Hemodialysis &

partial differential equation

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Course: MTH2245

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Abstract

- When the kidneys fail to remove sufficient impurities from the blood, a hemodialyzer for removing the impurities is used.

- Why did we pick this topic? The global prevalence of CKD was 9.1% (697.5 million cases). In 2017, CKD resulted in 1.2 million deaths and was the 12th leading cause of death worldwide. in 2017, CKD resulted in 35.8 million disability adjusted life years, so we wanted to go in depth of this serious issue and learn about all its aspects.

- The 1-D hemodialyzer uses the concept of the mass transfer between fluids we derived the equations till we reached a PDE expressed equations for this system. These equations are used to get the mass transfer coefficient (K_m) which indicates to the treatment cases.

- This coefficient (K_m) is obtained by solving the PDE system using a numerical solution called method of lines (MOL); at which we discretize the equations and convert them to ODE's.

- We found that as the mass transfer coefficient varies, the efficacy of the purification process varies accordingly. In this report we dive deeper into the implications of these variations.

1. Problem Definition

1.1. Chronic Kidney Disease Overview

Chronic kidney disease (CKD) is a condition in which the kidneys are damaged and cannot filter blood as well as they should. Because of this, excess fluid and waste from blood remain in the body.

CKD can get worse over time and eventually the kidneys may stop working altogether, but this is uncommon. Many people with CKD can live long lives with this condition.

1.2. Symptoms of CKD

There are usually no symptoms of kidney disease in the early stages. It may only be diagnosed if you have a blood or urine test for another reason and the results show a possible problem with your kidneys.

At a more advanced stage, symptoms can include:

- **Tiredness, swollen ankles, shortness of breath and blood in urine**

1.3. Statistics[1]

Worldwide, over 10% of people over twenty-year-old now have chronic kidney disease. The expected mortality (death rate) of an adult being on dialysis is 70% by five years, and 90% by 10 years.

The most recent available estimation for the prevalence of dialysis in Egypt is in 2019 and is reported to be 0.61 per 1,000 people with an incidence estimation of 0.192 per 1,000 people.

[1] Kidney360: Global Dialysis Perspectives (Egypt).

the burden of chronic kidney disease (CKD) has increased by (35.7%) in Egypt, ranking CKD as the 5th in leading causes of death from 2009 to 2019.

in 2020, Patients undergoing dialysis in Egypt were mostly males (58.7%) and half of them are aged 55 years and older.

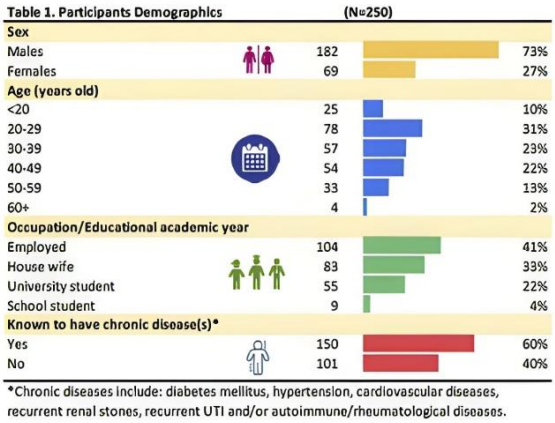


Table 1. Participants Demographics		(N=250)	
Sex			
Males		182	73%
Females		69	27%
Age (years old)			
<20		25	10%
20-29		78	31%
30-39		57	23%
40-49		54	22%
50-59		33	13%
60+		4	2%
Occupation/Educational academic year			
Employed		104	41%
House wife		83	33%
University student		55	22%
School student		9	4%
Known to have chronic disease(s)*			
Yes		150	60%
No		101	40%

*Chronic diseases include: diabetes mellitus, hypertension, cardiovascular diseases, recurrent renal stones, recurrent UTI and/or autoimmune/rheumatological diseases.

Fig. 1

The following study (Fig.1) included volunteers of different age groups and educational background who were not belonging to medical field. Results were expressed in numbers and percentages.

1.4. Treatment[2]

There's no cure for CKD, but treatment can help relieve the symptoms and stop it getting worse. The main treatments are:

- **lifestyle changes** – to help you remain as healthy as possible
- **medicine** – to control associated problems such as high blood pressure and high cholesterol
- **dialysis** – treatment to replicate some of the kidney's functions; this may be necessary in advanced CKD
- **kidney transplant** – this may also be necessary in advanced CKD

[2] CDC "Centers for Disease Control and prevention".

2. Literature Review

2.1. The historical basis of hemodialysis

The first scientific descriptions of dialysis procedures date back to the 19th century and were pioneered by Scottish chemist Thomas Graham, who became known as "the Father of Dialysis." Graham invented a process in which he used a solution to filter out impurities from another solution.

In 1858, Graham described his experiments with the use of cellulose as an insoluble fiber and how it could be used to clean up polluted water. Graham's work was expanded on by German scientist Rudolf Fitting, who developed a dialyzer that could be used to filter out the salts in the blood.

