

سنتر فیوچر

Subject: رياضيات ابتدائي

Chapter: نظرية ذات الصلة

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binomial نظریہ ذات المدید

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + {}^n C_3 a^{n-3} b^3 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n$$

d=6

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Expand $(2+3x)^4$

$$\begin{aligned} & 2^4 + {}^4 C_1 (2)^3 (3x) + {}^4 C_2 (2)^2 (3x)^2 + {}^4 C_3 (2)^1 (3x)^3 + \\ & {}^4 C_4 (3x)^4 \\ & = 16 + 4(8)(3x) + 6(4)(9x^2) + 4(2)(27x^3) + 81x^4 \end{aligned}$$

Expand

$$[(1+2x) + x^2]^4$$

$$\begin{aligned} & (1+2x)^4 + {}^4 C_1 (1+2x)^3 (x^2) + {}^4 C_2 (1+2x)^2 (x^2)^2 + \\ & {}^4 C_3 (1+2x) (x^2)^3 + {}^4 C_4 (x^2)^4 \\ & = (1+2x)^4 + 4x^2 (1+2x)^3 + 6x^4 (1+2x)^2 + x^8 \\ & \quad + 4(1+2x)^6 \end{aligned}$$

$$(1+2x)^4 = \underbrace{(1+2x)^3}_{\text{پس}} - \underbrace{(1+2x)^2}_{\text{لے جو 6 دلے}} + \sqrt{1+2x}$$

اذا كانت $\sqrt[n]{|a_n|}$ ملحوظ

$$(1+u)^n = 1 + nu + \frac{n(n-1)}{2!} u^2 + \frac{n(n-1)(n-2)}{3!} u^3 + \dots$$

$|u| < 1$ شرط صحة المبرهنة

Condition for convergence $|u| < 1$

Expand $(1+3x)^{-4}$

$$|3x| < 1 \quad |x| < \frac{1}{3}$$

$$(1+3x)^{-4} = 1 - 4(3x) + \frac{(-4)(-5)}{2!} (3x)^2 + \frac{(-4)(-5)(-6)}{3!} (3x)^3 + \dots$$

Expand $(1 - \frac{s}{x})^{\frac{1}{3}}$

$$|\frac{s}{x}| < |x| \quad |s| < |x| \quad \text{شرط صحة المبرهنة}$$

$$(1 - \frac{s}{x})^{\frac{1}{3}} = \left(1 + \frac{-s}{x}\right)^{\frac{1}{3}} = 1 + \frac{1}{3} \left(\frac{-s}{x}\right) + \frac{\frac{1}{3} \left(\frac{-2}{3}\right)}{2!} \left(\frac{-s}{x}\right)^2 + \frac{\left(\frac{1}{3}\right) \left(\frac{-2}{3}\right) \left(\frac{-5}{3}\right)}{3!} \left(\frac{-s}{x}\right)^3 + \dots$$

(2)

$$\text{Expand } (2+3x)^{-4}$$

① in power of x increasing

② in power of x decreasing

عنـ الـ زـ دـ لـ بـ مـ جـ مـ جـ بـ

$$③ (2+3x)^{-4} = 2^{-4} \left[1 + \frac{3x}{2} \right]^{-4}$$

Condition $\left| \frac{3x}{2} \right| < 1 \quad |x| < \frac{2}{3}$

$$\therefore 2^{-4} \left[1 + \frac{3x}{2} \right]^{-4} = \frac{1}{16} \left[1 - \frac{4(3x)}{2} + \frac{(-4)(-5)}{2!} \frac{9x^2}{4} + \frac{(-4)(-5)(-6)}{3!} \left(\frac{3x}{2} \right)^3 - \dots \right]$$

