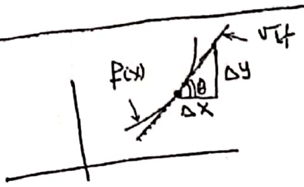
Deritvative الإشتقاق or Differentiation

* تعريفة الرياضى :

$$\boxed{\frac{d f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$



* تعريفة الهندسى : هو ميل المماس للدالة $f(x)$

وهو

$$\boxed{m = \frac{\Delta y}{\Delta x} = \tan(\theta)}$$

* رمز المشتقة الأولى :

$$y' \quad \text{or} \quad \frac{dy}{dx} \quad \text{or} \quad y^{(1)}$$

* رمز المشتقة الثانية :

$$y'' \quad \text{or} \quad \frac{d^2 y}{dx^2} \quad \text{or} \quad y^{(2)}$$

وهكذا ...

* بعض خصائص الإشتقاق

1. $\frac{d}{dx}(\overset{\text{ثابت}}{Const.}) = 0$ (تفاضل)

2. $\frac{d}{dx}(\overset{\text{ثابت}}{K} \cdot \overset{\text{دالة}}{f(x)}) = K * f'(x)$

3. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

4. $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

5. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$

$$= \frac{\text{المقام * تفاضل البسط} - \text{البسط * تفاضل المقام}}{(\text{المقام})^2}$$ ← عربى

6 $\frac{d}{dx}\left[f(x)\right]^{n} = n\left[f(x)\right]^{n-1} \cdot f'(x)$ ← عدد n

7 أكمل $\frac{d}{dx} X^{n} \longrightarrow n X^{n-1}$

8 $\frac{d}{dx} \ln(x) \longrightarrow \frac{1}{x}$

$\frac{d}{dx} \ln(u) \longrightarrow \frac{1}{u} \cdot u'$

9 $\frac{d}{dx} \log_a(u) = \frac{1}{u} \cdot u' \cdot \log_a(e)$

10 $\frac{d}{dx} e^{x} = e^{x}$

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$\frac{d}{dx} e^{u} = e^{u} \cdot u'$

11 $\frac{d}{dx} a^{u} = a^{u} \cdot u' \cdot \ln(a)$

نفسها * تفاضل أسها
* Ln(العدد).

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تفاضل (دالة)$^{\text{دالة}}$ اليشاتهم

12 $y = (f(x))^{g(x)}$

بأخذ Ln للطرفين

من خصائص Ln انه بتنزل الأس

$\therefore \ln(y) = \ln\left[f(x)\right]^{g(x)}$

بتفاضل الطرفين

$\therefore \ln(y) = \underline{g(x)} \cdot \underline{\ln(f(x))}$

بالضرب *y:

$\therefore \frac{1}{y} \cdot y' = g(x) \cdot \frac{1}{f(x)} \cdot f'(x) + \ln(f(x)) \cdot g'(x)$

$\boxed{\therefore y' = y\left[\frac{g(x)}{f(x)} \cdot f'(x) + \ln(f(x)) \cdot g'(x)\right]} = \boxed{(f)^{g}\left[\frac{g}{f} f' + \ln(f) \cdot g'\right]}$

Simple Examples

* Find $\frac{dy}{dx}$ for

1) $y = \underbrace{e^{x^2}}_{u} + \underbrace{\ln(x^2-1)}_{v} + \underbrace{\frac{\log(x)}{\ln(\sqrt{x})}}_{w}$

الحل

$\boxed{y' = u' + v' + w'} \Rightarrow ⊛$ $\quad \because u = e^{x^2}$ $\quad \boxed{\therefore u' = e^{x^2} \cdot 2x}$

$\because v = \ln(x^2-1)$ $\quad \boxed{\therefore v' = \frac{1}{x^2-1} \cdot 2x}$

$\because w = \frac{\log(x)}{\ln(\sqrt{x})}$ قسمة دالتين

$$\boxed{\therefore w' = \frac{\ln(\sqrt{x}) \cdot \frac{1}{x}\log(e) - \log(x) \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}{[\ln(\sqrt{x})]^2}}$$

بالتعويض في ⊛ #

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2) $y = \underbrace{7^{\sqrt{x\sqrt{x\sqrt{x}}}}}_{u} + \underbrace{\log_3(x^2+\ln(x))}_{v} - \underbrace{\frac{20}{x} \cdot e^{-3\sqrt{x}}}_{w}$

الحل

$\therefore y' = u' + v' - w' \longrightarrow ⊛$

$\because u = 7^{\sqrt{x\sqrt{x\sqrt{x}}}} = 7^{\sqrt{x\sqrt{x \cdot x^{1/2}}}} = 7^{\sqrt{x\sqrt{x^{3/2}}}} = 7^{\sqrt{x \cdot x^{3/4}}} = 7^{\sqrt{x^{7/4}}} = 7^{x^{7/8}}$

$\boxed{\therefore u' = 7^{x^{7/8}} \cdot \frac{7}{8} x^{\frac{-1}{8}} \cdot \ln(7)}$ #

$\because v = \log_3[x^2+\ln(x)] \Longrightarrow$ $\boxed{\therefore v' = \frac{1}{x^2+\ln(x)} \cdot \left(2x + \frac{1}{x}\right) \cdot \log_3(e)}$

$\because w = \frac{20\, e^{-3\sqrt{x}}}{x} \Longrightarrow$ $\boxed{\therefore w' = \frac{x \cdot 20 \cdot e^{-3\sqrt{x}} \cdot (-3)\frac{1}{\sqrt{x}} - 20\, e^{-3\sqrt{x}} \cdot 1}{(x)^2}}$

حاصل قسمة دالتين

بالتعويض في ⊛ #

$$\boxed{3}\quad y = \underbrace{[x^2+3x]^9 \cdot \log(\log(x))}_{u} + \underbrace{2\sqrt{x} \cdot \ln[\ln(\log_3 x)]}_{v}$$

الحل

$y' = u' + v' \to \boxed{*}$

$u =$ حاصل ضرب دالتين

$\therefore u' = [x^2+3x]^9 \cdot \frac{\log(e)}{\log(x)} \cdot \frac{1}{x}\log(e) + \log(\log(x)) \cdot 9[x^2+3x]^8 \cdot (2x+3)$

و $v =$ حاصل ضرب دالتين

$\therefore v' = 2\sqrt{x} \cdot \frac{1}{\ln(\log_3(x))} \cdot \frac{1}{\log_3(x)} \cdot \frac{1}{x}\log_3(e) + \ln[\ln(\log_3(x))] \cdot \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x}}$

عوض في $\boxed{*}$ #

$$\boxed{4}\quad y = \underbrace{e^{\sqrt{\frac{x^2-2}{\ln(x)}}}}_{u} - \underbrace{e^{\ln[\log\sqrt{x-1}]}}_{v} + \underbrace{x^{(x-2)}}_{w}$$

الحل

$y' = u' - v' + w' \to \boxed{*}$ $\quad \because u = e^{\sqrt{\frac{x^2-2}{\ln(x)}}}$

$\therefore u' = e^{\sqrt{\frac{x^2-2}{\ln x}}} \cdot \frac{1}{2\sqrt{\frac{x^2-2}{\ln x}}} \cdot \frac{\ln(x)(2x) - (x^2-2)\cdot\frac{1}{x}}{(\ln x)^2}$

$\because v = \cancel{e}^{\cancel{\ln}[\log\sqrt{x-1}]} = \log(\sqrt{x-1})$

$\therefore v' = \frac{1}{\sqrt{x-1}} \cdot \frac{1}{2\sqrt{x-1}} \cdot \log(e)$

$\because w = x^{(x-2)}$ دالة (دالة)

بأخذ $\ln$ للطرفين

$\therefore \ln(w) = \ln x^{(x-2)} = (x-2) \cdot \ln(x)$

بالتفاضل:

$\therefore \frac{1}{w} \cdot w' = [(x-2) \cdot \frac{1}{x} + \ln(x) \cdot 1]$

بالضرب $\times w$

$\therefore w' = w\left[\frac{x-2}{x} + \ln(x)\right]$

$$\boxed{\therefore w' = x^{x-2}\left[\frac{x-2}{x} + \ln(x)\right]}$$

عوض # *

$\boxed{5}$ $y = \underbrace{Ln[Ln[Ln[\sqrt{x^2+\log(x)}]]]}_{u} - \underbrace{x^{x^x}}_{v}$

$y' = u' + v' \rightarrow \boxed{*}$

$u' = \frac{1}{Ln[Ln[\sqrt{x^2+\log(x)}]]} \cdot \frac{1}{Ln[\sqrt{x^2+\log(x)}]} \cdot \frac{1}{\sqrt{x^2+\log(x)}} \cdot \frac{1}{2\sqrt{x^2+\log(x)}} \cdot (2x + \frac{1}{x}\log(e))$

$\therefore v = x^{x^x} \xrightarrow{} \omega = x^{\omega}$

بأخذ Ln للطرفين

$\therefore Ln(v) = Ln(x^{\omega})$

$Ln(v) = \omega \cdot Ln(x)$ بتفاضل الطرفين

$\therefore \frac{1}{v} \cdot v' = \omega \cdot \frac{1}{x} + Ln(x) \cdot \omega'$

بأخذ Ln للطرفين $\omega = x^x$

$\therefore Ln\,\omega = x \cdot Ln(x)$ بالتفاضل

$\frac{1}{\omega} \cdot \omega' = x \cdot \frac{1}{x} + Ln(x) \cdot 1$

$\omega' = \omega[1 + Ln(x)]$

$\boxed{\therefore \omega' = x^x [1 + Ln(x)]}$

بالضرب v *

$\therefore v' = v[\frac{\omega}{x} + Ln(x) \cdot \omega']$

$\therefore v' = x^{x^x}[\frac{x^x}{x} + Ln(x) \cdot x^x (1 + Ln(x))]$

عوض في $\boxed{*}$ #

$\boxed{6}$ find $\frac{dy}{dx}$ from

حل بإيدك :

$\log(x - \sqrt{x})$

$y = x^2 \cdot \log[\log[Ln(\sqrt{x^2+\sqrt{x}})]] + [Ln(x-1)]$ #

$\boxed{7}$ $y = [\frac{3x-2}{x^2-4}]^7 + Ln[\log[\frac{x^2-1}{\sqrt{x+3}}]]$ #

⑤

* تعريفة الرياضي :

{ التفاضل (الاشتقاقة) الضمني }

(Implicit Derivatives)

* عندما تكون x, y عبارة عن حدود مشتركة ولا يمكن جعل y لوحدها في طرف.

