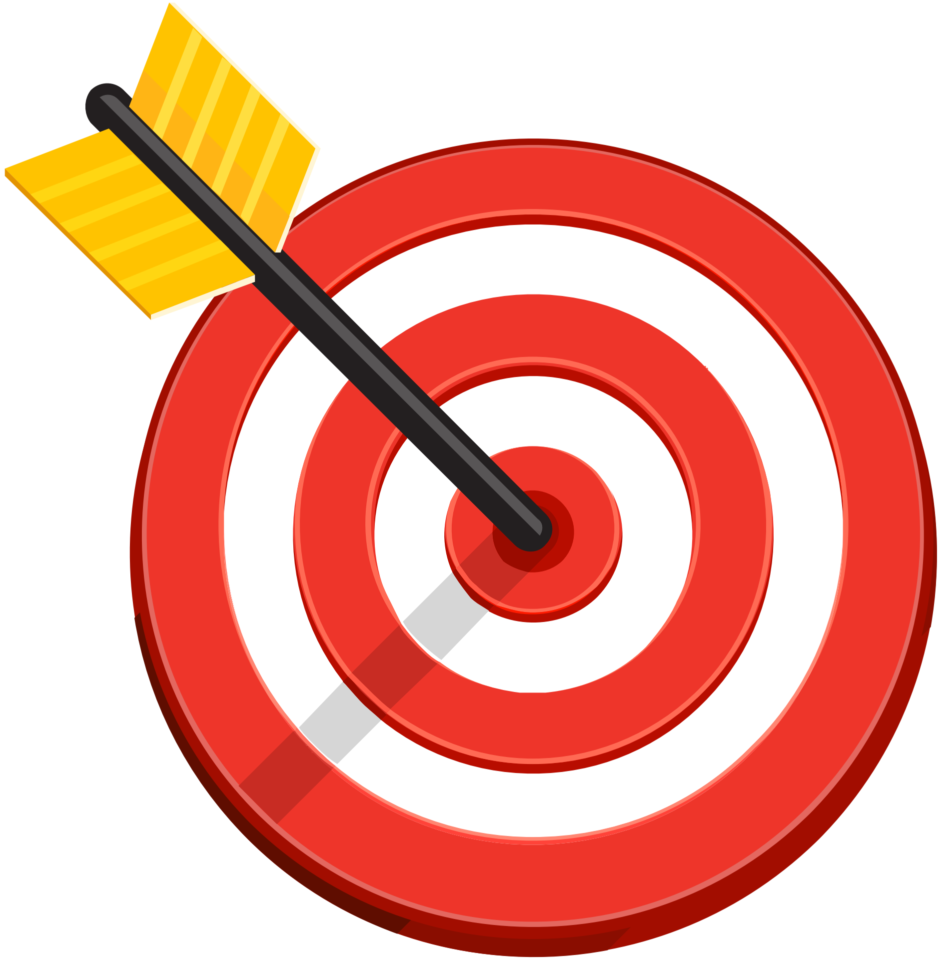
MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS DEPARTMENT

MTE 506 DIGITAL CONTROL

LAB 7 – SPRING 2019

Lab 7

Goals of The Lab



- Inverse Z-Transform
- Solved examples and assignment

Lab 7

Inverse Z- Transform

$$1 + 0.5z^{-1} + 0.25z^{-2} + \cdots$$

$$-0.5 \overline{\big) \, z \qquad\qquad\qquad}$$

$$z - 0.5$$

$$0.5$$

$$0.5 - 0.25z^{-1}$$

$$0.25z^{-1}$$

$$0.25z^{-1} - 0.125z$$

$$\vdots$$

Problem Approach

Difference Equation

$$y(k+n)+\cdots=u(k+m)+\cdots$$

Z-Transform

$$\frac{Y(z)}{z}=\frac{k}{z-a}+\cdots$$

$$\begin{array}{r l} & az^{-1}+bz^{-2}+\cdots \\ \hline z^{m}+\cdots \,\big) & z^{n}+\cdots \end{array}$$

