سنتر فيوتشر

Subject: رياضة إعدادي

Chapter: نظرية ذات الحدين

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binomial نظرية ذات الحدين

n عدد صحيح موجب

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \cdots$$
$$+ {}^nC_r a^{n-r} b^r + \cdots + b^n$$

حفظ

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

Expand $(2+3x)^4$

$$2^4 + {}^4C_1 (2)^3 (3x) + {}^4C_2 (2)^2 (3x)^2 + {}^4C_3 (2)^1 (3x)^3 + {}^4C_4 (3x)^4$$

$$= 16 + 4(8)(3x) + 6(4)(9x^2) + 4(2)(27x^2) + 81x^4$$

Expand

$$[(1+2x) + x^2]^4$$

$$(1+2x)^4 + {}^4C_1 (1+2x)^3 (x^2) + {}^4C_2 (1+2x)^2 (x^2)^2 + {}^4C_3 (1+2x)(x^2)^3 + {}^4C_4 (x^2)^4$$

$$= (1+2x)^4 + 4x^2(1+2x)^3 + 6x^4(1+2x)^2 + x^8 + 4(1+2x)^6$$

$(1+2x)^4$,   ① $(1+2x)^3$ — $(1+2x)^2$ نفكهم تاني — نكمل

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اذا كانت n كسر او عدد سالب

$$(1+u)^n = 1 + nu + \frac{n(n-1)}{2!}u^2 + \frac{n(n-1)(n-2)}{3!}u^3 + \cdots$$

شرط صحة المفكوك : $|u| < 1$

Condition for Convergance $|u| < 1$

Expand $(1+3x)^{-4}$

$|3x| < 1 \qquad |x| < 1/3$

$$(1+3x)^{-4} = 1 - 4(3x) + \frac{(-4)(-5)}{2!}(3x)^2 + \frac{(-4)(-5)(-6)}{3!}(3x)^3 + \cdots$$

Expand $\left(1-\frac{s}{x}\right)^{1/3}$

$\left|\frac{s}{x}\right| < 1 \qquad |s| < |x|$ شرط صحة المفكوك

$$\left(1-\frac{s}{x}\right)^{\frac{1}{3}} = \left(1+\frac{-s}{x}\right)^{\frac{1}{3}} = 1 + \frac{1}{3}\left(\frac{-s}{x}\right) + \frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{2!}\left(\frac{s}{x}\right)^2$$

$$+ \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)}{3!}\left(\frac{s}{x}\right)^3 + \cdots$$

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Expand $(2+3x)^{-4}$

① in power of x increasing

② in power of x decreasing ________ لـ

① عندما الأس يزداد من جهة الـ x بسط

$$(2+3x)^{-4} = 2^{-4}\left[1+\frac{3x}{2}\right]^{-4}$$

Condition $\left|\frac{3x}{2}\right| < 1$ $\quad |x| < 2/3$

$$\therefore 2^{-4}\left[1+\frac{3x}{2}\right]^{-4} = \frac{1}{16}\left[1 - 4\left(\frac{3x}{2}\right) + \frac{(-4)(-5)}{2!}\,\frac{9x^2}{4} + \frac{(-4)(-5)(-6)}{3!}\left(\frac{3x}{2}\right)^3 - - -\right.$$

② عندما يتناقص الـ x بتكون مقام x

$$(2+3x)^{-4} = (3x)^{-4}\left[1+\frac{2}{3x}\right]^{-4}$$

Condition for Convergance شرط تقارب المتسلسلة

$$\left|\frac{2}{3x}\right| < 1$$

$$\frac{2}{3} < |x|$$

$$(3x)^{-4}\left[1 - 4\left(\frac{2}{3x}\right) + \frac{(-4)(-5)}{2!}\left(\frac{2}{3x}\right) + \frac{(-4)(-5)(-6)}{3!}\left(\frac{2}{3x}\right)^2 - - -\right]$$

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Expand $\left(2+\frac{5}{X}\right)^{-4}$

in Power of x increasing

" " " x decreasing

② $\left(2+\frac{5}{2X}\right)^{-4} = 2^{-4}\left[1-4\left(\frac{5}{2X}\right)+\frac{(-4)(-5)}{2!}\left(\frac{5}{2X}\right)^{2}+----\right]$