الحل

نقوم بتفاضل الطرفين بالنسبة لـ x :

$\boxed{Ex. (8)}$: $y^3 + 3x.y + x^3 - 5 = 0$

لا يمكن جعل y لوحدها في طرف ∴ نفاضل الطرفين بالنسبة لـ x

$$\therefore \quad 3y^2 . \frac{dy}{dx} + 3\underbrace{\left[1 . \cancel{\frac{dx}{dx}} . y + \frac{dy}{dx} . x\right]} + 3x^2 . \frac{dx}{dx} = 0$$

حيث أن $x.y$ عبارة عن حاصل ضرب دالتين

$$\therefore \quad 3y^2 \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} + 3x^2 = 0$$

خذ $\frac{dy}{dx}$ عامل مشترك من الحدود اللي موجود فيها $\frac{dy}{dx}$

$$\therefore \quad \frac{dy}{dx}(3y^2 + 3x) + 3y + 3x^2 = 0$$

$$\therefore \quad \frac{dy}{dx} = \frac{-\cancel{3}(y + x^2)}{\cancel{3}(y^2 + x)} = \frac{-(y + x^2)}{(y^2 + x)}$$

#

$\boxed{9}$ $y^5+3x^2y^3-7x^6-8=0$

بتفاضل الطرفين بالنسبة لـ x :

$$\therefore 5y^4\cdot\frac{dy}{dx}+3\left(2x\cdot y^3+3y^2\frac{dy}{dx}\cdot x^2\right)-42x^5+0=0$$

$$\therefore 5y^4\cdot\frac{dy}{dx}+6xy^3+9y^2\frac{dy}{dx}\cdot x^2-42x^5=0$$

$$\therefore \frac{dy}{dx}\left[5y^4+9y^2x^2\right]+6xy^3-42x^5=0$$

$$\therefore \frac{dy}{dx}=\frac{-6xy^3+42x^5}{5y^4+9x^2y^2}=\tan(\theta)$$

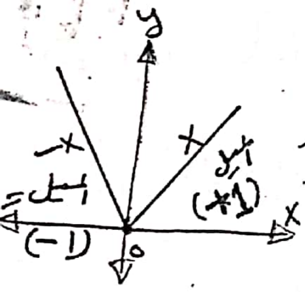
ممكن يعطيني الـ (x,y) اعوض في معادلة $\frac{dy}{dx}$ واجيب $\tan\theta$ ثم $\theta=\tan^{-1}(\ )$

ملاحظة هامة : إذا كانت الدالة لها مشتقة عند نقطة فلابد أن تكون متصلة عند هذه النقطة.

والعكس ليس بالضرورة أن يكون صحيح (أي إذا كانت الدالة متصلة عند نقطة فليس بالضرورة أن يكون لها مشتقة عند هذه النقطة).

مثال ذلك : الدالة $\boxed{f(x)=|x|}$ متصلة عند $x=0$

ولكن ليس لها مشتقة عند $x=0$



لأن ميل المماس عند الصفر $= +1$ و -1 وليس قيمة واحدة فقط

7

Ex 10 find $\frac{dy}{dx}$ from:

$$e^{x\cdot\sqrt{y^2-1}} - 5\ln(x^2-4y) + 7^{x\cdot y} = e^{x^2}$$

الحل: تفاضل ضمنى

نتفاضل الطرفين بالنسبة لـ x :

$$\therefore e^{x\cdot\sqrt{y^2-1}}\cdot\left[x\cdot\frac{1}{\cancel{2}\sqrt{y^2-1}}\cdot \cancel{2}y\cdot y' + \sqrt{y^2-1}\cdot 1\right] - 5\cdot\frac{1}{x^2-4y}\cdot(2x-4y')$$

$$+ 7^{x\cdot y}\cdot(xy'+y\cdot 1) = e^{x^2}\cdot 2x$$

$$\therefore \frac{xy\cdot e^{x\sqrt{y^2-1}}}{\sqrt{y^2-1}}\,y' + e^{x\sqrt{y^2-1}}\cdot\sqrt{y^2-1} - \frac{5(2x)}{x^2-4} + \frac{5(4)}{x^2-4y}\,y' + 7^{xy}\cdot x\,y'$$

$$+ 7^{xy}\cdot y = 2x\cdot e^{x^2}$$

$$y'\underbrace{\left[\frac{xy\,e^{x\sqrt{y^2-1}}}{\sqrt{y^2-1}} + \frac{20}{x^2-4y} + 7^{xy}\cdot x\right]}_{B} = \underbrace{-e^{x\sqrt{y^2-1}}\cdot\sqrt{y^2-1} + \frac{10x}{x^2-4} - 7^{xy}\cdot y + 2xe^{x^2}}_{A}$$

$$\therefore y' = \frac{A}{B} \quad \#$$

Ex 11: find $\frac{dy}{dx}$ from:

$$\ln[\log(x\cdot y)] - \frac{4x^2}{x^3-y^3} + 5\,7^{x} = e^{y}$$

الحل

نتفاضل الطرفين بالنسبة لـ x

$$\therefore \frac{1}{\log(xy)}\cdot\frac{\log(e)}{xy}\cdot(xy'+y\cdot 1) - \frac{[x^3-y^3]\cdot 8x - 4x^2(3x^2-3y^2\cdot y')}{[x^3-y^3]^2} + 5\cdot 7^{x}\ln 7 = e^{y}y'$$

$$\therefore \frac{x\log(e)\cdot y'}{xy\log(xy)} + \frac{y\log(e)}{xy\log(xy)} - \frac{8x(x^3-y^3)}{(x^3-y^3)^2} + \frac{12x^4}{(x^3-y^3)^2} - \frac{12x^2y^2\,y'}{(x^3-y^3)^2} + 5\cdot 7^{x}\ln 7 = e^{y}\,y'$$

عامل مشترك كما سبق ... . كمل

Chain Rule قاعدة السلسلة

معطى Given $y = f(x)$

و $x = g(t)$

مطلوب $\boxed{\dfrac{dy}{dt}} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt}$

$y \to x \to t$

Example 12: if $y = e^{\sin(x^2)} + \ln[\log\sqrt{x^2-1}]$

و $x = \underline{3t^2} \cdot \underline{\sin(t)} + \underline{2\cos(t)} \cdot \underline{\ln(\sqrt{t})}$

Find $\dfrac{dy}{dt}$ (Solution)

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

حيث: $\dfrac{dy}{dx} = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x + \dfrac{1}{\log(\sqrt{x^2-1})} \cdot \dfrac{\log(e)}{\sqrt{x^2-1}} \cdot \dfrac{1}{2\sqrt{x^2-1}} \cdot 2x$

حاصل ضرب دالتين

و $\dfrac{dx}{dt} = 3t^2 \cdot \cos(t) + \sin(t) \cdot (6t) + 2\cos(t) \cdot \dfrac{1}{\sqrt{t}} \cdot \dfrac{1}{2\sqrt{t}}$

$+ \ln(\sqrt{t})\,(-2\sin(t))$ #

EX: 13: if $y = (3x^2 + 5x - 1) \cdot 7^{x^2-1}$ حل بإيدك

و $x = 2t^2 \cdot \ln(3t)$

Find $\dfrac{dy}{dt}$ #

الدوال المثلثية

$\sin\theta$, $\cos\theta$, $\tan\theta$

, $\sec\theta = \dfrac{1}{\cos\theta}$

, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

, $\cot\theta = \dfrac{1}{\tan\theta}$

بعض أشكال الدوال المثلثية وخصائصها :

$\boxed{\text{I} \quad y = \sin(x)}$

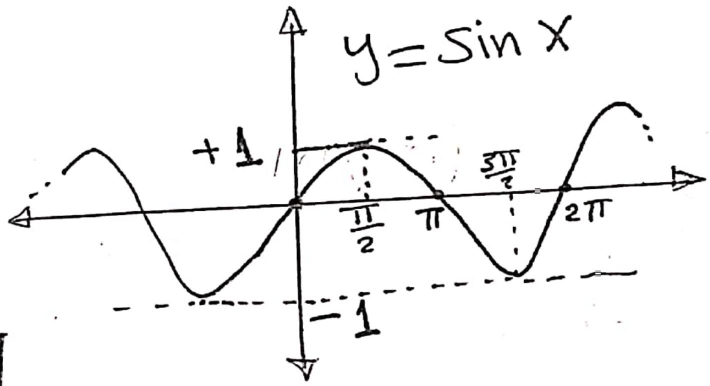
على محور X — Domain (المجال) = $\mathbb{R}$