② عنـ تـ قـ وـ نـ كـ وـ مـ بـ سـ بـ يـ تـ بـ عـ اـ لـ اـ لـ مـ بـ

$$(2+3x)^{-4} = (3x)^{-4} \left[1 + \frac{2}{3x} \right]^{-4}$$

Condition for Convergence سـ طـ تـ قـ دـ رـ بـ اـ لـ حـ دـ لـ دـ

$$(3x)^{-4} \left[1 - \frac{4(2)}{3x} + \frac{(-4)(-5)}{2!} \left(\frac{2}{3x} \right)^2 + \frac{(-4)(-5)(-6)}{3!} \left(\frac{2}{3x} \right)^3 - \dots \right]$$

$$\left| \frac{2}{3x} \right| < 1$$

$$\frac{2}{3} < |x|$$

#

③

$$\text{Expand } \left(2 + \frac{5}{x}\right)^{-4}$$

in power of x increasing

• • • x decreasing

$$\textcircled{2} \quad \left(2 + \frac{5}{x}\right)^{-4} = \overbrace{\left[1 - 4\left(\frac{5}{2x}\right) + \frac{(-4)(-5)}{2!} \left(\frac{5}{2x}\right)^2 + \dots\right]}^{\left(2^{-4}\right)}$$

$$\left|\frac{5}{2x}\right| < \frac{5}{2} < |x|$$

\textcircled{1} in power of x increasing

$$\left(\frac{5}{x}\right)^{-4} \left[1 + \frac{2x}{5}\right]^{-4}$$

$$\left(\frac{5}{x}\right)^{-4} \left[1 - 4\left(\frac{2x}{5}\right) + \frac{(-4)(-5)}{2!} \left(\frac{2x}{5}\right)^2 + \frac{(-4)(-5)(-6)}{3!} \left(\frac{2x}{5}\right)^3 + \dots\right]$$

Expand

$$\left(2 + \frac{3}{x}\right)^{-4}$$

\textcircled{1} $x \ggg$

\textcircled{2} $x \lll$

بخار البارometric

$$\left(2 + \frac{3}{x}\right)^{-4} = \overbrace{\left[1 + \frac{3}{2x}\right]^{-4}}^{\textcircled{1}} + \overbrace{\left[\frac{(-4)(-5)(-6)(-7)}{3!} \left(\frac{3}{2x}\right)^3\right]}^{\textcircled{2}} \dots$$

\textcircled{2}

if $x \ll$

$$\left(2 + \frac{3}{x}\right)^{-4} = \left(\frac{3}{x}\right)^{-4} \left[1 + \frac{2x}{3}\right]^{-4}$$

$$\left|\frac{2x}{3}\right| < 1 \quad \frac{2}{3} < |x| \text{ Condition for Conv.}$$

$$\therefore \left(2 + \frac{3}{x}\right)^{-4} = \left(\frac{3}{x}\right)^{-4} \left[1 - 4\left(\frac{2x}{3}\right) - \frac{4(-5)}{2!} \left(\frac{2x}{3}\right)^2 + \frac{(-4)(-5)(-6)}{3!} \left(\frac{2x}{3}\right)^3 \dots\right]$$

Expand $(s + 3x)^{y_2}$

② $x \ll$

① $\underline{\underline{x \gg}}$

$$\frac{1}{1} \text{ if } x \gg \quad (s + 3x)^{y_2} = \sqrt{3x} \left[1 + \frac{s}{3x}\right]^{y_2}$$
$$= \sqrt{3x} \left[1 + \frac{1}{2} \left(\frac{s}{3x}\right) + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(\frac{s}{3x}\right)^2 + \dots\right]$$

if $x \ll \quad (s + 3x)^{y_2} = \sqrt{s} \left[1 + \frac{3x}{s}\right]^{y_2}$

Condition for Conv $\left|\frac{3x}{s}\right| < 1 \quad |x| < s/3$

$$(s + 3x)^{y_2} = \sqrt{s} \left[1 + \frac{1}{2} \left(\frac{3x}{s}\right) + \frac{1}{2!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(\frac{3x}{s}\right)^2\right]$$

$$+ \frac{1}{3!} \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(\frac{3x}{s}\right)^3 \dots$$

ایجاد قیمت تقریبیتے لے لیز

$$\sqrt[3]{10}$$

ط

نوسرازب متنی ۱۰ نہ بزر دعییہ

$$\sqrt[3]{10} = (2+8)^{1/3}$$
$$= 8^{1/3} \left[1 + \frac{2}{8} \right]^{1/3}$$

$$= 2 \left[1 + \frac{1}{3} \binom{1}{4} + \frac{\binom{1}{3} \binom{-2}{3}}{2!} \left(\frac{1}{4} \right)^2 + \frac{\binom{1}{3} \binom{-2}{3} \binom{-5}{3}}{3!} \left(\frac{1}{4} \right)^3 \right]$$

