

سنتر فيوتشر

Subject: إعداد رفاه

Chapter: الـ والـ

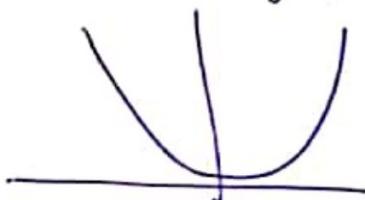
Mob: 0112 3333 122

0109 3508 204

# • Functions •

①

$$f(-x) = f(x)$$



even function  
الدالة الزوجية

مُعَادِنٌ مُوَلِّدٌ

$$\text{② odd function } f(-x) = -f(x)$$

مُعَادِنٌ مُوَلِّدٌ قُصْبَةٌ  
الدالة الفردية



Ex  $f(x) = \frac{x^2 + 4}{1 + \cos x}$

$$f(-x) = \frac{(-x)^2 + 4}{1 + \cos(-x)} = \frac{x^2 + 4}{1 + \cos x} = f(x)$$

even function

Ex  $f(x) = \frac{x + 3}{x^4 + 2}$

neither nor

$$f(-x) = \frac{-x + 3}{x^4 + 2} \neq f(x) \neq -f(x)$$

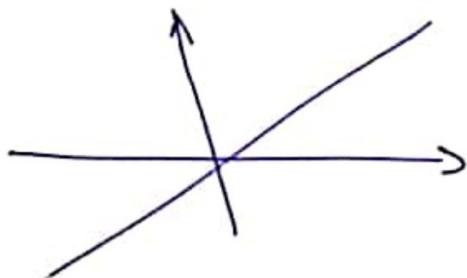
①

③ increasing function

$$x_1 > x_2$$

الدالة التصاعدية  
 $y_1 > y_2$

كل زوايا بين  $x$  زادت تبعت  $y$

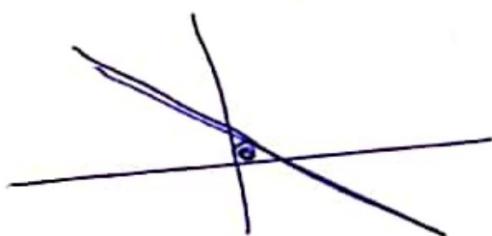


④ decreasing function

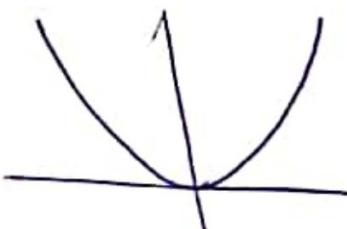
$$x_1 > x_2$$

الدالة التنازدية  
 $y_1 < y_2$

كل زوايا بين  $x$  قلت في  $y$



$$+ f(x) = f(x) = x^2$$



increasing  $[0, \infty[$

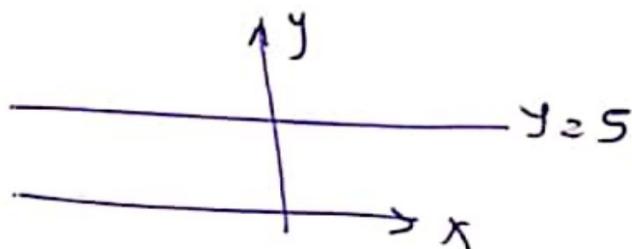
decreasing  $] -\infty, 0 ]$

⑤

$f(x) = \text{Constant}$

(الرالٰت التاٰبٰت)