المدى على محور y — $R_f = [-1, 1]$

- الدالة فردية (متماثلة حول نقطة الأصل)
odd F^n

$\boxed{\therefore \sin(-x) = -\sin x}$

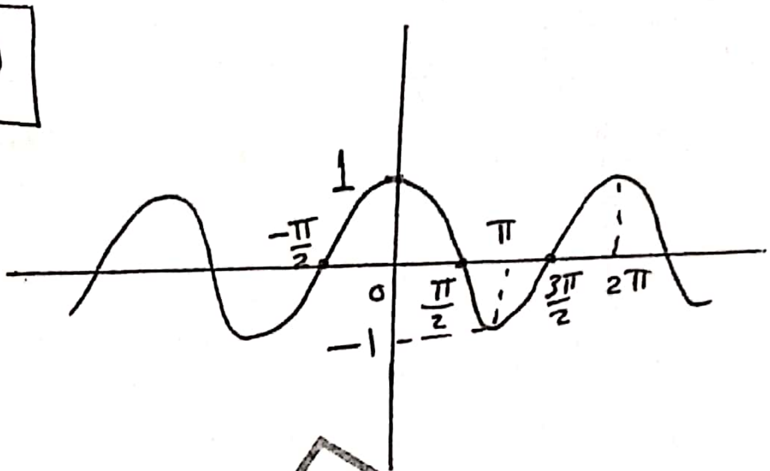
$(f(-x) = -f(x))$



الدالة دورية : أي أنها تكرر نفسها كل فترة (دورة)
Periodic

ودورتها هي 2π

$\boxed{2}$ $y = \cos(x)$



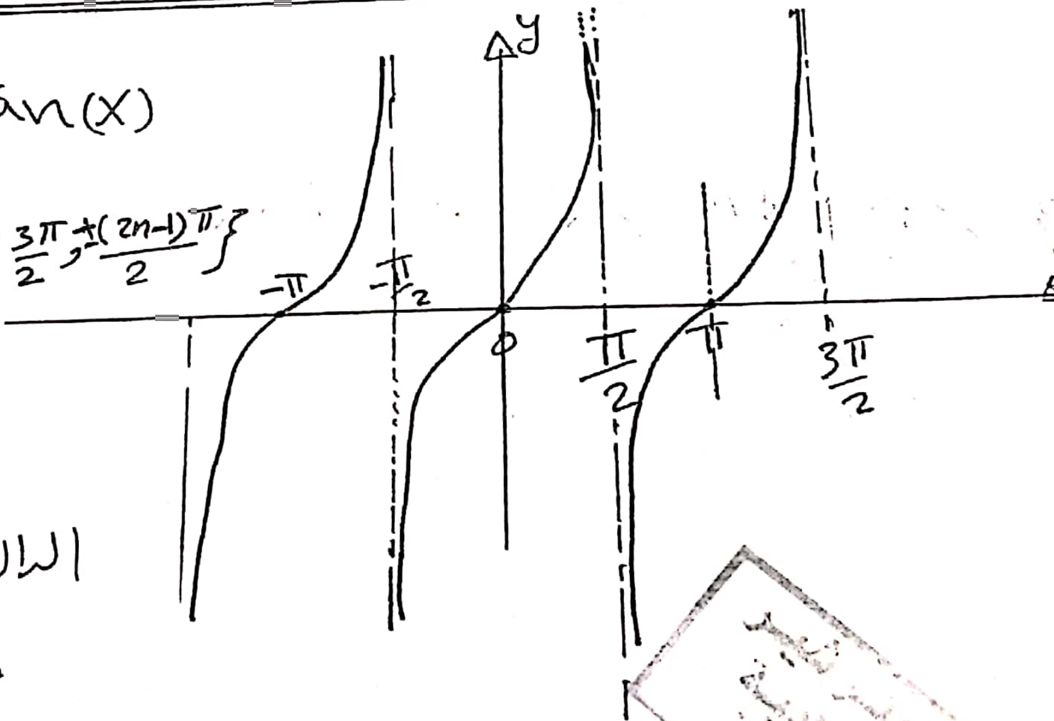
- المجال $= R$
- المدى $= [-1, 1]$
- الدالة دورية
- الدالة زوجية

أي أنها متماثلة حول محور y

$\therefore f(-x) = f(x)$

$$\boxed{\therefore \cos(-x) = \cos x}$$

$\boxed{3}$ $y = \tan(x)$



المجال $= R - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{(2n-1)\pi}{2}\}$

المدى $R_f = R$

الدالة فردية أي أنها متماثلة حول نقطة الأصل

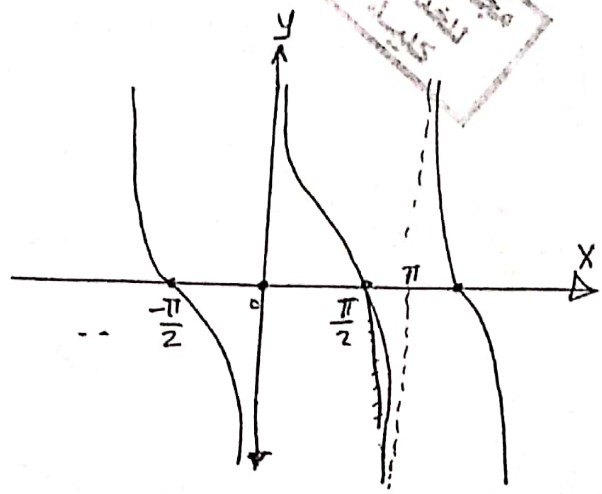
$$\therefore \boxed{\tan(-x) = -\tan(x)}$$

الدالة دورية أيضاً أي تتكرر

4 $y = \cot(x) = \dfrac{1}{\tan(x)}$

ال $D_f = R - \{\pm n\pi\}$

ال $R_f = R$

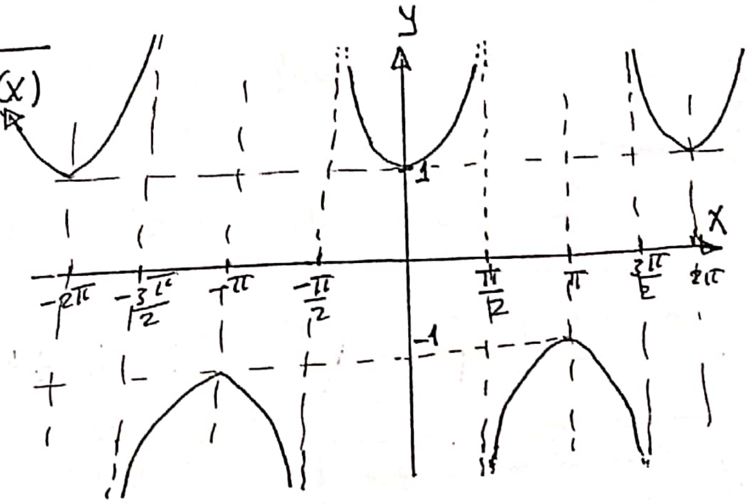


5 $y = \sec(x) = \dfrac{1}{\cos(x)}$

$D_f = R - \left\{\underbrace{\pm(2n+1)}\frac{\pi}{2}\right\}$

أي عدد فردي
مضروب $*90^\circ$

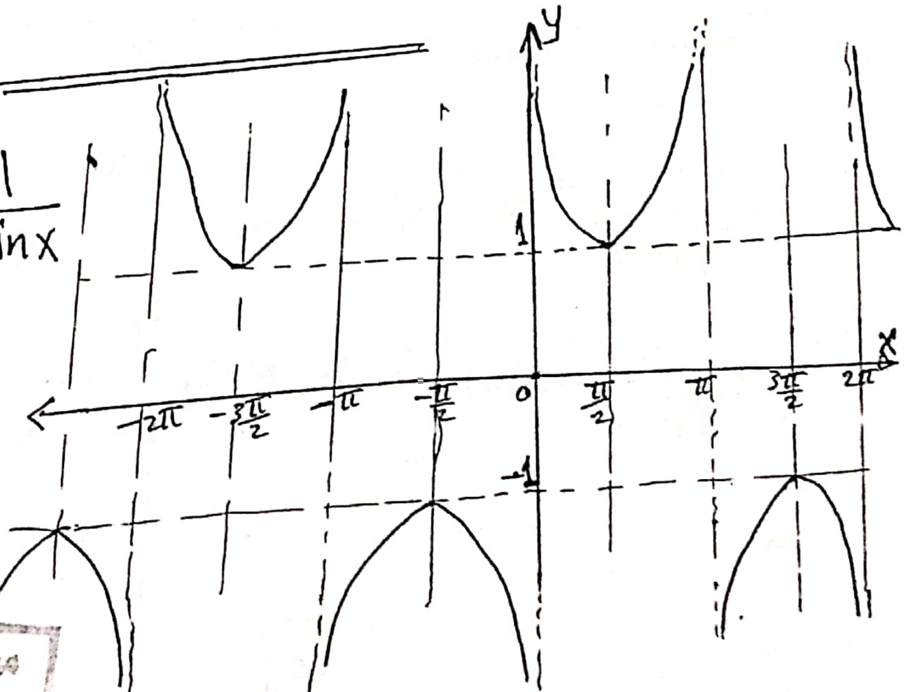
$R_f = R -]-1,1[$



6 $y = \csc(x) = \dfrac{1}{\sin x}$

$D_f = R - \{\pm n\pi\}$

$R_f = R -]-1,1[$



(حفظ!!)

* مشتقات الدوال المثلثية :-

$\sin(x) \xrightarrow{\text{تفاضل}} \cos(x)$

$\sin(u) \longrightarrow \cos(u) \cdot u'$ ← $u' = \frac{du}{dx}$ و $u = u(x)$

$\cos(u) \longrightarrow -\sin(u) \cdot u'$

$\tan(u) \longrightarrow \sec^2(u) \cdot u'$

$\cot(u) \longrightarrow -\operatorname{cosec}^2(u) \cdot u'$

$\sec(u) \longrightarrow \sec(u) \cdot \tan(u) \cdot u'$

$\operatorname{cosec}(u) \longrightarrow -\operatorname{cosec}(u) \cdot \cot(u) \cdot u'$

* الدوال المثلثية العكسية :-

1 $y = \sin^{-1}(x) \xrightarrow{\text{تفاضل}} \frac{1}{\sqrt{1-x^2}}$

$\sin^{-1}(u) \longrightarrow \frac{1}{\sqrt{1-u^2}} \cdot u'$

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2 $y = \cos^{-1}(x) \xrightarrow{\text{تفاضل}} \frac{-1}{\sqrt{1-u^2}} \cdot u'$

3 $y = \tan^{-1}(u) \xrightarrow{''} \frac{1}{1+u^2} \cdot u'$

4 $y = \cot^{-1}(u) \xrightarrow{''} \frac{-1}{1+u^2} \cdot u'$

5 $y = \sec^{-1}(x) \xrightarrow{\text{تفاضل}} \frac{1}{|u|\sqrt{u^2-1}} \cdot u'$

6 $y = \operatorname{cosec}^{-1}(u) \xrightarrow{''} \frac{-1}{|u|\sqrt{u^2-1}} \cdot u'$

#

$y = \sin^{-1}(x) \Longleftrightarrow x = \sin y$

إثبات مشتقات الدوال المثلثية العكسية

Find $\frac{dy}{dx}$

Solution نفرض $\frac{dx}{dy} = \cos y$

$\therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{1}{\cos y}$

$\because \cos^2 y + \sin^2 y = 1$

$\therefore \cos^2 y = 1 - \sin^2 y$

$\therefore \cos y = \sqrt{1 - \sin^2 y}$

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$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 - \sin^2 y}}$

$\because x = \sin y \Rightarrow \therefore \sin^2 y = x^2$

$$\boxed{\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}}$$

$\therefore y = \cos^{-1}(x) \Leftrightarrow x = \cos y$

Find $\frac{dy}{dx}$

$\sin^2 y + \cos^2 y = 1$

نوجد $\frac{dx}{dy} = -\sin y$

$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{-\sin y} = \frac{-1}{\sqrt{1-\cos^2 y}}$

$\because \cos y = x$

$\therefore \cos^2 y = x^2$

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$\therefore \frac{dy}{dx} = \frac{\boxed{-}1}{\sqrt{1-x^2}}$

الفرق بينه وبين $\sin^{-1}(x)$ هو إشارة السالب

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$y = \tan^{-1} x \Leftrightarrow x = \tan y$

$\frac{dx}{dy} = \sec^2 y = 1 + \tan^2 y = 1 + x^2$

$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \boxed{\frac{1}{1+x^2}}$

Ex

$\cot^{-1}(x) = y \Leftarrow x = \cot y$

$$\frac{dx}{dy} = -\operatorname{cosec}^2 y = -(1+\cot^2 y)$$

$$= -(1+x^2)$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{-1}{1+x^2}} = \frac{1}{\frac{dx}{dy}}$$

ملحوظة :

$$\boxed{\begin{array}{l} 1+\tan^2 x = \sec^2 x \\ 1+\cot^2 x = \operatorname{cosec}^2 x \end{array}}$$

$$y = \sec^{-1}(x) \Longleftrightarrow x = \sec y$$

$$\frac{dx}{dy} = \sec y . \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$\therefore \sec y = x$

$, 1 + \tan^2 y = \sec^2 y$

$\therefore \tan^2 y = \sec^2 y - 1$

$\therefore \tan y = \sqrt{\sec^2 y - 1} = \sqrt{x^2 - 1}$

$$\boxed{\therefore \frac{dy}{dx} = \frac{\pm 1}{x\sqrt{x^2-1}}}$$

$$\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$$

$$y = \operatorname{cosec}^{-1} x \longleftarrow \boxed{x = \operatorname{cosec} y}$$

$$\therefore \frac{dx}{dy} = -\operatorname{cosec} y \cdot \cot y$$

$$\because 1 + \cot^2 y = \operatorname{cosec}^2 y$$

$$\therefore \cot^2 y = \operatorname{cosec}^2 y - 1$$

$$\therefore \cot y = \sqrt{\operatorname{cosec}^2 y - 1} = \sqrt{x^2 - 1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{\mp 1}{x\sqrt{x^2-1}}$$

2

$$= \frac{-1}{|u|\sqrt{u^2-1}} u'$$

Hyperbolic f^{ns} الدوال الزائدية

← هي دوال تختص بدراسة نقاط ع القطع الزائد Hyperbola.

