MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS DEPARTMENT

MTE 506 DIGITAL CONTROL

LAB 3 – SPRING 2019

Lab 3

Goals of The Lab

Discretization of Analog Control Systems

- Converting differential model into algebraic model
- Computing steady state error and system types

Waleed El-Badry

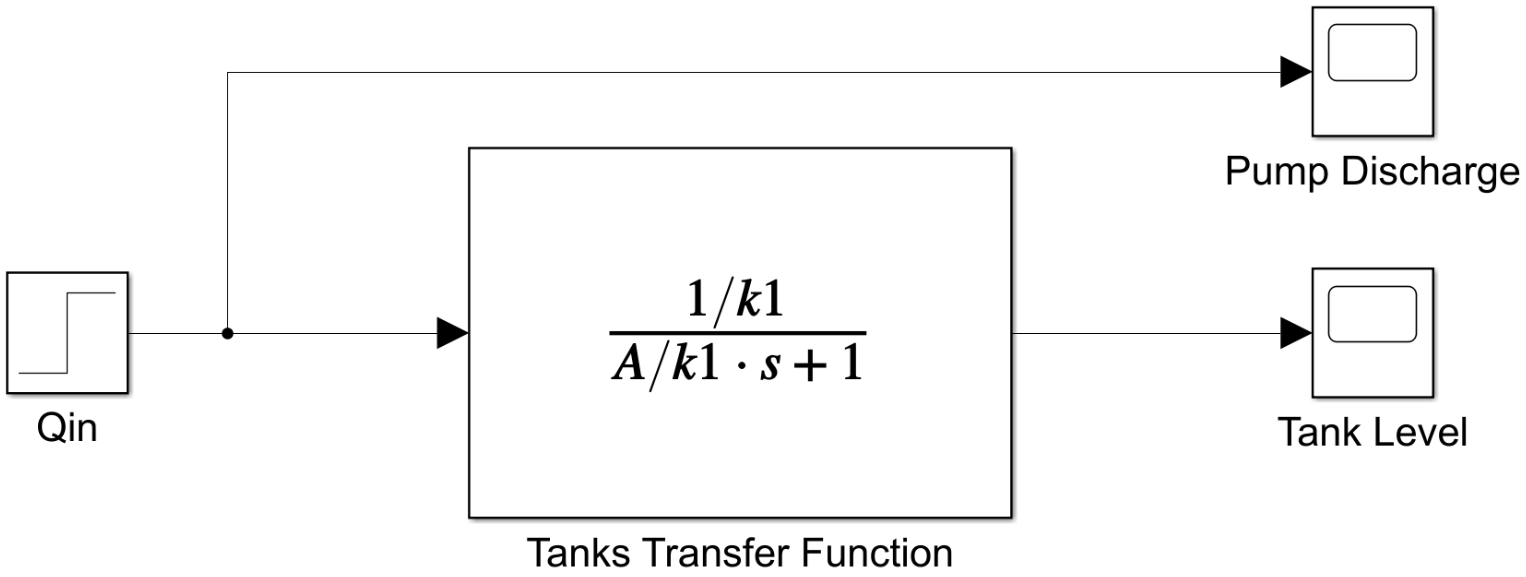
Lab 3

Automatic Control

Closed Loop System

Open Loop System

Using Simulink



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Closed Loop System

Simple example

Previously (***Tank Simulation***):

$$A\frac{dh}{dt} + k_1 h(t) = q_i(t)$$

Taking Laplace Transform

$$A(sH(s) - h_0) + k_1 H(s) = Q_i(s)$$

$$AsH(s) - Ah_0 + k_1 H(s) = Q_i(s)$$

$$\therefore H(s) = \frac{Q_i(s) + Ah_0}{As + k_1}$$

Open Loop System

$\therefore$ *Asuming* $h_0 = 0$ *(empty tank)*

$$\therefore \frac{H(s)}{Q_i(s)} = \frac{1}{As + k_1} = \frac{K}{\tau s + 1}$$

$$\therefore \frac{H(s)}{Q_i(s)} = G(s) = \frac{K}{\tau s + 1}$$

Can you convert diff. eqn. into Laplace Transform using MATLAB script ?

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Laplace table

PDF

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n, \quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}}, \quad n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$

