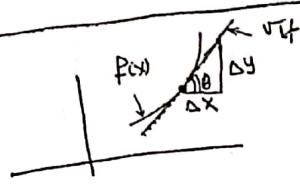


Derivative or ال微商

$$\frac{d f(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

* تعریف الرياضی:



* تعریف الممیز: هو مصطلح کامن للملائمة وله معنی

$$m = \frac{\Delta y}{\Delta x} = \tan(\theta)$$

* رمز الممیزة الاولی:

$$y' \text{ or } \frac{dy}{dx} \text{ or } \overset{(1)}{y}$$

$$y'' \text{ or } \frac{d^2 y}{dx^2} \text{ or } \overset{(2)}{y}$$

* رمز الممیزة الثانية:

وکذا

~~موجودة الممیزة الشیر~~
~~لخدمات الطالبیة~~
~~کیفیت الممیزة~~

* بعض خصائص الممیزة

1) $\left(\frac{d}{dx} \right) (\text{Const.}) = 0$

2) $\frac{d}{dx} (K \cdot f(x)) = K \cdot f'(x)$

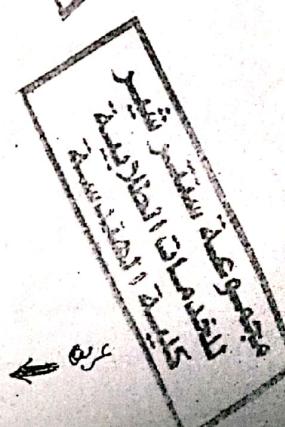
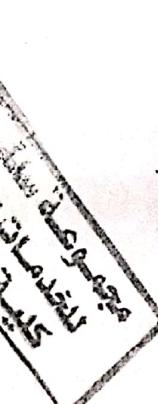
3) $\frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

4) $\frac{d}{dx} [f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

5) $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$

=
$$\frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

①



$$[6] \frac{d}{dx} [f(x)]^n = n [f(x)]^{n-1} \cdot f'(x)$$

$$[7] \frac{d}{dx} \cdot x^n \rightarrow n x^{n-1}$$

$$[8] \frac{d}{dx} \ln(x) \rightarrow \frac{1}{x}$$

مجموعة سنتر شير
لخدمات الطلابية
كلية الهندسة

$$\frac{d}{dx} \ln(u) \rightarrow \frac{1}{u} \cdot u'$$

$$[9] \frac{d}{dx} \log_a(u) = \frac{1}{u} \cdot u' \cdot \log_a(e)$$

$$[10] \frac{d}{dx} e^x = e^x$$

مجموعة سنتر شير
لخدمات الطلابية
كلية الهندسة

$$\frac{d}{dx} e^u = e^u \cdot u'$$

$$[11] \frac{d}{dx} \ln(a)$$

نضر * تفاضل لـ \ln
* (بعد).

$$[12] y = (f(x))^{g(x)}$$

(دالة) y

تفاضل

البيباتات

$$\therefore \ln(y) = \ln[f(x)]^{g(x)} \leftarrow \text{إليكرين } \ln$$

$$\therefore \ln(y) = g(x) \cdot \ln(f(x)) \quad \text{بتقاضل لطرفين}$$

$$\therefore \frac{1}{y} \cdot y' = g(x) \cdot \frac{1}{f(x)} \cdot f'(x) + \ln(f(x)) \cdot g'(x) \quad : y \neq 0$$

$$\therefore y' = y \left[\frac{g(x)}{f(x)} \cdot f'(x) + \ln(f(x)) \cdot g'(x) \right] = \boxed{\left[\frac{g}{f} \cdot f' + \ln(f) \cdot g' \right]}$$

(2)

Simple Examples

* Find $\frac{dy}{dx}$ for

$$\text{1) } y = \underbrace{e^{x^2}}_u + \underbrace{\ln(x^2-1)}_v + \underbrace{\frac{\log(x)}{\ln(\sqrt{x})}}_w$$

مكتبة شير
لخدمات الطالب
جامعة عجمان

$$y' = u' + v' + w' \Rightarrow \# \quad \therefore u = e^{x^2} \quad \therefore u' = e^{x^2} \cdot 2x$$

$$\therefore v = \ln(x^2-1) \quad \therefore v' = \frac{1}{x^2-1} \cdot 2x$$

$$\therefore w = \frac{\log(x)}{\ln(\sqrt{x})} \quad \therefore w' = \frac{\ln(\sqrt{x}) \cdot \frac{1}{x} \log(e) - \log(x) \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}{[\ln(\sqrt{x})]^2}$$

بالتحويل من *

$$\text{2) } y = \underbrace{7}_{u} + \underbrace{\log_{\frac{1}{3}}(x^2 + \ln(x))}_{v} - \underbrace{\frac{20}{x} \cdot e^{-3\sqrt{x}}}_{w}$$

$$\therefore y' = u' + v' - w' \rightarrow \#$$

$$\therefore u = 7^{\frac{\sqrt{x}\sqrt{x}\sqrt{x}}{8}} = 7^{\frac{\sqrt{x}\sqrt{x}\cdot x^{1/2}}{8}} = 7^{\frac{\sqrt{x}\sqrt{x^{3/2}}}{8}} = 7^{\frac{\sqrt{x}\cdot x^{3/4}}{8}} = 7^{\frac{\sqrt{x^{7/4}}}{8}} = 7^{\frac{x^{7/8}}{8}}$$

$$\therefore u' = 7^{\frac{7}{8}} \cdot \frac{1}{8} 7^{\frac{-1}{8}} \cdot \ln(7) \quad \#$$

مكتبة
لخدمات الطالب
جامعة عجمان

$$\therefore v = \log_{\frac{1}{3}}(x^2 + \ln(x)) \Rightarrow$$

$$\therefore v' = \frac{1}{x^2 + \ln(x)} \cdot \left(2x + \frac{1}{x}\right) \cdot \log_{\frac{1}{3}}(e)$$

$$\therefore w = \frac{20}{x} e^{-3\sqrt{x}}$$

دالة
جامعة عجمان

$$\therefore w' = \frac{x \cdot 20 \cdot -3\sqrt{x} \cdot \frac{1}{\sqrt{x}} - 20 e^{-3\sqrt{x}}}{(x)^2}$$

(3)

بالتحويل من *

$$③ y = \underbrace{[x^2 + 3x]^9}_{u} \cdot \underbrace{\log(\log(x)) + 2\sqrt{x} \cdot \ln[\ln(\log_3 x)]}_{v}$$

$$\hat{y} = u + v \rightarrow \boxed{\text{F}}$$

$u =$ حاصل ضرب دالستان

$$\therefore u' = [x^2 + 3x]^9 \cdot \frac{\log(e)}{\log(x)} \cdot \frac{1}{x} \log(e) + \frac{\log(\log(x)) \cdot 9 \cdot [x^2 + 3x] \cdot (2x+3)}{\log_3 x}$$

$v =$ حاصل ضرب دالستان

$$\therefore v' = 2\sqrt{x} \cdot \frac{1}{\ln(\log_3 x)} \cdot \frac{1}{\log_3 x} \cdot \frac{1}{x} \log(e) + \ln[\ln(\log_3 x)] \cdot \frac{2}{2\sqrt{x}} \quad \# \boxed{\text{F}} \text{ عميق}$$

$$④ y = \underbrace{e^{\frac{\sqrt{x^2-2}}{\ln(x)}}}_{u} - \underbrace{e^{\frac{\ln[\log\sqrt{x-1}]}{\ln(x)}}}_{v} + \underbrace{\frac{(x-2)}{x}}_{w}$$

$$\hat{y} = u - v + w \rightarrow \boxed{\text{F}} \quad \therefore u = e^{\frac{\sqrt{x^2-2}}{\ln(x)}}$$

$$\therefore u' = e^{\frac{\sqrt{x^2-2}}{\ln(x)}} \cdot \frac{1}{2\sqrt{\frac{x^2-2}{\ln(x)}}} \cdot \frac{\ln(x)(2x) - (x^2-2) \cdot \frac{1}{x}}{(\ln x)^2}$$

$$\therefore v = \underbrace{\cancel{e}^{\ln[\log\sqrt{x-1}]}}_{\cancel{e}} = \log(\sqrt{x-1})$$

$$\therefore v' = \frac{1}{\sqrt{x-1}} \cdot \frac{1}{2\sqrt{x-1}} \cdot \log(e)$$

$$\therefore w = x^{(x-2)} \ln^{(x-1)} \quad \text{بـ} \ln \text{ طرفي}$$

$$\therefore \ln(w) = \ln x^{(x-2)} = (x-2) \cdot \ln(x) \quad (A)$$

$$\therefore \frac{1}{w} \cdot w' = [(x-2) \cdot \frac{1}{x} + \ln(x) \cdot 1]$$

$$\therefore w' = w \left[\frac{x-2}{x} + \ln(x) \right]$$

$$\therefore w' = x^{(x-2)} \left[\frac{x-2}{x} + \ln(x) \right] \quad \# \text{ عميق}$$

$$5 \quad y = \underbrace{\ln[\ln[\ln[\sqrt{x^2 + \log(x)}]]]}_u - \frac{x}{v}$$

$$y' = u' + v' \rightarrow \boxed{*}$$

$$u' = \frac{1}{\ln[\ln[\sqrt{x^2 + \log(x)}]]} \cdot \frac{1}{\ln[\sqrt{x^2 + \log(x)}]} \cdot \frac{1}{\sqrt{x^2 + \log(x)}} \cdot \frac{2\sqrt{x^2 + \log(x)}}{x}$$

$$\therefore v = \frac{x}{\ln(x)} = \omega$$

لـ \ln لـ x يـ ω

$$\therefore \ln(v) = \ln(\omega)$$

$$\ln(v) = \omega \cdot \ln(x) \quad \text{لـ \ln لـ ω بـ \ln لـ x }$$

$$\therefore \frac{1}{v} \cdot v' = \omega \cdot \frac{1}{x} + \ln(x) \cdot \omega'$$

$v \neq 0$ لـ v يـ ω'

$$\therefore v' = v \left[\frac{\omega}{x} + \ln(x) \cdot \omega' \right]$$

$$\therefore v' = x \left[\frac{x}{x} + \ln(x) \cdot x (1 + \ln(x)) \right]$$

شیر استراتيوجرافيا

$w = x \ln(x)$ يـ w

$\ln w = x \cdot \ln(x)$ يـ $\ln w$

$\frac{1}{w} \cdot w' = x \cdot \frac{1}{x} + \ln(x) \cdot 1$

$w' = w [1 + \ln(x)]$

$\therefore w' = x [1 + \ln(x)]$

شیر استراتيوجرافيا

$\boxed{*}$ عـ y'

6 Find $\frac{dy}{dx}$ from

جـ y جـ y' جـ y''

$$y = x^2 \cdot \log[\log[\ln(\sqrt{x^2 + \sqrt{x}})]] + [\ln(x-1)]^{\log(x-\sqrt{x})} \quad \#$$

$$y = \left[\frac{3x-2}{x^2-4} \right]^7 + \ln \left[\log \left[\frac{x^2-1}{\sqrt{x+3}} \right] \right] \quad \#$$

حل بـ

طـ y'

التفاضل (التفاضل الخفية) لـ y

→ (Implicit Derivatives)

كذلك y عبارة عن دالة متعدلة

وـ y يمكن جعل y لوحدها في طرف.

