MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS DEPARTMENT

MTE 506 DIGITAL CONTROL

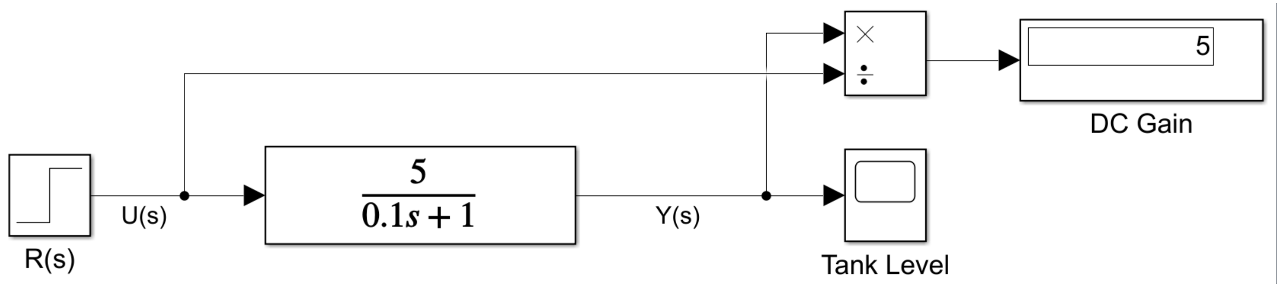
LAB 4 – SPRING 2019

Goals of The Lab

- Studying the PID terms on system response
- Computing steady state for each term

Open Loop DC Gain

$$G_p(s) = \frac{K}{\tau s + 1} = \frac{5}{0.1s + 1}$$



Open Loop DC Gain

$$DC\ Gain\ of\ G_p(s = j\omega = 0) = \frac{K}{(0.1)(j0) + 1} = \frac{5}{1} = 5$$

What is the importance of DC Gain ?