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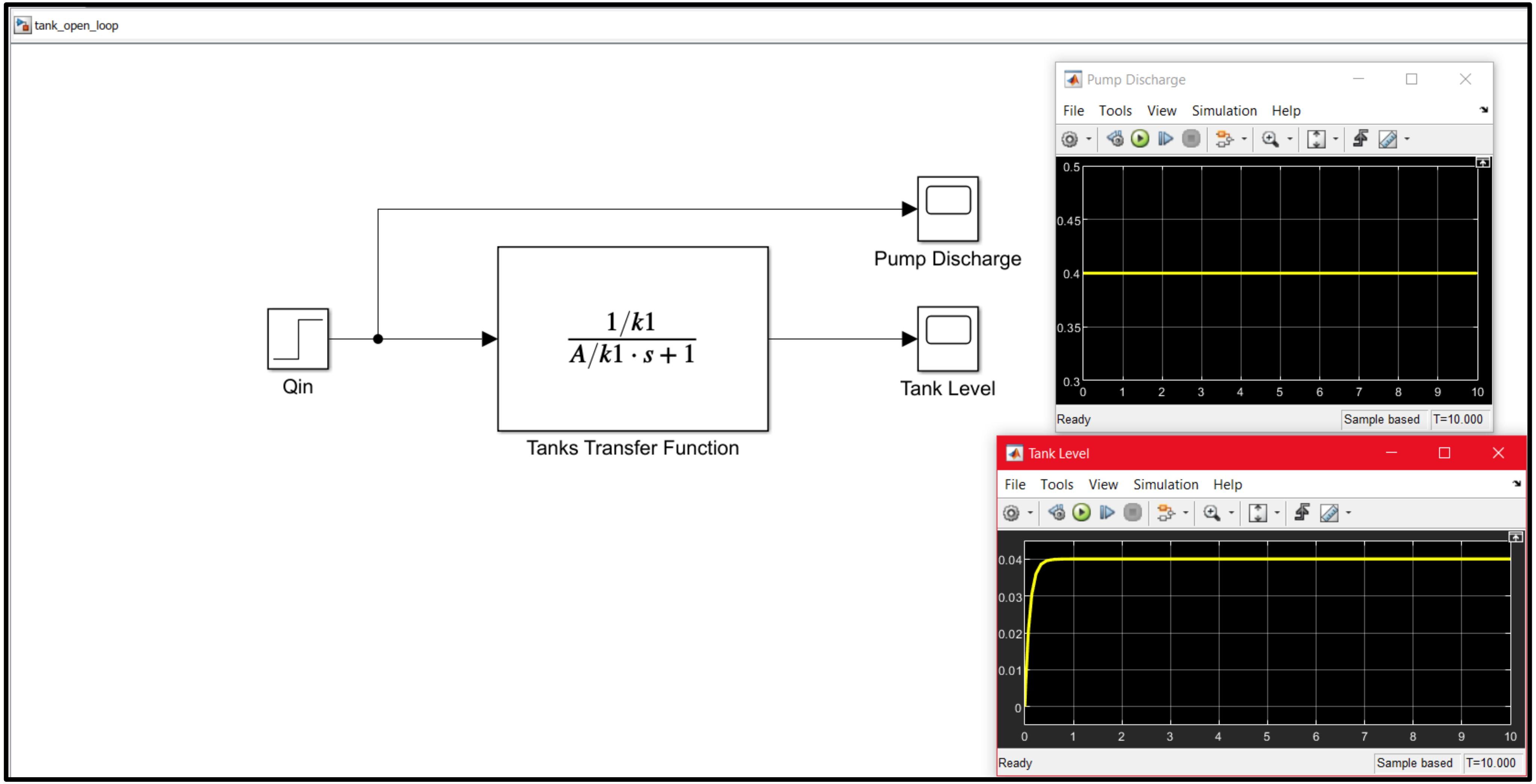
Open Loop Simulink Model

Open Loop



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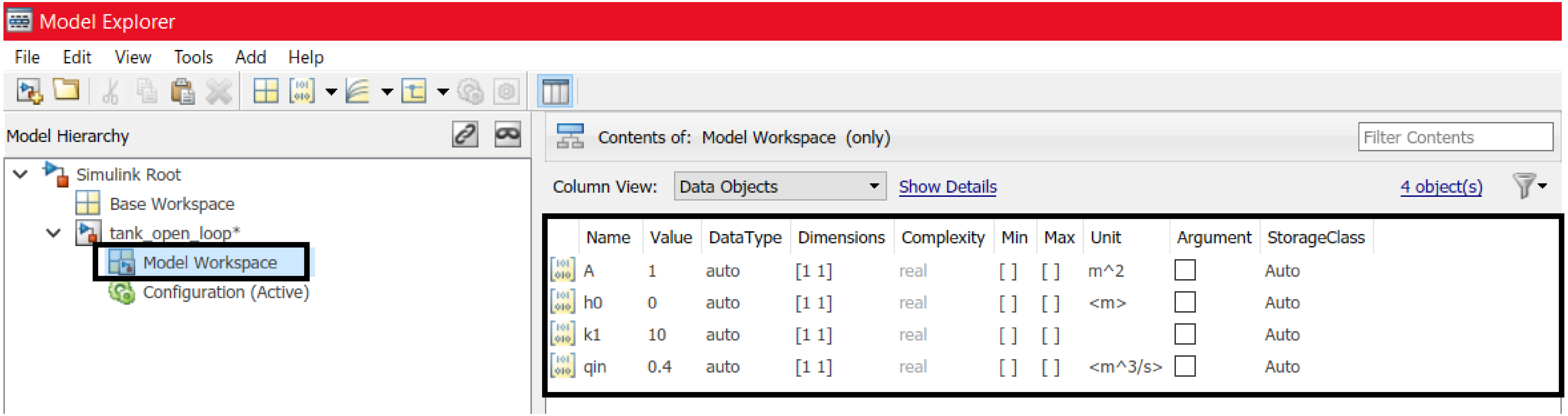
Model Workspace