$\left|\frac{5}{2X}\right| < \quad \frac{5}{2} < |X|$

① in power of x increasing

$\left(\frac{5}{X}\right)^{-4}\left[1+\frac{2X}{5}\right]^{-4}$

$\left(\frac{5}{X}\right)^{-4}\left[1-4\left(\frac{2X}{5}\right)+\frac{(-4)(-5)}{2!}\left(\frac{2X}{5}\right)^{2}+\frac{(-4)(-5)(-6)}{3!}\left(\frac{2X}{5}\right)^{3}\right.$ ---

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Expand

$\left(2+\frac{3}{X}\right)^{-4}$ ① $X >>>$ ② $x <<<$

بناخد الأكبر مشترك

$\left(2+\frac{3}{X}\right)^{-4} = 2^{-4}\left[1+\frac{3}{2X}\right]^{-4}$ $\quad \underline{\underline{x >>>}}$

$2^{-4}\left[1-4\left(\frac{3}{2X}\right)+\frac{(-4)(-5)}{2!}\left(\frac{3}{2X}\right)^{2}+\frac{(-4)(-5)(-6)}{3!}\left(\frac{3}{2X}\right)^{3}--\right.$

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$\because$ if $x <<<$

$$\left(2+\frac{3}{x}\right)^{-4} = \left(\frac{3}{x}\right)^{-4}\left[1+\frac{2x}{3}\right]^{-4}$$

$\left|\frac{2x}{3}\right| < 1 \qquad \frac{2}{3} < |x|$ Condition for Conv

$$\therefore \left(2+\frac{3}{x}\right)^{-4} = \left(\frac{3}{x}\right)^{-4}\left[1-4\left(\frac{2x}{3}\right)-\frac{4(-5)}{2!}\left(\frac{2x}{3}\right)^2 + \frac{(-4)(-5)(-6)}{3!}\left(\frac{2x}{3}\right)^3 \cdots\right.$$

Expand $(5+3x)^{1/2}$

① $x >>$ ② $x <<<$

[1] if $x >>>$ $\quad (5+3x)^{1/2} = \sqrt{3x}\left[1+\frac{5}{3x}\right]^{1/2}$

$$= \sqrt{3x}\left[1+\frac{1}{2}\left(\frac{5}{3x}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{3x}\right)^2+\cdots\right.$$

if $x <<$ $\quad (5+3x)^{1/2} = \sqrt{5}\left[1+\frac{3x}{5}\right]^{1/2}$

Condition for Conv $\quad \left|\frac{3x}{5}\right| < 1 \qquad |x| < 5/3$

$$(5+3x)^{1/2} = \sqrt{5}\left[1+\frac{1}{2}\left(\frac{3x}{5}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2!}\left(\frac{3x}{5}\right)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{3x}{5}\right)^3 \cdots\right.$$

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إيجاد قيمة تقريبية للجذر

$\sqrt[3]{10}$ حل

نوجد أقرب قيمة لـ ١٠ لها جذر تكعيبي

$$\sqrt[3]{10} = (2+8)^{1/3}$$

$$= 8^{1/3}\left[1+\frac{2}{8}\right]^{1/3}$$

$$= 2\left[1+\frac{1}{3}\left(\frac{1}{4}\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2!}\left(\frac{1}{4}\right)^2+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{-5}{3}\right)}{3!}\left(\frac{1}{4}\right)^3 \cdots\right]$$

ونجمع

Find approximation val—

$\sqrt[5]{30}$

$$\sqrt[5]{30} = (32-2)^{\frac{1}{5}} = (32)^{\frac{1}{5}}\left[1+\frac{-2}{32}\right]^{\frac{1}{5}}$$

$$= 2\left[1+\frac{-1}{16}\right]^{\frac{1}{5}} = 2\left[1+\frac{1}{5}\left(-\frac{1}{16}\right)+\frac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)}{2!}\left(-\frac{1}{16}\right)^2\right.$$

$$\left.+\frac{\frac{1}{5}\left(-\frac{4}{5}\right)\left(-\frac{9}{5}\right)}{3!}\left(-\frac{1}{16}\right)^3 \cdots\right]$$

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Find approximation value $\sqrt[4]{80}$

$$\sqrt[4]{80} = \sqrt[4]{81-1} = (81-1)^{\frac{1}{4}}$$

$$= (81)^{\frac{1}{4}}\left(1+\frac{-1}{81}\right)^{1/4}$$

$$= 3\left[1+\frac{1}{4}\left(\frac{-1}{81}\right)+\frac{\left(\frac{1}{4}\right)\left(\frac{-3}{4}\right)}{2!}\left(\frac{-1}{81}\right)^2+\frac{\left(\frac{1}{4}\right)\left(\frac{-3}{4}\right)\left(\frac{-7}{4}\right)}{3!}\left(\frac{-1}{81}\right)^3\right.$$