$$y = 5$$

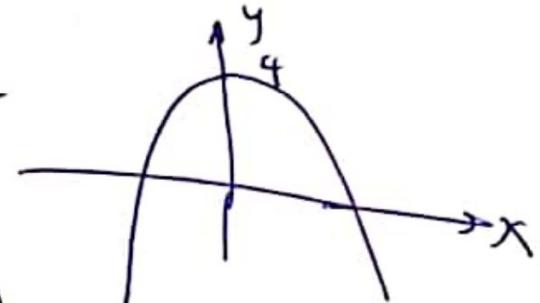


\* bounded function الرالٰت المحدودة

① bounded up المحدودة سراع

$$y = 4 - x^2$$

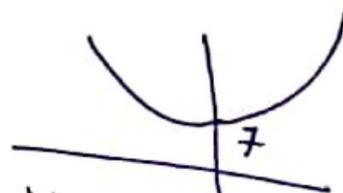
اعلى قيٰمة  $y$



فيم لا يزيد عما مقدر معين

② bounded down محدودة سافل

$$y = x^2 + 7$$



الرالٰت محدودة من سافل

أدنى محدودة  $y \geq 7$

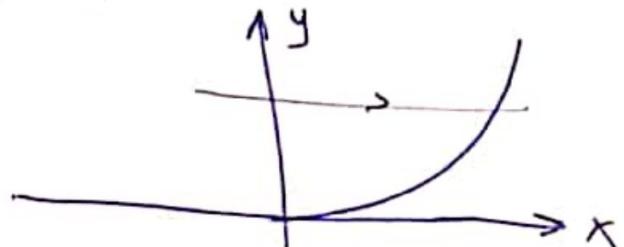


one to one

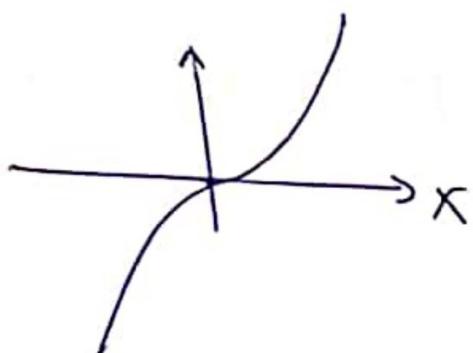
الرالٰت الاحاديت

كل قيٰمة لـ  $y$  ينكرها قيمة لـ  $x$

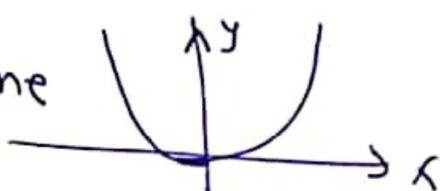
لما رسمت خط افق يقطع  
الرالٰت تتقاطع ايمان



not one to one



one to one



onto

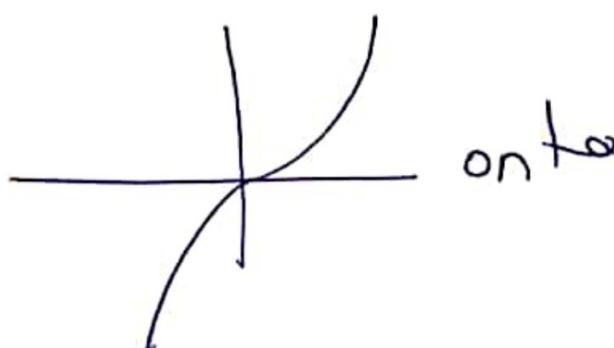
الرالٰت الغوٰية

$R$  هو

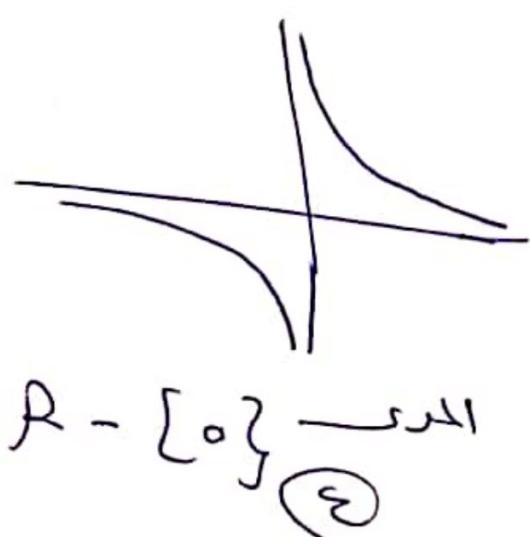
Range

اًدوكاًه (الدرى)

$R - \{0\}$  اد نفخ



$R$  ادرى



$R - \{0\}$  ادرى ③

المجال (مدى)  $x$  $R$ حال الرؤى لـ  $\sqrt[n]{x}$  المحدد $R$  ماعدا صفر المقدار