Store model parameters



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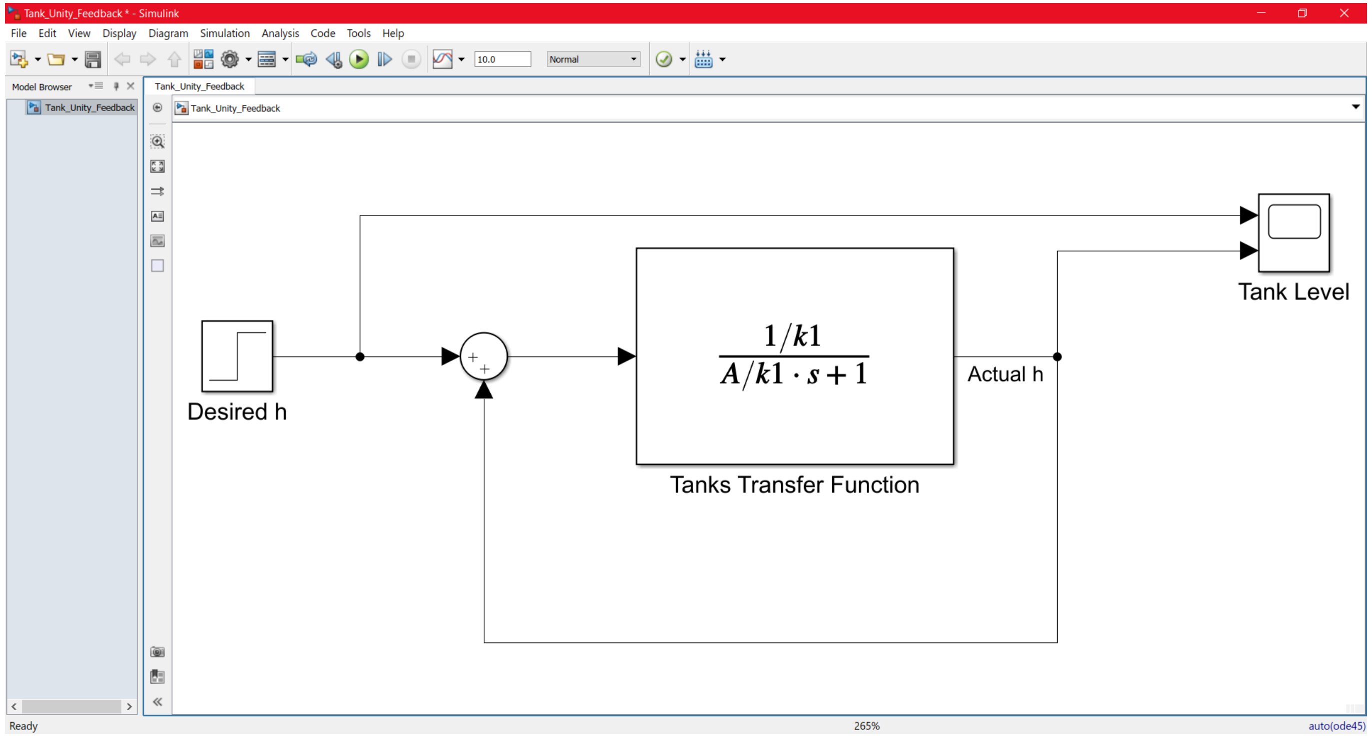
Automatic Control