← تعريفها الرياضي:

حفظ

1. $y = \sinh(x) = \dfrac{e^{x} - e^{-x}}{2}$
2. $y = \cosh(x) = \dfrac{e^{x} + e^{-x}}{2}$
3. $y = \tanh(x) = \dfrac{\sinh x}{\cosh x} = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$
4. $y = \coth(x) = \dfrac{1}{\tanh x} = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$
5. $y = \operatorname{sech}(x) = \dfrac{1}{\cosh(x)} = \dfrac{2}{e^{x} + e^{-x}}$
6. $y = \operatorname{cosech}(x) = \dfrac{1}{\sinh(x)} = \dfrac{2}{e^{x} - e^{-x}}$

مقارنة

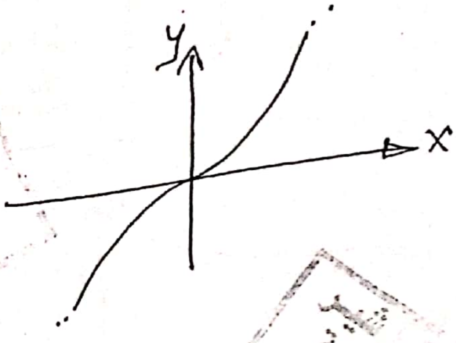
الدوال الثلثية	الدوال الزائدية
1. $\cos^2(x) + \sin^2(x) = 1$	$\cosh^2(x) - \sinh^2(x) = 1$
2. $1 + \tan^2(x) = \sec^2(x)$	$1 - \tanh^2(x) = \operatorname{sech}^2(x)$
3. $1 + \cot^2(x) = \operatorname{cosec}^2(x)$	$\coth^2(x) - 1 = \operatorname{cosech}^2(x)$
4. $\cos(0) = 1$, $\sin(0) = 0$	$\cosh(0) = 1$, $\sinh(0) = 0$

* Hyperbolic Functions

أهم الدوال الزائدية

نسبة للقطع الزائد.

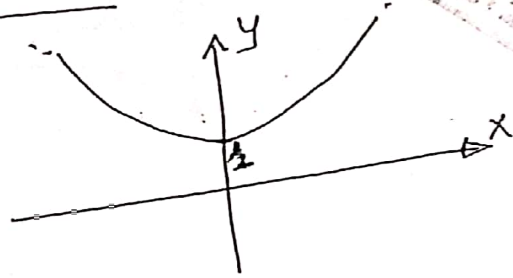
1. $y = \sinh(x) = \dfrac{e^{x} - e^{-x}}{2}$



$D_f = R$

$R_f = R$

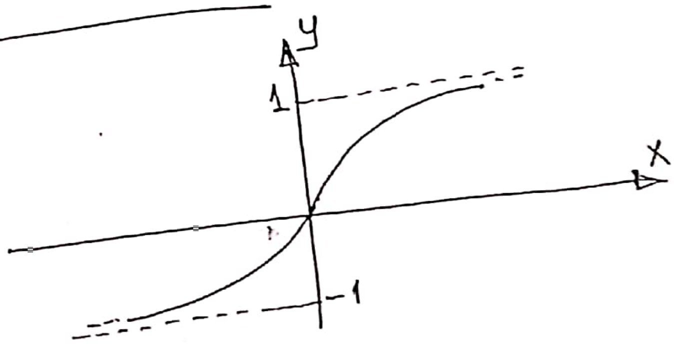
2. $y = \cosh(x) = \dfrac{e^{x} + e^{-x}}{2}$



$D_f = R$

$R_f = [1, \infty[$

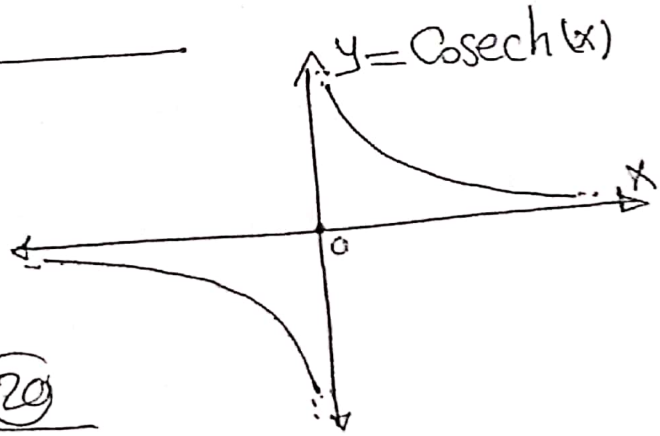
3. $y = \tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$



$D_f = R$

$R_f =]-1, 1[$

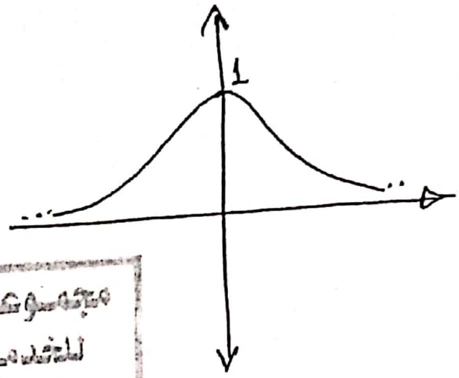
4. $y = \operatorname{Cosech}(x) = \dfrac{1}{\sinh(x)}$



$D_f = R - \{0\}$

$R_f = R - \{0\}$

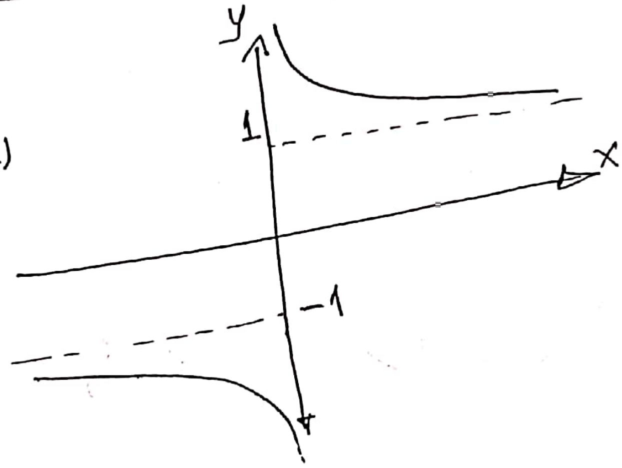
$\boxed{5}$ $y = \operatorname{Sech}(x) = \dfrac{1}{\operatorname{Cosh}(x)}$



$D_f = R$

$R_f =]0, 1]$

$\boxed{6}$ $y = \operatorname{Coth}(x) = \dfrac{1}{\tanh(x)}$



$D_f = R - \{0\}$

$R_f = R - [-1, 1]$

* بعض خصائص الدوال الزائدية ومقارنتها بالدوال المثلثية :

مثلثية | زائدية

$\boxed{1}$ $\sin^2(x) + \cos^2(x) = 1$ $\longleftrightarrow$ $\boxed{1}$ $\operatorname{Cosh}^2(x) - \operatorname{Sinh}^2(x) = 1$

الإثبات $\Rightarrow$ $\left(\dfrac{e^{x} + e^{-x}}{2}\right)^2 - \left(\dfrac{e^{x} - e^{-x}}{2}\right)^2$

$= \frac{1}{4}\left[\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2 - \cancel{e^{-2x}}\right]$

$= 1$

$\boxed{2}$ $1 + \tan^2(x) = \sec^2(x)$ $\longleftrightarrow$ $\boxed{2}$ $1 - \tanh^2(x) = \operatorname{Sech}^2(x)$

$\boxed{3}$ $1 + \cot^2(x) = \operatorname{Cosec}^2(x)$ $\longleftrightarrow$ $\boxed{3}$ $\operatorname{Coth}^2(x) - 1 = \operatorname{Cosech}^2(x)$

كتابة الدوال الزائدية العكسية بدلالة الدالة اللوغاريتمية

1) Prove that $\sinh^{-1}(x) = \ln(x+\sqrt{x^2+1})$

Proof: let $y = \sinh^{-1}(x)$

$$\therefore x = \sinh(y) = \frac{e^y - e^{-y}}{2}$$

$\therefore e^y - e^{-y} = 2x$ بالضرب $* e^y$:

$\therefore e^{2y} - 1 = 2xe^y$ $\therefore e^{2y} - 2xe^y - 1 = 0$

دالة تربيعية في المتغير e^y $(e^y)^2 - 2x(e^y)^1 - 1 = 0$

Put $e^y = z$ $\Longrightarrow \therefore \underset{A=1}{z^2} - \underbrace{2x}_{B=-2x}.z \underset{C=-1}{-1} = 0$

نحلها بالقانون العام
$Az^2 + Bz + C = 0$
$\therefore z = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$

$$\therefore z = \frac{2(x) \pm \sqrt{4x^2 - 4(1)(-1)}}{2(1)}$$

$$z = \frac{2x \pm \sqrt{4x^2+4}}{2} = \frac{\cancel{2}x \pm \cancel{2}\sqrt{x^2+1}}{\cancel{2}} = x \pm \sqrt{x^2+1}$$

$$e^y = x \pm \sqrt{x^2+1}$$ بأخذ Ln للطرفين

$$\therefore \ln e^y = \ln(x \pm \sqrt{x^2+1})$$

$$\boxed{\therefore y = \ln(x+\sqrt{x^2+1}) = \sinh^{-1}(x)}$$

السالب مرفوض لأن مابداخل Ln لازم يكون عدد موجب # (مجال دالة Ln)