نقطة $\frac{dy}{dx}$ بتفاضل الطرفين بالنسبة لـ x

$$\text{Ex. (8)}: y^3 + 3xy + x^3 - 5 = 0$$

كذلك y يمكن جعل y لوحدها في طرف $\frac{dy}{dx}$ بتفاضل الطرفين بالنسبة لـ x

$$\therefore 3y^2 \cdot \frac{dy}{dx} + 3 \left[1 \cdot \frac{dx}{dx} \cdot y + \frac{dy}{dx} \cdot x \right] + 3x^2 \cdot \frac{dx}{dx} = 0$$

حيث أن y عبارة عن دالة متعدلة

$$3y^2 \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} + 3x^2 = 0$$

$\frac{dy}{dx}$ عامل مشترك مع $3y$ فهو معجل خارج

$$\therefore \frac{dy}{dx} (3y^2 + 3x) + 3y + 3x^2 = 0$$

$$\therefore \frac{dy}{dx} = \frac{-3(y+x^2)}{3(y^2+x)} = \frac{-(y+x^2)}{(y^2+x)}$$

$$\textcircled{9} \quad y^5 + 3x^2 y^3 - 7x^6 - 8 = 0$$

بتفاصيل الظرفية بـ نسبة $\rightarrow X$

$$\therefore 5y^4 \cdot \frac{dy}{dx} + 3(2x \cdot y^3 + 3y^2 \frac{dy}{dx} \cdot x^2) - 42x^5 = 0$$

$$\therefore 5y^4 \cdot \frac{dy}{dx} + 6xy^3 + gy^2 \frac{dy}{dx} \cdot x^2 - 42x^5 = 0$$

$$\therefore \frac{dy}{dx} [5y^4 + 9yx^2] + 6xy^3 - 42x^5 = 0$$

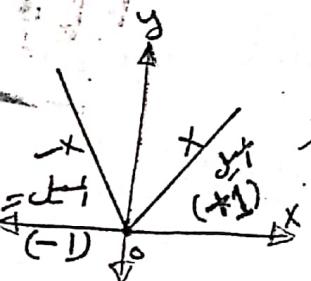
$$\therefore \frac{dy}{dx} = \frac{-6xy^3 + 42x^5}{-4} = \tan(\theta)$$

$$\theta = \tan^{-1} \left(\frac{dy}{dx} \right) \text{ and } \frac{d\theta}{dx} = \frac{1}{1+y^2} \cdot \frac{d}{dx}(y^2) = \frac{2y}{1+y^2}$$

(صراحته حادة) : «إذا كان الله (رسوله) عَلَيْكُمْ حِلٌّ

والعكس ليس بالضرورة $\{\text{ا}\}$ صحيح (أى ما زائدة
الإلة فتحلة عند نقله - فليس بالضرورة $\{\text{ا}\}$ صحيح $\{\text{فـ}\}$

• حنة النقطة ()



$x=0$ in \bar{M}

$$f(x) = |x|$$

مثال ذلك: الماء

$x = 0$ ist keine gute Stelle

- لا يصل الماء إلى الماء = 1 + 1 = 2
وليس قيمة واحدة فقط

Ex 10 find $\frac{dy}{dx}$ from:

$$e^{\frac{x\sqrt{y^2-1}}{2\sqrt{y^2-1}}} - 5 \ln(x^2 - 4y) + \frac{xy}{7} = e^{x^2}$$

جامعة سانت شير
لكلية التربية
جامعة الازهر

١٤٣١

تفاضل ضمن: ١٣١

تفاضل لطرmin بالنسبة ل x \therefore

$$= e^{\frac{x\sqrt{y^2-1}}{2\sqrt{y^2-1}}} \cdot \left[x \cdot \frac{1}{2\sqrt{y^2-1}} \cdot 2y \cdot y' + \sqrt{y^2-1} \cdot 1 \right] - 5 \cdot \frac{1}{x^2-4y} \cdot (2x-4y)' + \frac{xy}{7} \cdot (x^2+y \cdot 1) = e^{x^2} \cdot 2x$$

جامعة سانت شير
لكلية التربية
جامعة الازهر

$$= \frac{xy \cdot e^{\frac{x\sqrt{y^2-1}}{2\sqrt{y^2-1}}}}{\sqrt{y^2-1}} + e^{\frac{x\sqrt{y^2-1}}{2\sqrt{y^2-1}}} \cdot \sqrt{y^2-1} - \frac{5(2x)}{x^2-4} + \frac{5(4)}{x^2-4y} + \frac{xy}{7} + \frac{xy}{7} \cdot x \cdot y'$$

جامعة سانت شير
لكلية التربية
جامعة الازهر

$$y' \left[\frac{xy}{\sqrt{y^2-1}} e^{\frac{x\sqrt{y^2-1}}{2\sqrt{y^2-1}}} + \frac{20}{x^2-4y} + \frac{xy}{7} \cdot x \right] = -e^{\frac{x\sqrt{y^2-1}}{2\sqrt{y^2-1}}} + \frac{10x}{x^2-4} - \frac{xy}{7} \cdot y + 2x \cdot e^{x^2}$$

B

جامعة سانت شير
لكلية التربية
جامعة الازهر

$$\therefore y' = \frac{A}{B} \quad \#$$

Ex 11, find $\frac{dy}{dx}$ from:

$$\ln[\log(xy)] - \frac{4x^2}{x^3-y^3} + 5 \frac{x}{7} = e^y$$

جامعة سانت شير
لكلية التربية
جامعة الازهر

$$\therefore \frac{1}{\log(xy)} \cdot \frac{\log(e)}{xy} \cdot (x^2y + y \cdot 1) - \frac{[x^3-y^3] \cdot 8x - 4x^2(3x^2-3y^2 \cdot y')}{[x^3-y^3]^2} + 5 \cdot \frac{x}{7} \ln 7 = e^y$$

جامعة سانت شير
لكلية التربية
جامعة الازهر

$$\therefore \frac{x \log(e) \cdot y'}{xy \log(xy)} + \frac{y \log(e)}{xy \log(xy)} - \frac{8x(x^3-y^3)}{(x^3-y^3)^2} + \frac{12x^4}{(x^3-y^3)^2} - \frac{12x^2y^2 \cdot y'}{(x^3-y^3)^2} + 5 \cdot \frac{x}{7} \ln 7 = e^y$$

عامل مشترك كسب

جامعة سانت شير
لكلية التربية
جامعة الازهر

Given

Chain Rule قاعدة زائد

$$y = f(x)$$

$$\rightarrow x = g(t)$$

مطلوب

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$



Example 12: if $y = e^{\sin(x^2)} + \ln[\log(\sqrt{x^2-1})]$

$$\rightarrow x = 3t^2 \cdot \sin(t) + 2 \cos(t) \cdot \ln(\sqrt{t})$$

Find $\frac{dy}{dt}$ (solution)

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\text{Sup: } \frac{dy}{dx} = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x + \frac{1}{\log(\sqrt{x^2-1})} \cdot \frac{\log(e)}{\sqrt{x^2-1}} \cdot \frac{1}{2\sqrt{x^2-1}} \cdot 2x$$

$$\rightarrow \frac{dx}{dt} = 3t^2 \cdot \cos(t) + \sin(t) \cdot (6t) + 2\cos(t) \cdot \frac{1}{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}$$
$$+ \ln(\sqrt{t}) (-2 \sin(t))$$

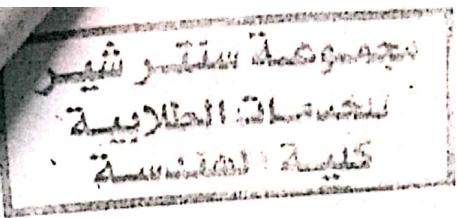
#

Ex: 13: if $y = (3x^2 + 5x - 1) \cdot 7^{x^2-1}$ حل بـ

$$\rightarrow x = 2t^2 \cdot \ln(3t)$$

Find $\frac{dy}{dt}$ #

(9)



الدوال المثلثية

$\sin \theta, \cos \theta, \tan \theta$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$



II $y = \sin(x)$

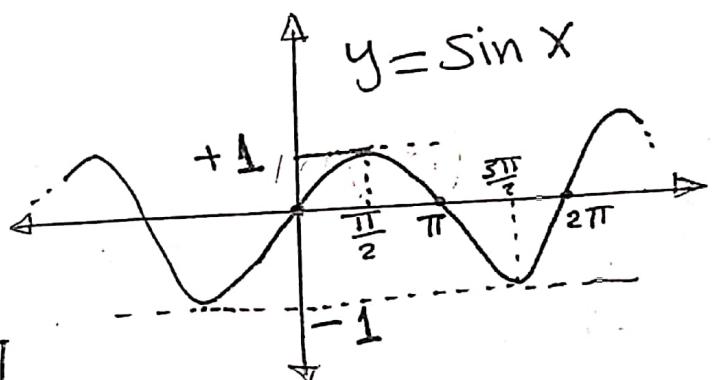
X من محور Domain (\cup) = \mathbb{R}

مدى الدالة $R_f = [-1, 1]$

- الدالة فردية (عانياها حول نقطة $x=0$ صر)

$$\sin(-x) = -\sin x$$

$$(f(-x) = -f(x))$$



الدالة دورية : أي زر تكرر نفس كل خترة (دورة)