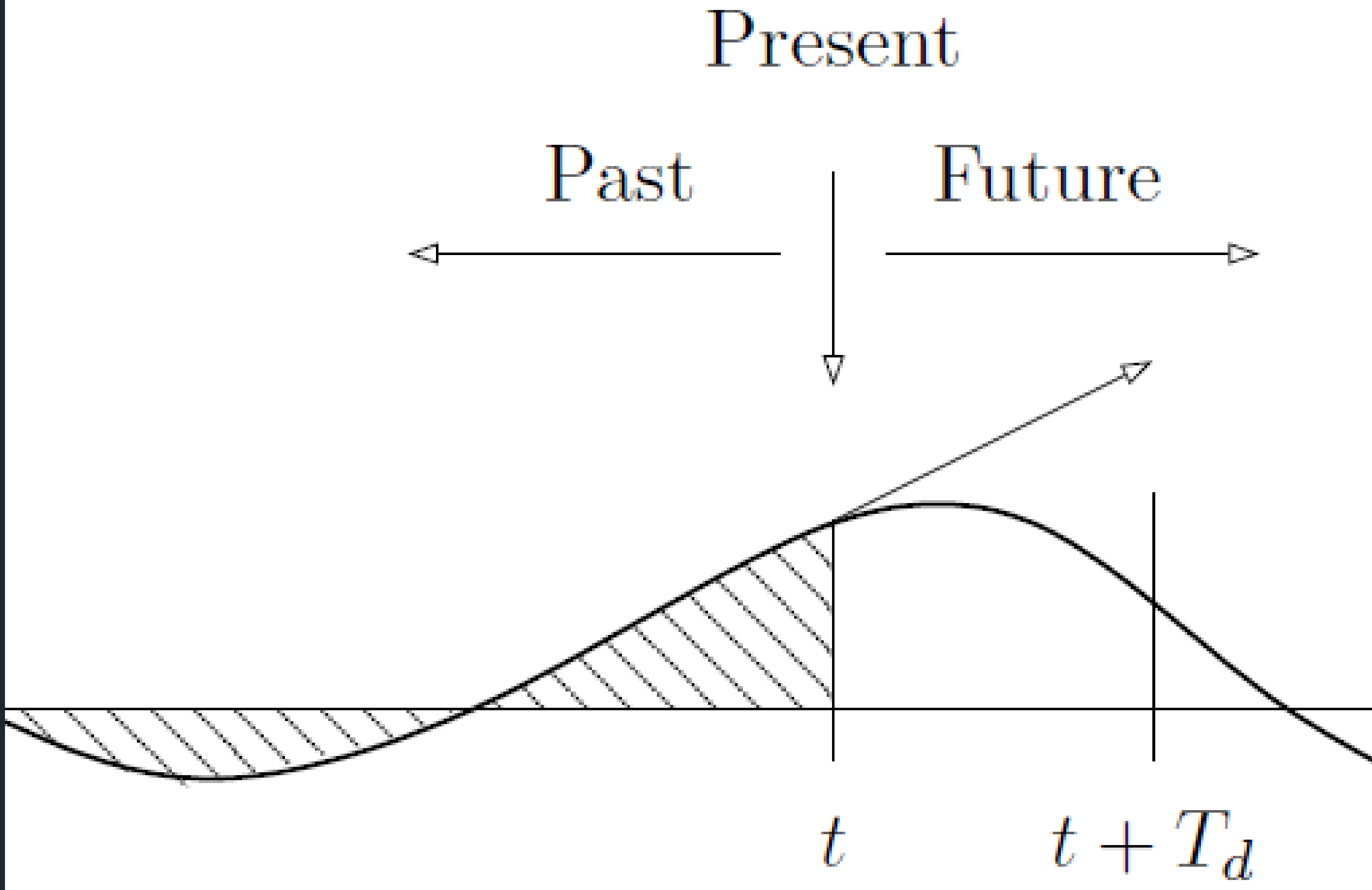
Lab 4

Automatic Control

P AND PI CONTROLLER

PID Controller

Using Simulink



PID Controller

Simple example

Standard Form

$$u(t) = k_p e(t) + k_i \int_0^t e(t).dt + k_d \frac{de}{dt}$$

$$u(t) = k_p(e(t) + \frac{1}{T_i} \int_0^t e(t).dt + T_d \frac{de}{dt}$$

PID Controller

Simple example

Standard Form

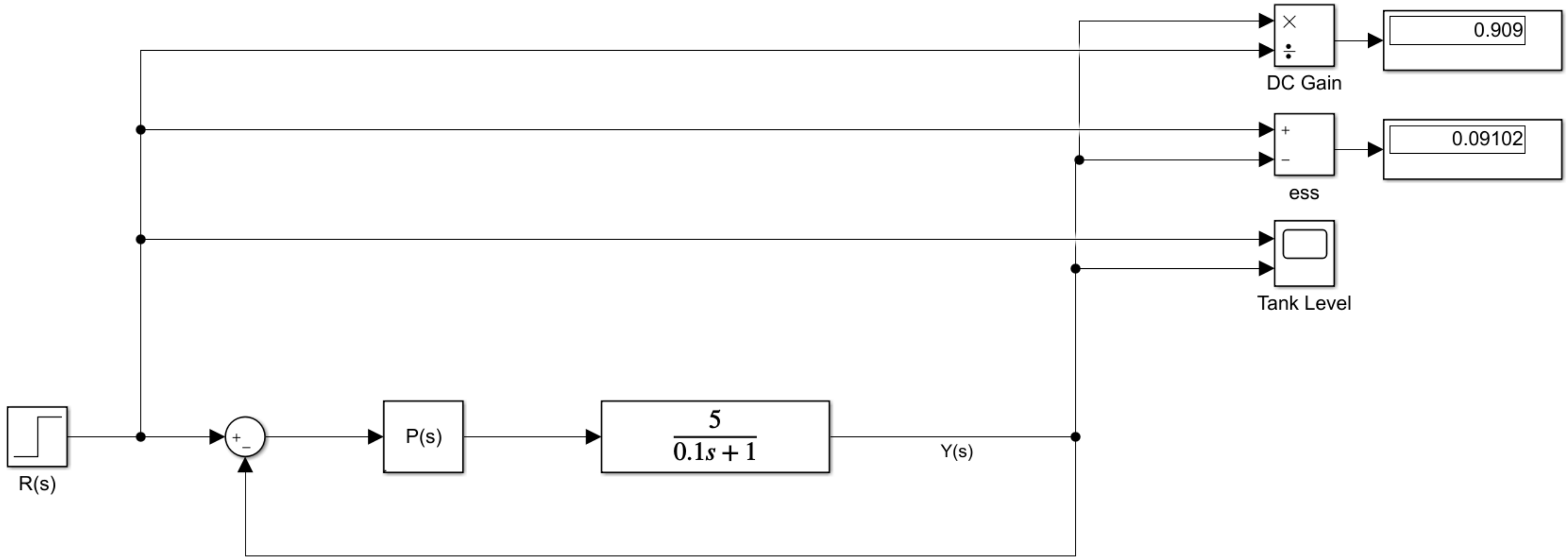
$$u(t) = \boxed{k_p e(t)} + \boxed{k_i \int_0^t e(t).\,dt} + \boxed{k_d \frac{de}{dt}}$$