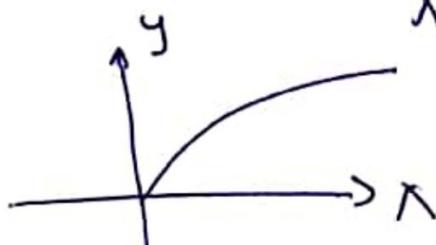
حال الرؤى المسمى

 $f \geq 0$  $\sqrt{f}$ 

حال الرؤى المسمى

 $R \ni f \in \sqrt[3]{f}$ 

حال الرؤى اللوغاريتمية

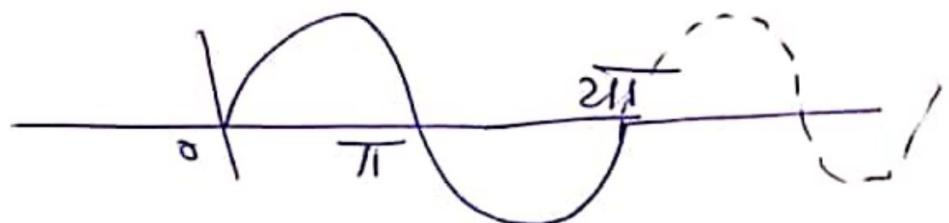
 $\ln f$  $\log f$ Range المعنىمدى  $y$  أو المدى المطلوب لـ  $x$  لـ  $y$ Range  $[0, \infty [$ Domaine  $[0, \infty [$ 

(٦)