لو طلب error

$$= 3\left[\frac{\left(\frac{1}{4}\right)\left(\frac{-3}{4}\right)\left(\frac{-7}{4}\right)\left(\frac{-11}{4}\right)}{4!}\left(\frac{-1}{81}\right)^4\right]$$

Find $\sqrt[3]{\frac{97}{101}}$

$$\left(\frac{97}{101}\right)^{\frac{1}{3}} = \left(\frac{101-4}{101}\right)^{\frac{1}{3}} = \left(1+\frac{-4}{101}\right)^{1/3}$$

$$= 1+\frac{1}{3}\left(\frac{-4}{101}\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\frac{-4}{101}\right)^2$$

$$+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}\left(\frac{-4}{101}\right)^3+\cdots$$

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Find $\sqrt[5]{\frac{41}{17}}$ ط

$$\left(\frac{41}{17}\right)^{\frac{1}{5}} = \left(\frac{17+24}{17}\right)^{\frac{1}{5}} = \cancel{\left(1+\frac{24}{17}\right)^{\frac{1}{5}}}$$

غلط لأن البسط اكبر

$$\therefore \sqrt[5]{\frac{41}{17}} = \left(\frac{17}{41}\right)^{-\frac{1}{5}} = \left(\frac{41-24}{41}\right)^{-1/5}$$

$$= \left[1+\frac{-24}{41}\right]^{-\frac{1}{5}} = 1 + \left(-\frac{1}{5}\right)\left(\frac{-24}{41}\right) + \frac{\left(-\frac{1}{5}\right)\left(\frac{-6}{5}\right)}{2!}\left(-\frac{24}{41}\right)^2$$

$$+ \frac{\left(-\frac{1}{5}\right)\left(\frac{-6}{5}\right)\left(-\frac{11}{5}\right)}{3!}\left(-\frac{24}{41}\right)^3 + \cdots$$

Find approximation ل $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ ط

$$\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{4+3+4\sqrt{3}}{4-3}$$

$$= 7+4\sqrt{3} = 7+4\left[4-1\right]^{1/2} = 7+8\left(1-\frac{1}{4}\right)^{1/2}$$

$$= 7+(4)(2)\left[1+\frac{1}{2}\left(\frac{-1}{4}\right)+\frac{\frac{1}{2}\left(-3/2\right)}{2!}\left(\frac{-1}{4}\right)^2 \cdots\right.$$

A

ونحو

<u>مهم جداً</u>

$$\frac{1}{1-u} = 1 + u + u^2 + u^3 + \cdots + u^n$$

$$\frac{1}{1+u} = 1 - u + u^2 - u^3 + \cdots + (-u)^n$$

$$\frac{1}{(1-u)^2} = 1 + 2u + 3u^2 + 4u^3 \cdots + (n+1)u^n$$

$$\frac{1}{(1+u)^2} = 1 - 2u + 3u^2 - 4u^3 \cdots + (n+1)(-u)^n$$

<u>حالة عامة</u>

$$(1+u)^{-k} = 1 - ku - \frac{k(-k-1)}{2!}u^2 + \cdots + {}^{n+k-1}C_n\,(-u)^n$$

Expand $(2+3x)^{-4}$ and find Coefficient x^n

$$(2+3x)^{-4} = 2^{-4}\left[1 + \frac{3x}{2}\right]^{-4}$$

شرط صحة المفكوك

$$\left|\frac{3x}{2}\right| < 1 \qquad |x| < 2/3$$

$$(2+3x)^{-4} = 2^{-4}\left[1 - 4\left(\frac{3x}{2}\right) + \frac{(-4)(-5)}{2!}\left(\frac{3x}{2}\right)^2 + \cdots + {}^{n+4-1}C_n\left(\frac{-3x}{2}\right)^n\right]$$

$\therefore$ Coefficient x^n is $2^{-4} \cdot {}^{n+3}C_n \left(\frac{-3}{2}\right)^n$

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Expand $\frac{2}{(5+4x)^2}$ and find Coefficient x^n

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$$\frac{2}{(5+4x)^2} = \frac{2}{25}\left(\frac{1}{\left(1+\frac{4x}{5}\right)^2}\right)$$