نحو

Find approximation val

$$\sqrt[5]{30}$$

$$\sqrt[5]{30} = (32 - 2)^{1/5} = (32)^{1/5} \left[1 + \frac{-2}{32} \right]^{1/5}$$

$$= 2 \left[1 + \frac{-1}{16} \right]^{1/5} = 2 \left[1 + \frac{1}{5} \left(-\frac{1}{16} \right) + \frac{\binom{1}{5} \binom{-4}{5} \binom{-7}{5}}{2!} \left(-\frac{1}{16} \right)^2 \right.$$
$$\left. + \frac{\binom{1}{5} \binom{-4}{5} \binom{-9}{5}}{3!} \left(-\frac{1}{16} \right)^3 \right]$$

②

Find approximation value $\sqrt[4]{80}$

$$\begin{aligned}\sqrt[4]{80} &= \sqrt[4]{81 - 1} = (81 - 1)^{\frac{1}{4}} \\ &= (81)^{\frac{1}{4}} \left[1 + \frac{-1}{81} \right]^{\frac{1}{4}} \\ &= 3 \left[1 + \frac{1}{4} \left(\frac{-1}{81} \right) + \frac{\left(\frac{1}{4} \right) \left(-\frac{3}{4} \right)}{2!} \left(\frac{-1}{81} \right)^2 + \frac{\left(\frac{1}{4} \right) \left(-\frac{3}{4} \right) \left(-\frac{7}{4} \right)}{3!} \left(\frac{-1}{81} \right)^3 \right]\end{aligned}$$

$$= 3 \underbrace{\left[\left(\frac{1}{4} \right) \left(-\frac{3}{4} \right) \left(-\frac{7}{4} \right) \left(-\frac{19}{4} \right) \right]}_{4!} \left(\frac{-1}{81} \right)^{\frac{1}{4}} \quad \text{لحوظة}$$

Find $\sqrt[3]{\frac{97}{101}}$

$$\begin{aligned}\left(\frac{97}{101} \right)^{\frac{1}{3}} &= \left(\frac{101 - 4}{101} \right)^{\frac{1}{3}} = \left(1 + \frac{-4}{101} \right)^{\frac{1}{3}} \\ &= 1 + \frac{1}{3} \left(-\frac{4}{101} \right) + \frac{\left(\frac{1}{3} \right) \left(-\frac{2}{3} \right) \left(-\frac{4}{101} \right)}{2!} \\ &\quad + \frac{\left(\frac{1}{3} \right) \left(-\frac{2}{3} \right) \left(-\frac{5}{3} \right) \left(-\frac{4}{101} \right)^3}{3!} + \dots\end{aligned}$$

Find $\sqrt[5]{\frac{41}{17}}$

$$\left(\frac{41}{17}\right)^{\frac{1}{5}} = \left(\frac{17+24}{17}\right)^{\frac{1}{5}} = \left(1 + \frac{\cancel{24}}{17}\right)^{\frac{1}{5}}$$

using binomial

$$\begin{aligned} \therefore \sqrt[5]{\frac{41}{17}} &= \left(\frac{17}{41}\right)^{-\frac{1}{5}} = \left(\frac{41-24}{41}\right)^{-\frac{1}{5}} \\ &= \left[1 + \frac{-24}{41}\right]^{-\frac{1}{5}} = 1 + \left(-\frac{1}{5}\right)\left(\frac{-24}{41}\right) + \frac{\left(-\frac{1}{5}\right)\left(\frac{-6}{5}\right)}{2!} \\ &\quad + \frac{\left(-\frac{1}{5}\right)\left(\frac{-6}{5}\right)\left(-\frac{9}{5}\right)}{3!} \left(\frac{-24}{41}\right)^3 + \dots \end{aligned}$$