In 1913, Abel and Rowntree published a description of the first procedure involving blood dialysis. They used tubes made of cellulose-based material (called Collodion) to send anesthetized animals' blood outside their bodies—a practice later called hemodialysis.

2.2. Model #1- The movement of solutes across Semipermeable membranes.

The fundamental description of fluid and solute transport is usually based on linear non-equilibrium thermodynamics

The pore models that are widely used in physiology research and have been applied to peritoneal dialysis modeling have a wide range of applications. The pore models are based on a description of the membrane as a porous medium in which the pores are connected. In this case, the transport of solutes across the membrane is described by three-dimensional diffusion through a series of interconnected pores.

2.2.1. Thermodynamic model[3]

A new thermodynamic model is developed for water and solute transport through reverse osmosis membranes. This model describes the transport of water, as well as other solutes, through these membrane filters.

The model has solid theoretical development derivations and precisely established parameters for membrane transport characteristics.

The proposed model is capable of accurately describing the nonlinear relationship between water flux and pressure as well as the dependency of salt rejection on pressure and salt concentration.

The new model's parameters are concentration-independent, as shown by comparisons between simulations and the published reverse osmosis tests. This work shows that a system's inability to become more orderly under the application of energy can be applied to understand water and salt movement through reverse osmosis membranes.

2.2.2. Three-pore model[4]

The capillary membrane is considered the main barrier in the three-pore model of peritoneal transport because it controls how much solute can pass from the interstitial fluid into the peritoneal cavity.

According to the three-pore model, water and water-soluble substances travel through a protein-restricted pore pathway of radius 40–55 angstroms (A), accounting for roughly 99% of the total exchange area. This is also about 90% of all peritoneal ultrafiltration (UF) coefficient (LPs).

Proteins are restricted to so-called "large pores"—with a radius of around 250 A for their passage through the peritoneal membrane. They're incredibly rare (0.01% of the total pore population) and mostly

[3] Taylor and Francis: Chemical Engineering Communications
[4] National Library of Medicine: [https://pubmed.ncbi.nlm.nih.gov/8399608/]

unrestrictive in terms of protein transport, though some proteins may also be able to pass through smaller holes that have yet unfound as well.

In the three-pore hypothesis, a third channel called the "water-only" (transcellular) pathway is thought to make up only about 2% of total LP pores and be permeable to water but not solutes.

The three-pore model, in contrast to the traditional Pyle-Popovich (P&P) model, is able to predict with some degree of accuracy not only the transport of water and "small solutes" and "intermediate-size" solutes, but also the transport of albumin and larger molecules.

2.3. Model #2- Kinetic modeling in hemodialysis

In hemodialysis, blood flows through an out-of-body circuit and is cleaned in dialyzers made of synthetic membranes that separate the blood from chemicals. The treatment lasts about four hours every two or three days.

Mathematical modeling can be used to estimate the changes that take place in small metabolites (such as urea and creatinine) during hemodialysis and inter-dialysis periods. These estimates are then compared to measured values, allowing for the determination of adequacy indices such as Kt/V.

2.3.1 Urea Kinetic modeling

Urea kinetic modeling was originally proposed as a guide to optimally adjust the dialysis prescription in uremic patients by F. Gotch and J Sargent (1983).

In a recent report, the US National Cooperative Dialysis Study group demonstrated the power of this approach compared to conventional methods. They also reached conclusions about dialysis adequacy: three parameters—

The uremia (or Urea Time Averaged Concentration) was calculated from the Dialysis dose, defined as KT/V ratio and dietary protein intake. In spite of its potential usefulness, UKM has not gained clinical acceptance among nephrologists since it has always appeared complicated due to its mathematical formulation. It is concluded that UKM, by using very simple and basic parameters, is a practical and very powerful tool for assessing the dialysis adequacy and nutritional status of dialyzed patients.

The model accurately describes urea concentration as it exponentially decays during dialysis and also accounts for the phenomenon of urea rebound —the sudden spike in blood-urea levels observed following treatment.

This new model proposed the discrimination between organs with high and low blood flow, and it was shown to yield the same numerical description of urea removal as the standard two-compartment urea model. So far, no definitive answer has been found for the basic physiological mechanisms of urea transport during dialysis.

In addition, the new studies on the problem of dialysis dose did not find any benefit from an increased level of KT/V over that which is currently accepted as minimal for patients undergoing hemodialysis or peritoneal dialysis. These observations have increased interest in alternative dialysis adequacy parameters, such as fractional solute removal.

2.4. Model #3- Models of peritoneal transport[5]

Peritoneal transfusion is a procedure that uses the lining of the abdomen to filter toxins from the blood, unlike dialysis which takes place in an external device.

The process is done by adding a substance to the dialysate that travels in the bloodstream, causing high osmotic pressure within the peritoneal cavity and thus drawing excess water out of the blood.

A mathematical model may help in analyzing peritoneal transport in four aspects:

1- The separation of peritoneal transport components, such as water ultrafiltration from blood and absorption to tissue in fluid transport (along with convective transport) is separate from bulk absorption with absorbed fluid for the solute.