$\left|\frac{4x}{5}\right| < 1$ شرط صحة التحول

$\therefore |x| < 4/5$

$$\frac{2}{(5+4x)^2} = \left(\frac{2}{25}\right)\left[1 \mp 2\left(\frac{4x}{5}\right) + (3)\left(\frac{4x}{5}\right)^2 + (4)\left(\frac{4x}{5}\right)^3 + \cdots (n+1)\left(-\frac{4x}{5}\right)^n\right]$$

$\therefore$ Coefficient x^n is $\frac{2}{25}(n+1)\left(-\frac{4}{5}\right)^n$

Expand $\frac{3}{5-x}$

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$$\frac{3}{5}\left[\frac{1}{1-x/5}\right] = \frac{3}{5}\left[1+\frac{x}{5}+\left(\frac{x}{5}\right)^2+\cdots+\left(\frac{x}{5}\right)^n\right]$$

$\therefore$ معامل x^n هو

$$\frac{3}{5}\left(\frac{1}{5}\right)^n$$

لو طلب معامل x^{2n} هو

$$\frac{3}{5}\left(\frac{1}{5}\right)^{2n}$$

عوضت عن n بـ $2n$

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Factorize to partial Fraction and Expand

Find Coefficient x^n

$$f(x) = \frac{12x}{(x-2)(x-3)}$$

$$\frac{12x}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

let $x = 2$ $\qquad A = \left.\frac{12x}{x-3}\right|_{x=2} = \frac{24}{-1}$

let $x = 3$ $\qquad B = \left.\frac{12x}{x-2}\right|_{x=3} = 36$

$$\therefore f(x) = \frac{36}{x-3} - \frac{24}{x-2}$$

$$= \frac{36}{-3}\left[\frac{1}{1-x/3}\right] + \frac{24}{2}\left[\frac{1}{1-x/2}\right]$$

$$-12\left[1 + \frac{x}{3} + \left(\frac{x}{3}\right)^2 + --- + \left(\frac{x}{3}\right)^n\right] +$$

$$12\left[1 + \frac{x}{2} + \left(\frac{x}{2}\right)^n + -- + \left(\frac{x}{2}\right)^n\right]$$

معامل x^n هو :

$$12\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n\right]$$

شروط التحويل

$\therefore |x| < 2$ $\qquad \left|\frac{x}{3}\right| < 1 \ , \ \left|\frac{x}{2}\right| < 1$

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Expand and find Coefficient x^7

$$f(x) = \frac{12x}{(x-2)^2(x+3)}$$

$$\frac{12x}{(x-2)^2(x+3)} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

let $x=-3$ $\qquad A = \frac{-36}{25}$

let $x=2$ $\qquad C = \frac{24}{5}$

let $x=0$ $\qquad 0 = \frac{A}{3} - \frac{B}{2} + \frac{C}{4}$

$$\frac{B}{2} = \frac{-12}{25} + \frac{6}{5} = \frac{30-12}{25} = \frac{18}{25}$$

$$B = 36/25$$

$$f(x) = \frac{-36/25}{x+3} + \frac{36/25}{x-2} + \frac{24/5}{(x-2)^2}$$

$$= \frac{-12}{25}\left[\frac{1}{1+\frac{x}{3}}\right] - \frac{18}{25}\left[\frac{1}{1-x/2}\right] + \frac{6}{5}\left[\frac{1}{(1-\frac{x}{2})^2}\right]$$

شرط صحة المفكوك

$\left|\frac{x}{3}\right| < 1 \longrightarrow |x| < 3$,

$\left|\frac{x}{2}\right| < 1 \qquad |x| < 2$

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$|x| < 2$ شرط صحة المفكوك :

$$\therefore \frac{12x}{(x-2)^2(x+3)} = \frac{-12}{25}\left[1 - \frac{x}{3} + \left(\frac{x}{3}\right)^2 - \left(\frac{x}{3}\right)^3 \cdots + \left(-\frac{x}{3}\right)^n\right]$$

$$+\frac{18}{25}\left[1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 - - + \left(\frac{x}{2}\right)^n\right]$$

$$+\frac{6}{5}\left[1 + 2\left(\frac{x}{2}\right) + 3\left(\frac{x}{2}\right)^2 + - - - + (n+1)\left(\frac{x}{2}\right)^n\right]$$

$\therefore$ معامل x^n هو

$$\frac{-12}{25}\left[-\frac{1}{3}\right]^n - \frac{18}{25}\left(\frac{1}{2}\right)^n + \frac{6}{5}(n+1)\left(\frac{1}{2}\right)^n$$