Inverse Z-Transform

$$y(k)=u(k)+\ e^{-3k}+\cdots$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$Disctinct\ poles \rightarrow Y(z) = \frac{z}{z-0.4} + \frac{z}{z-e^{-2T}}$$

$$Repeated\ poles \rightarrow Y(z) = \frac{z}{(z-1)^2} + \frac{z}{z-0.3}$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z)=\frac{z+1}{z^2+0.3z+0.02}$$

$$\mathit{Solutin\ pattern}\ \rightarrow F(z)\rightarrow\frac{F(z)}{z}$$

$$F(z)=\frac{z}{z}\frac{z+1}{z^2+0.3z+0.02}\rightarrow\frac{F(z)}{z}=\frac{A}{z}+\frac{B}{z+0.1}+\frac{C}{z+0.2}$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z}\frac{z+1}{z^2+0.3z+0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

Finding partial fraction coefficients

$$A = z\frac{F(z)}{z}\Bigg]_{z=0} = F(0) = \frac{0+1}{0^2+0.3(0)+0.02} = \frac{1}{0.02} = 50$$

$$A = 50$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z)=\frac{z}{z}\frac{z+1}{z^2+0.3z+0.02}\rightarrow\frac{F(z)}{z}=\frac{A}{z}+\frac{B}{z+0.1}+\frac{C}{z+0.2}$$

Finding partial fraction coefficients

$$B=(z+0.1)\left.\frac{F(z)}{z}\right]_{z=-0.1}=(z+0.1)\left.\frac{z+1}{z(z+0.1)(z+0.2)}\right]_{z=-0.1}$$

$$B=\frac{-0.1+1}{(-0.1)\,(-0.1+0.2)}=\frac{0.9}{-0.01}=-90$$

$$B=-90$$

$$A=50$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z)=\frac{z}{z}\frac{z+1}{z^2+0.3z+0.02}\rightarrow\frac{F(z)}{z}=\frac{A}{z}+\frac{B}{z+0.1}+\frac{C}{z+0.2}$$

Finding partial fraction coefficients

$$C=(z+0.2)\left.\frac{F(z)}{z}\right]_{z=-0.2}=(z+0.2)\left.\frac{z+1}{z(z+0.1)(z+0.2)}\right]_{z=-0.2}$$

$$C=\frac{-0.2+1}{(-0.2)\,(-0.2+0.1)}=\frac{0.8}{0.02}=40$$

$$C=40$$

$$B=-90$$

$$A=50$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z)=\frac{z}{z}\frac{z+1}{z^2+0.3z+0.02}\rightarrow\frac{F(z)}{z}=\frac{A}{z}+\frac{B}{z+0.1}+\frac{C}{z+0.2}$$

$$\frac{F(z)}{z}=50\frac{1}{z}-90\frac{1}{z+0.1}+40\frac{1}{z+0.2}$$

$$F(z)=50\frac{z}{z}-90\frac{z}{z+0.1}+40\frac{z}{z+0.2}$$

$$C=40$$

$$B=-90$$

$$f(t)=50\delta(k)-90(-0.1)^k+40(-0.2)^k$$

$$A=50$$

Finding Inverse Z-Transform using *partial fraction expansion*

a^k	$\frac{1}{1-az^{-1}}$

Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1

From Z-Transform Table

$$F(z) = 50\frac{z}{z} - 90\frac{z}{z+0.1} + 40\frac{z}{z+0.2}$$

$$C = 40$$

$$B = -90$$

$$f(t) = 50\delta(k) - 90(-0.1)^k + 40(-0.2)^k$$

$$A = 50$$

Finding Inverse Z-Transform using *partial fraction expansion*

For repeated roots

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\Big|_{z\to z_l}, j = 1, 2, \dots, r$$