Find approximation of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

$$\begin{aligned} \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} &= \frac{4+3+4\sqrt{3}}{4-3} \\ &= 7+4\sqrt{3} = 7+4\left[4-1\right]^{\frac{1}{2}} = 7+8\sqrt{1-\frac{1}{4}}^{\frac{1}{2}} \\ &= 7+(4)^{\frac{1}{2}}\left[1+\frac{1}{2}\left(-\frac{1}{4}\right)+\frac{1}{2}\frac{(-3/2)}{2!}\left(-\frac{1}{4}\right)^2\right] \\ &\quad \text{A} \end{aligned}$$

$$\frac{1}{1-u} = 1 + u + u^2 + u^3 + \dots + u^n$$

$$\frac{1}{1+u} = 1 - u + u^2 - u^3 + \dots + (-u)^n$$

$$\frac{1}{(1-u)^2} = 1 + 2u + 3u^2 + 4u^3 + \dots + (n+1)u^n$$

$$\frac{1}{(1+u)^2} = 1 - 2u + 3u^2 - 4u^3 + \dots + (n+1)(-u)^n$$

$$(1+u)^{-k} = 1 - ku - \frac{k(-k-1)}{2!}u^2 + \dots + c_n (-u)^n$$

Expand $\underbrace{(2+3x)^{-4}}_{x^n}$ and find Coefficient

$$(2+3x)^{-4} = \frac{1}{2^4} \left[1 + \frac{3x}{2} \right]^{-4}$$

$\left| \frac{3x}{2} \right| < 1 \quad \frac{\text{حيث المجموع}}{|x| < 2/3}$

$$(2+3x)^{-4} = \frac{1}{2^4} \left[1 - 4 \frac{(3x)}{2} + \frac{(-4)(-5)}{2!} \left(\frac{3x}{2} \right)^2 + \dots + \frac{n+4-1}{c_n} \left(-\frac{3x}{2} \right)^n \right]$$

$$\therefore \text{Coefficient of } x^7 \text{ is } \frac{1}{2^4} \cdot \frac{n+3}{c_n} \left(-\frac{3}{2} \right)^n$$

(9)

Expand $\frac{2}{(s+4x)^2}$ and find coefficient x^2

$$\frac{2}{(s+4x)^2} = \frac{2}{2s} \left(\frac{1}{\left(1 + \frac{4x}{s}\right)^2} \right)$$

$\left| \frac{4x}{s} \right| < 1$ سرطان محتوى s

$$\therefore |x| < \frac{4}{s}$$

$$\frac{2}{(s+4x)^2} = \left[\frac{2}{2s} \right] \left[1 - 2\left(\frac{4x}{s}\right) + (3)\left(\frac{4x}{s}\right)^2 + (4)\left(\frac{4x}{s}\right)^3 \right. \\ \left. + \dots (n+1)\left(-\frac{4x}{s}\right)^n \right]$$

Coefficient x^n is $\frac{2}{2s} (n+1) \left(-\frac{4}{s}\right)^n$

Expand $\frac{3}{s-x}$

$$\frac{3}{s} \left[\frac{1}{1-\frac{x}{s}} \right] = \frac{3}{s} \left[1 + \frac{x}{s} + \left(\frac{x}{s}\right)^2 + \dots + \left(\frac{x}{s}\right)^n \right]$$

$$\frac{3}{s} \left(\frac{1}{s}\right)^n$$

$$\frac{3}{s} \left(\frac{1}{s}\right)^{2n}$$

معلم x^n ..
مطلوب معلم x^{2n}
عرضت ١٠

Factorize to partial fraction and Expand.

Find Coefficient x^7

$$f(x) = \frac{12x}{(x-2)(x-3)}$$

$$\frac{12x}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{\beta}{x-3}$$

$$\text{let } x=2 \quad A = \left. \frac{12x}{x-3} \right|_{x=2} = \frac{24}{-1}$$

$$\text{let } x=3 \quad \beta = \left. \frac{12x}{x-2} \right|_{x=3} = 36$$

$$f(x) = \frac{36}{x-3} - \frac{24}{x-2}$$

$$= \frac{36}{-3} \left[\frac{1}{1-\frac{x}{3}} \right] + \frac{24}{2} \left[\frac{1}{1-\frac{x}{2}} \right]$$

$$-12 \left[1 + \frac{x}{3} + \left(\frac{x}{3}\right)^2 + \dots + \left(\frac{x}{3}\right)^n \right] +$$

$$12 \left[1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \dots + \left(\frac{x}{2}\right)^n \right]$$

for $x > 0$ \therefore

$$12 \left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n \right] \quad \therefore$$

