COLLEGE
Tanta Engineering

إعدادي 2020

تفاضل

مفكوك تايلور

سنتر فيوتشر

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Subject: ادارى رياضه

Chapter: مفكوك تيلور

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مفكوك تيلور/ماكلورين

$$f(x) = f(a) + (x-a)y' + \frac{(x-a)^2}{2!}y'' + \frac{(x-a)^3}{3!}y'''$$

اذا طلب مفكوك ماكلورين $a = 0$

$$f(x) = f(0) + x \cdot y' + \frac{x^2}{2!}y'' + \frac{x^3}{3!}y''' + \cdots$$

* لو طلب المفكوك بدلالة قوى X يعني $a = 0$

بدلالة قوى $(x-a)$ يعني تيلور

Expand near $X = 5$ $\qquad a = 5$

Expand about $x = 4$ $\qquad a = 4$

Find taylor series in power of $(x-3)$

يعني $a = 3$

Ex

Prove that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

hence find expansion e^{-x}, e^{x^2}, e^{2x}

$\sinh x$, $\cosh x$

$y = e^{x} \xrightarrow{\text{عوض } x=0} y(0) = 1$

$y' = e^{x}$ $\qquad y' = 1$

$y'' = e^{x}$ $\qquad y'' = 1$

$y''' = e^{x}$ $\qquad y''' = 1$

$y^{(4)} = e^{x}$ $\qquad y^{(4)} = 1$

$y^{(5)} = 1$

$$\therefore e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$$

لإيجاد مفكوك e^{2x} نعوض عن x بـ $2x$

$$\therefore e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!}$$

لإيجاد مفكوك e^{x^2} نعوض عن x في المعادلة بـ x^2

$$\therefore e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!}$$

لإيجاد مفكوك e^{-x} نعوض عن الـ x بـ $-x$

$$\therefore e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}$$

$$\because e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \longrightarrow ①$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} \longrightarrow ②$$

$$\because \sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

بطرح ① ، ②

$$\therefore \sinh x = \cancel{\frac{1}{2}}\left[\cancel{2}x + \frac{\cancel{2}x^3}{3!} + \frac{\cancel{2}x^5}{5!} + \frac{\cancel{2}x^7}{7!} \dots\right.$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!}$$

$$\because \cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

بجمع ① ، ②

$$= \cancel{\frac{1}{2}}\left[\cancel{2} + \frac{\cancel{2}x^2}{2!} + \frac{\cancel{2}x^4}{4!} + \frac{\cancel{2}x^6}{6!} \dots\right]$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} \dots$$

Find maclorine expansion $f(x) = \cos x$

<u>OR</u> <u>Prove</u> $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

hence find expansion, $\cos x^2$, $\cos^2 x$ ③

$y = \cos x \quad \text{———} \quad y(0) = 1$

$y' = -\sin x \qquad y' = 0$

$y'' = -\cos x \qquad y'' = -1$

$y''' = \sin x \qquad y''' = 0$

$y^{(4)} = \cos x \qquad y^{(4)} = 1$

$y^{(5)} = -\sin x \qquad y^{(5)} = 0$

$y^{(6)} = -\cos x \qquad y^{(6)} = -1$

$$\therefore \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots$$

ايجاد مفكوك $\cos x^2$ نضع x^2 مكان ال x

$$\therefore \cos x^2 = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} \cdots$$

يعني لو جات $\cos x^2$ تقدر تحسب $\cos x$ ثم نعوض مكان ال x ب x^2

$$\therefore \cos 2x = 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \frac{64x^6}{6!}$$

عوضنا عن x ب $2x$

$$\therefore \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$ ٤

$$\cos^2 x = \frac{1}{2}\left(1 + 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \frac{64x^6}{6!}\right.$$

$$\cos^2 x = \frac{1}{2}\left[2 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} \cdots\right.$$