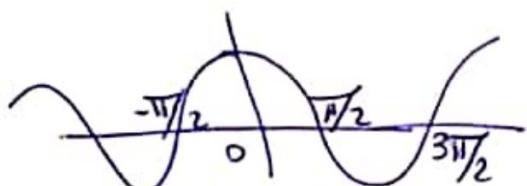
الدالة الدورية

و الدالة تكررت كل فترة

$$y = \sin x$$

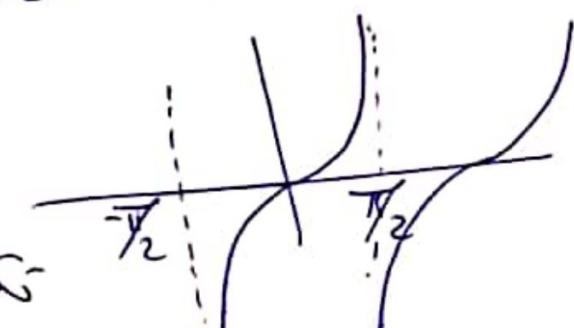


$$y = \cos x$$



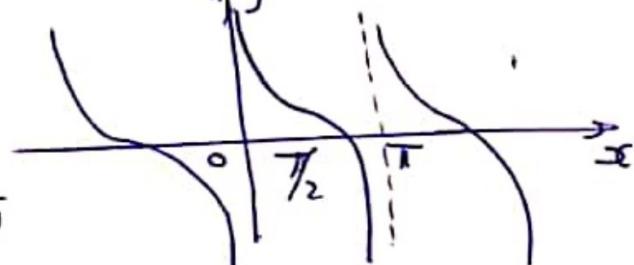
2π تكررت كل

$$y = \tan x$$



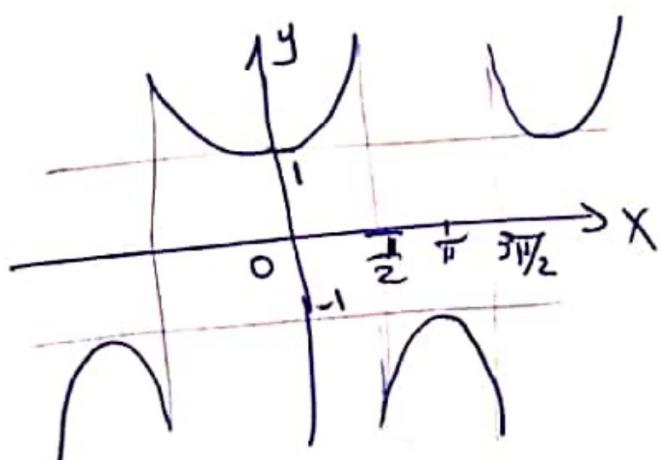
π تكررت كل

$$y = \cot x$$



π تكررت كل

$$y = \sec x$$

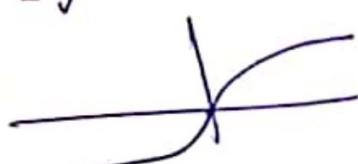
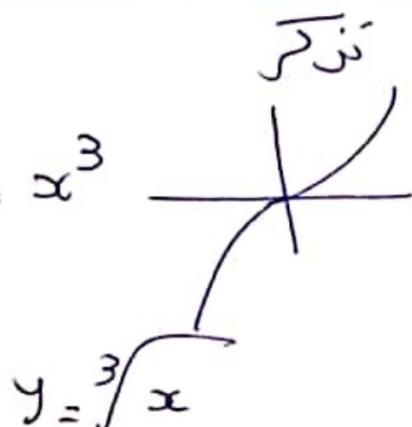
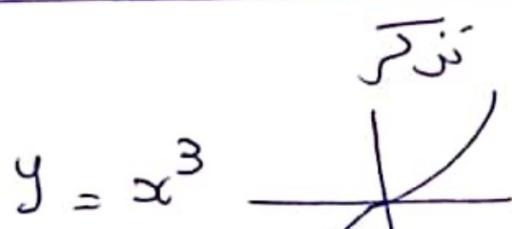
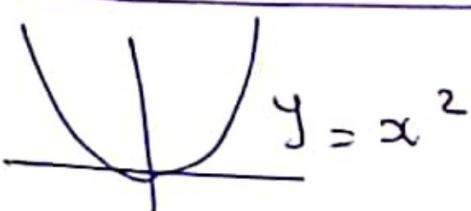
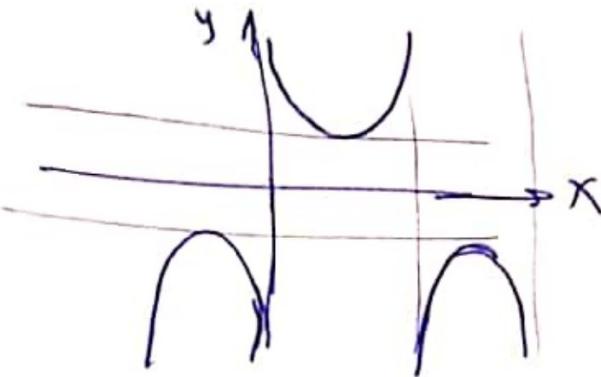


π تكررت كل

2

$$y = \alpha \ln x$$

متریک



Sketch the function and find its properties

$$y = \sqrt{4 - x^2}$$

① نوع الدالة  
الدالة زوجية

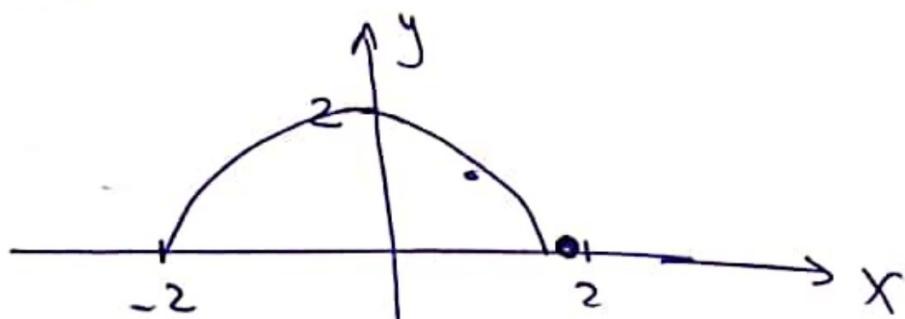
$$f(-x) = \sqrt{4 - (-x)^2} = \sqrt{4 - x^2}$$