2π دورة

[2]

$$y = \cos(x)$$

$$J_{\text{ب}} = R$$

$$\sigma_{\text{ب}} = [-1, 1]$$

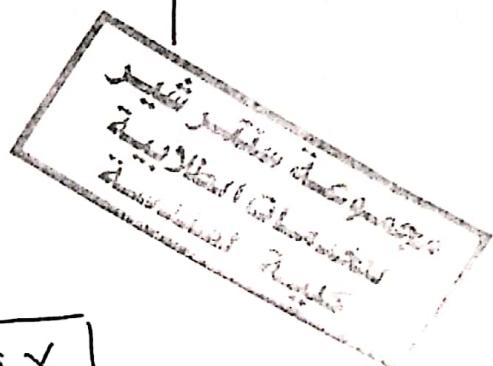
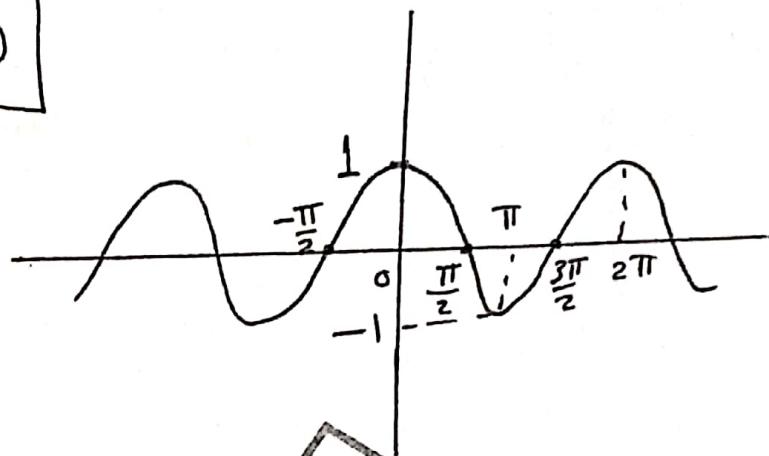
الدالة دورية

الدالة زوجية

y متحركة حول محور x $\Rightarrow f(-x) = f(x)$

$$\therefore f(-x) = f(x)$$

$$\therefore \cos(-x) = \cos x$$



[3] $y = \tan(x)$

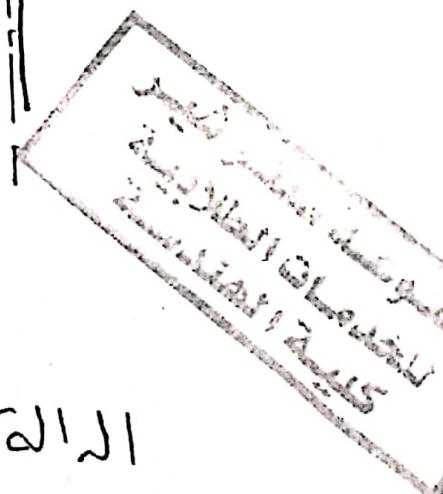
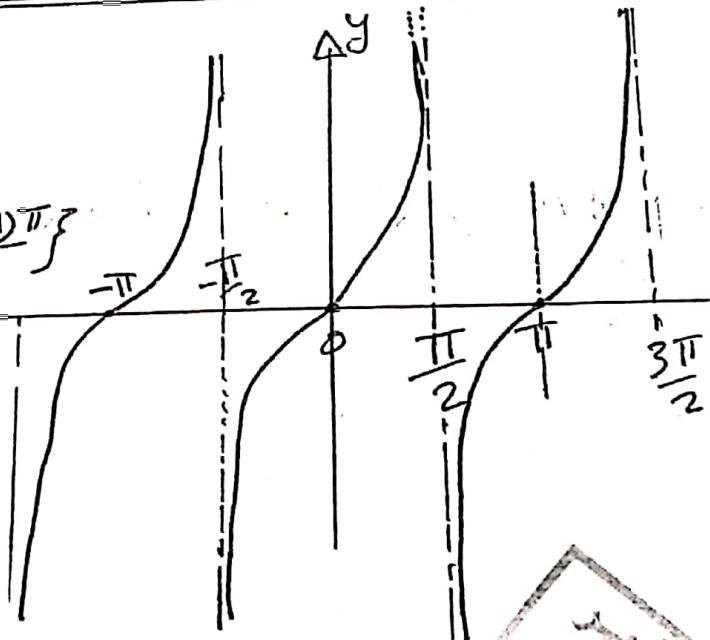
$$J_{\text{ب}} = R - \left\{ \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{(2n-1)\pi}{2} \right\}$$

$$J_{\text{Rf}} = R$$

الدالة فردية
متحركة حول نقطة $x = 0$

$$\therefore [\tan(-x) = -\tan(x)]$$

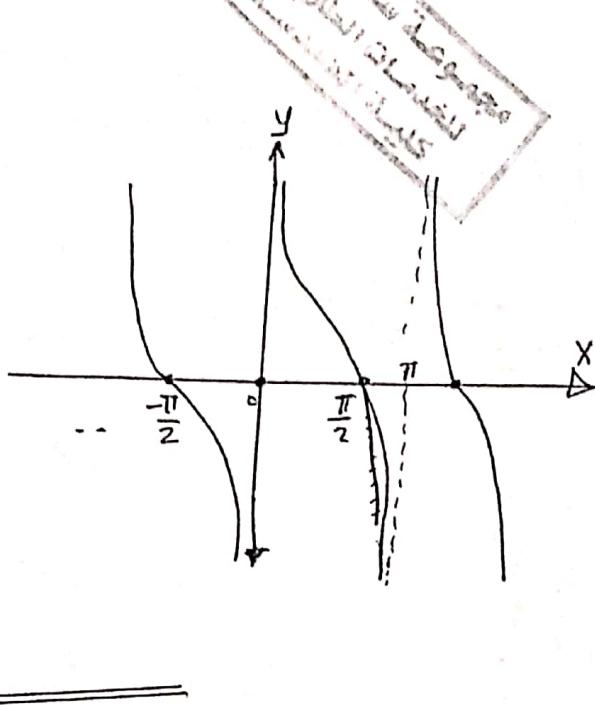
الدالة دورية \Rightarrow سكرر



$$\boxed{4} \quad y = \cot(x) = \frac{1}{\tan(x)}$$

$$Df = R - \left\{ \pm n\pi \right\}$$

$$R_f = R$$

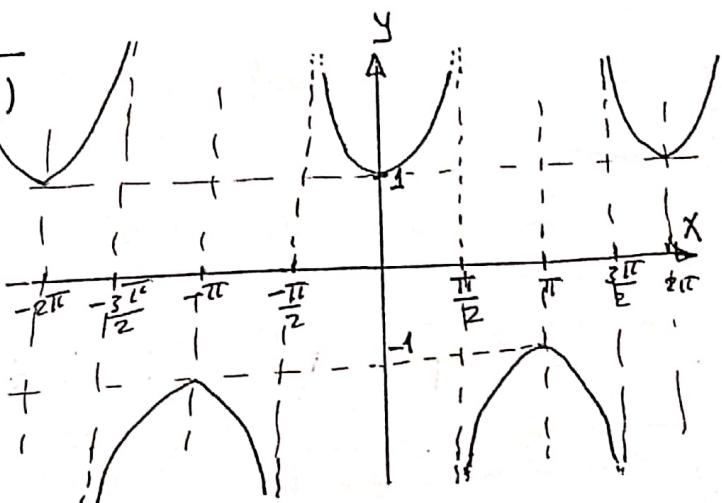


$$\boxed{5} \quad y = \operatorname{Sec}(x) = \frac{1}{\cos(x)}$$

$$Df = R - \left\{ \pm \frac{(2n+1)\pi}{2} \right\}$$

Cosecant
 90° series

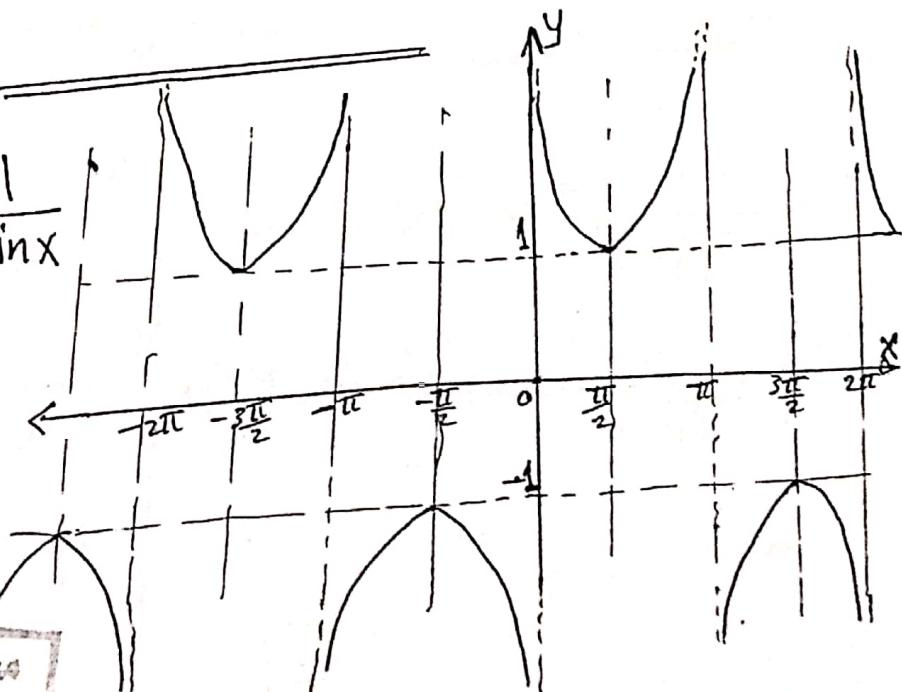
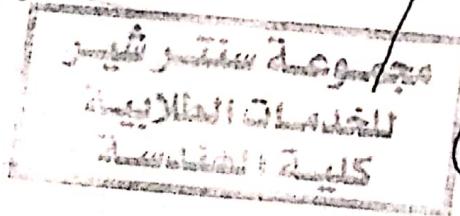
$$R_f = R - [-1, 1]$$



$$\boxed{6} \quad y = \operatorname{Cosec}(x) = \frac{1}{\sin(x)}$$

$$Df = R - \left\{ \pm n\pi \right\}$$

$$R_f = R - [-1, 1]$$



(١٠٢)

-:- الحالات المثلثية *

$$\sin(x) \xrightarrow{\text{نهاية}} \cos(x)$$

$$\sin(u) \rightarrow \cos(u) \cdot u' \quad u' = \frac{du}{dx}, u = \underline{\underline{u(x)}}$$

$$\cos(u) \rightarrow -\sin(u) \cdot u'$$

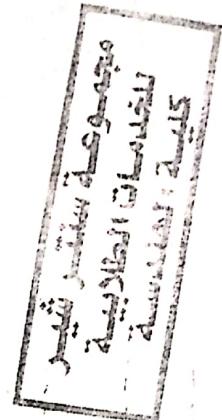
$$\tan(u) \rightarrow \sec^2(u) \cdot u'$$

$$\cot(u) \rightarrow -\operatorname{cosec}^2(u) \cdot u'$$

$$\sec(u) \rightarrow \sec(u) \cdot \tan(u) \cdot u'$$

$$\operatorname{cosec}(u) \rightarrow -\operatorname{cosec}(u) \cdot \cot(u) \cdot u'$$