[2] $\cosh^{-1}(x) = \ln\left(x+\sqrt{x^2-1}\right)$ أثبت أن الحل

[3] Prove that: $\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

الحل

let $y = \tanh^{-1}(x) \Longrightarrow \therefore \frac{x}{1} = \tanh(y) = \frac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$

طرفين * وسطين

$\therefore x e^{y} + x e^{-y} = e^{y} - e^{-y}$ (بالضرب $*e^{y}$)

$\therefore x e^{2y} + x = e^{2y} - 1$ $\therefore x e^{2y} - e^{2y} + x + 1 = 0$

$e^{2y}(x-1) = -(x+1)$ $\therefore e^{2y} = \frac{-(x+1)}{x-1} = \frac{1+x}{1-x}$ بأخذ $\ln$ للطرفين

$\therefore \ln e^{2y} = \ln\left(\frac{1+x}{1-x}\right)$ $\therefore 2y = \ln\left(\frac{1+x}{1-x}\right)$

$$\boxed{\therefore y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = \tanh^{-1}(x)}$$ #

[4] Prove that $\coth^{-1}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ Proof

let $y = \coth^{-1}(x)$

$\therefore x = \coth(y) = \frac{\cosh(y)}{\sinh(y)}$

$\therefore x = \frac{e^{y}+e^{-y}}{e^{y}-e^{-y}}$ طرفين * وسطين

$\therefore x e^{y} - x e^{-y} = e^{y} + e^{-y}$ (بالضرب $*e^{y}$)

$\therefore x e^{2y} - x = e^{2y} + 1$

$\therefore x e^{2y} - e^{2y} = x + 1$

$\therefore e^{2y}(x-1) = x+1$

$\therefore e^{2y} = \frac{x+1}{x-1}$ (بأخذ $\ln$ للطرفين)

$\therefore 2y = \ln\left(\frac{x+1}{x-1}\right)$

$$\boxed{\therefore y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) = \coth^{-1}(x)}$$ #

5 Prove that $\operatorname{sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ (Proof)

let $y = \operatorname{sech}^{-1}(x)$

$\therefore x = \operatorname{sech}(y) = \frac{1}{\cosh(y)}$

$\therefore x = \frac{2}{e^y + e^{-y}}$ طرفين * وسطين

$\therefore x e^y + x e^{-y} = 2$ (بالضرب $e^y *$)

$\therefore x e^{2y} + x = 2e^y$

$$\boxed{\therefore \underbrace{x}_{A} e^{2y} \underbrace{-2}_{B} e^y + \underbrace{x}_{C} = 0}$$

معادلة تربيعية تُحل بالقانون العام

$\therefore e^y = \frac{-(-2) \pm \sqrt{4 - 4(x)(x)}}{2x}$

$e^y = \frac{\cancel{2} \pm \cancel{2}\sqrt{1-x^2}}{\cancel{2}x}$

$e^y = \frac{1 \pm \sqrt{1-x^2}}{x}$ بأخذ ln للطرفين

$\therefore y = \ln\left(\frac{1 \pm \sqrt{1-x^2}}{x}\right)$

$\downarrow$

$\therefore \operatorname{sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

والسالب مرفوض لعدم وجود ln لعدد سالب كما سبق في $\cosh^{-1}$, $\sinh^{-1}$

6 Prove that $\operatorname{cosech}^{-1}(x) =$ (Proof)

الحل

let $y = \operatorname{cosech}^{-1}(x)$

$\therefore x = \operatorname{csech}(y) = \frac{1}{\sinh(y)}$

$\therefore x = \frac{2}{e^y - e^{-y}}$

$\therefore x e^y - x e^{-y} = 2$ $(* e^y)$

$\therefore x e^{2y} - x = 2e^y$

$$\boxed{\therefore \underbrace{x}_{A} e^{2y} \underbrace{-2}_{B} e^y \underbrace{-x}_{C} = 0}$$

$\therefore e^y = \frac{2 \pm \sqrt{4 + 4(x)(+x)}}{2x}$

$\therefore e^y = \frac{\cancel{2} \pm 2\sqrt{1+x^2}}{\cancel{2}x}$

بأخذ ln للطرفين

$\therefore y = \ln\left(\frac{1 \pm \sqrt{1+x^2}}{x}\right)$

$\downarrow$

$\therefore \operatorname{csech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)$

والسالب مرفوض لعدم وجود ln لعدد سالب

#

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dPerentiation

إثباتات

$y = \sin(x)$

* أوجد مشتقة

Prove that $\frac{d}{dx} \sin(x) = \cos x$ أو

الحل

$$\therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2\cos(x+\frac{\Delta x}{2}) . \sin(\frac{\Delta x}{2})}{\Delta x}$$

مقام المقام

$$= \lim_{\Delta x \to 0} \cos(x+\frac{\Delta x}{2}) . \lim_{\Delta x \to 0} \frac{\sin(\frac{\Delta x}{2})}{(\frac{\Delta x}{2})}$$

$= 1$

$$= \cos(x+0) =$$

$$= \cos(x)$$

لاحظ

$$\sin(x+\Delta x) - \sin(x) = 2\cos(x+\frac{\Delta x}{2}) . \sin(\frac{\Delta x}{2})$$

الإثبات ...

$$\sin(x+\Delta x) - \sin x =$$

$$= \sin x \cos\Delta x + \sin\Delta x \cos x - \sin x$$

$$= \sin x\,[\cos\Delta x - 1] + \cos x \sin\Delta x$$

$$= \cos x \sin\Delta x - 2 * \tfrac{1}{2}(1-\cos\Delta x).\sin x$$

$$= \cos x\left(2\sin\tfrac{\Delta x}{2}.\cos\tfrac{\Delta x}{2}\right) - 2\sin^2\left(\tfrac{\Delta x}{2}\right).\sin(x)$$

عامل مشترك

$$= 2\sin\left(\tfrac{\Delta x}{2}\right)\left[\cos x.\cos\tfrac{\Delta x}{2} - \sin x.\sin\left(\tfrac{\Delta x}{2}\right)\right]$$

$$= 2\sin\left(\tfrac{\Delta x}{2}\right).\cos\left(x+\tfrac{\Delta x}{2}\right) \quad \#$$

* Prove that $\frac{d}{dx}\cos x = -\sin(x)$

(الحل)

لاحظ أن

$$\cos(x) = \sin\left(\tfrac{\pi}{2} - x\right)$$

$$\therefore \frac{d}{dx}\cos(x) = \frac{d}{dx}\left[\sin\left(\tfrac{\pi}{2}-x\right)\right] = \cos\left(\tfrac{\pi}{2}-x\right)(-1)$$

$$= -\sin(x)$$

* $y = \tan(x) = \dfrac{\sin(x)}{\cos x} = \dfrac{\text{بسط}}{\text{مقام}}$

$$\therefore \frac{dy}{dx} = \frac{\cos x.\cos x - \sin x.(-\sin x)}{\cos^2 x}$$

$$\therefore \frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = \boxed{-\operatorname{cosec}^2 x}$$

$$y = \sec x = \frac{1}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cancel{(\cos x) \cdot 0} - (1) \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x \cdot \cos x} = \tan x \cdot \frac{1}{\cos x}$$

$$= \sec x \cdot \tan x$$

$$y = \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\frac{dy}{dx} = \frac{(\sin x \cdot)(0) - (1)\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x \cdot \sin x}$$

$$\because \frac{\cos x}{\sin x} = \cot x \quad , \quad \frac{1}{\sin x} = \operatorname{cosec} x$$

$$\boxed{\therefore \frac{dy}{dx} = -\operatorname{cosec} x \cdot \cot x}$$

إثباتات تفاضلات الدوال الزائدية

1 Prove that $\frac{d}{dx}\sinh(x)=\cosh x$

الحل

$$\because \sinh(x)=\frac{e^{x}-e^{-x}}{2}$$

بتفاضل الطرفين

$$\therefore \frac{d}{dx}\sinh(x)=\frac{d}{dx}\left(\frac{1}{2}(e^{x}-e^{-x})\right)$$

$$=\frac{1}{2}(e^{x}+e^{-x})=\cosh x$$

2 Prove that $\frac{d}{dx}\cosh x=\sinh x$

الحل

$$\because \cosh(x)=\frac{e^{x}+e^{-x}}{2}$$

بتفاضل الطرفين

$$\therefore \frac{d}{dx}\cosh(x)=\frac{1}{2}(e^{x}-e^{-x})=\sinh(x)$$

3 Prove that $\frac{d}{dx}\tanh(x)=\operatorname{sech}^2(x)$

الحل

$$\because \tanh(x)=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

بتفاضل الطرفين

$$\therefore \frac{d}{dx}\tanh(x)=\frac{[e^{x}+e^{-x}][e^{x}+e^{-x}]-[e^{x}-e^{-x}][e^{x}-e^{-x}]}{[e^{x}+e^{-x}]^2}$$

$$=\frac{[e^{x}+e^{-x}]^2-[e^{x}-e^{-x}]^2}{[e^{x}+e^{-x}]^2}$$

$$=\frac{\cancel{e^{2x}}+2+\cancel{e^{-2x}}-\cancel{e^{2x}}+2+\cancel{e^{-2x}}}{[e^{x}+e^{-x}]^2}$$

$$=\frac{4}{[e^{x}+e^{-x}]^2}=\left[\frac{2}{e^{x}+e^{-x}}\right]^2$$

$$=\left(\frac{1}{\cosh x}\right)^2=\operatorname{sech}^2(x)$$

4 Prove that $\frac{d}{dx}\coth(x)=-\operatorname{cosech}^2 x$

الحل

$$\because \coth(x)=\frac{1}{\tanh(x)}=\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