2- Quantitative correlation between flows and their driving forces, which are: osmotic pressure for fluid ultrafiltration, hydrostatic pressure for fluid absorption, concentration gradient for solute diffusion, ultrafiltrate flow for convective solute transport, and absorptive fluid flow for solute absorption; the correlations are described by the so-called transport parameters.

3- Quantitative relationship between the transport parameters for various solutes and between fluid and solute transport parameters.

4- Quantitative relationship between the structure and physiological state of peritoneal tissue and its transport characteristics.

As four aspects of peritoneal transport are considered in these models, the three main models applied for evaluation include the membrane model (objectives 1 and 2), the 3-pore model (objective 3), and distributed model (objective 4).

[5] Waniewski, J. (2006). Mathematical modeling of fluid and solute transport in hemodialysis and peritoneal dialysis. Journal of Membrane Science, 274(1-2), 24-37.

3. Mathematical Modelling

3.1. Hemodialyzer dynamics

When the kidneys fail to remove sufficient impurities from the blood, a device for removing the impurities is used, which is termed a **hemodialyzer**. Basically, it transfers the impurities from the blood to another fluid termed the dialysate by mass transfer through a membrane.

We now consider the derivation of a PDE model based on mass conservation.

The configuration of a 1D hemodialyzer model is explained in **(Fig. 2)**, primarily with words.

• The model is one dimensional (1D) with distance along the dialyzer, z, as the spatial (boundary value) independent variable. Time t is an initial value independent variable.

• Two PDE-dependent variables, $u_1(z,t)$, $u_2(z,t)$, represent the impurity concentrations in the blood and dialysate, respectively. The PDEs that define these dependent variables are derived subsequently.

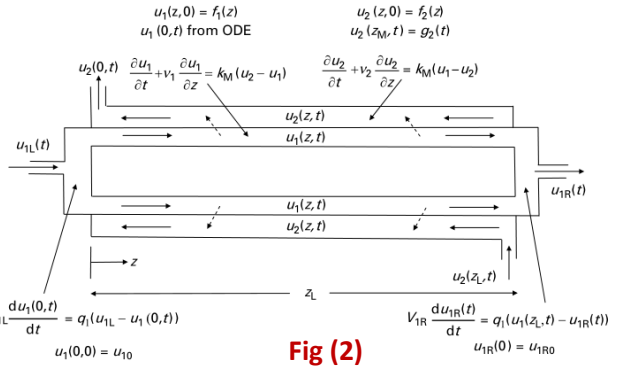


Fig (2)

• Blood enters the left end at concentration $u_{1L}(t)$. This BC is not designated as $u_1(z = 0,t)$ because of a header volume at the left end (explained next).

• Similarly, the exiting blood concentration at the right end is designated as $u_{1R}(t)$ rather than $u_1(Z = Z_L,t)$ (again, because of a header volume).

•The entering and exiting dialysate concentrations are $u_2(Z = Z_L,t)$ and $u_2(z = 0,t)$, respectively.

• The overall objective in formulating the model and computing numerical solutions is to determine $u_1(z,t)$,$u_2(z,t)$ and how effective the dialyzer is in reducing the exiting (outflow) blood concentration $u_{1R}(t)$ below its entering value $u_{1L}(t)$.

3.2. Mass Balance Equation

The mass balance equation simply states that the total mass in any system is always conserved. That is,

$\{rate\ of\ change\ in\ mass\} = \{rate\ of\ mass\ in\} - \{rate\ of\ mass\ out\} + \{rate\ of\ production\ of\ mass\}$

$$Accumulation = In - Out + Generation/Consumption$$

In this case, we'll assume our container has a square cross-section and the slice will have a thickness of Δx **(Fig.3)**. Based on our starting point, we know that there is no generation or consumption occurring but flux in through the slice (inflow) and out of it must still be accounted for. So,

$$Accumulation = In - Out$$

If you have more stuff coming in than going out, you will be accumulating stuff, and vice versa

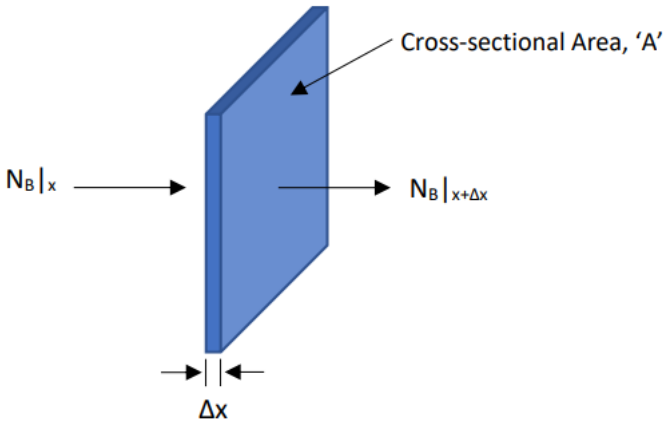


Fig. 3

Where N_i. This represents the number of mass/moles of 'i' that go through a unit area in a unit time.