-:- الحالات المثلثية *



II $y = \sin^{-1}(x) \xrightarrow{\text{نهاية}} \frac{1}{\sqrt{1-x^2}}$

مقدمة في الكثافة والشhir
للمعلمات والتحليلية
جامعة الملك عبد الله

$$\sin^{-1}(u) \rightarrow \frac{1}{\sqrt{1-u^2}} \cdot u'$$

III $y = \cos^{-1}(x) \xrightarrow{\text{نهاية}} \frac{-1 \cdot u'}{\sqrt{1-u^2}}$

IV $y = \sec^{-1}(x) \xrightarrow{\text{نهاية}} \frac{1}{|u|\sqrt{u^2-1}} \cdot u'$

V $y = \tan^{-1}(u) \rightarrow \frac{1}{1+u^2} \cdot u'$

VI $y = \operatorname{cosec}^{-1}(u) \rightarrow \frac{-1}{|u|\sqrt{u^2-1}} \cdot u'$

VII $y = \cot^{-1}(u) \rightarrow \frac{-1}{1+u^2} \cdot u'$

⑬

#

$$y = \sin^{-1}(x) \Leftrightarrow x = \sin y$$

$$\text{Find } \frac{dy}{dx}$$

$$\text{Solution} \Rightarrow \frac{dx}{dy} = \cos y$$

$$\therefore \frac{dy}{dx} = \frac{1}{(\frac{dx}{dy})} = \frac{1}{\cos y}$$

$$\therefore \cos^2 y + \sin^2 y = 1$$

$$\therefore \cos^2 y = 1 - \sin^2 y$$

$$\therefore \cos y = \sqrt{1 - \sin^2 y}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 - \sin^2 y}}$$

$$\therefore x = \sin y \Rightarrow \sin^2 y = x^2$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

✓ //

الدالة
العلاقة
النهايات
الستقيات

مكتبة شيرش
لخدمات الطلاب
جامعة الملك عبد الله

Yalla! Welcome to Giga
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$$y = \cos^{-1}(x) \Leftrightarrow x = \cos y$$

Find $\frac{dy}{dx}$

نوجي $\frac{dx}{dy} = -\sin y$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{-\sin y} = \frac{-1}{\sqrt{1 - \cos^2 y}}$$

$$\therefore \cos y = x$$

$$\therefore \cos^2 y = x^2$$

$$\therefore \frac{dy}{dx} = \frac{\boxed{-1}}{\sqrt{1 - x^2}}$$

الفرق بين

$\sin^{-1}(x)$ وبين

الفرق بين

$$y = \tan^{-1} x \Leftrightarrow x = \tan y$$

$$\frac{dx}{dy} = \sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \boxed{\frac{1}{1+x^2}} \checkmark$$

$$\text{Ex} \quad \cot^{-1}(x) = y \iff x = \cot y$$

$$\frac{dx}{dy} = -\operatorname{cosec}^2 y = -(1 + \cot^2 y)$$

$$= -(1 + x^2)$$

$$\therefore \frac{dy}{dx} = \frac{-1}{1+x^2} = \frac{1}{\frac{dx}{dy}} = -\operatorname{cosec} y$$

$$\boxed{1 + \tan^2 x = \sec^2 x}$$

$$\boxed{1 + \cot^2 x = \operatorname{cosec}^2 x}$$

$$y = \sec^{-1}(x) \iff x = \sec y$$

$$\frac{dx}{dy} = \sec y \cdot \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \neq$$

$$\therefore \sec y = x$$

$$\therefore 1 + \tan^2 y = \sec^2 y$$

$$\therefore \tan^2 y = \sec^2 y - 1$$

$$\therefore \tan y = \sqrt{\sec^2 y - 1} = \sqrt{x^2 - 1}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{\pm 1}{x \sqrt{x^2 - 1}}} \quad \#$$

$$\frac{dy}{dx} = \frac{1}{|x| \sqrt{x^2 - 1}}$$

$$y = \operatorname{cosec}^{-1} x \leftarrow \boxed{x = \operatorname{cosec} y}$$

$$\frac{dx}{dy} = -\operatorname{cosec} y \cdot \cot y$$

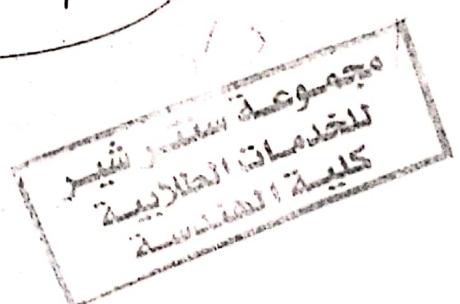
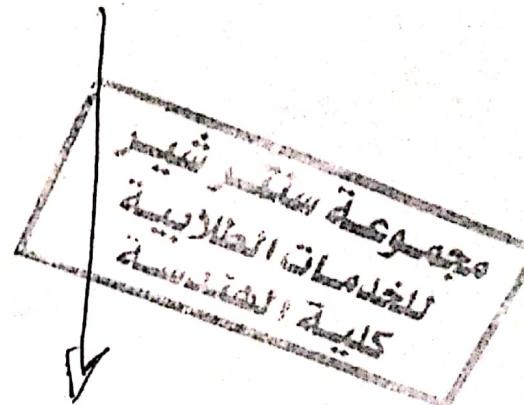
$$\therefore 1 + \cot^2 y = \operatorname{cosec}^2 y$$

$$\therefore \cot^2 y = \operatorname{cosec}^2 y - 1$$

$$\therefore \cot y = \sqrt{\operatorname{cosec}^2 y - 1} = \sqrt{x^2 - 1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{\mp 1}{x \sqrt{x^2 - 1}}$$

$$= \frac{-1}{1 \operatorname{ul} \sqrt{u^2 - 1}} u' \cancel{\neq 0}$$



Hyperbolic f^{ns}

الموال لزائريه

الخدمات المعاشرة للغير
لخدمات الطلاقية
كلية لخدمات الطلاقية

لـ Hyperbola هي من تمثيلها بـ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\text{II) } y = \sinh(x) = \frac{e^x - e^{-x}}{2}$$

لـ تعریفیت لرستان

$$\boxed{2} \quad y = \cosh(x) = \frac{e^x + e^{-x}}{2}$$

John

$$\boxed{3} \quad y = \tanh(x) = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$4 \quad y = \operatorname{Gth}(x) = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$[5] y = \operatorname{Sech}(x) = \frac{1}{\operatorname{Cosh}(x)} = \frac{2}{e^x + e^{-x}}$$

$$6 \quad y = \text{Gsech}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}$$

$$\text{④ } \cos^2(x) + \sin^2(x) = 1$$

$$\boxed{2} \quad 1 + \tan^2(x) = \sec^2(x)$$

$$\boxed{3} \quad 1 + \cot^2(x) = \csc^2(x)$$

$$\boxed{4} \cos(0) = 1, \quad \sin(0) = 0$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

19. 2 12

$$\coth^2(x) - 1 = \operatorname{csch}^2(x)$$

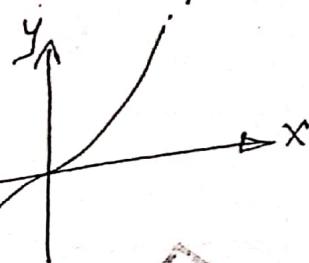
$$\cosh(0) = 1 \quad , \quad \sinh(0) = 0$$

* HYPERBOLIC FUNCTIONS

الحال المزدوجة

نسبة المقطوع لزاوية

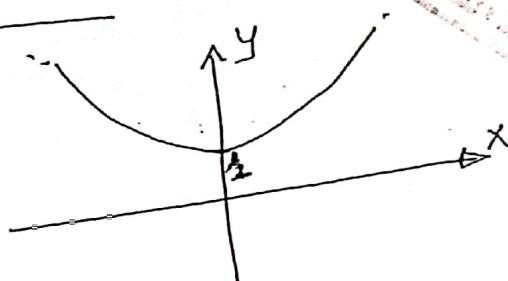
$$\boxed{1} \quad y = \sinh(x) = \frac{e^x - e^{-x}}{2}$$



$$D_f = \mathbb{R}$$

$$R_f = \mathbb{R}$$

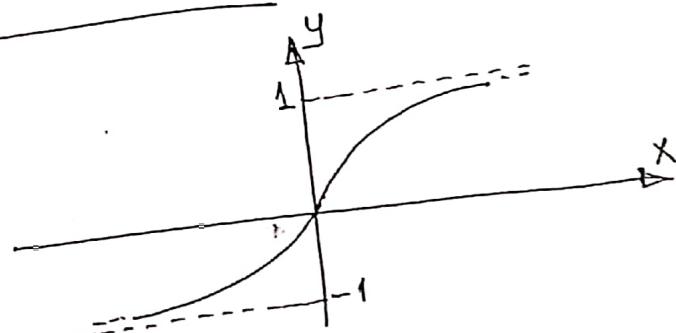
$$\boxed{2} \quad y = \cosh(x) = \frac{e^x + e^{-x}}{2}$$



$$D_f = \mathbb{R}$$

$$R_f = [1, \infty]$$

$$\boxed{3} \quad y = \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$



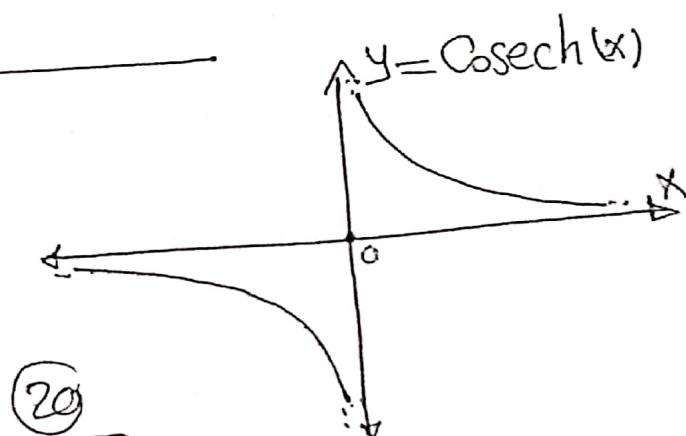
$$D_f = \mathbb{R}$$

$$R_f = [-1, 1]$$

$$\boxed{4} \quad y = \operatorname{Cosech}(x) = \frac{1}{\sinh(x)}$$

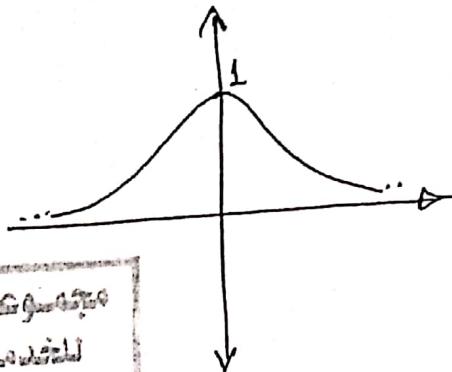
$$D_f = \mathbb{R} - \{0\}$$

$$R_f = \mathbb{R} - \{0\}$$



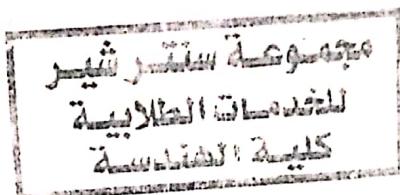
(20)