Closed Loop System

Closed Loop System

Using Simulink



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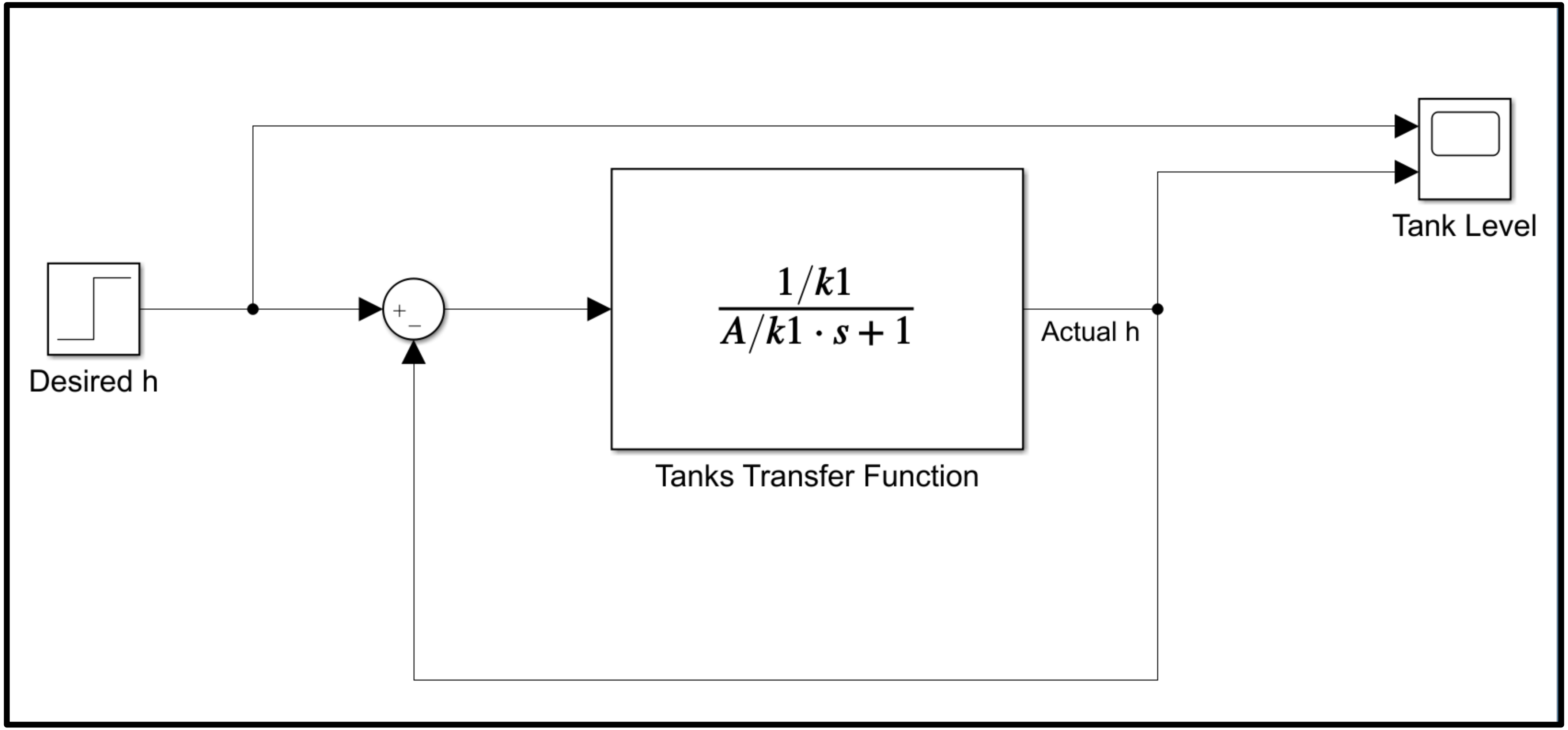
Closed Loop System

Unity Feedback



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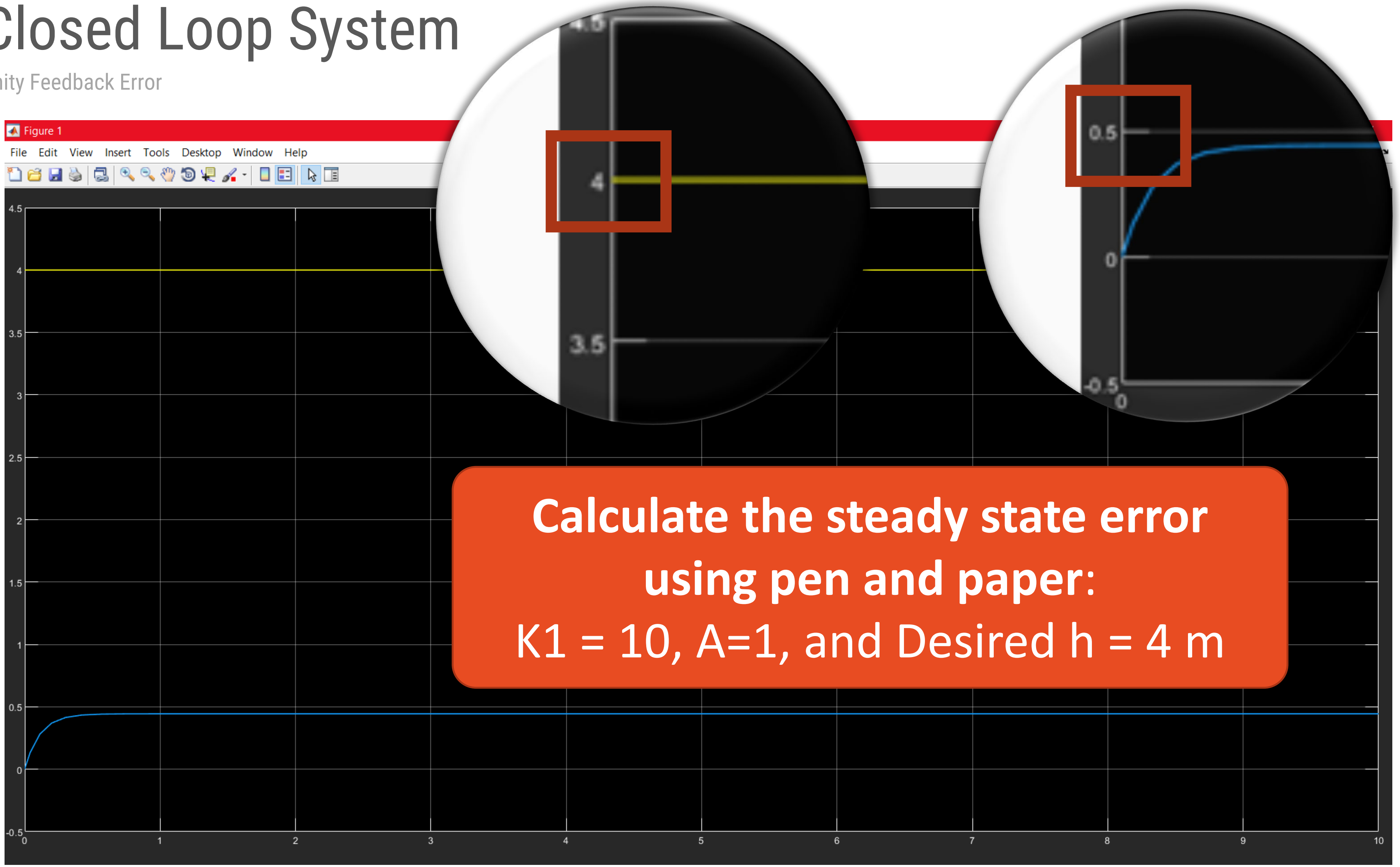
Closed Loop System