$$A\Delta x\left[\frac{C_B|_{t+\Delta t} - C_B|_t}{\Delta t}\right] = AN_B|_x - AN_B|_{x+\Delta x}$$

Change in concentration in volume over a short increment of time

Flux-In at the surface x

Flux-Out at the surface x → Δx

divide both sides by Δx and take the limit as Δx → 0 and Δt → 0

$$\lim_{x\to 0}\left(\lim_{t\to 0} A\left[\frac{C_B|_{t+\Delta t}-C_B|_t}{\Delta t}\right]\right)=\lim_{x\to 0}\left(\lim_{t\to 0}\frac{AN_B|_x-AN_B|_{x+\Delta x}}{\Delta x}\right)$$

$$A\frac{\partial c_B}{\partial t}=-A\frac{\partial N_B}{\partial x}$$

3.3. Assumptions

I. Only impurities pass through the membrane and that the principal components of blood remain in the blood stream.

II. We will take v_2 and v_2 as independent of z so that it can be taken outside the derivative in z.

3.4. A mass balance on the blood gives

$$\epsilon A\Delta z\frac{\partial u_1}{\partial t}=\epsilon A v_1 u_1|_z-\epsilon A v_1 u_1|_{z+\Delta z}+A_M\Delta z k_M(u_2-u_1) \quad \textbf{(1)}$$

Variable	Interpretation
u_1	concentration of impurities in blood
u_2	concentration of impurities in dialyzate
t	time
z	axial position along the dialyzer
A	cross sectional area of dialyzer (transverse to z)
ϵ	fraction of A for blood flow
V_1	superficial velocity[6] of blood flow
A_M	area for mass transfer per unit length in z
K_M	membrane mass transfer coefficient

[6] **Superficial velocity** is a hypothetical flow velocity calculated as if the given phase or fluid were the **only one flowing** or present in each cross-sectional area. The velocity of the given phase is calculated as if the second phase was ignored.

As shown in figure (2):

$$\epsilon A \Delta z \frac{\partial u_1}{\partial t} = \epsilon A v_1 u_1|_z - \epsilon A v_1 u_1|_{z+\Delta z} + A_M \Delta z k_M (u_2 - u_1) \quad (1)$$

- **LHS-1**: $\epsilon A \Delta z \, (\partial u1 / \partial t)$ – Accumulation of impurities in an incremental volume
- **RHS-1**: $\epsilon A v1 u1|z$ – Flow of impurities into the incremental volume at z
- **RHS-2**: $-\epsilon A v1 u1|z + \Delta z$ –Flow of impurities out of the incremental volume at z $+ \Delta z$
- **RHS-3**: $+A_M \Delta z k_M \, (u2 - u1)$ – Mass transfer of impurities between blood and dialysate into or out of the incremental volume at z

If eq. **(1)** is divided by $\epsilon A \Delta z$,

$$\frac{\partial u_1}{\partial t} = -\frac{v_1 u_1|_{z+\Delta z} - v_1 u_1|_z}{\Delta z} + \frac{A_M \Delta z k_M}{\epsilon A}(u_2 - u_1)$$

For $\Delta z \rightarrow 0$

$$\frac{\partial u_1}{\partial t} = -\frac{\partial (v_1 u_1)}{\Delta z} + \frac{A_M k_M}{\epsilon A}(u_2 - u_1) \quad (2)$$

Equation **(2)** is the PDE for the calculation of $\mathbf{u_1(z, t)}$.

3.5. Mass Balance for the Dialysate

$$(1-\epsilon)\mathrm{A}\Delta\mathrm{z}\frac{\partial u_2}{\partial t} = (1-\epsilon)Av_2u_2|_z - (1-\epsilon)Av_2u_2|_{z+\Delta z} + \mathrm{A}_M\Delta z k_M(u_2-u_1).$$

Division by $(1-\epsilon)\,\mathrm{A}\Delta\mathrm{z}$ followed by $\Delta\mathrm{z} \rightarrow 0$ gives:

$$\frac{\boldsymbol{\partial u_2}}{\boldsymbol{\partial t}} = -\frac{\boldsymbol{\partial(v_2u_2)}}{\boldsymbol{\partial z}} + \frac{\boldsymbol{A_Mk_M}}{\boldsymbol{(1-\epsilon)A}}(\boldsymbol{u_1-u_2}) \quad \textbf{(3)}$$

Equation **(3)** is the PDE for the calculation of $\mathbf{u_2(z,t)}$.

3.6. Initial and Boundary Conditions

Equations **(2)** and **(3)** are first-order, hyperbolic (PDEs), which means that they have an initial condition and a boundary condition associated with each of their variable's, **t** and **z**.