$A_{1j} \dots j^{th}$ *coefficient of repeated root*

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z-0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\bigg|_{z\to z_l}, j = 1, 2, \ldots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z-0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z-0.5}$$

$$A_{11} = \frac{1}{0!} z^3 \frac{F(z)}{z}\bigg]_{z=0} = z^3 \frac{1}{zz^2(z-0.5)}\bigg]_{z=0} = \frac{1}{0-0.5} = -2$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z)=\frac{1}{z^2(z-0.5)}$$

$$A_{1j}=\frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\bigg|_{z\to z_l},\ j=1,2,\dots,r$$

$$\frac{F(z)}{z}=\frac{1}{z^3(z-0.5)}\rightarrow\frac{A_{11}}{z^3}+\frac{A_{12}}{z^2}+\frac{A_{13}}{z}+\frac{B}{z-0.5}$$

$$A_{12}=\frac{1}{1!}\frac{d}{dz}z^3\frac{F(z)}{z}\bigg]_{z=0}=\frac{d}{dz}z^3\frac{1}{z^3(z-0.5)}\bigg]_{z=0}=$$

$$\frac{(0-0.5)(0)-(1)(1)}{(0-0.5)^2}=\frac{1}{0.25}=4$$

$$A_{12}=4$$

$$A_{11}=-2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z-0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\Big|_{z\to z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z-0.5)} \to \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z-0.5}$$

$$A_{13} = \frac{1}{2!}\frac{d^2}{dz^2} z^3 \frac{F(z)}{z}\Big]_{z=0} = \frac{1}{2!}\frac{d^2}{dz^2} z^3 \frac{1}{z^3(z-0.5)}\Big|_{z=0} = \frac{1}{2}\frac{d}{dz}\frac{1}{(z-0.5)^2}\Big|_{z=0}$$

$$= \frac{1}{2}\frac{(-1)(-2)}{(0-0.5)^3} = \frac{2}{0.125} = 8$$

$$A_{13} = 8$$

$$A_{12} = 4$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z-0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\bigg|_{z\to z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z-0.5)} \to \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z-0.5}$$

$$B = (z-0.5)\frac{F(z)}{z}\bigg]_{z=0.5} = (z-0.5)\frac{1}{z^3(z-0.5)}\bigg]_{z=0.5}$$

$$= \frac{1}{0.5^3} = 8$$

$$B = 8$$

$$A_{13} = 8$$

$$A_{12} = 4$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z-0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\Big|_{z\to z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = -2\frac{1}{z^3} + 4\frac{1}{z^2} + 8\frac{1}{z} + 8\frac{1}{z-0.5}$$

$$F(z) = -2\frac{1}{z^2} + 4\frac{1}{z} + 8 + 8\frac{z}{z-0.5}$$

$$F(z) = -2(1)z^{-2} + 4(1)z^{-1} + 8(1) + 8\frac{z}{z-0.5}$$

$$f(k) = -2\delta(k-2) + 4\delta(k-1) + 8\delta(k) + 8(0.5)^k$$

$$B = 8$$

$$A_{13} = 8$$

$$A_{12} = 4$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *long division*

$$F(z) = \frac{z+1}{z^2 + 0.2z + 0.1}$$

$$\begin{array}{r l}
 & z^{-1} + 0.8\,z^{-2} - 0.26\,z^{-3} \\
\hline
z^2 + 0.2z + 0.1 \;\big) & z \quad + \quad 1 \\
 & z \quad + \quad 0.2 \quad + \quad 0.1z^{-1} \\
\hline
 & \qquad 0.8 \quad - \quad 0.1 \quad z^{-1} \\
 & \qquad 0.8 \quad + \quad 0.16\; z^{-1} \quad + \quad 0.08\; z^{-2} \\
\hline
 & \qquad\qquad -0.26\,z^{-1} \quad + \quad \ldots
\end{array}$$

$$f(k) = \{1, 0.8, -0.26, \ldots\}$$

Problems

$\boldsymbol{Solve\ the\ following\ difference\ equations}$:

$$y(k+1) - 0.8\,y(k) = 0, y(0) = 1$$

$$y(k+1) - 0.8\,y(k) = 1(k), y(0) = 1$$

$$y(k+1) - 0.8\,y(k) = 1(k), y(0) = 0$$

$$y(k+2) + 0.7\,y(k+1) + 0.006\,y(k) = \delta(k), y(0) = 0, y(1) = 2$$

Don’t forget to pull the lab update from.

http://github.com/wbadry/mte506

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END OF Lab 7