Find in maclorine Series $f(x) = \ln(1+x)$

and find $\tanh^{-1} x$, and approximation value

$\ln(1.3)$

$$y = \ln(1+x) \qquad y(0) = 0$$

$$y' = (1+x)^{-1} \qquad y' = 1$$

$$y'' = -(1+x)^{-2} \qquad y'' = -1$$

$$y''' = 2(1+x)^{-3} \qquad y''' = 2$$

$$y^{(4)} = -6(1+x)^{-4} \qquad y^{(4)} = -6$$

$$\therefore \ln(1+x) = 0 + x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{6x^4}{4!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots \quad (1)$$

لو جالك دفعولك $\ln(1-x)$ عوض في ال x بـ $-x$ (0)

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \cdots \quad (2)$$

$\because \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$

$= \frac{1}{2}\left[\ln(1+x) - \ln(1-x)\right]$

دمج ① ②

$= \frac{1}{2}\left[2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} \cdots\right.$

$\therefore \tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} \cdots$

$\ln(1.3)$ ايجاد قيمة تقريب

$\because \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$

$\ln(\cancel{1+x}\ 1.3) = 0.3 - \frac{0.09}{2} + \frac{0.027}{3} - \frac{(0.3)^4}{4}$

ونجمع

Expand $\sin x$ in taylor series near $x = \pi/3$

and find an approximation value $\sin 62°$

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$$y = \sin x \qquad y(\pi/3) = \frac{\sqrt{3}}{2}$$

$$y' = \cos x \qquad y' = \frac{1}{2}$$

$$y'' = -\sin x \qquad y'' = -\frac{\sqrt{3}}{2}$$

$$y''' = -\cos x \qquad y''' = -1/2$$

$$\therefore \sin x = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{2}\frac{\left(x - \pi/3\right)^2}{2!} - \frac{1}{2}\frac{\left(x - \frac{\pi}{3}\right)^3}{3!}$$

لإيجاد قيمة تقريب $\sin 62^\circ$.

① نوجد مفكوك تيلور للدالة حول $x = \pi/3$

ثم نعوض $x = 62^\circ$

$$\therefore \sin(62^\circ) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left[62^\circ - 60^\circ\right]\frac{\pi}{180} - \frac{\sqrt{3}}{2}\frac{\left(2^\circ \times \frac{\pi}{180}\right)^2}{2!}$$

ونحل

Prove that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$$

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$$y = (1+x)^n \quad \text{———} \quad y(0) = 1$$

$$y' = n(1+x)^{n-1} \qquad y' = n$$

$$y'' = n(n-1)(1+x)^{n-2} \qquad y'' = n(n-1)$$

$$y''' = n(n-1)(n-2)(1+x)^{n-3} \qquad y''' = n(n-1)(n-2)$$

$$\therefore (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 - -$$

Expand $\ln x$ in taylor in power of $(x-1)$

$$y = \ln x \qquad y(1) = 0$$

$$y' = x^{-1} \qquad y' = 1$$

$$y'' = -x^{-2} \qquad y'' = -1$$

$$y''' = 2x^{-3} \qquad y''' = 2$$

$$y^{(4)} = -6x^{-4} \qquad y^{(4)} = -6$$

$$\ln x = 0 + (x-1) - \frac{(x-1)^2}{2!} - \frac{2(x-1)^3}{3!}$$

$$\ln x = (x-1) - \frac{(x-1)^2}{2} - \frac{(x-1)^3}{3} + \frac{(x-1)^4}{4}$$

Expand $\tan x$ in maclorine series

$y = \tan x$ $\quad y(0) = 0$

$y' = \sec^2 x$ $\quad y' = 1$

$y'' = 2\sec x \cdot \sec x \tan x$ $\quad y'' = 0$

$= 2\sec^2 x \tan x$

$y''' = 2\sec^2 x \cdot \sec^2 x + 4 \cdot \sec x \sec x \tan x \tan x$

$= 2\sec^4 x + 4\sec^2 x \tan^2 x$ $\quad y''' = 2$

$y^{(4)} = 8\sec^3 x \cdot \sec x \tan x + 4\sec x \cdot \sec x \tan x + 8\sec^2 x \cdot \tan x \sec^2 x$