65) $y = \operatorname{Sech}(x) = \frac{1}{\cosh(x)}$



$$D_f = R$$

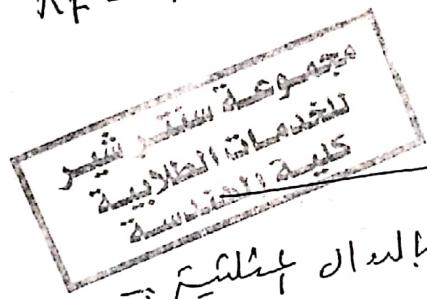
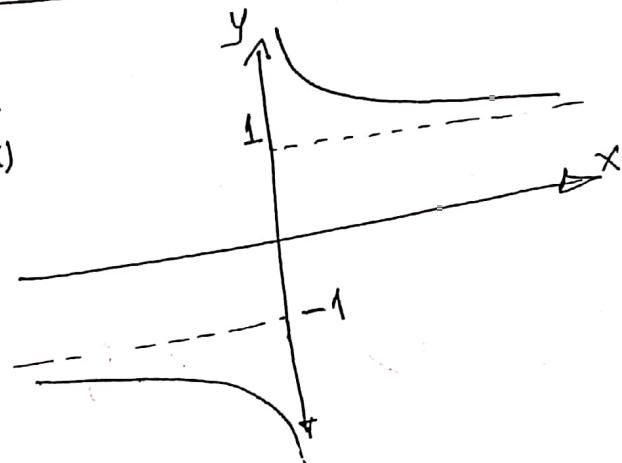
$$R_f = [0, 1]$$



6) $y = \operatorname{Coth}(x) = \frac{1}{\tanh(x)}$

$$D_f = R - \{0\}$$

$$R_f = R - [-1, 1]$$



لعمري المراكز التراثية ومقارنتها بالمعارف العالمية *

الكتاب

مكتبة

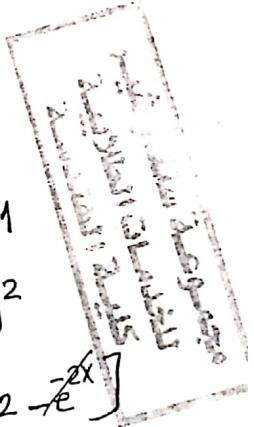
$$\text{III} \quad \cosh^2(x) - \sinh^2(x) = 1$$

↓ ↓

$$\Rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{1}{4} [e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}]$$

$$= 1$$



II) $\sin^2(x) + \cos^2(x) = 1$

2) $1 + \tan^2(x) = \sec^2(x)$

3) $1 + \cot^2(x) = \operatorname{cosec}^2(x)$

2) $1 - \tanh^2(x) = \operatorname{sech}^2(x)$

3) $\operatorname{coth}^2(x) - 1 = \operatorname{cosech}^2(x)$

(21)

لما زالت المطالعات العكسيّة بدلالة المطالع
اللوغاريتميّ

٦٤

□ Prove that $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$

Proof: let $y = \sinh^{-1}(x)$

$$\therefore x = \sinh(y) = \frac{e^y - e^{-y}}{2}$$

$$\therefore e^y - e^{-y} = 2x \quad : e^y \neq 0 \text{ لغيره}$$

$$\therefore e^{2y} - 1 = 2x e^y \quad : e^{2y} - 2x e^y - 1 = 0$$

e^y دالة تبعية لا تتغير

$$\text{Put } e^y = z \Rightarrow z^2 - 2x \cdot z - 1 = 0$$

$$\therefore z = \frac{2x \pm \sqrt{4x^2 - 4(1)(-1)}}{2(1)}$$

حلقة بالقانون

$$A = 1, B = -2x, C = -1$$

$$AZ^2 + BZ + C = 0$$

$$\therefore z = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$z = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = \frac{2x \pm 2\sqrt{x^2 + 1}}{2} = x \pm \sqrt{x^2 + 1}$$

$$e^y = x \pm \sqrt{x^2 + 1} \quad \text{نأخذLn لغيره}$$

$$\therefore \ln e^y = \ln(x \pm \sqrt{x^2 + 1})$$

$$\boxed{\therefore y = \ln(x + \sqrt{x^2 + 1}) = \sinh^{-1}(x)}$$

الآن مرفوض لأن مابدا في \ln يكون عدد موجب $\# (\ln \text{ موجب})$

$$\text{② } \cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$$

الله أعلم رأيت أن

③ Prove that: $\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$

$$\text{let } y = \tanh^{-1}(x) \implies x = \tanh(y) = \frac{e^y - e^{-y}}{e^y + e^{-y}}$$

$$\therefore x e^y + x e^{-y} = e^y - e^{-y} \quad (\text{طرفین را جمع کنید})$$

$$\therefore x e^{2y} + x = \frac{2y}{e} - 1 \quad \therefore x e^{2y} - \frac{2y}{e} + x + 1 = 0$$

$$e^{2y} (x-1) = -(x+1) \Rightarrow e^{2y} = \frac{-(x+1)}{x-1} = \frac{1+x}{1-x}$$

$$\therefore \sqrt{\ln \frac{2y}{e}} = \ln \left(\frac{1+x}{1-x} \right) \quad \therefore 2y = \ln \left(\frac{1+x}{1-x} \right)$$

$$\therefore y = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) = \tanh^{-1}(x) \quad \#$$

4) Prove that $\operatorname{Coth}^{-1}(x) = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right)$

Proof

$$\text{let } y = \operatorname{Cth}^{-1}(x)$$

$$\therefore x = \operatorname{cth}(y) = \frac{\operatorname{csh}(y)}{\operatorname{sinh}(y)}$$

$$\therefore x = \frac{e^y + \bar{e}^{-y}}{e^y - \bar{e}^{-y}}$$

$$\therefore x^y e^{-x \bar{e}^y} = \bar{e}^y e^{-\bar{e}^y}. (e^{y * -j\theta})$$

$$\therefore x e^{2y} - \cancel{x} = e^{2y} + 1$$

$$\therefore x^2 e^{-\frac{xy}{x}} = x + 1$$

$$\therefore e^y(x-1) = x+1$$

$$\therefore \frac{2y}{e} = \frac{x+1}{x-1} \quad (\text{طريق Ln tot})$$

$$\therefore 2y = \ln\left(\frac{x+1}{x-1}\right)$$

$$\therefore y = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right) = G^{-1}(x)$$

□ Prove that $\operatorname{Sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ (Proof)

$$\text{let } y = \operatorname{Sech}^{-1}(x)$$

$$\therefore x = \operatorname{Sech}(y) = \frac{1}{\cosh(y)}$$

$$\therefore x = \frac{2}{e^y + e^{-y}}$$

طريقتين

$$\therefore x e^y + x e^{-y} = 2 \quad (e^y \times \text{طرفين})$$

$$\therefore x e^{2y} + x = 2 e^y$$

$$\therefore x e^{2y} - 2 e^y + x = 0$$

حلحلة

A بـ B C

$$\therefore e^y = \frac{-1 \pm \sqrt{4 - 4(x)(x)}}{2x}$$

$$e^y = \frac{x \pm \sqrt{1-x^2}}{2x}$$

$$e^y = \frac{1 \pm \sqrt{1-x^2}}{x}$$

طريقتين

$$\therefore y = \ln\left(\frac{1 \pm \sqrt{1-x^2}}{x}\right)$$

$$\therefore \operatorname{Sech}^{-1}(x) = \ln\left(\frac{1 + \sqrt{1-x^2}}{x}\right)$$

لـ $\operatorname{Sech}^{-1}(x) = \ln\left(\frac{1 + \sqrt{1-x^2}}{x}\right)$
 $\operatorname{cosech}^{-1}(x) = \ln\left(\frac{1 - \sqrt{1-x^2}}{x}\right)$
 $\operatorname{cosech}'(x) = \frac{1}{x \sinh^2(x)}$

□ Prove that $\operatorname{Cosech}^{-1}(x) =$

$$\text{let } y = \operatorname{Cosech}^{-1}(x)$$

$$\therefore x = \operatorname{Cosech}(y) = \frac{1}{\sinh(y)}$$

$$\therefore x = \frac{2}{e^y - e^{-y}}$$

$$\therefore x e^y - x e^{-y} = 2 \quad (* e^y)$$

$$\therefore x e^{2y} - x = 2 e^y$$

$$\therefore x e^{2y} - 2 e^y - x = 0$$

A B C

$$\therefore e^y = \frac{2 \pm \sqrt{4 + 4(x)(-x)}}{2x}$$

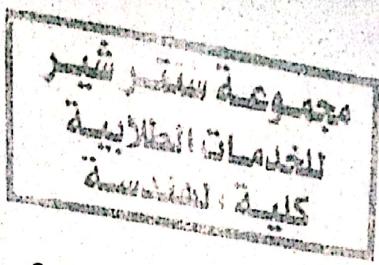
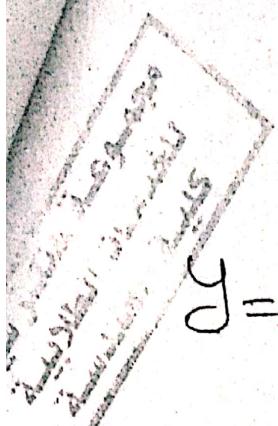
$$\therefore e^y = \frac{2 \pm 2\sqrt{1+x^2}}{2x}$$

طريقتين

$$\therefore y = \ln\left(\frac{1 \pm \sqrt{1+x^2}}{x}\right)$$

$$\therefore \operatorname{Cosech}^{-1}(x) = \ln\left(\frac{1 + \sqrt{1+x^2}}{x}\right)$$

لـ $\operatorname{cosech}^{-1}(x) = \ln\left(\frac{1 - \sqrt{1+x^2}}{x}\right)$

الكل

نهاية درجة

$$y = \sin(x)$$

Prove that $\frac{d}{dx} \sin(x) = \cos x$

証明

$$\therefore \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



$$= \lim_{\Delta x \rightarrow 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$$

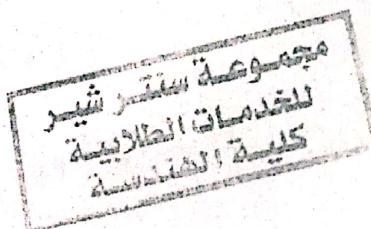
$$= \lim_{\Delta x \rightarrow 0} \frac{2 \cos(x + \frac{\Delta x}{2}) \cdot \sin(\frac{\Delta x}{2})}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \cos(x + \frac{\Delta x}{2}) \left(\lim_{\Delta x \rightarrow 0} \frac{\sin(\frac{\Delta x}{2})}{(\frac{\Delta x}{2})} \right)$$