Domain ايجل ②

$$4 - x^2 \geq 0$$

$$4 \geq x^2$$

$$-2 \leq x \leq 2$$



Range  $[0, 2]$

الدالة زوجية من الـ 2nd

③

الراله تناهية

[0, 2] تناهية

الراله لبيت one to one

on to

درب

- -

- -

$$f(x) = \sqrt{x^2 - 4}$$

$$f(-x) = \sqrt{x^2 - 4} = f(x)$$

الراله زوجي

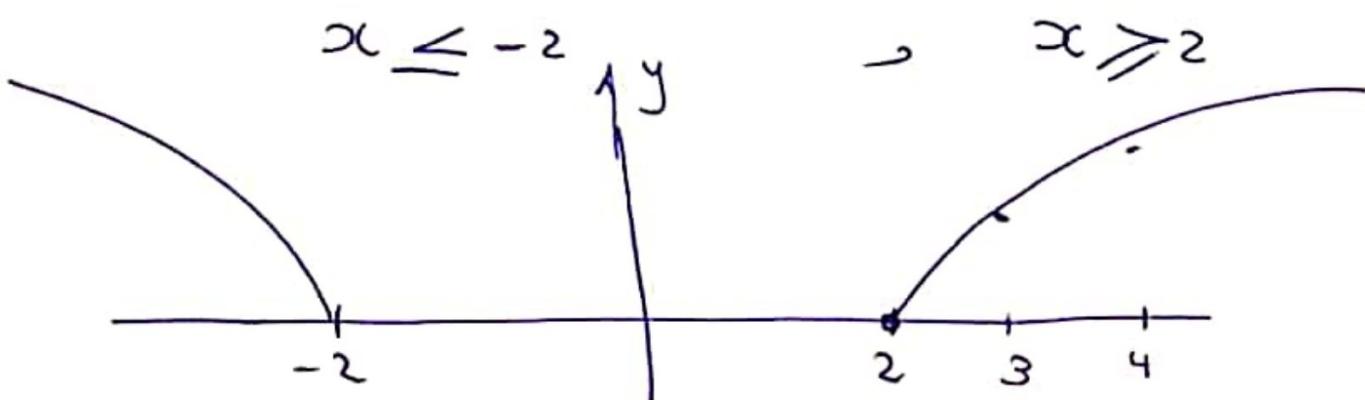
١٠

Domaine

$$x^2 - 4 \geq 0$$

الحل ⑥

$$x^2 \geq 4$$



Range [0, ∞[

٩

الرالك عربة من كفر

الرالك تزامن [2, ∞) دوافعه [-2, 0]

الرالك ليت one to one

onto

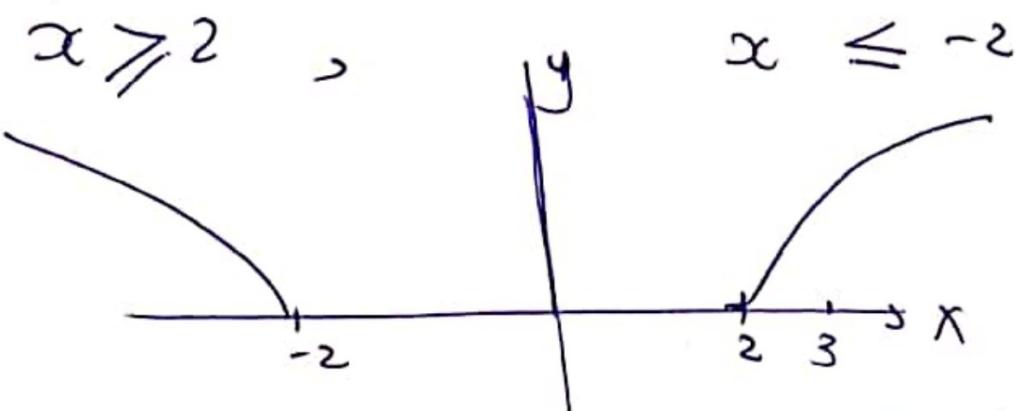
درست درست درست

$$f(x) = \sqrt{|x| - 2}$$

$$f(-x) = \sqrt{|-x| - 2} = \sqrt{|x| - 2}$$

الرالك زوجي

Domain  $|x| - 2 \geq 0$



تقس خواص الباقي

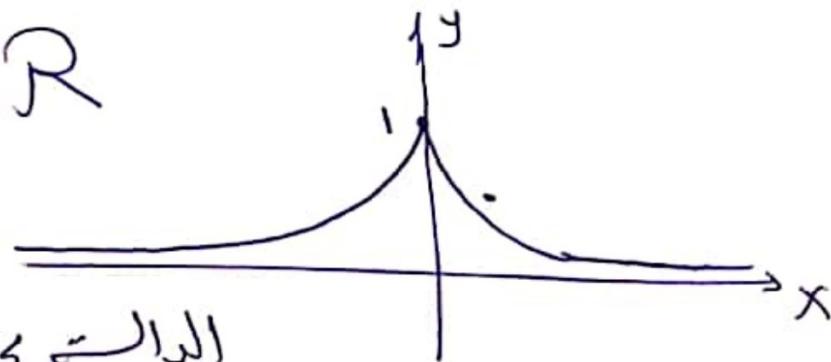
1.

$$\text{Ex} \quad f(x) = \frac{1}{1+x^2}$$

$$f(-x) = \frac{1}{1+x^2} \quad \text{even function}$$

Domain

$\mathbb{R}$



الدالة محددة على  $\mathbb{R}$

$$y = 1$$

Range

$$\mathbb{R} \setminus \{1\}$$

الدالة ليست one-to-one  
onto " "  
ذيل " "