$\underbrace{4x_1 < 2}$

$$\left| \frac{x}{3} \right| < 1, \left| \frac{x}{2} \right| < 1 \quad (1)$$

Expand and find Coefficient a^7

$$f(x) = \frac{12x}{(x-2)^2(x+3)}$$

$$\frac{12x}{(x-2)^2(x+3)} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\text{let } x = -3 \quad A = \frac{-36}{25}$$

$$\text{let } x = 2 \quad C = \frac{24}{5}$$

$$\text{let } x = 0 \quad 0 = \frac{A}{3} - \frac{B}{2} + \frac{C}{4}$$

$$\frac{B}{2} = \frac{-12}{25} + \frac{6}{5} = \frac{30 - 12}{25} = \frac{18}{25}$$

$$(B = \frac{36}{25})$$

$$P(x) = \frac{-36/25}{x+3} + \frac{36/25}{x-2} + \frac{24/5}{(x-2)^2}$$

$$= -\frac{12}{25} \left[\frac{1}{1+\frac{x}{3}} \right] - \frac{18}{25} \left[\frac{1}{1-\frac{x}{2}} \right] + \frac{6}{5} \left[\frac{1}{(1-\frac{x}{2})^2} \right]$$

$$\begin{aligned} \left| \frac{x}{3} \right| < 1 &\rightarrow \frac{s}{\left| x \right|} < 3 \rightarrow \\ \left| \frac{x}{2} \right| < 1 &\quad \left| x \right| < 2 \end{aligned}$$

(c)

$|x| < 2$ مُنْظَرِي مُصْطَبَتِ الْمُنْجَوِي ..

$$\begin{aligned} \frac{12x}{(x-2)^2(x+3)} &= \frac{-12}{25} \left[1 - \frac{x}{3} + \left(\frac{x}{3}\right)^2 - \left(\frac{x}{3}\right)^3 + \cdots + \left(-\frac{x}{3}\right)^n \right. \\ &\quad \left. + \frac{18}{25} \left[1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 - \cdots + \left(\frac{x}{2}\right)^n \right] \right. \\ &\quad \left. + \frac{6}{5} \left[1 + 2\left(\frac{x}{2}\right) + 3\left(\frac{x}{2}\right)^2 + \cdots + (n+1)\left(\frac{x}{2}\right)^n \right] \right] \\ &= \frac{-12}{25} \left[-\frac{1}{3} \right]^n - \frac{18}{25} \left(\frac{1}{2} \right)^n + \frac{6}{5} (n+1) \left(\frac{1}{2} \right)^n \end{aligned}$$

Expand $\frac{2x+3}{x^3+x^2+x+1}$ and find coefficient

$$x^n \qquad \qquad \qquad \overbrace{\qquad\qquad\qquad}$$

$$\frac{2x+3}{x^3+x^2+x+1} = \frac{2x+3}{x^2(x+1)+(x+1)}$$

$$\frac{2x+3}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$\text{Let } x = -1 \qquad A = \frac{1}{2}$$

(٢)

$$\text{if } x = 0 \\ 3 = A + C \quad C = 5/2$$

let $x = 1$

$$\frac{5}{4} = \frac{A}{2} + \frac{B}{2} + \frac{C}{2}$$

$$\frac{5}{2} = A + B + C \quad B = -1/2$$

$$f(x) = \frac{1/2}{x+1} + \frac{-\frac{1}{2}x + 5/2}{x^2+1}$$

$$= \frac{1}{2} \left[\frac{1}{1+x} \right] + (5-x) \left(\frac{1}{1+x^2} \right)$$

$$\frac{1}{2} \left[1 - x + x^2 + x^3 + \dots + (-x)^n \right]$$

$$+ (5-x) \left[1 - x^2 + x^4 - x^6 + \dots + (-x)^{2n} \right]$$

$$\frac{1}{2} (-1)^n \text{ معنی } x^n \text{ معنی}$$

$$\frac{1}{2} (-1)^5 \text{ معنی } x^{2n} \text{ معنی}$$

$$-\frac{1}{2} (-1)^n \text{ معنی } x^{2n+1} \text{ معنی}$$

$$\text{implies } \frac{1}{2}(1) + \frac{1}{2}(-1)^{2n} \rightarrow x^{2n} \text{ معنی} \therefore$$