بتفاضل الطرفين

$$\therefore \frac{d}{dx}(\coth(x))=\frac{[e^{x}-e^{-x}][e^{x}-e^{-x}]-[e^{x}+e^{-x}][e^{x}+e^{-x}]}{[e^{x}-e^{-x}]^2}$$

$$=\frac{\cancel{e^{2x}}-2+\cancel{e^{-2x}}-\cancel{e^{2x}}-2-\cancel{e^{-2x}}}{[e^{x}-e^{-x}]^2}$$

$$=\frac{-4}{[e^{x}-e^{-x}]^2}=-\left[\frac{2}{e^{x}-e^{-x}}\right]^2$$

$$=-\left[\frac{1}{\sinh x}\right]^2=-\operatorname{cosech}^2(x)$$

بالمثل نستنتج أن تفاضل

$$\frac{d}{dx}(\operatorname{sech} x)=-\operatorname{sech} x.\tanh x$$

و $$\frac{d}{dx}(\operatorname{cosech} x)=-\operatorname{cosech} x.\coth x.$$

مشتقات الدوال الزائدية العكسية

الإثبات مشابه تماماً إثبات مشتقات الدوال المثلثية العكسية

$$\boxed{1} \quad \boxed{y = \tanh^{-1} x} \longleftrightarrow \boxed{x = \tanh y}$$

$$\frac{dx}{dy} = \operatorname{sech}^2 y$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\operatorname{sech}^2 y} = \frac{1}{1-\tanh^2 y}$$

$$\therefore \frac{d}{dx}(\tanh^{-1}(x)) = \frac{1}{1-x^2}$$

قاعدة عامة

$$\therefore \frac{d}{dx} \tanh^{-1}(u) = \frac{1}{1-u^2} \cdot u'$$

$$\boxed{2} \quad y = \sinh^{-1}(x) \longleftrightarrow x = \sinh y$$

$$\therefore \frac{dx}{dy} = \cosh y$$

$$\frac{dy}{dx} = \frac{1}{\cosh y}$$

$$\because \cosh^2 y - \sinh^2 y = 1$$

$$\cosh y = \sqrt{1+\sinh^2 y} = \sqrt{1+x^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$$

$y = \cosh^{-1} x \iff x = \cosh y$

$$\frac{dx}{dy} = \sinh y$$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh x = \sqrt{\cosh^2 x - 1}$

$= \sqrt{x^2 - 1}$

$$\therefore \frac{dy}{dx} = \frac{1}{\sinh y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 1}}$$

7) $y = \coth^{-1} x \iff x = \coth y$

$$\frac{dx}{dy} = -\operatorname{cosech}^2 y$$

$\because 1 - \tanh^2 x = \operatorname{sech}^2 x$

$\coth^2 x - 1 = \operatorname{cosech}^2 x$

$$\frac{dy}{dx} = \frac{-1}{\operatorname{cosech}^2 y}$$

$$= \frac{-1}{\coth^2 x - 1} = \frac{-1}{x^2 - 1}$$

$\boxed{5}$ $y = \operatorname{sech}^{-1} x \iff x = \operatorname{sech} y$

$$\frac{dx}{dy} = -\operatorname{sech} y \cdot \tanh y$$

$\because 1 - \tanh^2 y = \operatorname{sech}^2 y$

$\therefore \tanh y = \sqrt{1 - \operatorname{sech}^2 y}$

$= \sqrt{1 - x^2}$

$$\frac{dy}{dx} = \frac{-1}{\operatorname{sech} y \cdot \tanh y}$$

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{1 - x^2}}$$

$\boxed{6}$ $y = \operatorname{cosech}^{-1} x \iff x = \operatorname{cosech} y$

$$\frac{dx}{dy} = -\operatorname{cosech} y \cdot \coth y$$

$\because \coth^2 y - 1 = \operatorname{cosech}^2 y$

$\therefore \coth y = \sqrt{\operatorname{cosech}^2 y + 1}$

$= \sqrt{x^2 + 1}$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{-1}{\operatorname{cosech} y \cdot \coth y}$$

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2 + 1}}$$

$$\therefore e^{2y} = \frac{-(x+1)}{(x-1)} = \frac{1+x}{1-x}$$

بأخذ $\ln$ للطرفين

$$\therefore 2y = \ln \frac{1+x}{1-x}$$

$$\boxed{\therefore y = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) = \tanh^{-1} x}$$

$\boxed{4}$ $y = \coth^{-1} x \longleftrightarrow x = \coth y = \frac{e^{y}+e^{-y}}{e^{y}-e^{-y}}$

$$\therefore x e^{y} - x e^{-y} = e^{y} + e^{-y}$$

$$\therefore e^{y}(x-1) - e^{-y}(x+1) = 0$$

بالضرب $\times e^{y}$

$$\therefore e^{2y}(x-1) - (x+1) = 0$$

$$\therefore e^{2y} = \frac{x+1}{x-1}$$

بأخذ $\ln$ للطرفين

$$\therefore 2y = \ln\left(\frac{x+1}{x-1}\right)$$

$$\boxed{\therefore y = \coth^{-1}(x) = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right)}$$

اكتب $\sech^{-1} x$ و $\operatorname{cosech}^{-1} x$ بدلالة الدالة $(\ln)$

واجب بنفسك

مسائل على الاشتقاقة (التفاضل) 1

* Find $\frac{dy}{dx}$ for

(1) $y = \underbrace{(x^3-5)}\cdot\underbrace{\tan^3(x^2+\cos^3(2x))}$ حاصل ضرب دالتين

Solution:

$$\frac{dy}{dx} = (x^3-5)\cdot(3)\cdot\tan^2(x^2+\cos^3(2x))\cdot\sec^2(x^2+\cos^3(2x))\cdot(2x+3\cos^2(2x)\cdot(-\sin(2x))\cdot 2) + \tan^3(x^2+\cos^3(2x))\cdot(3x^2)$$

(2) $x^2 - \sin(xy) - \tan^2 y^2 = 0$ دالة ضمنية : بتفاضل الطرفين بالنسبة إلى x (w.r.t. x)

$$\therefore 2x - \cos(x\cdot y)\cdot\left(x\frac{dy}{dx}+y\right) - 2\tan y^2\cdot\sec^2 y^2\cdot 2y\frac{dy}{dx} = 0$$

$$\therefore 2x - y\cos(xy) = x\cos(xy)\frac{dy}{dx} + 4y\tan(y^2)\sec^2 y^2\cdot\frac{dy}{dx}$$

$$2x - y\cos(xy) = \frac{dy}{dx}\left[x\cos xy + 4y\tan y^2\sec^2 y^2\right]$$

$$\therefore \frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos xy + 4y\tan y^2\sec^2 y^2}$$ #

$\boxed{3}$ $y = \underbrace{e^{-x}}\cdot\underbrace{\cos 2x}$ find $y'' + 2y' + 5y$

Solution :

$$y' = -e^{-x}\cdot 2\sin 2x - e^{-x}\cos 2x$$

$$y' = -2e^{-x}\cdot\sin 2x - e^{-x}\cdot\cos 2x$$

$$y'' = -2e^{-x}\cdot 2\cos 2x + 2e^{-x}\cdot\sin 2x + 2e^{-x}\cos 2x + e^{-x}\cdot\cos 2x$$

$$y'' = -4e^{-x}\cos 2x + 2\cdot e^{-x}\cdot\sin 2x + 3e^{-x}\cos 2x$$

$$\therefore y'' = 2e^{-x}\cdot\sin 2x - e^{-x}\cos 2x$$

$$\therefore y'' + 2y' + 5y = 2e^{-x}\sin 2x - e^{-x}\cos 2x - 4e^{-x}\sin 2x - 2e^{-x}\cos 2x + 5\cdot e^{-x}\cdot\cos 2x$$

$$= -2e^{-x}\sin 2x + 2e^{-x}\cos 2x$$

#

$$* \quad y = \frac{x^p}{x^m - a^m} \qquad a, p, m = \text{Consts.}$$

حاصل قسمة دالتين

$$\therefore \frac{dy}{dx} = \frac{(x^m - a^m) \cdot p x^{p-1} - x^p (m x^{m-1})}{(x^m - a^m)^2}$$

$$* \quad y = \tan(ax+b)$$

$$\frac{dy}{dx} = \sec^2(ax+b) \cdot a$$

$$* \quad y = \underbrace{\sin^3 t} \cdot \underbrace{\cos t}$$

$$\frac{dy}{dt} = \sin^3 t \cdot (-\sin t) + 3 \sin^2 t \cdot (\cos t) \cdot (\cos t)$$

$$= -\sin^4 t + 3 \sin^2 t \cdot \cos^2 t$$

$$* \quad y = \sqrt[3]{x^2+x+1} = (x^2+x+1)^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3}(x^2+x+1)^{-\frac{2}{3}}(2x+1) = \frac{2x+1}{3\sqrt[3]{(x^2+x+1)^2}}$$

* $y = \cos(x+y)$ دالة ضمنية

بتفاضل الطرفين بالنسبة لـ (x)

$$\therefore \frac{dy}{dx} = -\sin(x+y)\cdot\left(1+\frac{dy}{dx}\right) = -\sin(x+y) - \frac{dy}{dx}(\sin(x+y))$$

$$\therefore \frac{dy}{dx} + \frac{dy}{dx}\sin(x+y) = -\sin(x+y)$$

$$\therefore \frac{dy}{dx}(1+\sin(x+y)) = -\sin(x+y)$$

$$\therefore \frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)} \quad \#$$

---

* $\cos(xy) = x$ بتفاضل الطرفية بالنسبة لـ(x) :

$$\therefore -\sin(xy)\cdot\left[x\frac{dy}{dx}+y\right] = 1$$

$$\therefore -x\sin(xy)\cdot\frac{dy}{dx} - y\sin(xy) = 1$$

$$\therefore \frac{dy}{dx} = \frac{1+\cancel{y}\sin(xy)}{-x\sin(xy)} \quad \#$$

---

* $x^{1/2}+y^{1/2}=a^{1/2}$ بتفاضل الطرفين بالنسبة لـ(x) :

, $a = \text{const}$

$$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{-\frac{1}{2}x^{-1/2}}{\frac{1}{2}y^{-1/2}} = \frac{y^{1/2}}{x^{1/2}} = \sqrt{\frac{y}{x}}$$

$*$ if $x = a\cos t$, $y = b\sin t$

find $\frac{dy}{dx}$ Solution

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

Exp where $\frac{dy}{dt} = b\cos t$

$$\frac{dt}{dx} = \frac{1}{\left(\frac{dx}{dt}\right)} = \frac{1}{-a\sin t}$$

$$\therefore \frac{dy}{dt} = b\cos t \cdot \frac{-1}{a\sin t} = \frac{-b}{a}\frac{\cos t}{\sin t} = \frac{-b}{a}\cot(t)$$

$*$ $x = a\cos^3 t$, $y = b\sin^3 t$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\cancel{3}b\sin^2 t \cdot \cos t}{-\cancel{3}a\cos^2 t \cdot \sin t}$$

$$= \frac{-b}{a}\left(\frac{\sin t}{\cos t}\right)^2 \cdot \frac{\cancel{\cos t}}{\cancel{\sin t}} = \frac{-b}{a}\frac{\sin t}{\cos t}$$

$$= \frac{-b}{a}\tan t \quad \#$$

---

* Calculate The Slop of the Tangent for the Curve : $X \cdot \cos y + y \cdot \sin x = \frac{\pi}{2}$ at Point $(\frac{\pi}{2}, \frac{\pi}{2})$

احسب ميل المماس المنحنى عند النقطة

اوجد ميل المماس للمنحنى $x\cos y + y\sin x = \frac{\pi}{2}$ عند النقطة $(\frac{\pi}{2}, \frac{\pi}{2})$