الدالة تزيل  $\{1\}$  وتأخذ  $[0, \infty)$   $\cup (-\infty, 0]$

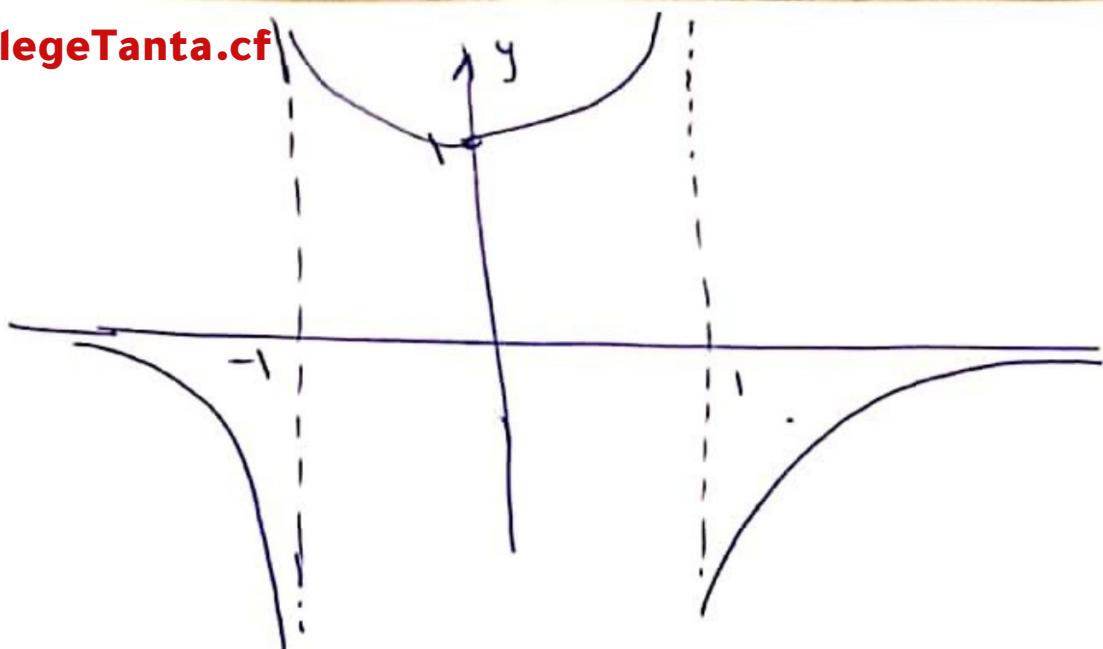
$$f(x) = \frac{1}{1-x^2}$$

$$f(-x) = f(x) = \text{even}$$

Domain

$$\mathbb{R} - \{-1, 1\}$$

(ii)