Present **Past** **Future**

P-Controller

Simple example

Steady State Error with P − Controller



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P-Controller

Simple example

$$Steady\ State\ Error\ with\ P - Controller$$

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)K_p} = \lim_{s\to 0} \frac{\cancel{s}\frac{1}{\cancel{s}}}{1 + \frac{5}{0.1s + 1}K_p} = \frac{1}{1 + \frac{5}{1}K_p} = \frac{1}{1 + \mathbf{5}K_p}$$

$$e(\infty)_{K_p=2} = \frac{1}{1 + \mathbf{5}K_p} = \frac{1}{1 + (5)(2)} = \frac{1}{11} = 0.0909$$

Lab 4

Classwork

Simple example

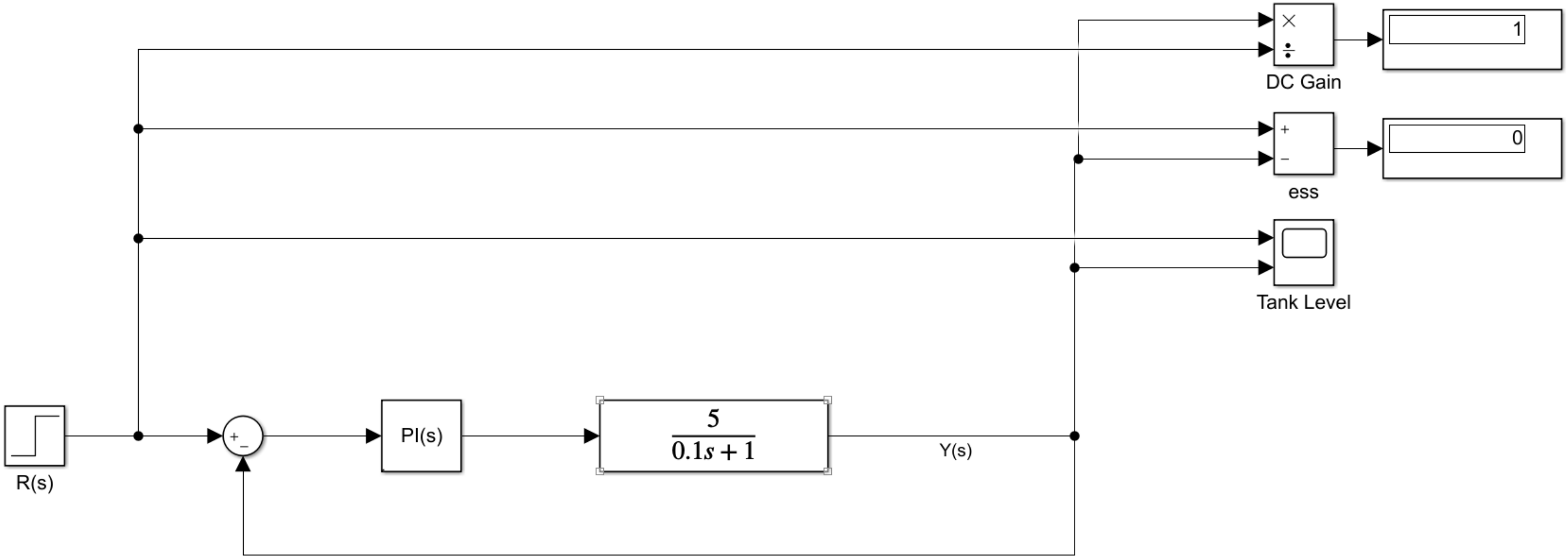
$Compute\ the\ DC\ gain\ and\ steady\ state\ error\ for:$

$$G(s) = \frac{6}{(2s+1)(4s+1)(6s+1)}\ for\ Kp = 10\ (\boldsymbol{use\ simulink\ zero - pole\ block})$$