ميل المماس هو المشتقة الأولى

نحسب اولاً $\frac{dy}{dx}$ له

<< Solution >>

$$X\cos y + y \sin x = \frac{\pi}{2}$$

نتفاضل الطرفين بالنسبة لـ x :

$$\therefore -X \cdot \sin y \cdot \frac{dy}{dx} + \cos y + y\cos x + \frac{dy}{dx}\sin x = 0$$

$$\frac{dy}{dx}\left[\sin x - x\sin y\right] = -\cos y - y\cos x$$

$$\therefore \frac{dy}{dx} = \frac{-\cos y - y\cos x}{\sin x - x\sin y} = \frac{\cos y + y\cos x}{x\sin y - \sin x}$$

$\therefore$ ميل المماس عند النقطة $(\overset{x}{\frac{\pi}{2}}, \overset{y}{\frac{\pi}{2}})$ هو :

$$\frac{dy}{dx} = \frac{\cos(\frac{\pi}{2}) + \frac{\pi}{2}\cos(\frac{\pi}{2})}{(\frac{\pi}{2})(\sin(\frac{\pi}{2})) - \sin(\frac{\pi}{2})}$$

$$\tan\theta = \frac{\cos(90^\circ) + \frac{3.14}{2}\cos(90^\circ)}{\frac{3.14}{2}(\sin 90^\circ) - \sin(90^\circ)} = \text{Zero}$$

$\therefore \theta = 0 = \tan^{-1}(0)$

>> أى أن المنحنى يوازى محور X <<

$$y^2 \cdot \ln(\cosh^{-1} x^2) + e^{\frac{1}{2}\ln y} \cdot \tan\left[\sqrt{x \cdot \sqrt{x \cdot \sqrt{x}}}\right] = 0$$

find $\dfrac{dy}{dx}$: <Solution>

$$e^{\frac{1}{2}\ln y} = e^{\ln y^{1/2}} = y^{1/2} = \sqrt{y}$$

$$\sqrt{x \cdot \sqrt{x \cdot \sqrt{x}}} = \sqrt{x \sqrt{x \cdot x^{1/2}}} = \sqrt{x \cdot \sqrt{x^{3/2}}}$$

$$= \sqrt{x \cdot x^{3/4}} = \sqrt{x^{7/4}} = x^{7/8}$$

$$\Rightarrow y^2 \cdot \ln(\cosh^{-1} x^2) + \sqrt{y} \cdot \tan[x^{7/8}] = 0$$

بتفاضل الطرفين بالنسبة لـ x

$$\therefore 2y \frac{dy}{dx} \cdot \ln(\cosh^{-1} x^2) + y^2 \cdot \frac{1}{\cosh^{-1} x^2} \cdot \frac{-1}{\sqrt{1+x^4}} \cdot 2x$$

$$+ \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} \cdot \tan[x^{7/8}] + \sec^2[x^{7/8}] \cdot \frac{7}{8} x^{-1/8} \cdot \sqrt{y} = 0$$

$$\therefore \frac{dy}{dx}\left[2y \ln(\cosh^{-1} x^2) + \frac{\tan(x^{7/8})}{2\sqrt{y}}\right]$$

$$= \left[\frac{2xy}{\cosh^{-1} x^2 \cdot \sqrt{1+x^4}} - \frac{\frac{7}{8}\sqrt{y}\sec^2 x^{7/8}}{x^{1/8}}\right]$$

$$\therefore \frac{dy}{dx} = \left[\frac{2xy}{\cosh^{-1} x^2 \sqrt{1+x^4}} - \frac{\frac{7}{8}\sqrt{y}\sec^2 x^{7/8}}{x^{1/8}}\right] \Big/ \left[2y \ln(\cosh^{-1} x^2) + \frac{\tan x^{7/8}}{2\sqrt{y}}\right]$$

$$y = \sqrt{e^{\sinh^{-1}\sqrt{x}}} + 7^{x^2 \cdot \sin^2 x} \qquad \text{find } y'$$

<Solution>

$$y' = \frac{1}{2\cdot\sqrt{e^{\sinh^{-1}(\sqrt{x})}}} \cdot e^{\sinh^{-1}\sqrt{x}} \cdot \frac{1}{\sqrt{1+x}} \cdot \frac{1}{2\sqrt{x}}$$

$$+ 7^{x^2 \cdot \sin^2 x} \cdot \ln(7) \cdot [x^2 \cdot 2\sin x \cdot \cos x + 2x\sin^2 x]$$

$$u = 2\ln[\cot(s)] \quad , \quad v = \tan(s) + \cot(s)$$

find $\frac{du}{dv}$ <Solution>

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$$\frac{du}{dv} = \frac{du}{ds} \cdot \frac{ds}{dv}$$

where $\frac{du}{ds} = 2 \cdot \frac{1}{\cot(s)} \cdot (-\csc^2 s)$

$$\frac{dv}{ds} = \sec^2(s) \cancel{+} \csc^2(s)$$

$$\therefore \frac{ds}{dv} = \frac{1}{\sec^2(s) - \csc^2(s)}$$

$$\therefore \frac{du}{dv} = \left[\frac{-2\csc^2 s}{\cot(s)}\right] \cdot \left[\frac{1}{\sec^2(s) - \csc^2 s}\right] \#$$

* $y = \operatorname{Log}\left(\underbrace{10^{\sin x}}_{u} \cdot \underbrace{\tan(3x^2)}_{V}\right) + \sin\left(\cos\left(\underbrace{e^{\sqrt{x}}}_{W}\right)\right)$

Solution:-

let $y = \operatorname{Log}(U \cdot V) + \sin(\cos(W))$

Where $U = 10^{\sin x}$, $V = \tan(3x^2)$

$W = e^{\sqrt{x}}$

نحتاج W', V', U'

$U' = 10^{\sin x} \cdot \ln(10) \cdot \cos x$

$V' = \sec^2(3x^2) \cdot 6x = 6x \cdot \sec^2(3x^2)$

$W' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{1}{U \cdot V} \cdot \operatorname{Log}(e) \cdot (U \cdot V' + V \cdot U')$

$+ \cos(\cos(e^{\sqrt{x}})) \cdot [-\sin(e^{\sqrt{x}}) \cdot W']$

$\therefore \frac{dy}{dx} = \frac{1 * \operatorname{Log}(e) \cdot [10^{\sin x} \cdot (6x \cdot \sec^2(3x^2)) + \tan(3x^2)}{(10^{\sin x} \cdot \tan(3x^2))}$

$\cdot (10^{\sin x} \cdot \ln(10) \cdot \cos x)]$

$+ \cos(\cos(e^{\sqrt{x}})) \cdot [-\sin(e^{\sqrt{x}}) \cdot (e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}})]$ #

* $(\tan x)^y = (\tan y)^x$ find $\frac{dy}{dx} = y'$

الحل: بأخذ Ln للطرفين

$\therefore\ y \cdot \ln(\tan x) = x \cdot \ln(\tan y)$

بتفاضل الطرفين بالنسبة لـ (x):

$\therefore\ y \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot y' = x \cdot \frac{1}{\tan y} \cdot \sec^2 y \cdot y' + \ln(\tan y) \cdot (1)$

$y' \ln(\tan x) - \frac{x \sec^2 y}{\tan y} \cdot y' = \ln(\tan y) - \frac{y \sec^2 x}{\tan x}$

$\therefore\ y' \left[\ln(\tan x) - \frac{x \sec^2(y)}{\tan y}\right] = \ln(\tan y) - \frac{y \sec^2 x}{\tan x}$

$$\therefore\ y' = \frac{\ln(\tan y) - \frac{y \sec^2 x}{\tan x}}{\ln(\tan x) - \frac{x \sec^2(y)}{\tan y}}$$

#

$$y = x^{(x^x)}$$ find $\frac{dy}{dx}$

Solution:-

بأخذ ln الطرفين :-

$$\therefore \ln y = x^x \cdot \ln x$$

Put $u = x^x$

$$\therefore \ln y = u \cdot \ln x \Rightarrow$$ بتفاضل الطرفين بالنسبة لـ (x)

$$\therefore \frac{1}{y} \cdot y' = u \cdot \frac{1}{x} + \ln(x) \cdot u'$$

$$\therefore y' = y\left[\frac{u}{x} + \ln(x) \cdot u'\right]$$

باقي لدينا إيجاد u' :

$$\because u = x^x$$ بأخذ ln الطرفين

$$\therefore \ln u = x \cdot \ln x \Rightarrow$$ بتفاضل الطرفين بالنسبة لـ x

$$\therefore \frac{1}{u} \cdot u' = \cancel{x} \cdot \frac{1}{\cancel{x}} + \ln x$$

$$\therefore u' = u[1 + \ln x] = x^x[1 + \ln x]$$

$$\therefore \frac{dy}{dx} = y' = \underset{y}{x^{x^x}}\left[\frac{x^x}{x} + \ln x[x^x(1 + \ln x)]\right]$$

* Find $\dfrac{dy}{dx}$ for :-

[1] $y = 7^{x^2 \cdot \operatorname{cosec}^2 x} + x^2 \cdot \tan^3(3x^2 + \sin 2x)$

<Solution>

$$\frac{dy}{dx} = 7^{x^2 \cdot \operatorname{cosec}^2 x} \cdot \ln(7) \cdot \left(x^2 \cdot 2\operatorname{cosec} x \cdot (-\operatorname{cosec} x \cdot \cot x) + 2x \operatorname{cosec}^2 x\right) +$$

$$x^2 \cdot \left[3\tan^2(3x^2 + \sin 2x) \cdot \sec^2(3x^2 + \sin 2x) \cdot (6x + 2\cos 2x)\right] + 2x \cdot \tan^3(3x^2 + \sin 2x).$$

حل آخر للمثال السابق للتبسيط:

$$y = 7^{(u)} + \underbrace{x^2 \cdot \tan^3(V)}_{\text{حاصل ضرب دالتين}} = 7^u + x^2 \cdot (\tan(V))^3$$

where $u = x^2 \cdot \operatorname{cosec}^2 x$ و $V = 3x^2 + \sin 2x$

$$\frac{dy}{dx} = 7^u \cdot \ln(7) \cdot u' + \underset{\text{الأولى}}{x^2} \cdot \underline{3[\tan(V)]^2} \cdot \underbrace{[\sec^2(V) \cdot V']}_{\text{مشتقة الثانية}}$$

$$+ \underbrace{2x}_{\text{مشتقة الأولى}} \cdot \underbrace{[\tan(V)]^3}_{\text{والثانية}}$$