$$R - \{1, -1\}$$

$$R - [0, 1]$$

خواص اليمى

الى

الراله ذاتي

لست ذردة

$\leftarrow$   $\rightarrow$   
one to one  
onto

الراله تابعية

$$]-\infty, 0]$$

لما



$$f(x) = \sqrt{\frac{x}{1-x^2}}$$

$$f(-x) = \frac{-x}{\sqrt{1-x^2}} = -f(x)$$

odd function  
دالة فردية

$$1-x^2 > 0$$

حل

$$1 > x^2$$

$$-1 < x < 1$$

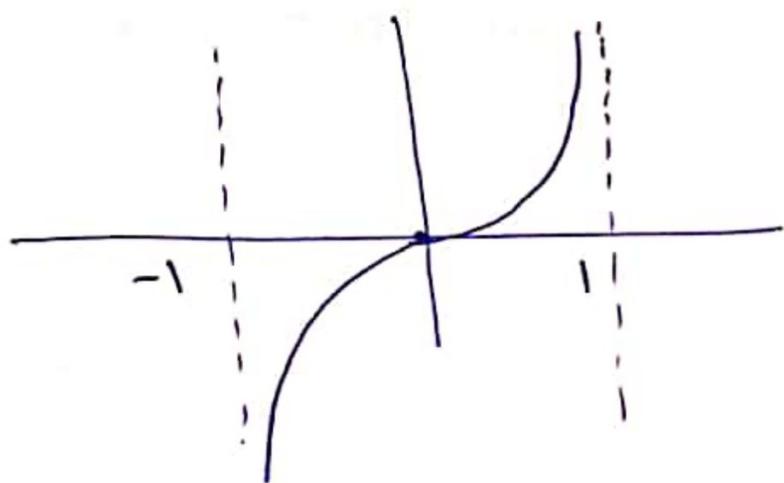
Range  $R$

الدالة غير معرفة

one to one الدالة

onto

الدالة تزايديه



$$f(x) = \sqrt{\frac{x}{x^2-1}}$$

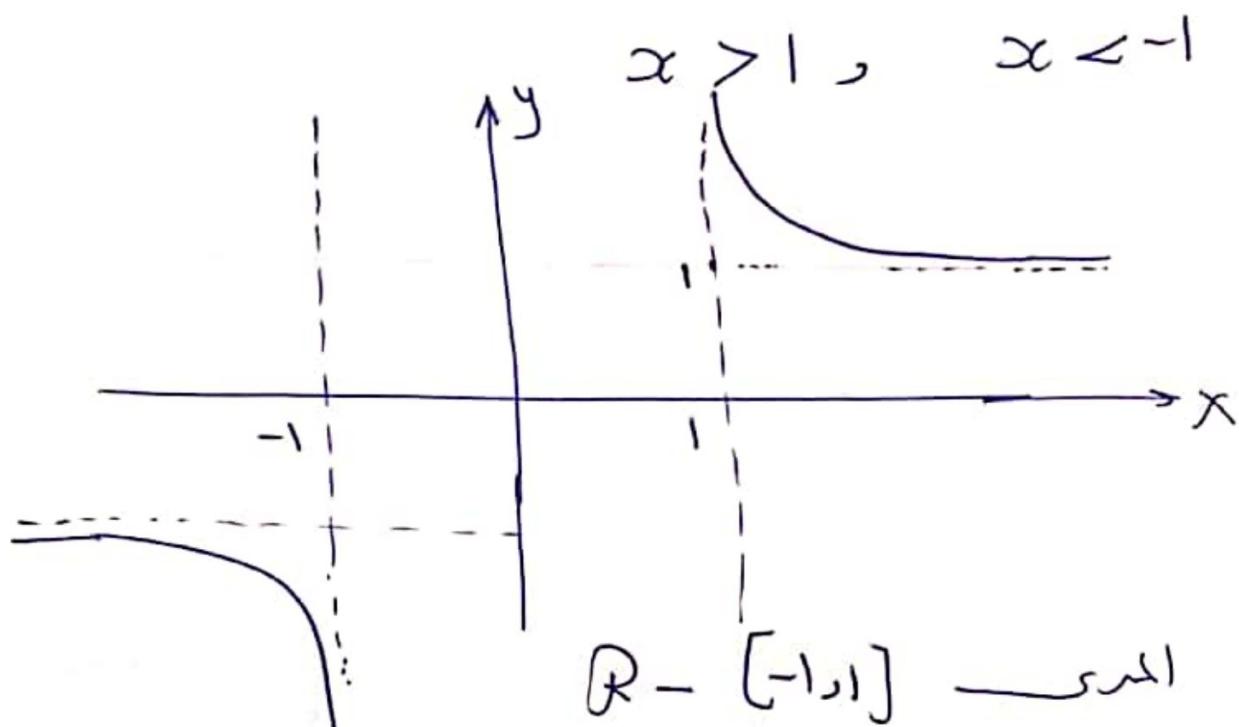
$$f(-x) = \frac{-x}{\sqrt{x^2-1}} = -f(x)$$

odd function.

Domaine  $x^2-1 > 0$

(١٤)

$$\therefore x > 1$$



$R - [-1]$  المدى

الدالة غير قياسية

الدالة متاقيمية

one to one  
لبيت  
onto

لبيت دوريّة

$$y_1 = y_2 \rightarrow x_1 = x_2 \quad \text{one to one} \quad \text{الدالة}$$

$$P(x) = \frac{2x}{3+5x}$$

$$\frac{2x_1}{3+5x_1} = \frac{2x_2}{3+5x_2} \quad x_1 \rightarrow x \rightarrow \text{نهاية} \\ x_2 \rightarrow x \rightarrow \text{نهاية}$$

one to one  $x_2 = x_1$  دلائل يوضح دوام المحت

(٤)

$$\frac{2x_1}{3+5x_2} = \frac{2x_2}{3+5x_1}$$

$$3x_1 + 5x_1 x_2 = 3x_2 + 5x_1 x_2$$

$$x_1 = x_2 \quad \therefore f(x) \text{ one to one}$$


---

$$\underline{y = x^2}$$

$$x_1^2 = x_2^2 \quad x_1 = \pm x_2$$

not one to one

$$y = x^2 \quad x > 0$$

$$x_1^2 = x_2^2 \quad x_1 = x_2$$

$\therefore f(x)$  one to one

الدالة逆 للدالة  $\rightarrow$  دالة معكوس (Inverse function)  $\leftarrow$  الدالة