The ICs are

$$\mathrm{u}_1(\mathrm{z},\mathrm{t}=0) = \mathrm{f}_1(\mathrm{z});\ \ \mathrm{u}_2(\mathrm{z},\mathrm{t}=0) = \mathrm{f}_2(\mathrm{z}).$$

The BCs

Because of header volumes which are a result of the design of the dialyzer, in each volume of blood, we assume complete mixing. Hence, the concentration is described by a first-order ordinary differential equation (ODE) in t (z variations due to flow conditions are not possible because perfect mixing assumption has been made).

- ODE for the left header:

$$\frac{V_{1L}du_{1R}(t)}{dt} = q_1(u_{1L}(t) - u_1(z=0,t); u_1(z=0,t=0) = u_{10}.$$

- ODE for the right header:

$$\frac{\mathrm{V}_1 du_{1R}(t)}{dt} = q_1\big(u_1(z=z_L,t) - u_{1R}(t)\big); u_{1R}(t=0) = u_{1R0}$$

- The BC for eq. **(3)** is

$$\mathrm{u}_2(z=z_L,t) = g_2(t)$$

Where $\mathrm{q}_1\ (\mathrm{m3/s}) = \mathrm{v}_1 \epsilon \mathrm{A}$ which relates the blood volumetric and linear flow rates.

3.7. Methodology of Solution:

- Applying the mass balance concept between Impurities concentration in blood and in dialysate to obtain formula of rate of change of impurities.

- Then using the MOL (method of lines) to solve the ODE/PDE system to obtain the mass transfer coefficient and then compute the numerical values of u1 and u2.

3.8. MOL Overview[7]

The method of lines (or MOL) is a technique for solving partial differential equations (PDEs), which are used to describe physical phenomena that occur in more than one dimension, by reducing it to a single continuous dimension. The MOL allows solutions be computed via methods and software developed.

To relate to PDE's: It's a method of analyzing partial differential equations in which spatial derivatives are first discretized and the time variable is left continuous. This results in a system of ordinary differential equations that can be solved using an initial value ordinary equation method.

[7] **Schiesser WE.** The Numerical Method of Lines Integration of Partial Differential Equations. San Diego: Academic Press; 1991.

3.9 Experimental work[8]

The mass transfer coefficient (Km) varies depending on the method of solution used. If the dialyzer's area remains constant, then its clearance (ϵ) (Fraction of Area available for blood flow ($0 \leq \epsilon \leq 1$)) will vary depending on how much blood flows through it. The more blood that passes through, the larger fraction of A is filled with blood.

By changing the amount of fluid entering the dialyzer which is blood (ϵ), then the dialysate will fill the remaining Area of the dialyzer (1- ϵ).

3.9.1 The effect of changing mass transfer coefficient (K_m):

- When $K_m = 0$: we expect that $U_{1R}(t) = U_2(0,t)$ (no mass transfer happens).
- When $K_m > 0$: we expect that $U_2(0,t) > U_{1R}(t)$.
- As K_m increases: $U_2(0,t)$ increases (directly proportional with K_m) and at the same time $U_{1R}(t)$ decreases (inversely proportional with K_m).

3.9.2 The effect of changing the diameter of dialyzer (D) (Assume K_m constant):

- When the diameter increases, the velocity of the fluid and thus the flow rate decreases, as a result, the period needed for purification increases.
- Conversely, when the diameter decreases, the velocity increases, as a result the period needed for purification will decrease, but each cycle won't perform as it would initially.
- As (D) decreases past a certain point, the velocity increases significantly, which impairs the purification process.

3.8.3 As the length of the device increases we will need to decrease the diameter to reach the same objective and vice versa.

[8] Considering all the parameters of the dialyzer are constants (same design of dialyzer.

3. Results & Analysis

4.1. Test cases and results[9][10]

Objective: determine $U_{1R}(t)$, $U_2(0, t)$

$$\frac{\partial u_1}{\partial t} = -\frac{\partial (v_1 u_1)}{\partial z} + \frac{A_M k_M}{\epsilon A}(u_2 - u_1).$$

$$\frac{\partial u_2}{\partial t} = -\frac{\partial (v_2 u_2)}{\partial z} + \frac{A_M k_M}{(1-\epsilon) A}(u_1 - u_2).$$

- We consider all the parameters of the dialyzer are constants (same design of dialyzer)

- Initial given data is as follows and further changes will be noted:

$$D = 5.0 \quad A = 19.6 \quad q_1 = 0..25 \quad q_2 = 0.25 \quad v_1 = 0.025 \quad v_2 = -0.025$$
$$u_{1L} = 1.000 \quad u_{2zL} = 1.00 \quad u_{10} = 0.00 \quad u_{20} = 0.00 \quad z_L = 50.0$$

Where,

- D (cm) = constant for geometry shape of machine equal to 4 cm
- A (cm^2) = $\pi * D^2$
- ϵ (dimensionless): fraction of A available for blood flow,
  Hence, $(1-\epsilon)$ is the fraction of A available for dialysate flow
- v_1 (cm^2/s) = $q_1/(\epsilon * A)$, v_2 (cm^2) = $-q_2/[(1-\epsilon) * A]$
- q_1, q_2 (cm^3/s): volumetric flow rate of blood, dialysate respectively
- Z_M (cm): dialyzer length equal to 50 cm