Unity Feedback Error



Calculate the steady state error using pen and paper:

K1 = 10, A=1, and Desired h = 4 m

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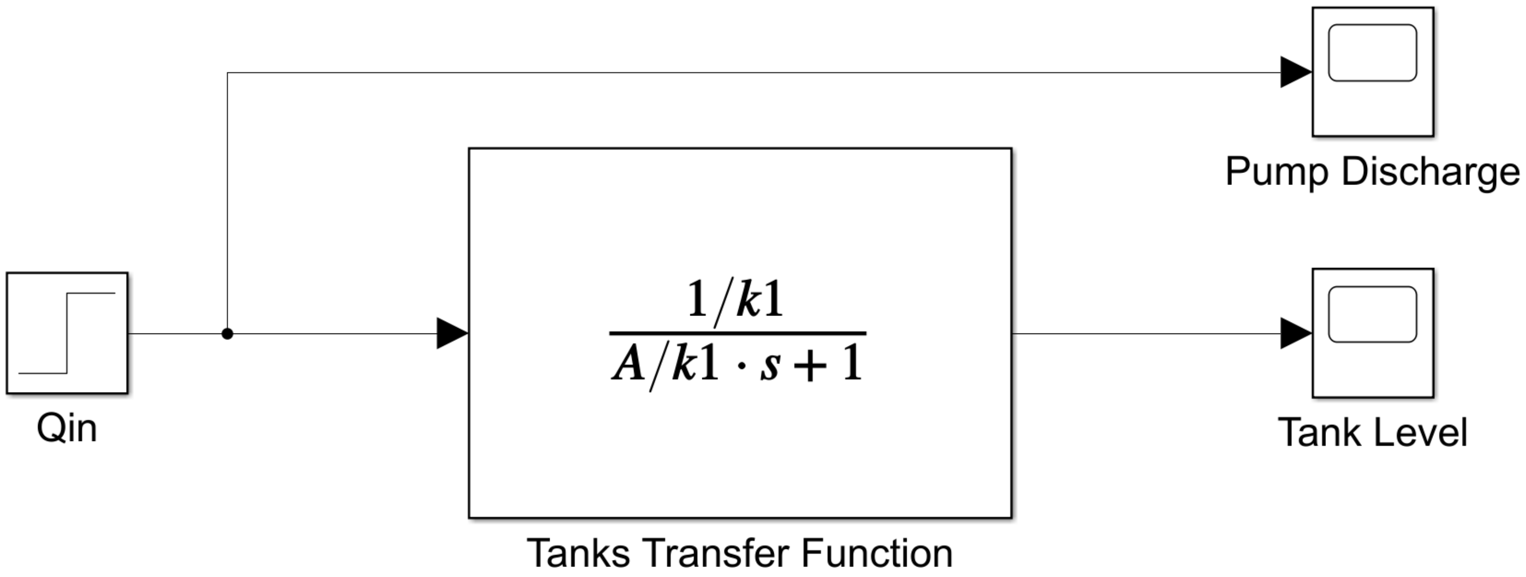
Automatic Control