(E) $\frac{1}{2}(-1)^{2n+1} = x^{2n+1} \text{ معنی}$

$$\frac{5}{2}(-1)^n + \frac{1}{2}(-1)^{2n} \xrightarrow{2n \rightarrow 0} x^n \quad \text{معنی:} \\ \text{عطفتى الطر} \xrightarrow{x^n} \text{معنی:}$$

$$= \frac{5}{2}(-1)^n + \frac{1}{2}$$

$$-\frac{1}{2}(-1)^n + \frac{1}{2}(-1)^{2n-1}$$

$$= x^{n+1} \quad \text{معنی:}$$

$$= -\frac{1}{2}(-1)^n + \frac{1}{2} = -\frac{1}{2} [1 + (-1)^n]$$

Find $\sqrt[3]{2}$

$$\sqrt[3]{2} =$$

$$\begin{aligned} \sqrt[3]{2} &= \frac{2^{1/3}}{3^{1/2}} = \frac{(2^2)^{1/6}}{(3^3)^{1/6}} = \left(\frac{4}{27}\right)^{1/6} \\ &= \left(\frac{27 - 23}{27}\right)^{1/6} = \left[1 - \frac{23}{27}\right]^{1/6} \\ &= 1 + \frac{1}{6} \left(-\frac{23}{27}\right) + \frac{1}{6} \left(\frac{-5}{6}\right) \left(-\frac{23}{27}\right)^2 + \frac{1}{6} \left(\frac{1}{6}\right) \left(\frac{-5}{6}\right) \left(-\frac{11}{6}\right)^2 \\ &\quad \text{اعنى: } \left(\frac{-23}{27}\right)^2 \end{aligned}$$

⑩

Expand $\left(\frac{1+x}{1-x}\right)^2$ Find Coefficient x^n

$$\left(\frac{1+x}{1-x}\right)^2 = (1+x)^2 (1-x)^{-2}$$

$$= [1 + 2x + x^2] \left(1 + 2x + 3x^2 + \dots + (n+1)x^n\right)$$

Coeffic x^n is

$$(n+1) + 2(n) + (n-1) = 4n$$

نحوی مع جمع محدث عبیر (الجبر)

$$a, ar, ar^2, \dots, ar^{n-1}$$

$$S = a \frac{(1-r^n)}{1-r}$$

Expand $(1+x+x^2+x^3+x^4+x^5)^4$

$$1+x+x^2+x^3+x^4+x^5$$

$$= \frac{1-x^6}{1-x}$$

$$\therefore (1+x+x^2+x^3+x^4+x^5)^4 = \left(\frac{1-x^6}{1-x}\right)^4$$

(17)

$$\left(\frac{1-x^6}{1-x}\right)^4 = (1-x^6)^4 [1-x]^{-4}$$

Since

$$\begin{aligned} & \left[-4c_1x^6 + 4c_2(x^6)^2 - 4c_3(x^6)^3 + 4c_4(x^6)^4 \right] (1-x)^{-4} \\ & \left[-4x^6 + 6x^{12} - 4x^{18} + x^{24} \right] \left(1 + 4x + \frac{4}{2}x^2 + \dots + c_n x^n \right) \end{aligned}$$

$$c_n = \frac{n+3}{4}c_{n-6} + 6\frac{n-9}{c_{n-12}} - 4\frac{n-15}{c_{n-18}} + \frac{n-21}{c_{n-24}}$$

$$= \frac{2}{4}c_{17} - 4c_{11} + 6c_5 - 4c_{-1} + c_{-7}$$

لوكس

↓
↓
↓
↓
↓

جذب = قياس

$$\text{Expand } (x^2 + x^4 + x^6 + \dots + x^{12})^3$$

. Find Coefficient x^n and coeff x^{29}, x^{30}

$$(x^2)^3 \left[1 + x^2 + x^4 + \dots + x^{10} \right]^3$$

$$= x^6 \left[\frac{(1-(x^2)^6)}{1-x^2} \right]^3$$

(W)