Expand $\dfrac{2x+3}{x^3+x^2+x+1}$ and find coefficient

x^n ـــــــــــــــ

$$\frac{2x+3}{x^3+x^2+x+1} = \frac{2x+3}{x^2(x+1)+(x+1)}$$

$$= \frac{2x+3}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

let $x = -1$ $\qquad A = \frac{1}{2}$

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if $x = 0$

$3 = A + C$ $\qquad C = 5/2$

let $x = 1$

$\frac{5}{4} = \frac{A}{2} + \frac{B}{2} + \frac{C}{2}$

$\frac{5}{2} = A + B + C$ $\qquad B = -1/2$

$$f(x) = \frac{1/2}{x+1} + \frac{-\frac{1}{2}x + 5/2}{x^2+1}$$

$$= \frac{1}{2}\left[\frac{1}{1+x}\right] + (5 - x)\left(\frac{1}{1+x^2}\right)$$

$$\frac{1}{2}\left[1 - x + x^2 + x^3 - - - - + (-x)^n\right]$$

$$+ (5 - x)\left[1 - x^2 + x^4 - x^6 - - - + (-1)^n x^{2n}\right]$$

معامل x^n هو $\frac{1}{2}(-1)^n$

معامل x^{2n} هو $\frac{1}{2}(-1)^n 5$

معامل x^{2n+1} هو $\frac{-1}{2}(-1)^n$

$\therefore$ ~~معامل x^{2n} هو $\frac{1}{2}(-1)^{2n} + \frac{1}{2}(-1)^n$~~

معامل $x^{2n+1} = \frac{1}{2}(-1)^{2n+1}$

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∴ معامل x^{2n} هو

$$\frac{5}{2}(-1)^n + \frac{1}{2}(-1)^{2n}$$

عوضنا الحد x^n بجد n بـ $2n$

$$= \frac{5}{2}(-1)^n + \frac{1}{2}$$

معامل x^{2n+1} =

$$-\frac{1}{2}(-1)^n + \frac{1}{2}(-1)^{2n-1}$$

$$= -\frac{1}{2}(-1)^n + \frac{1}{2} = -\frac{1}{2}\left[1 + (-1)^n\right]$$

Find

$$\frac{\sqrt[3]{2}}{\sqrt{2}}$$ ط

$$\frac{\sqrt[3]{2}}{\sqrt{3}} = \frac{2^{1/3}}{3^{1/2}} = \frac{(2^{2})^{\frac{1}{6}}}{(3^3)^{\frac{1}{6}}} = \left(\frac{4}{27}\right)^{\frac{1}{6}}$$

$$= \left(\frac{27-23}{27}\right)^{\frac{1}{6}} = \left[1 - \frac{23}{27}\right]^{\frac{1}{6}}$$

$$= 1 + \frac{1}{6}\left(\frac{-23}{27}\right) + \frac{\frac{1}{6}\left(\frac{-5}{6}\right)}{2!}\left(\frac{-23}{27}\right)^2 + \frac{\left(\frac{1}{6}\right)\left(\frac{-5}{6}\right)\left(\frac{-11}{6}\right)}{3!}\left(\frac{-23}{27}\right)^3$$

ونتج

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Expand $\left(\frac{1+x}{1-x}\right)^2$ Find Coefficient x^n

$$\left(\frac{1+x}{1-x}\right)^2 = (1+x)^2(1-x)^{-2}$$

$$= \left[1+2x+x^2\right]\left(1+2x+3x^2+ \cdots (n+1)x^n\right]$$

Coeffic x^n is

$$(n+1)+2(n)+(n-1) = 4n$$

تذكر مجموع حدود متتابعة الهندسية

$$a, ar, ar^2, \cdots ar^{n-1}$$

$$S = a\,\frac{(1-r^n)}{1-r}$$

Expand $(1+x+x^2+x^3+x^4+x^5)^4$

$$1+x+x^2+x^3+x^4+x^5$$

$$= \frac{1-x^6}{1-x}$$

$$\therefore (1+x+x^2+x^3+x^4+x^5)^4 = \left(\frac{1-x^6}{1-x}\right)^4$$

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$$\left(\frac{1-x^6}{1-x}\right)^4 = (1-x^6)^4\,(1-x)^{-4}$$

نفك عادي

$$\left[1-{}^4C_1x^6+{}^4C_2(x^6)^2-{}^4C_3(x^6)^3+{}^4C_4(x^6)^4\right](1-x)^{-4}$$

$$\left[1-4x^6+6x^{12}-4x^{18}+x^{24}\right]\left[1+4x+\frac{(4)(5)}{2}x^2 --- + {}^{n+4-1}C_n\,x^n\right]$$