Closed Loop System

Open Loop System

Steady State Error



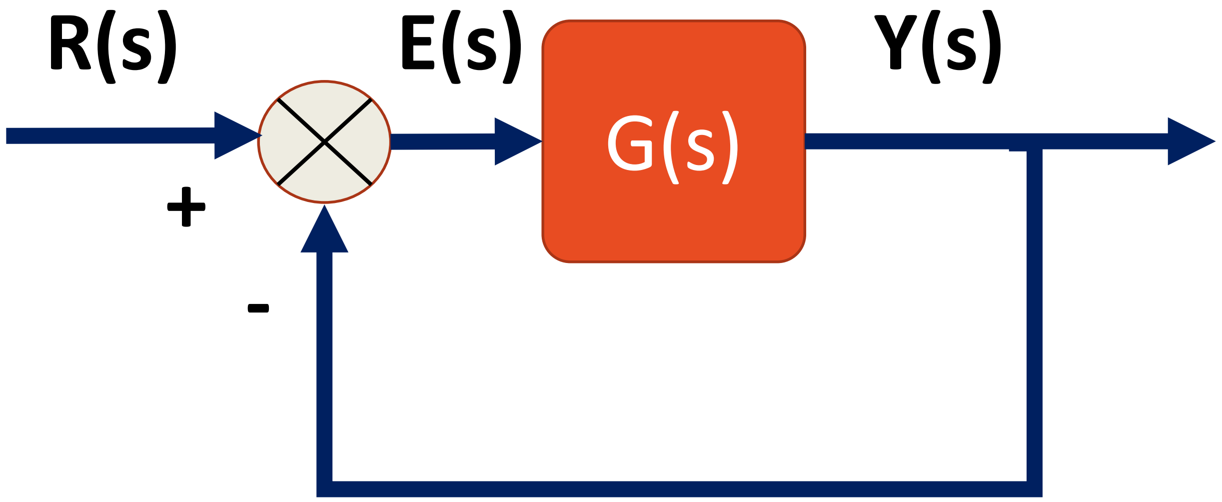
Lab 3

Closed Loop System

Steady State Error

Final Value Theorem

$$e(\infty) = \lim_{s\to 0} sE(s) = \frac{sR(s)}{1+G(s)}$$



$$e(\infty) = \lim_{s\to 0} \frac{s\,\frac{4}{s}}{1+\frac{K}{\tau s+1}} = \frac{4}{1+\frac{\left(\frac{1}{10}\right)}{1}} = 3.6 \rightarrow (4 - 0.4)$$

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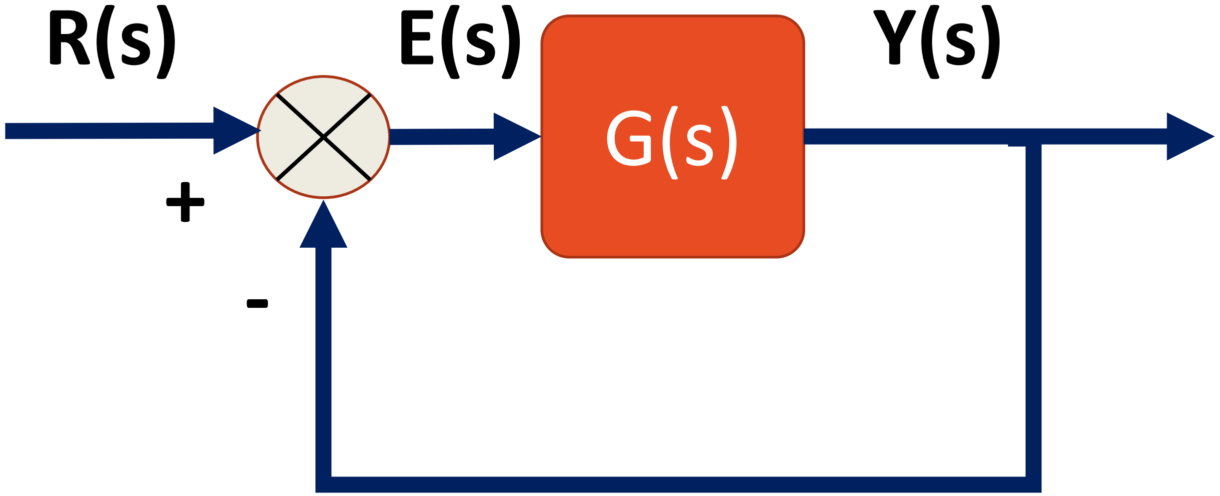
Closed Loop System