= 1

$$= \cos(x+0) =$$

$$= \cos(x)$$



لهم

$$\sin(x+\Delta x) - \sin x = 2 \cos(x + \frac{\Delta x}{2}) \cdot \sin(\frac{\Delta x}{2})$$

الإثبات

$$\sin(x + \Delta x) - \sin x =$$

$$= \underbrace{\sin x \cos \Delta x}_{\text{---}} + \underbrace{\sin \Delta x \cos x}_{\text{---}} - \sin x$$

$$= \sin x [\cos \Delta x - 1] + \cos x \sin \Delta x$$

$$= \cos x \sin \Delta x - 2 * \frac{1}{2} (1 - \cos \Delta x) \cdot \sin x$$

$$= \cos x \left(2 \sin \frac{\Delta x}{2} \cdot \cos \frac{\Delta x}{2} \right) - 2 \sin^2 \left(\frac{\Delta x}{2} \right) \cdot \sin x$$

$$= 2 \sin \left(\frac{\Delta x}{2} \right) \left[\cos x \cdot \cos \frac{\Delta x}{2} - \sin x \cdot \sin \left(\frac{\Delta x}{2} \right) \right]$$

$$= 2 \sin \left(\frac{\Delta x}{2} \right) \cdot \cos \left(x + \frac{\Delta x}{2} \right) \quad \#$$

* Prove that $\frac{d}{dx} \cos x = -\sin x$

$$\cos(x) = \sin \left(\frac{\pi}{2} - x \right)$$

$$\therefore \frac{d}{dx} \cos(x) = \frac{d}{dx} \left[\sin \left(\frac{\pi}{2} - x \right) \right] = \cos \left(\frac{\pi}{2} - x \right) (-1)$$

$$= -\sin(x)$$

* $y = \tan(x) = \frac{\sin(x)}{\cos x} = \frac{\sin(x)}{\sqrt{1 - \sin^2 x}}$

$$\therefore \frac{dy}{dx} = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

(26)

$$\therefore \frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = \boxed{-\operatorname{cosec}^2 x}$$

$$y = \sec x = \frac{1}{\cos x}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(\cos x) \cdot 0 - (1) \cdot (-\sin x)}{\cos^2 x} \\ &= \frac{\sin x}{\cos x \cdot \cos x} = \tan x \cdot \frac{1}{\cos x} \\ &= \sec x \cdot \tan x\end{aligned}$$

$$y = \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\frac{dy}{dx} = \frac{(\sin x \cdot 0) - (1) \cos x}{\sin^2 x} = \frac{-\cos x}{\sin x \cdot \sin x}$$

$$\therefore \frac{\cos x}{\sin x} = \cot x \quad , \quad \frac{1}{\sin x} = \operatorname{cosec} x$$

$$\therefore \frac{dy}{dx} = -\operatorname{cosec} x \cdot \cot x$$

(27)

#

٣) تفاضلات لدوال لـ زائدات

١) Prove that

$$\frac{d}{dx} \sinh(x) = \cosh x$$

$$= \frac{\frac{2x}{e^x + e^{-x}} - \frac{-2x}{e^x - e^{-x}}}{[e^x + e^{-x}]^2}$$

$$= \frac{4}{[e^x + e^{-x}]^2} = \left[\frac{2}{e^x + e^{-x}} \right]^2$$

$$= \left(\frac{1}{\cosh x} \right)^2 = \operatorname{sech}^2(x)$$

٤) Prove that $\frac{d}{dx} \coth(x) = -\operatorname{csch}^2(x)$

$$\therefore \coth(x) = \frac{1}{\tanh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\therefore \frac{d}{dx} (\coth(x)) = \frac{[e^x - e^{-x}][e^x - e^{-x}] - [e^x + e^{-x}][e^x + e^{-x}]}{[e^x - e^{-x}]^2}$$

$$= \frac{2x}{e^x - e^{-x}} - \frac{-2x}{e^x - e^{-x}} - \frac{2x}{e^x - e^{-x}} = \frac{-2x}{[e^x - e^{-x}]^2}$$

$$= -\frac{4}{[e^x - e^{-x}]^2} = -\left[\frac{2}{e^x - e^{-x}} \right]^2$$

$$= -\left[\frac{1}{\sinh x} \right]^2 = -\operatorname{cosech}^2(x)$$

٢) Prove that

$$\frac{d}{dx} \cosh x = \sinh x$$

ج ٣

$$\therefore \cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{نحوه}$$

$$\therefore \frac{d}{dx} \cosh(x) = \frac{1}{2} (e^x - e^{-x}) = \sinh(x)$$

٣) Prove that $\frac{d}{dx} \tanh(x) = \operatorname{sech}^2(x)$

ج ١

$$\therefore \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{نحوه}$$

$$\therefore \frac{d}{dx} \tanh(x) = \frac{[e^x + e^{-x}][e^x - e^{-x}] - [e^x - e^{-x}][e^x + e^{-x}]}{[e^x + e^{-x}]^2}$$

$$= \frac{[e^x + e^{-x}]^2 - [e^x - e^{-x}]^2}{[e^x + e^{-x}]^2}$$

ج

$$\frac{d}{dx} (\operatorname{sech} x) = -\operatorname{sech} x \cdot \tanh x$$

$$\therefore \frac{d}{dx} (\operatorname{cosech} x) = -\operatorname{cosech} x \cdot \coth x.$$

ج

٢٨

مسارات الموال

الإيجات مسارات الموال

$$\boxed{1} \quad y = \tanh^{-1} x \leftrightarrow x = \underline{\tanh y}$$

$$\frac{dx}{dy} = \operatorname{Sech}^2 y$$

$$\operatorname{Sech}^2 y = \frac{1}{1 - \tanh^2 y}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\operatorname{Sech}^2 y} = \frac{1}{1 - \tanh^2 y}$$

$$\therefore \frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2}$$

$$\therefore \frac{d}{dx} \tanh^{-1}(u) = \frac{1}{1 - u^2} \cdot u'$$

$$\boxed{2} \quad y = \sinh^{-1} x \leftrightarrow x = \sinh y$$

$$\therefore \frac{dx}{dy} = \cosh y$$

$$\therefore \cosh^2 y - \sinh^2 y = 1$$

$$\frac{dy}{dx} = \frac{1}{\cosh y}$$

$$\cosh y = \sqrt{1 + \sinh^2 y}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 + x^2}}$$

$$= \sqrt{1 + x^2}$$

$$y = \cosh^{-1} x \iff x = \cosh y$$

$$\frac{dx}{dy} = \sinh y$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sinh y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 1}}$$

$$\begin{aligned}\cosh^2 x - \sinh^2 x &= 1 \\ \sinh x &= \sqrt{\cosh^2 x - 1} \\ &= \sqrt{x^2 - 1}\end{aligned}$$

ممکن است این اگرچه
 این نتیجه باشد
 اما این نتیجه

$$y = \coth^{-1} x \iff x = \coth y$$

$$\frac{dx}{dy} = -\operatorname{csch}^2 y$$

$$\frac{dy}{dx} = \frac{-1}{\operatorname{csch}^2 y}$$

$$= \frac{-1}{\coth^2 x - 1} = \frac{-1}{x^2 - 1}$$

اگرچه این نتیجه
 این نتیجه باشد

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

$$[5] \quad y = \operatorname{Sech}^{-1} x \Leftrightarrow x = \operatorname{sech} y$$

$$\frac{dx}{dy} = -\operatorname{sech} y \cdot \tanh y$$

$$\frac{dy}{dx} = \frac{-1}{\operatorname{sech} y \cdot \tanh y}$$

$$\frac{dy}{dx} = \frac{-1}{x \sqrt{1-x^2}}$$

$$\begin{aligned} &= 1 - \tanh^2 y = \operatorname{sech}^2 y \\ &\therefore \tanh y = \sqrt{1 - \operatorname{sech}^2 y} \\ &= \sqrt{1 - x^2} \end{aligned}$$

$$[6] \quad y = \operatorname{cosech}^{-1} x \Leftrightarrow x = \operatorname{Cosech} y$$

$$\frac{dx}{dy} = -\operatorname{cosech} y \cdot \coth y$$

$$\frac{dy}{dx} = \frac{1}{(\frac{dx}{dy})} = \frac{-1}{\operatorname{cosech} y \cdot \coth y}$$

$$\frac{dy}{dx} = \frac{-1}{x \sqrt{x^2 + 1}}$$

$$\begin{aligned} &= \coth^2 y - 1 = \operatorname{cosech}^2 y \\ &\therefore \coth y = \sqrt{\operatorname{cosech}^2 y + 1} \\ &= \sqrt{x^2 + 1} \end{aligned}$$

(31)

$$\text{O.C. } e^{2y} = \frac{-(x+1)}{(x-1)} = \frac{1+x}{1-x}$$

$$\therefore 2y = \ln \frac{1+x}{1-x}$$

$$\boxed{\therefore y = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) = \tanh^{-1} x}$$

$$4 \quad y = \coth^{-1} x \iff x = \coth y = \frac{e^y + e^{-y}}{e^y - e^{-y}}$$

$$x e^y - x e^{-y} = e^y + e^{-y}$$

$$\therefore e^y (x-1) - e^{-y} (x+1) = 0$$

$$e^{2y} (x-1) - (x+1) = 0$$

$$\therefore e^{2y} = \frac{x+1}{x-1}$$

ln ist
nur bei

$$\therefore 2y = \ln \left(\frac{x+1}{x-1} \right)$$

$$\boxed{\therefore y = \coth^{-1}(x) = \frac{1}{2} \cdot \ln \left(\frac{x+1}{x-1} \right)}$$

(ln) الـ 1 الـ 2 الـ 3 cosech⁻¹ x → sech⁻¹ x ←

↓ جمعي

↑ جمعي

مكمل لـ التفاضل (الإسقاط)

* Find $\frac{dy}{dx}$ for

$$(1) \quad y = (x^3 - 5) \cdot \tan^3(x^2 + \cos^3(2x))$$

حاصد صرب دالنبر

Solution:

$$\frac{dy}{dx} = (x^3 - 5) \cdot (3) \cdot \tan^2(x^2 + \cos^3(2x)) \cdot \sec^2(x^2 + \cos^3(2x))$$

$$+ \tan^3(x^2 + \cos^3(2x)) \cdot (3x^2)$$

$$(2) \quad x^2 - \sin(xy) - \tan^2(y^2) = 0$$

دالة مختبطة

(w.r.t. x)

$$\therefore 2x - \cos(xy) \cdot (x \frac{dy}{dx} + y) - 2\tan^2 y \cdot \sec^2 y \cdot 2y$$

$$\frac{dy}{dx} = 0$$

$$\therefore 2x - y \cos(xy) = x \cos(xy) \frac{dy}{dx} + 4y \tan^2 y \sec^2 y \cdot \frac{dy}{dx}$$

$$2x - y \cos(xy) = \frac{dy}{dx} [x \cos(xy) + 4y \tan^2 y \sec^2 y]$$

$$\therefore \frac{dy}{dx} = \frac{2x - y \cos(xy)}{x \cos(xy) + 4y \tan^2 y \sec^2 y}$$