معامل x^n هو

$${}^{n+3}C_n-4\,{}^{n-3}C_{n-6}+6\,{}^{n-9}C_{n-12}-4\,{}^{n-15}C_{n-18}+{}^{n-21}C_{n-24}$$

لو طلب معامل x^{17}

$$={}^{20}C_{17}-4\,{}^{14}C_{11}+6\,{}^{8}C_{5}-4\,\cancel{{}^{2}C_{-1}}+\cancel{{}^{-4}C_{-7}}$$

قيمته = صفر

Expand $\left[x^2+x^4+x^6+ --- +x^{12}\right]^3$

. Find Coefficient x^n and Coeff x^{29}, x^{30}

$$(x^2)^3\left[\overline{1+x^2+x^4 -- +x^{10}}^{\,6}\right]^3$$

$$=x^6\left[\frac{1-(x^2)^6}{1-x^2}\right]^3$$

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$$= x^6 \left[\frac{1-x^{12}}{1-x^2}\right]^3 = x^6 (1-x^{12})^3 (1-x^2)^{-3}$$

$$= x^6 \left[1-3x^{12}+3x^{24}-x^{36}\right]\left(1+3x^2+\frac{(3)(4)x^4}{2!} \cdots + {}^{n+3-1}C_n\, x^{2n}\right]$$

$$= \left[x^6-3x^{18}+3x^{30}-x^{42}\right]\left(1+3x^2 - - + {}^{n+2}C_n\, x^{2n}\right)$$

معامل x^{2n} هو

$${}^{n-1}C_{n-3} - 3\,{}^{n-7}C_{n-9} + 3\,{}^{n-13}C_{n-15} - {}^{n-19}C_{n-21}$$

لإيجاد معامل x^{30} نضع $n=15$

$$\text{coeff } x^{30} = {}^{14}C_{12} - 3\,{}^{8}C_{6} + 3\,{}^{2}C_{0} - \cancel{{}^{-4}C_{-6}}$$

صفر

معامل x^{29} = صفر لأن n مينفعش نعوض بكسر لأنه
n رتبة الحد

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Expand $\left(\frac{1+x}{1-x}\right)^4$

$(1+x)^4(1-x)^{-4}$

$= \left[1+4x+6x^2+4x^3+x^4\right]\left[1+4x+ --- + {}^{n+3}C_n (x^n)\right]$

معامل x^n هو

$${}^{n+3}C_n + 4\,{}^{n+2}C_{n-1} + 6\,{}^{n+1}C_{n-2} + 4\,{}^{n}C_{n-3} + {}^{n-1}C_{n-4}$$

Expand $\frac{1-x}{(1+x+x^2)^2}$ Find Coefficient x^n

$\because (1+x+x^2) = \frac{1-x^3}{1-x}$

$(1+x+x^2)^2 = \left(\frac{1-x^3}{1-x}\right)^2$

$\therefore \frac{1-x}{(1+x+x^2)^2} = (1-x)^3(1-x^3)^{-2}$

$= \left[1-3x+3x^2-x^3\right]\left(1+2x^3+3x^6 --- + (n+1)x^{3n}\right)$

معامل x^{3n} هو $(n+1)$

معامل x^{3n+1} هو $-3(n+1)$

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معامل x^{3n+2} هو $3(n+1)$

معامل x^{3n+3} هو $-(n+1)$

معوض x^{3n} نستبدل $(n+1)$ بـ n

معامل x^{3n+1} = $-3(n+1)$

معامل x^{3n+2} هو $3(n+1)$

معامل x^{3n} هو $+(n+1)-n = 1$

Expand $\frac{2+3x}{4-x^2}$ Find cofficent x^n

$$(2+3x)\left(\frac{1}{4-x^2}\right) = (2+3x)\frac{1}{4}\left[\frac{1}{1-\frac{x^2}{4}}\right]$$

$$= \left(\frac{1}{2}+\frac{3x}{4}\right)\left[1+\frac{x^2}{4}+\left(\frac{x^2}{4}\right)^2+\cdots+\left(\frac{x^2}{4}\right)^n\right]$$

∴ معامل x^{2n} هو $\frac{1}{2}\left(\frac{1}{4}\right)^n$

معامل x^{2n+1} هو $\frac{3}{4}\left(\frac{1}{4}\right)^n$

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