[9] Eloot, S. (2004), Experimental and Numerical Modeling of Dialysis, PhD dissertation, Ghent University, Ghent, Belgium

[10] Bazaev, N.A., Grinvald, V. M., and Selishchev, S. V. (2010), A mathematical model for a biotechnological hemodialysis system, Biomed. Eng

4.1.1 Case 1: K_m= 0 (no mass transfer)

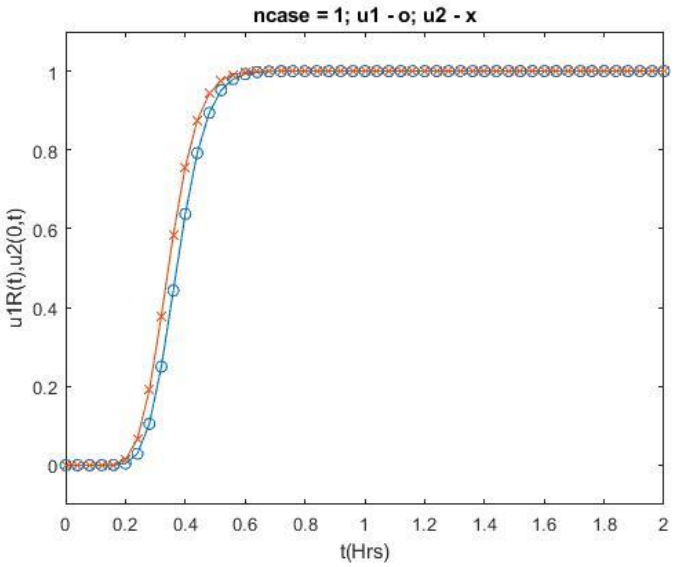
I/O	T = 0 (Trivial)	T = 0.4H	T = 1.8H	T = 2H
$u_{1R}(t)$	0	0.6373	1	1
$u_2(0,t)$	0	0.7571	1	1

Analysis:

As shown in fig(4) In this case, no mass transfer happens, so at the end of the process (t = 2H):

- $u_{1R}(t) = 1 * u_1(z,t)$
- $u_2(0,t) = u_2(z_m,0) * 1$

  (Output = Input)



Conc. of impurities in blood and dialysate with time

Fig (4) ɔurities in blood and dialysate with time

4.1.2. Case 2: $\mathrm{Km} = 0.001$

I/O	$\mathrm{T} = 0$ (Trivial)	$\mathrm{T} = 0.4\mathrm{H}$	$\mathrm{T} = 1.8\mathrm{H}$	$\mathrm{T} = 2\mathrm{H}$
$u_{1R}(t)$	0	0.00068	0.1867	0.1918
$u_2(0,t)$	0	0.5743	0.7815	0.7863

Analysis:

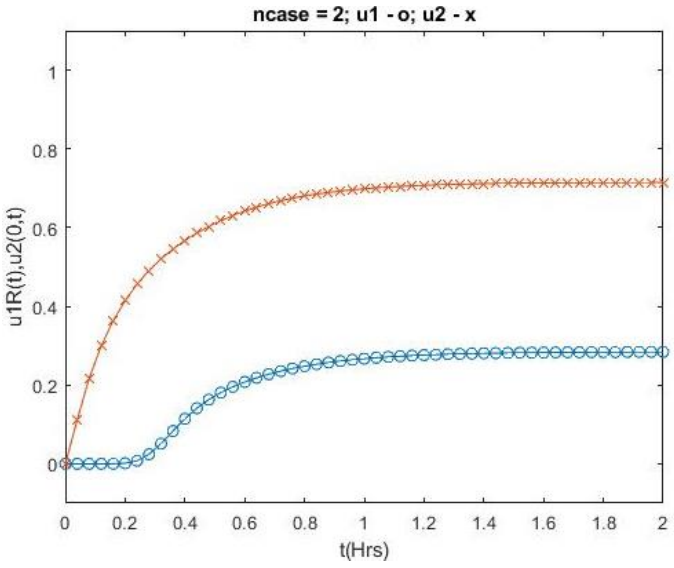
As shown in fig (5) In this case, the dialyzer purified blood by

$(1 - 0.1918) * 100 = 81\%$ of the blood impurities.

- If k_m is increased, e.g., to
  $\mathrm{km} = 0.0025$, the exiting blood impurities are reduced further.

- The u_2 response is immediate (just after t = 0) because the exiting dialysate sees the unit entering blood concentration at $z = 0$ immediately (across the membrane) and responds accordingly.