#

(33)

$$③ y = \underbrace{e^{-x}}_{\text{[}} \cdot \cos 2x \quad \text{Find } y'' + 2y' + 5y$$

Solution : $y' = -e^{-x} \cdot 2 \sin 2x - e^{-x} \cos 2x$

$$y' = -2e^{-x} \sin 2x - e^{-x} \cos 2x$$

$$y'' = -2e^{-x} \cdot 2 \cos 2x + 2e^{-x} \sin 2x + 2e^{-x} \cos 2x \\ + e^{-x} \cos 2x$$

$$y'' = -4e^{-x} \cos 2x + 2e^{-x} \sin 2x + 3e^{-x} \cos 2x$$

$$\therefore y'' = 2e^{-x} \sin 2x - e^{-x} \cos 2x$$

$$\therefore y'' + 2y' + 5y = 2e^{-x} \sin 2x - e^{-x} \cos 2x \\ - 4e^{-x} \sin 2x - 2e^{-x} \cos 2x \\ + 5e^{-x} \cos 2x$$

$$= -2e^{-x} \sin 2x + 2e^{-x} \cos 2x$$

#

$$* y = \frac{x^P}{x^m - a^m} \quad \begin{array}{l} \text{جامعة الملك عبد الله} \\ \text{جامعة الطائف} \\ \text{جامعة حفر الباطن} \end{array} \quad a, P, m = \text{consts.}$$

حامل قسمة دالين

$$\therefore \frac{dy}{dx} = \frac{(x^m - a^m) \cdot P x^{P-1} - x^P (m x^{m-1})}{(x^m - a^m)^2}$$

$$* y = \tan(ax+b)$$

$$\frac{dy}{dx} = \sec^2(ax+b) \cdot a$$

$$* y = \underbrace{\sin t}_3 \cdot \underbrace{\cos t}_1$$

$$\begin{aligned} \frac{dy}{dt} &= \sin^3 t \cdot (-\sin t) + 3 \sin^2 t \cdot (\cos t) \cdot (\cos t) \\ &= -\sin^4 t + 3 \sin^2 t \cdot \cos^2 t \end{aligned}$$

$$* y = \sqrt[3]{x^2+x+1} = (x^2+x+1)^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3} (x^2+x+1)^{-\frac{2}{3}} (2x+1) = \frac{2x+1}{3 \sqrt[3]{(x^2+x+1)^2}}$$

١٦

٢٢

للمراجعة والذكر
ذكرياتي

* $y = \cos(x+y)$ دالة خصائص
بنهايات المعرفة بالنسبة لـ (x)

$$\therefore \frac{dy}{dx} = -\sin(x+y) \cdot (1 + \frac{dy}{dx}) = -\sin(x+y) - \frac{dy}{dx}(\sin(x+y))$$

$$\therefore \frac{dy}{dx} + \frac{dy}{dx} \sin(x+y) = -\sin(x+y)$$

$$\therefore \frac{dy}{dx} (1 + \sin(x+y)) = -\sin(x+y)$$

$$\therefore \frac{dy}{dx} = \frac{-\sin(x+y)}{1 + \sin(x+y)}$$

* $\cos(xy) = x$ بنهايات المعرفة بالنسبة لـ (x)

$$\therefore -\sin(xy) \cdot [x \frac{dy}{dx} + y] = 1$$

$$\therefore -x \sin(xy) \cdot \frac{dy}{dx} - y \sin(xy) = 1$$

$$\therefore \frac{dy}{dx} = \frac{1 + y \sin(xy)}{-x \sin(xy)}$$

* $x^{1/2} + y^{1/2} = a^{1/2}$ بنهايات المعرفة بالنسبة لـ (x)

$$\frac{1}{2} x^{-1/2} + \frac{1}{2} y^{-1/2} \frac{dy}{dx} = 0$$

(36)

$$\therefore \frac{dy}{dx} = \frac{-\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}} = \frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}} = \sqrt{\frac{y}{x}}$$

* if $x = a \cos t$, $y = b \sin t$

find $\frac{dy}{dx}$ solution

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

Expt where $\frac{dy}{dt} = b \cos t$

$$\frac{dt}{dx} = \frac{1}{(\frac{dx}{dt})} = -a \sin t$$

$$\therefore \frac{dy}{dx} = b \cos t \cdot \frac{-1}{a \sin t} = \frac{-b}{a} \frac{\cos t}{\sin t} = \frac{-b}{a} \cot t$$

* $x = a \cos^3 t$, $y = b \sin^3 t$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{(\frac{dy}{dt})}{(\frac{dx}{dt})} = \frac{3b \sin^2 t \cdot \cos t}{-3a \cos^2 t \cdot \sin t} \\ &= -\frac{b}{a} \left(\frac{\sin t}{\cos t} \right)^2 \cdot \frac{\cos t}{\sin t} = -\frac{b}{a} \frac{\sin t}{\cos t} \end{aligned}$$

$$= -\frac{b}{a} \tan t$$

↗

(37)

* Calculate the slope of the Tangent for the
^{given} curve : $x \cdot \cos y + 4 \cdot \sin x = \pi$ at Point $(\frac{\pi}{2}, 0)$

جامعة بغداد
كلية التربية
لخدمات الملاحة
قسم الدراسات العليا

Curve : $x \cdot \cos y + y \cdot \sin x = \frac{\pi}{2}$ at Point $(\frac{\pi}{2}, \frac{\pi}{2})$

$$\text{لوجه حيل متساوٍ} \quad x\cos y + y\sin x = \frac{\pi}{2} \quad \text{في} \quad \left(\frac{\pi}{2}, \frac{\pi}{2} \right)$$

<< Solution >>

$$x \cos y + y \sin x = \frac{\pi}{2}$$

$$x \cos y + y \sin x = \frac{\pi}{2}$$

جواب ایسا کریں کہ

$$\therefore x \cos y + y \sin x + \frac{dy}{dx} \sin x = 0$$

$$\therefore -x \cdot \sin y \cdot \frac{dy}{dx} + \cos y + y \cos x + \frac{dy}{dx} \sin x = 0$$

$$\cos y - y \cos x$$

$$\frac{dy}{dx} \left[\sin x - x \sin y \right] = -\cos y - \frac{-\cos y - y \cos x}{x \sin y - \sin x}$$

$$\sin x = \frac{dy}{dx} = \left(\frac{\pi}{2}, \frac{\pi}{2} \right)$$

-ve, i.e. 0 < x <

$$\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{2}\right) + \frac{\pi}{2} \cos\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)}$$

$$\tan \theta = \frac{\cos(90^\circ) + \frac{3.14}{2} \cos(90^\circ)}{\frac{3.14}{2} (\sin 90^\circ) - \sin(90^\circ)} = \frac{\text{Zero}}{\text{Zero}}$$

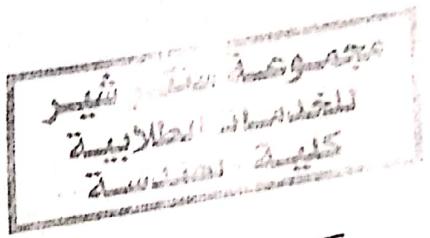
$\Leftrightarrow \theta = 0 = \tan^{-1}(0)$

<< X جو اسی جھوڑی میں نہیں >>

$$y^2 \ln(\cosh^{-1} x^2) + \frac{1}{e^{\frac{1}{2} \ln y}} \cdot \tan[\sqrt{x \cdot \sqrt{x} \cdot \sqrt{x}}] = 0$$

find $\frac{dy}{dx}$: *< solution >*

$$\frac{1}{e^{\frac{1}{2} \ln y}} = e^{\ln y^{\frac{1}{2}}} = y^{\frac{1}{2}} = \sqrt{y}$$



$$\begin{aligned} \sqrt{x \cdot \sqrt{x} \cdot \sqrt{x}} &= \sqrt{x \sqrt{x \cdot x^{\frac{1}{2}}}} = \sqrt{x \cdot \sqrt{x^{3/2}}} \\ &= \sqrt{x \cdot x^{\frac{3}{4}}} = \sqrt{x^{\frac{7}{4}}} = x^{\frac{7}{8}} \end{aligned}$$

$$\Rightarrow y^2 \ln(\cosh^{-1} x^2) + \sqrt{y} \cdot \tan[x^{\frac{7}{8}}] = 0$$

$$\therefore 2y \frac{dy}{dx} \cdot \ln(\cosh^{-1} x^2) + y^2 \cdot \frac{1}{\cosh^{-1} x^2} \cdot \frac{-1}{\sqrt{1+x^4}} \cdot 2x$$

$$+ \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} \cdot \tan[x^{\frac{7}{8}}] + \sec^2[x^{\frac{7}{8}}] \cdot \frac{7}{8} x^{-\frac{1}{8}} \cdot \sqrt{y} = 0$$

$$\therefore \frac{dy}{dx} \left[2y \ln(\cosh^{-1} x^2) + \frac{\tan(x^{\frac{7}{8}})}{2\sqrt{y}} \right] = \left[\frac{2xy}{\cosh^{-1} x^2 \cdot \sqrt{1+x^4}} - \frac{\frac{7}{8} \sqrt{y} \sec^2 x^{\frac{7}{8}}}{x^{1/8}} \right]$$

$$\therefore \frac{dy}{dx} = \left[\frac{2xy}{\cosh^{-1} x^2 \sqrt{1+x^4}} - \frac{\frac{7}{8} \sqrt{y} \sec^2 x^{\frac{7}{8}}}{x^{1/8}} \right] / \left[2y \ln(\cosh^{-1} x^2) + \frac{\tan x^{\frac{7}{8}}}{2\sqrt{y}} \right]$$

(39)

$$y = \sqrt{e^{\sinh^{-1} \sqrt{x}} + 7^{x^2 \cdot \sin^2 x}}$$

Find y'