$y^{(4)} = 16\sec^4 x \tan x + 4\sec^2 x \tan x$ $\quad y^{(4)}(0) = 0$

$y^{(5)} = 64\sec^3 x \sec x \tan x \cdot \tan x + 16\sec^6 x + 4\sec^2 x \sec^2 x + 8\sec x \cdot \sec x \tan x \cdot \tan x$

$y^{(5)} = 20$

$\therefore \tan x = x + \frac{2x^3}{3!} + \frac{20x^5}{5!} \ldots$ ⓐ

$= x + \frac{x^3}{3} + \frac{x^5}{6} - - -$

Expand e^{2X} in taylor near $X=1$

OR Expand e^{2X} in power of $(x-1)$

$y = e^{2X} \xrightarrow[X=1]{\text{عوض}} y(1) = e^2$

$y' = 2e^{2X}$ $\quad y' = 2e^2$

$y'' = 4e^{2X}$ $\quad y'' = 4e^2$

$y''' = 8e^{2X}$ $\quad y''' = 8e^2$

$$\therefore e^{2X} = e^2 + 2e^2(X-1) + 4e^2\frac{(X-1)^2}{2!} + 8e^2\frac{(X-1)^3}{3!} + \cdots$$

Expand $f(X) = \cos X$ near $X = \frac{\pi}{4}$

$y = \cos X \xrightarrow{X=\frac{\pi}{4} \text{ عوض}} y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$y' = -\sin X$ $\quad y' = \frac{-1}{\sqrt{2}}$

$y'' = -\cos X$ $\quad y'' = -\frac{1}{\sqrt{2}}$

$y''' = +\sin X$ $\quad y''' = \frac{1}{\sqrt{2}}$

$y^{(4)} = \cos X$

$$\therefore \cos X = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right) - \frac{1}{\sqrt{2}}\frac{\left(X-\frac{\pi}{4}\right)^2}{2!} + \frac{1}{\sqrt{2}}\frac{\left(X-\pi/4\right)^3}{3!}$$

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by using maclorine series find approximation value $\sqrt{50}$ ط

$$\sqrt{50} = (49+1)^{\frac{1}{2}}$$

$$= 7\left[1+\frac{1}{49}\right]^{\frac{1}{2}}$$

$$= 7\left[1+\frac{1}{2}\left(\frac{1}{49}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{1}{49}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{1}{49}\right)^3\right.$$

دنجح

Ex Find approximation value $e^{1.2}$ ط

نو هرسقطوع حول $x=1$

$y = e^x$

$y = e^x \longrightarrow y(1) = e$

$y' = e^x \qquad y' = e$

$y'' = e^x \qquad y'' = e$

$$\therefore e^x = e + e(x-1) + e\frac{(x-1)^2}{2!} + \frac{e(x-1)^3}{3!}$$

put $x = 1.2$

$$\therefore e^{1.2} = e + e(0.2) + e\frac{(0.2)^2}{2!} + e\frac{(0.2)^3}{3!}$$

$$= e\left[1+0.2+\frac{0.04}{2!}+\frac{0.008}{3!}\ldots\right]$$

by using maclorine Find

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

$$\because e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - - -$$

$$\therefore \frac{e^x - 1}{x} = \frac{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - - - - 1}{x}$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} 1 + \frac{x}{2!} + \frac{x^2}{3!} + - - -$$

$$= 1$$

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by maclorine prove that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$y = \sin x \xrightarrow{x=0} y(0) = 0$$

$$y' = \cos x \qquad y' = 1$$

$$y'' = -\sin x \qquad y'' = 0$$

$$\therefore \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}}{x}$$

$$= \lim_{x \to 0} 1 - \frac{x^2}{3!} + \frac{x^4}{5!} = 1 \ \#$$