- Note also that for this case, the entering u_2 concentration $(u_2(z = z_m, t))$ is $u_{2zL} = 0$, so that the exiting u_2 concentration at $z = 0$ is determined entirely by the blood concentration on the other side of the membrane



Conc. of impurities in blood and dialysate with time

Fig (5)

At the end of the process (t = 2H):

- $u_{1R}(t) = 0.1918 * u_1(z,t)$
- $u_2(0,t) = 0.7863 * u_1(z,t)$

4.1.3 Case 3: $\mathrm{K_m} = 0.001, D = 2\ cm$

I/O	T = 0 (Trivial)	T = 0.4H	T = 1.8H
$u_{1R}(t)$	0	0.6141	0.6141
$u_2(0,t)$	0	0.3858	0.3858

Analysis:

As shown in fig(6) In this case, the dialyzer purified blood by

$(1 - 0.6141) * 100 = 39\%$ of the blood impurities.

- As expected from section **(3.9.2)**, when we decrease the diameter of the dialyzer then the contact time between blood & dialysate will decrease due to increase in velocity of fluids.

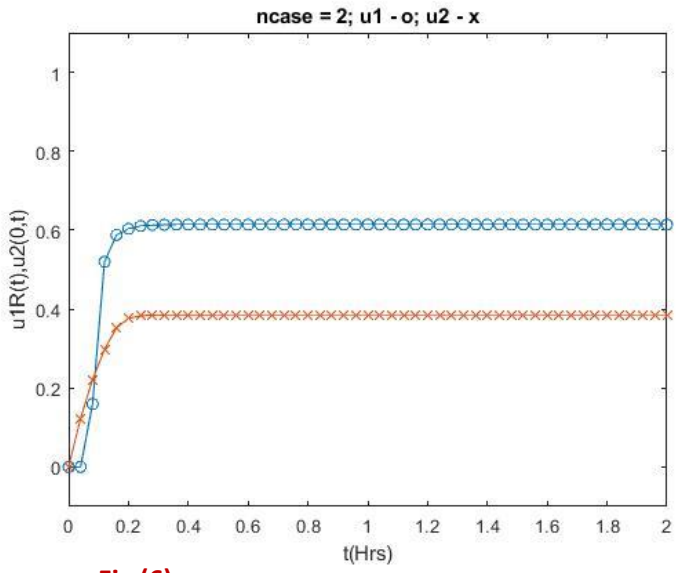


Fig (6)

5. Conclusions & Future work

5.1. Conclusions

In this research we dove into the complications of the 1-D Hemodialyzer model and derived a system of PDEs through which we could deduce the mass transfer coefficient (Km) of impurities during the hemodialysis process and we found that:

- As expected, the response of the blood concentration, $u_1(z,t)$ (with headers), slightly lags that for the dialysate, $u_2(z,t)$ (no headers). Both solutions approach 1 with increasing t as expected (the unit entering concentrations move through and exit the system when there is no mass transfer ($K_M = 0$ for case=1).
- The transfer of impurities was from blood to dialysate when $u_1 > u_2$, which caused the derivative in t (the LHS of eq. (1)) to become more negative, as expected, that is, for u_1 to decrease with t. This should be the usual case since the purpose of the dialyzer is to purify the blood.
- As the mass transfer coefficient increased, the amount of blood impurities was reduced accordingly, meaning they are inversely related.
- The development of an effective (efficient) membrane is the key step in the production of the dialyzer.
- Essentially only impurities pass through the membrane and that the principal components of blood remain in the blood stream.

5.2. What's next?

We have understood the relationship of the hemodialysis device to the partial differential equations, but there are more topics related to this research, such as how can we increase the speed of the hemodialysis process with higher efficiency by changing the dimensions of the parts responsible for transporting fluids, etc., and also studying the different types of disinfectant fluids used in the hemodialysis process and their effect on the speed of movement of substances from the blood to these fluids.

