2 iterations required if the solution lies on the trust-region boundary
real_wp_	obj	the value of the quadratic function
real_wp_	multiplier	the Lagrange multiplier corresponding to the trust-region constraint
real_wp_	mnormx	the M-norm of x
real_wp_	piv	the latest pivot in the Cholesky factorization of the Lanczos tridiagona
real_wp_	curv	the most negative cuurvature encountered
real_wp_	rayleigh	the current Rayleigh quotient
real_wp_	leftmost	an estimate of the leftmost generalized eigenvalue of the pencil (H, M)
bool	negative_curvature	was negative curvature encountered ?
bool	hard_case	did the hard case occur ?

3.1.2 Function Documentation

3.1.2.1 gltr_initialize()

```
void gltr_initialize (
            void ** data,
            struct gltr_control_type * control,
            int * status )
```

Set default control values and initialize private data

Parameters

in,out	*data*	holds private internal data
out	*control*	is a struct containing control information (see gltr_control_type)
out	*status*	is a scalar variable of type int, that gives the exit status from the package. Possible values are (currently): • 0. The import was succesful.

Examples

gltrt.c.

3.1.2.2 gltr_read_specfile()

```
void gltr_read_specfile (
            struct gltr_control_type * control,
            const char specfile[] )
```

Read the content of a specification file, and assign values associated with given keywords to the corresponding control parameters

Parameters

in,out	*control*	is a struct containing control information (see gltr_control_type)
in	*specfile*	is a character string containing the name of the specification file

3.1.2.3 gltr_import_control()

```
void gltr_import_control (
            struct gltr_control_type * control,
            void ** data,
            int * status )
```

Import control parameters prior to solution.

Parameters

in	*control*	is a struct whose members provide control paramters for the remaining prcedures (see gltr_control_type)
in,out	*data*	holds private internal data
in,out	*status*	is a scalar variable of type int, that gives the exit status from the package. Possible values are (currently): • 1. The import was succesful, and the package is ready for the solve phase

Examples

gltrt.c.

3.1.2.4 gltr_solve_problem()

```
void gltr_solve_problem (
            void ** data,
            int * status,
            int n,
            const real_wp_ radius,
            real_wp_ x[ ],
            real_wp_ r[ ],
            real_wp_ vector[ ] )
```

Solve the trust-region problem using reverse communication.

Parameters

in,out	*data*	holds private internal data

Parameters

in,out	*status*	is a scalar variable of type int, that gives the entry and exit status from the package. This must be set to • 1. on initial entry. Set r (below) to c for this entry. • 4. the iteration is to be restarted with a smaller radius but with all other data unchanged. Set r (below) to c for this entry. Possible exit values are: • 0. the solution has been found • 2. the inverse of M must be applied to vector with the result returned in vector and the function re-entered with all other data unchanged. This will only happen if control.unitm is false • 3. the product $H *$ vector must be formed, with the result returned in vector and the function re-entered with all other data unchanged • 5. The iteration must be restarted. Reset r (below) to c and re-enter with all other data unchanged. This exit will only occur if control.steihaug_toint is false and the solution lies on the trust-region boundary • -1. an array allocation has failed • -2. an array deallocation has failed • -3. n and/or radius is not positive • -15. the matrix M appears to be indefinite • -18. the iteration limit has been exceeded • -30. the trust-region has been encountered in Steihaug-Toint mode • -31. the function value is smaller than control.f_min
in	*n*	is a scalar variable of type int, that holds the number of variables
in	*radius*	is a scalar of type double, that holds the trust-region radius, Δ, used. radius must be strictly positive
in,out	*x*	is a one-dimensional array of size n and type double, that holds the solution x. The j-th component of x, j = 0, ... , n-1, contains x_j.
in,out	*r*	is a one-dimensional array of size n and type double, that that must be set to c on entry (status = 1) and re-entry ! (status = 4, 5). On exit, r contains the resiual $Hx + c$.
in,out	*vector*	is a one-dimensional array of size n and type double, that should be used and reset appropriately when status = 2 and 3 as directed.

