# Growth of Functions

[1] Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of $\Theta$-notation, prove that $max(f(n), g(n)) = \Theta(f(n) + g(n))$.

[2] Show that for any real constants $a$ and $b$, where $b > 0$,

$$(n+a)^b = \Theta(\mathrm{n}^{\mathrm{b}})$$

[3] Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

[4] We can extend our notation to the case of two parameters $n$ and $m$ that can go to infinity independently at different rates. For a given function $g(n, m)$, we denote by $O(g(n, m))$ the set of functions

$O(g(n, m)) = \{f(n, m)$: there exist positive constants $c$, $n_0$, and $m_0$ such that $0 \leq f(n, m) \leq cg(n, m)$ for all $n \geq n_0$ and $m \geq m_0\}$.

Give corresponding definitions for $\Omega(g(n, m))$ and $\Theta(g(n, m))$.

[5] Prove that $\mathrm{f(n)} = \mathrm{O(g(n))}$ iff $\mathrm{g(n)} = \Omega(\mathrm{f(n)})$.

[6] For each of the following statements, decide whether it is **always true**, **never true**, or **sometimes true** for asymptotically nonnegative functions f and g. If it is **always true** or **never true**, explain why. If it is **sometimes true**, give one example for which it is true, and one for which it is false.

(a) $\mathrm{f(n)} = \mathrm{O(f(n)^2)}$
(b) $\mathrm{f(n)} + \mathrm{g(n)} = \Theta(\max(\mathrm{f(n)}, \mathrm{g(n)}))$
(c) $\mathrm{f(n)} + \mathrm{O(f(n))} = \Theta(\mathrm{f(n)})$

*Good Luck* ☺