# الدوال العكسيّة

١) تأمين الدوال أحاديّة

٢) مات  $x$  لمعنون طرف

٣) كنيد  $x \rightarrow f(x)$  دلالة

بالدلالة  $f(x)$  هو مرد الدالة  
والمدى صحيح

$$\text{Ex} \quad y = \frac{2x}{x+1}$$

Find inverse function

$$\frac{2x_1}{x_1+1} = \frac{2x_2}{x_2+1}$$

$$\frac{x_1}{x_1+1} = \frac{x_2}{x_2+1}$$

$$\cancel{x_1 x_2 + x_1} = \cancel{x_1 x_2 + x_2}$$

$$\therefore x_1 = x_2$$

$\therefore f(x)$  is one to one

(٢)

$$y = \frac{2x}{x+1} \quad \text{إيجاد الراتب العكسي}$$

$$yx + y = 2x$$

$$y = 2x - yx = x(2-y)$$

$$\therefore x = \frac{y}{2-y}$$

$$\boxed{f^{-1}(x) = \frac{x}{2-x}} \quad \text{الراتب العكسي}$$

٤٦

مدى الراتب العكسي

$$\text{if } f(x) = \ln\left(\frac{x+1}{x-1}\right)$$

Prove that  $y$  is one to one

Find inverse function, prove that

$$f(x) \circ f^{-1}(x) = x$$



$$\ln\left(\frac{x_1+1}{x_1-1}\right) = \cancel{\ln\left(\frac{x_2+1}{x_2-1}\right)}$$

$$\cancel{x_1x_2 - x_1 + x_2 - 1} = \cancel{x_1x_2 - x_2 + x_1 - 1}$$

$$2x_2 = 2x_1 \quad x_1 = x_2$$

$\therefore f(x)$  is one to one

$$y = \ln\left(\frac{x+1}{x-1}\right)$$

which e if "

$$e^y = \frac{x+1}{x-1}$$

$$xe^y - e^y = x+1$$

$$xe^y - x = 1 + e^y$$

$$x(e^y - 1) = e^y + 1$$

$$\therefore x = \frac{e^y + 1}{e^y - 1}$$

$$f^{-1}(x) = \frac{e^x + 1}{e^x - 1} \neq$$

(11)

$f \circ f^{-1}(x) =$   $f(x)$  مخصوص للدالة  
 $f^{-1}(f(x))$  عند  $x$  ب الدالة

$$f \circ f^{-1}(x) = \ln \left[ \frac{\frac{e^x + 1}{e^x - 1} + 1}{\frac{e^x + 1}{e^x - 1} - 1} \right]$$

$$= \ln \left[ \frac{e^x + 1 + e^x - 1}{e^x + 1 - e^x + 1} \right] = \ln \left( \frac{2e^x}{2} \right)$$

$$\therefore \ln e^x = x$$


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الدالة المضادة

$f \circ g(x)$  مخصوص للدالة  
 $g(x) = x$   $f(x)$

$$* f(x) = x + 2 \qquad g(x) = e^x$$

$$f \circ g = e^x + 2$$

$$g \circ f = e^{x+2}$$

(19)

$$f(x) = \sqrt{x-1}$$

$$g(x) = \frac{1}{\sqrt{x+2}}$$

$$f \circ g = \sqrt{\frac{1}{\sqrt{x+2}} - 1}$$

$$g \circ f = \frac{1}{\sqrt{2 + \sqrt{x-1}}}$$

$$f \circ f = \sqrt{\sqrt{x-1} - 1}$$

$$g \circ g = \frac{1}{\sqrt{2 + \frac{1}{\sqrt{x+2}}}}$$

b-u if  $y = \sqrt{|x| - 2}$

$$\textcircled{2} \quad y = \sqrt{2 - |x|}$$

$$\textcircled{3} \quad y = \frac{1}{x^2}$$

$$\textcircled{4} \quad y = \frac{x}{x^2 + 1}$$

Sketch and find properties

(c)