Steady State Error

*Tank is Type 0 (**why**?)*

$$e(\infty) = \frac{step\ value\ (h_{desired})}{1 + \lim_{s\to 0} G(s)}$$



$$e(\infty) = \frac{step\ value}{1 + K_p} \to K_p = \lim_{s\to 0} G(s)$$

$$e(\infty) = \frac{4}{1 + 0.1} = 3.6$$

$$K_p = 0.1\ (Static\ Error\ Constant)$$

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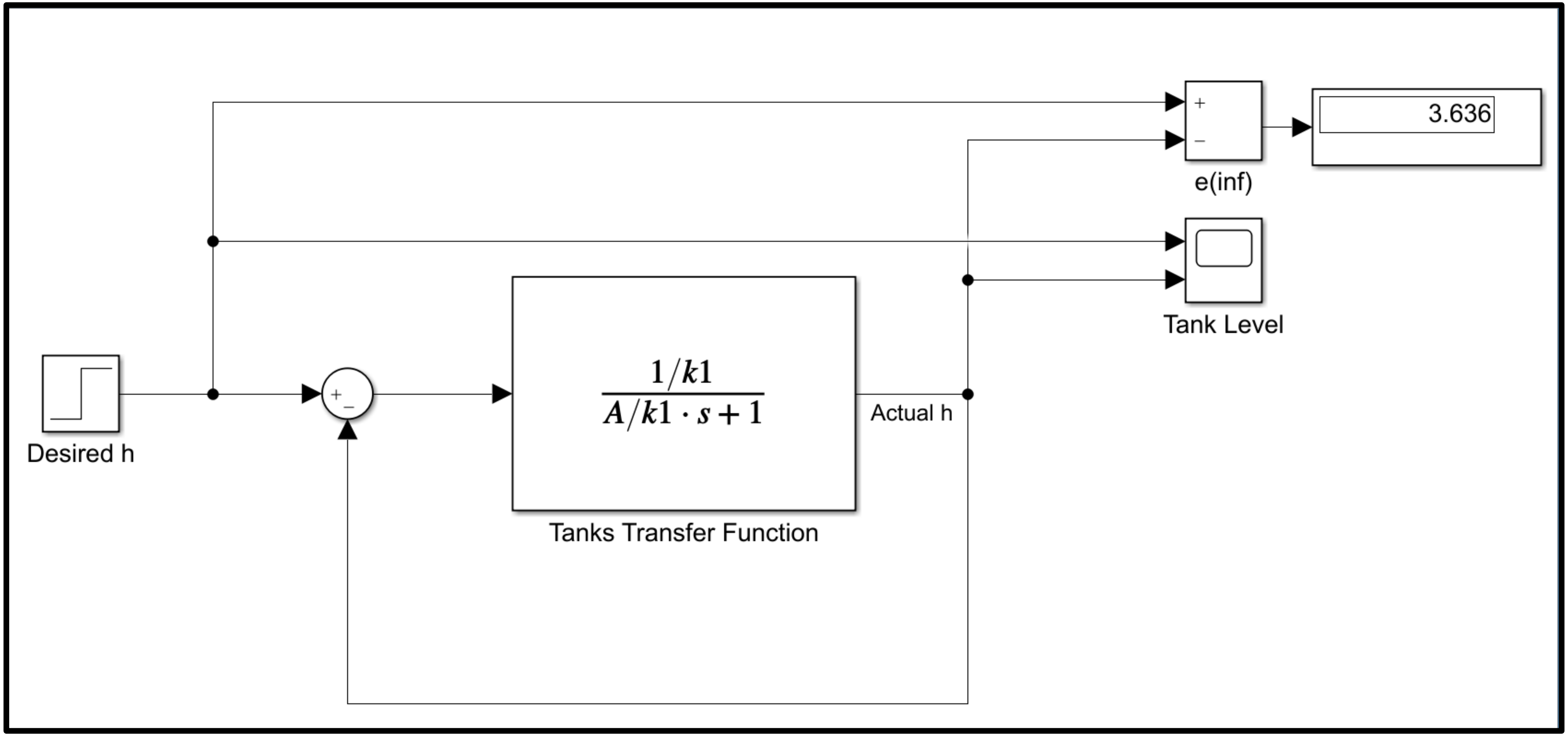
Steady State Error



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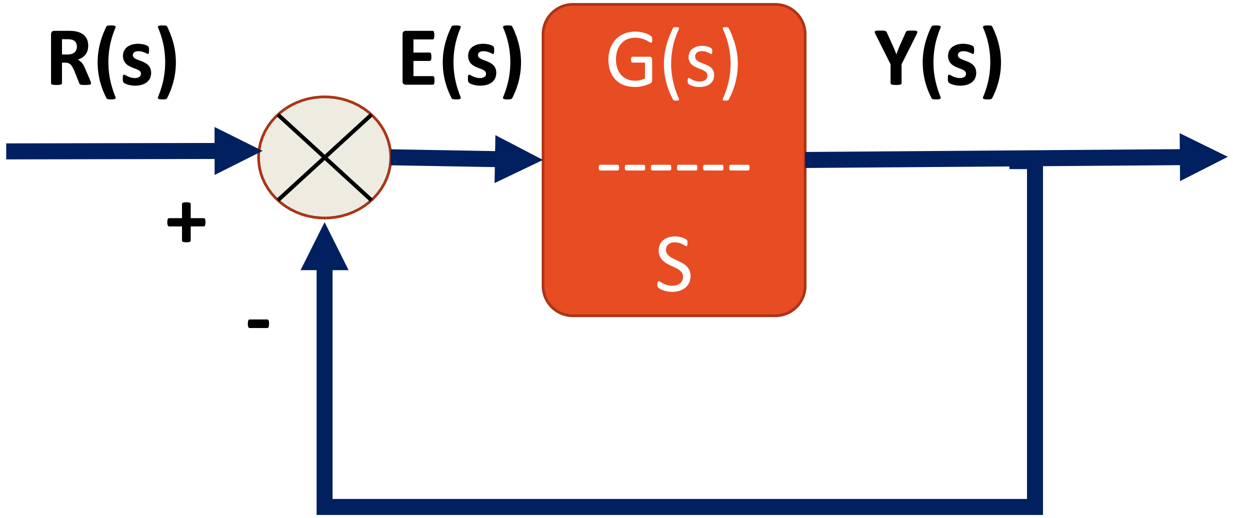
Closed Loop System