PI-Controller

Simple example

Steady State Error with PI − Controller



PI-Controller

Simple example

$Steady\ State\ Error\ with\ PI - Controller$

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)(K_p + \frac{K_i}{s})} = \lim_{s \to 0} \frac{\cancel{s}\frac{1}{\cancel{s}}}{1 + \frac{5}{0.1s + 1}(K_p + \frac{K_i}{s})} = \frac{1}{\infty} = 0$$

$\boldsymbol{What\ is\ the\ DC\ gain\ of\ PI\ Controller?\ Why?}$

Lab 4

SOLVING DIFFERENTAIL EQUATIONS

Open and Closed Loop

The Laplace transform

The most commonly used transform pairs

Original	Image
a	$\frac{a}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
$t^n, n \in N$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
te^{at}	$\frac{1}{(s-a)^2}$
t^2e^{at}	$\frac{2}{(s-a)^3}$
$t^n e^{at}, n \in N$	$\frac{n!}{(s-a)^{n+1}}$

Original	Image
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$
$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$
$t\sin(\omega t)$	$\frac{2s\omega}{(s^2+\omega^2)^2}$
$t\cos(\omega t)$	$\frac{s^2-\omega}{(s^2+\omega^2)^2}$
$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at}\cos(\omega t)$	$\frac{s-a}{(s-a)^2+\omega^2}$

Lab 4

Solving LTI differential equations

Excited systems

Find the ***unit step response*** $\boldsymbol{y(t)}$ *if applied on the below transfer function*

$$\mathbf{m\ddot{x}(t) + c\dot{x}(t) + kx(t)}$$

SOLUTION

Reformulating the problem:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

$$\because f(t) \text{ is unit step}$$

$$\therefore m\ddot{x}(t) + c\dot{x}(t) + kx(t) = u(t)$$

Taking Laplace of the system

$$m[s^2X(s) - sX(0) - \acute{X}(0)] + c[sX(s) - x(0)] + kX(s) = \frac{1}{s}$$

Solving LTI differential equations

Excited systems

Find the ***unit step response x(t)*** *if applied on the below transfer function*

$$\mathbf{m\ddot{x}(t) + c\dot{x}(t) + kx(t)}$$

SOLUTION

Assuming ***system starts from 0***

$$ms^2X(s) + csX(s) + kX(s) = \frac{1}{s}$$

$$X(s) = \frac{1}{ms^2 + cs + k}, \qquad m = 1, c = 7, k = 10$$

$$X(s) = \frac{1}{s(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5} \; (Partial\ Fractions)$$

Solving LTI differential equations

Excited systems

Find the ***unit step response*** $\boldsymbol{x(t)}$ *if applied on the below transfer function*

$$\mathbf{m\ddot{x}(t) + c\dot{x}(t) + kx(t)}$$

SOLUTION

Assuming ***system starts from 0***

$$X(s) = \frac{1}{s(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5} \text{ (Partial Fractions Decomposition)}$$

$$X(s) = 1 = A(s+2)(s+5) + B(s)(s+5) + C(s)(s+2)$$

$$Solve\ for\ s = 0, s = -2, s = -5\ \rightarrow A = 0.1, B = -0.17, C = 0.07$$

Lab 4

Solving LTI differential equations

Excited systems

Find the ***unit step response*** $\boldsymbol{x(t)}$ *if applied on the below transfer function*

$$\mathbf{m\ddot{x}(t) + c\dot{x}(t) + kx(t)}$$

SOLUTION

$$X(s) = \frac{1}{10}\frac{1}{s} - \frac{1}{6}\frac{1}{s+2} + \frac{1}{15}\frac{1}{s+5}$$

$$\boldsymbol{x(t) = \frac{1}{10} - \frac{1}{6}e^{-2t} + \frac{1}{15}e^{-5t}}$$

Table of Laplace Transforms

$) = \mathfrak{L}\{f(t)\}$	$f(t) = \mathfrak{L}^{-1}\{F(s)\}$	$F(s) = \mathfrak{L}\{f(t)\}$
$\frac{1}{s}$	2. $\mathbf{e}^{at}$	$\frac{1}{s-a}$
$\frac{n!}{s^{n+1}}$	4. $t^p,\ p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}},\quad n = 1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
$\frac{2as}{(s^2+a^2)^2}$	10. $t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$

Don’t forget to pull the lab update from.

http://github.com/wbadry/mte506

Waleed El-Badry

END OF Lab 4