6. References

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7. Appendix

Code for Method of lines solution

```
clc
clear
% 1D dialyzer model
% The ODE/PDE system is
% u1_t = -v1*u1_z + kM1*(u2 - u1) (PDE)(1a)
% u2_t = -v2*u2_z + kM2*(u1 - u2) (PDE)(1b)
% V1L*u1_t(z=0,t) = q1*(u1L - u1(z=0,t)) (ODE)(1c)
% V1R*u1R_t(t) = q1*(u1(z=zL,t) - u1R(t)) (ODE)(1d)
% For the countercurrent case (v1 > 0, v2 < 0).
% The primary outputs are u1R(t), u2(z=0,t) as a function t.
% The method of lines (MOL) solution for eqs. (1) is coded
% below. Specifically, the spatial derivatives in the fluid
% balances, u1_z, u2_z, are replaced by one of six approximations
% as selected by the variable ifd.
% The following cases are programmed:
% ncase = 1: kM = 0 (no mass transfer)
% ncase = 2: kM = 0.001 (blood to dialysate mass transfer)
global zL q1 q2 v1 v2 V1L V1R kM1 kM2 u1L u2zL u1 u2 ifd n...
ncase ncall
% Step through cases
for ncase=1:2
% Model parameters
D=2; A=pi*D^2/4; AM=A; q1=0.25; q2=0.25; eps=0.5;
v1=q1/(eps*A); v2=-q2/((1-eps)*A); u1L=1;
V1L=A; V1R=A; u10=0; u20=0; zL=50; n=21;
% Set parameters for each case
if(ncase==1)
    kM=0; u2zL=1; end
if(ncase==2)
    kM=0.01; u2zL=0; end
```

```
kM1=AM*kM/(A*eps);

kM2=AM*kM/(A*(1-eps));

% Display parameter summary

fprintf('\n\n D = %3.1f A = %4.1f q1 = %4.2f q2 = %4.2f eps =
%4.2f\n',D,A,q1,q2,eps);

fprintf('\n v1 = %5.3f v2 = %5.3f u1L = %5.3f u2zL = %4.2f kM = %5.3f\n',
v1,v2,u1L,u2zL,kM);

fprintf('\n u10 = %4.2f u20 = %4.2f zL = %4.1f n = %3d\n',u10,u20,zL,n);

% Select an approximation for the convective derivatives

% u1z, u2z

% ifd = 1: Two point upwind approximation


ifd=1;

% Level of output

% Detailed output - ip = 1

% Brief (IC) output - ip = 2

ip=1;

% Parameters for fourth order Runge Kutta integration

nsteps=10;

h=14.4;

% Initial condition

for i=1:n

u(i)=u10;

u(i+n)=u20;

end

u(2*n+1)=u10;

t=0;

% Display ifd, ncase, h, CFL

fprintf('\n ifd = %2d ncase = %2d h = %10.3e CFL =
%4.2f\n\n',ifd,ncase,h,v1*h/(zL/(n-1)));

% Display heading

fprintf(' t u1R(t) u2(0,t)\n');

% Display numerical solution at t = 0

fprintf('%5.2f%10.4f%10.4f\n',t/3600,u(2*n+1),u(n+1));

% Store solution for plotting
```

```
u1plot(1)=u(2*n+1);
u2plot(1)=u(n+1);
tplot(1)=t;
% nout output points
nout=51;
ncall=0;
for iout=2:nout
% Fourth order Runge Kutta integration
u0=u; t0=t;
[u,t]=rk4(u0,t0,h,nsteps);
% Numerical solutions
if(ip==1)
fprintf('%5.2f%10.4f%10.4f\n',t/3600,u(2*n+1),u(n+1));
end
% Store solution for plotting
u1plot(iout)=u(2*n+1);
u2plot(iout)=u(n+1);
tplot(iout)=t/3600;
% Next output
end
% Plots for u1R(t), u2(z=0,t)
figure(ncase);
plot(tplot,u1plot,'-o');
axis([0 2 -0.1 1.1]);
ylabel('u1R(t),u2(0,t)');xlabel('t(Hrs)');
if(ncase==1)
    title('ncase = 1; u1 - o; u2 - x'); end
if(ncase==2)
    title('ncase = 2; u1 - o; u2 - x'); end
hold on
plot(tplot,u2plot,'-x');
% Next case
end
```

Within the main code ,helper function will be called

```
function ut=pde_1(u,t)
% Function pde_1 computes the t derivative vector of the u
% vector
global zL q1 q2 v1 v2 V1L V1R kM1 kM2 u1L u2zL u1 u2 ifd n...
ncase ncall
% One vector to two PDEs, one ODE
for i=1:n
u1(i)=u(i);
u2(i)=u(i+n);
end
u1R=u(2*n+1);
%
% Boundary condition
u2(n)=u2zL;
%
% First order spatial derivative
% ifd = 1: Two point upwind finite difference (2pu)
if(ifd==1)
    [u1z]=dss012(0.0,zL,n,u1,v1); end
if(ifd==1)
    [u2z]=dss012(0.0,zL,n,u2,v2); end
%
% Temporal derivatives
% u1t
u1t(1)=(1/V1L)*q1*(u1L-u1(1));
for i=2:n
u1t(i)=-v1*u1z(i)+kM1*(u2(i)-u1(i));
end
u1Rt=(1/V1R)*q1*(u1(n)-u1R);
%
% u2t
u2t(n)=0.0;
```

```
for i=1:n-1
u2t(i)=-v2*u2z(i)+kM2*(u1(i)-u2(i));
end
%
% Two PDEs, one ODE to one vector
for i=1:n
ut(i)=u1t(i);
ut(i+n)=u2t(i);
end
ut(2*n+1)=u1Rt;
%
% Increment calls to pde_1
ncall=ncall+1;
```

Within the helper function ,another function will be called (runge kutta function helper)

```
function [u,t]=rk4(u0,t0,h,nsteps)
%
% nsteps Runge Kutta steps
for i=1:nsteps
%
% Runge Kutta integration
k1=pde_1(u0,t0);
u1=u0+k1*h/2;
t=t0+h/2;
k2=pde_1(u1,t);
u1=u0+k2*h/2;
k3=pde_1(u1,t);
u1=u0+k3*h;
t=t0+h;
k4=pde_1(u1,t);
u=u0+(1/6)*(k1+2*k2+2*k3+k4)*h;
% Next Runge Kutta step
u0=u; t0=t;
end
```