$$= x^6 \left[\frac{1 - x^{12}}{1 - x^2} \right]^3 = x^6 (1 - x^2)^3 (1 - x^2)^{-3}$$

$$= x^6 \left[1 - 3x^{12} + 3x^{24} - x^{36} \right] \left(1 + 3x^2 + \frac{(3)(+)(+)}{2!} x^4 \right. \\ \left. + \frac{n+3-1}{C_n} x^{2n} \right)$$

$$= \left[x^6 - 3x^{18} + 3x^{30} - x^{42} \right] \left(1 + 3x^2 + \dots + \frac{n+2}{C_n} x^{2n} \right)$$

$$\underset{n-1}{C_{n-3}} - 3 \underset{n-7}{C_{n-9}} + 3 \underset{n-13}{C_{n-15}} - \underset{n-19}{\cancel{C_{n-21}}} \xrightarrow{x^{2n}} \text{معادل}$$

$n = 15$ x^{30} معادل

$$\text{coeff } x^{30} = \underset{C_{12}}{14} - 3 \underset{C_6}{8} + 3 \underset{C_0}{2} - \cancel{\underset{C_{-6}}{-4}}$$

المعادل x^{29} معادل x^{29} معادل
نسبة المئوية صفر = صفر

(١١)

$$\text{Expand } \frac{(1+x)^n}{(1-x)}$$

$$(1+x)^n (1-x)^{-n}$$

$$= [1 + nx + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots] [1 + nx - \dots + \binom{n+3}{n}x^n]$$

و x^n معنی

$$\binom{n+3}{n} + \binom{n+2}{n-1} + 6 \binom{n+1}{n-2} + 4 \binom{n}{n-3} + \binom{n-1}{n-4}$$

$$\text{Expand } \frac{1-x}{(1+x+x^2)^2} \quad \text{Find coefficient } x^n$$

$$\therefore (1+x+x^2) = \frac{1-x^3}{1-x}$$

$$(1+x+x^2)^2 = \left(\frac{1-x^3}{1-x}\right)^2$$

$$\therefore \frac{1-x}{(1+x+x^2)^2} = (1-x)^3 (1-x^3)^{-2}$$

$$= [1 - 3x + 3x^2 - x^3] \left[1 + 2x^3 + 3x^6 - \dots + (n+1)x^{3n} \right]$$

$(n+1)$ و x^{3n} معنی
 $-3(n+1)$ و x^{3n+1} معنی
 (10)

$$\begin{aligned}
 & 3(n+1) \rightarrow x^{3n+2} \text{ معنی} \\
 & - (n+1) \rightarrow x^{3n+3} \text{ معنی} \\
 & n \rightarrow (1+n) \rightarrow x^{3n} \text{ معنی} \\
 & - 3(n+1) = x^{3n+1} \text{ معنی} \quad \left. \begin{array}{l} \text{معنی} \\ x \end{array} \right\} \\
 & 3(n+1) \rightarrow x^{3n+2} \\
 & \downarrow = + (n+1)-n \rightarrow x^{3n} \text{ معنی}
 \end{aligned}$$

Expand $\frac{2+3x}{4-x^2}$ Find coefficient x^n

$$(2+3x)\left(\frac{1}{4-x^2}\right) = (2+3x)\frac{1}{4} \left[\frac{1}{1-\frac{x^2}{4}}\right]$$

$$= \left(\frac{1}{2} + \frac{3x}{4}\right) \left[1 + \frac{x^2}{4} + \left(\frac{x^2}{4}\right)^2 + \dots + \left(\frac{x^2}{4}\right)^n\right]$$

$$\frac{1}{2} \left(\frac{1}{4}\right)^n \rightarrow x^{2n} \text{ معنی} \quad \therefore$$

$$\frac{3}{4} \left(\frac{1}{4}\right)^n \rightarrow x^{2n+1} \text{ معنی}$$

(20)