Steady State Error

Adding Integrator to the tank



$$e(\infty) = \lim_{s\to 0} sE(s) = \frac{sR(s)}{1+\frac{G(s)}{s}}$$

$$e(\infty) = \lim_{s\to 0} \frac{s\frac{4}{s}}{1+\frac{1}{s}\frac{K}{(\tau s+1)}} = \frac{4}{1+\frac{(\frac{1}{10})}{0}} = \frac{4}{\infty} = 0 \ \ (Type\ 1)$$

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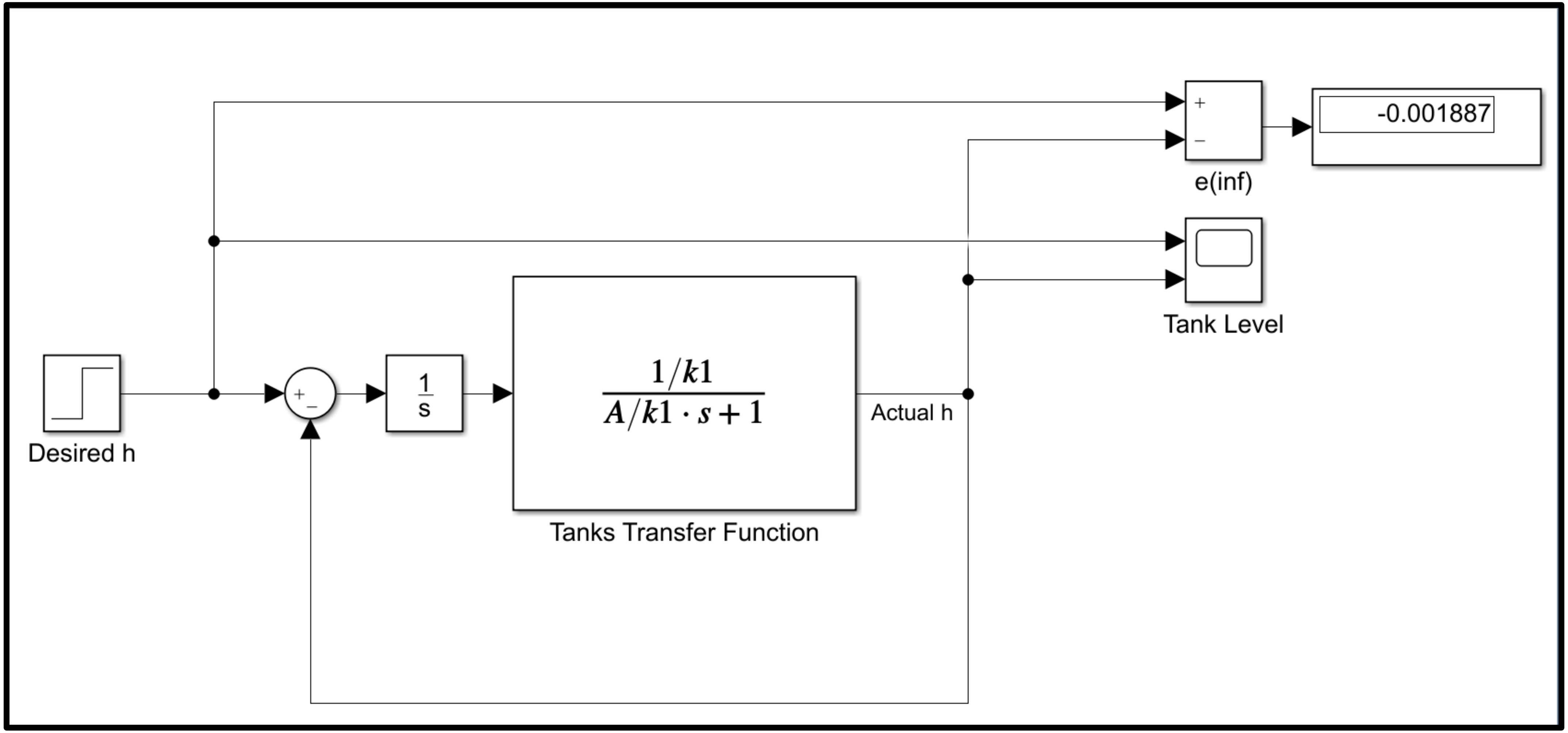
Steady State Error



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Closed Loop System

Steady State Error

Closed Loop Systems

- Steady State Error Analysis

System Type :

System Input :	zero (0)	One (1)	Two (2)
Step Input u (t)	$1 / (1 + k_P)$	Zero	Zero
Ramp Input r (t)	Infinity	$1 / k_V$	Zero
Parabolic Input p (t)	Infinity	Infinity	$1 / k_A$

Don’t forget to pull the lab update from.

http://github.com/wbadry/mte506

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END OF Lab 3