باقي لدينا إيجاد u' و V' ونعوضهم في المعادلة السابقة ⇐

$$u = \underbrace{x^2}\cdot\underbrace{\text{Cosec}^2 x} = x^2 \cdot (\text{Cosec}\, x)^2$$

$$\therefore \frac{du}{dx} = u' = 2x \cdot \text{cosec}^2 x + x^2 \cdot [(2)\cdot(\text{Cosec}x)^1 \cdot (-\text{Cosec}x \cdot \cot x)] \Longrightarrow (1)$$

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$$V = 3x^2 + \sin 2x$$

$$\therefore V' = 6x + 2\cos(2x) \Longrightarrow (2)$$

بالتعويض نقيم V', u' , u, v في $\frac{dy}{dx}$

$$\therefore \frac{dy}{dx} = 7^{x^2 \cdot \text{Cosec}^2 x} \cdot \ln(7) \cdot [2x\,\text{Cosec}^2 x + x^2 (2\,\text{cosec}\,x \cdot (-\text{Cosec}x \cdot \cot x))]$$

$$+ x^2 \cdot 3[\tan(3x^2+\sin 2x)]^2 \cdot [\sec^2(3x^2+\sin 2x) \cdot (6x + 2\cos(2x))] + 2x\tan^3(3x^2+\sin 2x).$$

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$y = (\sin x)^{x}$ Find the first derivative w.r.t. x $\equiv \frac{dy}{dx}$

(Solution)

بأخذ ln الطرفين

$\ln y = x \cdot \ln(\sin x) \Rightarrow$ بتفاضل الطرفين بالنسبة لـ x

$\therefore \frac{1}{y} \cdot y' = x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot (1)$

$\therefore y' = \underbrace{y}_{(\sin x)^{x}} \left[x \cdot \underbrace{\left(\frac{\cos x}{\sin x}\right)}_{\cot x} + \ln(\sin x) \right]$

$\therefore y' = (\sin x)^{x} \left[x \cdot \cot x + \ln(\sin x) \right]$

---

* $y = \cos x + \ln(x^2 - 1) + \underbrace{(\cos x)^{x}}_{u}$, then $\frac{dy}{dx}$ is ??

solution

$\frac{dy}{dx} = -\sin x + \frac{1}{x^2 - 1} \cdot (2x) + u'$

لإيجاد u' :-

where $u = (\cos x)^{x} \Rightarrow$ بأخذ ln الطرفين

$\ln u = x \cdot \ln(\cos x) \Rightarrow$ «بتفاضل الطرفين بالنسبة لـ x»

$\therefore \frac{1}{u} \cdot u' = x \cdot \frac{1}{\cos x} \cdot (-\sin x) + \ln(\cos x) \cdot (1)$

$\therefore u' = [-x \cdot \tan x + \ln(\cos x)] \cdot (\cos x)^{x}$ ← u

$\therefore \frac{dy}{dx} = -\sin x + \frac{2x}{x^2 - 1} + (\cos x)^{x} [\ln(\cos x) - \cancel{x \tan} \; x \tan x]$

#

$$y = Ln\left[\sqrt{\frac{1+\sin x}{1-\sin x}}\right] \quad \text{find } \frac{dy}{dx}$$

Solution

$$y = Ln\left[\frac{1+\sin x}{1-\sin x}\right]^{1/2} = \frac{1}{2} Ln\left[\frac{1+\sin x}{1-\sin x}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\left[\frac{1+\sin x}{1-\sin x}\right]} \cdot \left[\frac{(1-\sin x)\cdot(\cos x) - (1+\sin x)(-\cos x)}{(1-\sin x)^2}\right]$$

$$= \frac{(\cancel{1-\sin x})}{2(1+\sin x)} \cdot \frac{\cos x - \cancel{\sin x \cos x} + \cos x + \cancel{\sin x \cos x}}{(1-\sin x)^{\cancel{2}}}$$

$$= \frac{\cancel{2}\cos x}{\cancel{2}(1+\sin x)(1-\sin x)} = \frac{\cos x}{1-\sin^2 x}$$

$$= \frac{\cancel{\cos x}}{\cos^2 x} = \frac{1}{\cos x} = \boxed{\sec x}$$

$$* \quad y = e^{2\cdot \ln x} + 3^x + (\cos x)^{\sin x}$$

$$y = \cancel{e}^{\cancel{Ln}\, x^2} + 3^x + u$$

$$y = x^2 + 3^x + u$$

$$\therefore \frac{dy}{dx} = 2x + 3^x \ln(3) + u'$$

نوجد ال u' كالاتي

where $\quad u = (\cos x)^{\sin x}$

$\therefore \ln(u) = \sin x \cdot \ln(\cos x)$ $\longrightarrow$ باخد ln للطرفين

$\therefore \frac{1}{u} \cdot u' = \left(\sin x \cdot \frac{1}{\cos x} \cdot (-\sin x) + \ln(\cos x) \cdot \cos x\right.$ $\longrightarrow$ نفاضل الطرفين w.r.t. (x)

$\therefore u' = u\left[-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)\right]$

$$u' = (\cos x)^{\sin x}\left[-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)\right]$$

$$\therefore \frac{dy}{dx} = 2x + 3^x \ln(3) + (\cos x)^{\sin x}\left[-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)\right] \quad \#$$

$*\quad y = x^{-e} \cdot \tan^2(x^2)$ $\quad$ ($-e$ $\longrightarrow$ رقم ثابت)

$$\therefore \frac{dy}{dx} = x^{-e} \cdot 2\tan(x^2) \cdot \sec^2(x^2) \cdot 2x + (-e)\, x^{-e-1} \cdot \tan^2(x^2)$$

$$\frac{dy}{dx} = \frac{4\tan x^2 \sec^2(x^2)}{x^e} - \frac{\tan^2(x^2) \cdot e}{x^{e+1}} \quad \#$$

$$y = e^{-x} \cdot \cos(2x) \qquad \text{Find } y'' + 2y' + 5y$$

Solution

$$y' = e^{-x}(-2\sin 2x) + \cos(2x) \cdot (-e^{-x})$$

$$y' = -2 \cdot e^{-x} \cdot \sin 2x - e^{-x} \cdot \cos(2x).$$

$$y'' = -2e^{-x} \cdot 2\cos 2x + 2e^{-x} \cdot \sin 2x + 2e^{-x} \sin 2x + e^{-x} \cos(2x)$$

$$\therefore y'' + 2y' + 5y = \underline{-4 \cdot e^{-x} \cdot \cos 2x} + 2e^{-x} \sin 2x + 2e^{-x} \sin 2x + \underline{e^{-x} \cos 2x} - 4e^{-x} \sin 2x - \underline{2 \cdot e^{-x} \cdot \cos 2x} + \underline{5e^{-x} \cos 2x}$$

$$= \underline{\text{Zero}}$$

* **Report**: Find $\frac{dy}{dx}$ for

$$y = \tan^2\left[\sec\left(\operatorname{sech}^{-1}\sqrt{1+x^2}\right)\right]$$

$$\underbrace{x^{(\sin y)}}_{u} + \underbrace{y^{\cos(x)}}_{v} = 0$$

$$u = x^{\sin(y)} \Rightarrow u' = x^{\sin(y)}\left[\frac{\sin y}{x} + \ln(x)\cdot\cos(y)\, y'\right]$$

$$v = y^{\cos(x)} \Rightarrow v' = y^{\cos(x)}\left[\frac{\cos x}{y}\, y' + \ln(y)\cdot(-\sin(x))\right]$$

$$u' + v' = 0$$

$$\therefore\ x^{\sin(y)}\left[\frac{\sin y}{x} + \ln(x)\cos(y)\, y'\right] + y^{\cos(x)}\left[\frac{\cos x}{y}\, y' - \ln(y)\cdot\sin(x)\right] = 0$$

عمل توحيد لـ y' في طرف لوحده

$$y' = \frac{-x^{\sin y}\,\frac{\sin y}{x} + y^{\cos(x)}\ln(y)\sin(x)}{x^{\sin y}\ln(x)\cos(y) + y^{\cos x}\,\frac{\cos x}{y}} \quad \#$$

{جدول التفاضلات}

الدالة	← المشتقة	الدالة	← المشتقة
$\sin(u)$	$\cos(u) \cdot u'$	$\sin^{-1}(u)$	$\frac{u'}{\sqrt{1-u^2}}$
$\cos(u)$	$-\sin(u) \cdot u'$	$\cos^{-1}(u)$	$\frac{-u'}{\sqrt{1-u^2}}$
$\sec(u)$	$\sec(u) \cdot \tan(u) \cdot u'$	$\sec^{-1}(u)$	$\frac{u'}{u\sqrt{u^2-1}}$
$\operatorname{cosec}(u)$	$-\operatorname{cosec}(u) \cdot \cot(u) \cdot u'$	$\operatorname{cosec}^{-1}(u)$	$\frac{-u'}{u\sqrt{u^2-1}}$
$\tan(u)$	$\sec^2(u) \cdot u'$	$\tan^{-1}(u)$	$\frac{u'}{1+u^2}$
$\cot(u)$	$-\operatorname{cosec}^2(u) \cdot u'$	$\cot^{-1}(u)$	$\frac{-u'}{1+u^2}$
$\sinh(u)$	$\cosh(u) \cdot u'$	$\sinh^{-1}(u)$	$\frac{u'}{\sqrt{u^2+1}}$
$\cosh(u)$	$\sinh(u) \cdot u'$	$\cosh^{-1}(u)$	$\frac{u'}{\sqrt{u^2-1}}$
$\operatorname{sech}(u)$	$-\operatorname{sech}(u) \cdot \tanh(u) \cdot u'$	$\operatorname{sech}^{-1}(u)$	$\frac{-u'}{u\sqrt{1-u^2}}$
$\operatorname{cosech}(u)$	$-\operatorname{cosech}(u) \cdot \coth(u) \cdot u'$	$\operatorname{cosech}^{-1}(u)$	$\frac{-u'}{u\sqrt{1+u^2}}$
$\tanh(u)$	$\operatorname{sech}^2(u) \cdot u'$	$\tanh^{-1}(u)$	$\frac{u'}{1-u^2}$
$\coth(u)$	$-\operatorname{cosech}^2(u) \cdot u'$	$\coth^{-1}(u)$	$\frac{-u'}{1-u^2}$
a^u	$a^u \cdot \ln(a) \cdot u'$	e^u	$e^u \cdot u'$
$\log_p(u)$	$\frac{1}{u \cdot \ln p} \cdot u' = \frac{1}{u} \cdot \log_p e \cdot u'$	$\ln(u)$	$\frac{1}{u} \cdot u'$
$[f]^n$	$n \cdot [f]^{n-1} \cdot f'$	$[f]^g$	$[f]^g \cdot [\frac{g}{f} \cdot f' + g' \cdot \ln f]$

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