Examples

gltrt.c.

3.1.2.5 gltr_information()

```
void gltr_information (
            void ** data,
```

```
            struct gltr_inform_type * inform,
            int * status )
```

Provides output information

Parameters

in,out	*data*	holds private internal data
out	*inform*	is a struct containing output information (see gltr_inform_type)
out	*status*	is a scalar variable of type int, that gives the exit status from the package. Possible values are (currently): • 0. The values were recorded succesfully

Examples

gltrt.c.

3.1.2.6 gltr_terminate()

```
void gltr_terminate (
            void ** data,
            struct gltr_control_type * control,
            struct gltr_inform_type * inform )
```

Deallocate all internal private storage

Parameters

in,out	*data*	holds private internal data
out	*control*	is a struct containing control information (see gltr_control_type)
out	*inform*	is a struct containing output information (see gltr_inform_type)

Examples

gltrt.c.

Chapter 4

Example Documentation

4.1 gltrt.c

This is an example of how to use the package to solve a trus-region problem. A variety of supported Hessian, scaling and constraint matrix storage formats are shown.

```
/* gltrt.c */
/* Full test for the GLTR C interface */
#include <stdio.h>
#include <math.h>
#include "gltr.h"
int main(void) {
    // Derived types
    void *data;
    struct gltr_control_type control;
    struct gltr_inform_type inform;
    // Set problem data
    int n = 3; // dimension
    int status;
    double radius;
    double x[n];
    double r[n];
    double vector[n];
    double h_vector[n];
    // Initialize gltr
    gltr_initialize( &data, &control, &status );
    // use a unit M ?
    // for( int unit_m=0; unit_m <= 1; unit_m++){
    for( int unit_m=0; unit_m <= 0; unit_m++){
      if ( unit_m == 0 ){
        control.unitm = false;
      } else {
        control.unitm = true;
      }
      gltr_import_control( &control, &data, &status );
      printf("unitm %s\n", control.unitm ? "true" : "false");
      // resolve with a smaller radius ?
      // for( int new_radius=0; new_radius <= 1; new_radius++){
      for( int new_radius=0; new_radius <= 0; new_radius++){
        if ( new_radius == 0 ){
           radius = 1.0;
           status = 1;
        } else {
           radius = 0.1;
           status = 4;
        }
        for( int i = 0; i < n; i++) r[i] = 1.0;
        // iteration loop to find the minimizer
        while(true){ // reverse-communication loop
          gltr_solve_problem( &data, &status, n, radius, x, r, vector );
          printf(" status = %i\n", status );
          if ( status == 0 ) { // successful termination
              break;
          } else if ( status < 0 ) { // error exit
              break;
          } else if ( status == 2 ) { // form the preconditioned vector
```

```
          for( int i = 0; i < n; i++) vector[i] = vector[i] / 2.0;
        } else if ( status == 3 ) { // form the Hessian-vector product
          h_vector[0] =  2.0 * vector[0] + vector[1];
          for( int i = 1; i < n-1; i++){
            h_vector[i] = vector[i-1] + 2.0 * vector[i] + vector[i+1];
          }
          h_vector[n-1] = vector[n-2] + 2.0 * vector[n-1];
          for( int i = 0; i < n; i++) vector[i] = h_vector[i];
        } else if ( status == 5 ) { // restart
          for( int i = 0; i < n; i++) r[i] = 1.0;
        }else{
            printf(" the value %1i of status should not occur\n",
              status);
            break;
        }
      }
      gltr_information( &data, &inform, &status );
      printf("MR = %1i%1i gltr_solve_problem exit status = %i,"
           " f = %.2f\n", unit_m, new_radius, inform.status, inform.obj );
    }
  }
  // Delete internal workspace
  gltr_terminate( &data, &control, &inform );
}
```

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