(Solution)

$$y' = \frac{1}{2 \cdot \sqrt{e^{\sinh^{-1} \sqrt{x}}}} \cdot e^{\sinh^{-1} \sqrt{x}} \cdot \frac{1}{\sqrt{1+x}} \cdot \frac{1}{2\sqrt{x}}$$

$$+ 7^{x^2 \cdot \sin^2 x} \cdot \ln(7) \cdot [x^2 \cdot 2 \sin x \cdot \cos x + 2x \sin^2 x]$$

$$u = 2 \ln[\cot(s)] \Rightarrow v = \tan(s) + \cot(s)$$

find $\frac{du}{dv}$. *(Solution)*

$$\frac{du}{dv} = \frac{du}{ds} \cdot \frac{ds}{dv}$$

$$\text{where } \frac{du}{ds} = 2 \cdot \frac{1}{\cot(s)} \cdot (-\csc^2 s)$$

$$\frac{ds}{dv} = \sec^2(s) \neq \csc^2(s)$$

$$\therefore \frac{ds}{dv} = \frac{1}{\sec^2(s) - \csc^2(s)}$$

$$\therefore \frac{du}{dv} = \left[\frac{-2 \csc^2 s}{\cot(s)} \right] \cdot \left[\frac{1}{\sec^2(s) - \csc^2(s)} \right] \quad \# \quad (40)$$

$$y = \log(10^{\sin x} \cdot \tan(3x^2)) + \sin(\cos(e^{\sqrt{x}}))$$

U V W

Solution:-

$$\text{let } y = \log(U \cdot V) + \sin(W)$$

$$\text{Where } U = 10^{\sin x} \quad V = \tan(3x^2)$$

$$W = e^{\sqrt{x}}$$

U' , V' , W' \rightarrow 2 Z. Lösung

$$U' = 10^{\sin x} \cdot \ln(10) \cdot \cos x$$

$$V' = \sec^2(3x^2) \cdot 6x = 6x \cdot \sec^2(3x^2)$$

$$W' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{U \cdot V} \cdot \log(e) \cdot (U \cdot V' + V \cdot U')$$

$$+ \cos(\cos(e^{\sqrt{x}})) \cdot [-\sin(e^{\sqrt{x}}) \cdot W']$$

$$\therefore \frac{dy}{dx} = \frac{1 * \log(e) \cdot [10^{\sin x} \cdot (6x \cdot \sec^2(3x^2)) + \tan(3x^2)]}{(10^{\sin x} \cdot \tan(3x^2))}$$

$$\cdot (10^{\sin x} \cdot \ln(10) \cdot \cos x)$$

$$+ \cos(\cos(e^{\sqrt{x}})) \cdot [-\sin(e^{\sqrt{x}}) \cdot (e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}})] \neq$$

(41)

$$*(\tan x)^y = (\tan y)^x$$

find $\frac{dy}{dx} = y'$

use Ln $\ln \equiv \underline{\underline{y}}$

$$\therefore y \cdot \ln(\tan x) = x \cdot \ln(\tan y)$$

y بـ x بـ \ln بـ \ln بـ y بـ \ln بـ \ln بـ y

$$\therefore y \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot y' = \underbrace{x \cdot \frac{1 \cdot \sec^2 y}{\tan y} \cdot y'}_{\leftarrow}$$

$$+ \ln(\tan y) \cdot (1)$$

$$y' \ln(\tan x) - \frac{x \sec^2 y}{\tan y} \cdot y' = \ln(\tan y) - \frac{y \sec^2 x}{\tan x}$$

$$\therefore y' \left[\ln(\tan x) - \frac{x \sec^2 y}{\tan y} \right] = \ln(\tan y) - \frac{y \sec^2 x}{\tan x}$$

$$\therefore y' = \frac{\ln(\tan y) - \frac{y \sec^2 x}{\tan x}}{\ln(\tan x) - \frac{x \sec^2 y}{\tan y}}$$

$$y = x^x$$

Solution:-

Find $\frac{dy}{dx}$

- معرفة \ln

$$\therefore \ln y = x \cdot \ln x$$

Put

$$u = x^x$$

$$\therefore \ln y = u \cdot \ln x \Rightarrow \text{متضمنة البينة}(x)$$

$$\therefore \frac{1}{y} \cdot y' = u \cdot \frac{1}{x} + \ln x \cdot u'$$

$$\therefore y' = y \left[\frac{u}{x} + \ln x \cdot u' \right]$$

ما هو معندي ديار u'

$$\therefore u = x^x \quad \text{معرفة } \ln$$

$$\therefore \ln u = x \cdot \ln x \Rightarrow x \text{ متضمنة في } u$$

$$\therefore \frac{1}{u} \cdot u' = x \cdot \frac{1}{x} + \ln x$$

$$\therefore u' = u [1 + \ln x] = x^x [1 + \ln x]$$

$$\therefore \frac{dy}{dx} = y' = x^x \left[\frac{x^x}{x} + \ln x [x^x (1 + \ln x)] \right]$$

* Find $\frac{dy}{dx}$ for :-

$$\text{II} \quad y = 7^{x^2 \cdot \operatorname{cosec}^2 x} + x^2 \cdot \tan^3(3x^2 + \sin 2x)$$

<Solution>

$$\frac{dy}{dx} = 7^{x^2 \cdot \operatorname{cosec}^2 x} \cdot \ln(7) \cdot (x^2 \cdot 2 \operatorname{cosec} x \cdot (-\operatorname{cosec} x \cdot \cot x) + 2x \operatorname{cosec}^2 x) +$$

$$x^2 \cdot [3 \tan^2(3x^2 + \sin 2x) \cdot \sec^2(3x^2 + \sin 2x)]$$

$$\cdot (6x + 2 \cos 2x) + 2x \cdot \tan^3(3x^2 + \sin 2x) \cdot$$

↙ خر لحال سایه للتبیط:

$$y = 7^{(U)} + x^2 \cdot \tan^3(V) = 7^U + x^2 \cdot (\tan(V))^3$$

حاصف داشت

where $U = x^2 \cdot \operatorname{cosec}^2 x$ and $V = 3x^2 + \sin 2x$

$$\begin{aligned} \frac{dy}{dx} &= 7^U \cdot \ln(7) \cdot U' + x^2 \cdot 3[\tan(V)]^2 \cdot [\sec^2(V) \cdot V'] \\ &\quad + 2x \cdot [\tan(V)]^3 \cdot \underbrace{\text{صيغة اذرل}}_{\text{صيغة}} \end{aligned}$$

جاهي لدينا ديار ونفعنا في طبع U' و V'

(44)

$$u = \boxed{x^2} \cdot \boxed{\csc^2 x} = x^2 \cdot (\csc x)^2$$

$$\therefore \frac{du}{dx} = u' = 2x \cdot \csc^2 x + x^2 [2] \cdot (\csc x)^1 \cdot (-\csc x \cdot \cot x)$$

$$\left. \begin{array}{l} \\ \\ \hookrightarrow V = 3x^2 + \sin 2x \end{array} \right]$$

$$\Rightarrow (1) \quad \boxed{\begin{array}{l} \\ \\ \end{array}}$$

(2)

$$\therefore V' = 6x + 2 \cos(2x) \Rightarrow V, u'$$

$$\frac{dy}{dx} \ni u, v$$

$$\therefore \frac{dy}{dx} = f \cdot \left[\ln(f) \cdot [2x \csc x + \right.$$

$$\left. x^2 \csc x \cdot (-\csc x \cdot \cot x)] \right]$$

$$+ x^2 \cdot 3 \left[\tan(3x^2 + \sin 2x) \right]^2 \cdot [\sec(3x^2 + \sin 2x)]$$

$$(6x + 2 \cos(2x)) + 2x \tan^3(3x^2 + \sin 2x) \cdot$$

$$y = (\sin x)^x$$

(Solution)

نريد إيجاد

Find the first derivative w.r.t. x

$$\equiv \frac{dy}{dx}$$

$$\ln y = x \cdot \ln(\sin x) \Rightarrow \text{الآن نريد إيجاد}$$

$$\therefore \frac{1}{y} \cdot y' = x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot (1)$$

$$\therefore y' = y \left[x \cdot \underbrace{\frac{\cos x}{\sin x}}_{\cot x} + \ln(\sin x) \right]$$

$$\therefore y' = (\sin x)^x \left[x \cdot \cot x + \ln(\sin x) \right]$$

$$\star y = \cos x + \ln(x^2 - 1) + (\cos x)^x \quad | \text{ then } \frac{dy}{dx} \text{ is?}$$

Solution

$$\frac{dy}{dx} = -\sin x + \frac{1}{x^2 - 1} \cdot (2x) + u'$$

$$\text{where } u = (\cos x)^x \Rightarrow \text{نريد إيجاد}$$

$$\ln u = x \cdot \ln(\cos x) \Rightarrow x \text{ (إيجاد مشتق)}$$

$$\therefore \frac{1}{u} \cdot u' = x \cdot \frac{1}{\cos x} \cdot (-\sin x) + \ln(\cos x) \cdot (1)$$

$$\therefore u' = [x \cdot \tan x + \ln(\cos x)] \cdot (\cos x)^x$$

$$\therefore \frac{dy}{dx} = -\sin x + \frac{2x}{x^2 - 1} + (\cos x)^x \left[\ln(\cos x) - \cancel{x \tan x} \right]$$

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#

$$y = \ln \left[\sqrt{\frac{1+\sin x}{1-\sin x}} \right] \quad \text{find } \frac{dy}{dx}$$

Solution

$$y = \ln \left[\frac{1+\sin x}{1-\sin x} \right]^{\frac{1}{2}} = \frac{1}{2} \ln \left[\frac{1+\sin x}{1-\sin x} \right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\left[\frac{1+\sin x}{1-\sin x} \right]} \cdot \left[\frac{(1-\sin x) \cdot (\cos x) - (1+\sin x)(-\cos x)}{(1-\sin x)^2} \right]$$

$$= \frac{(1-\sin x)}{2(1+\sin x)} \cdot \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1-\sin x)^2}$$

$$= \frac{2 \cos x}{\sqrt{2}(1+\sin x)(1-\sin x)} = \frac{\cos x}{1 - \sin^2 x} = \sec x$$

$$= \frac{\cos x}{\cos^2 x} = \frac{1}{\cos x} = \boxed{\sec x}$$

$$* \quad y = e^{2 \ln x} + 3^x + (\cos x)^{\sin x}$$

$$y = e^{\ln x^2} + 3^x + u$$

$$y = x^2 + 3^x + u$$

$$\therefore \frac{dy}{dx} = 2x + 3^x \ln(3) + u'$$

or LI u' $\rightarrow 1 \rightarrow$

(47)

where

$$u = (\cos x)^{\sin x}$$

$$\therefore \ln(u) \rightarrow \text{right side}$$

$$\therefore \frac{1}{u} \cdot u' = \frac{\sin x \cdot \ln(\cos x)}{\sin x} \rightarrow \text{left side w.r.t. } y$$

$$\therefore u' = u [-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)]$$

$$u' = (\cos x)^{\sin x}$$

$$[-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)]$$

$$\therefore \frac{dy}{dx} = 2x + 3^x \ln(3) + (\cos x)^{\sin x} [-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)] \quad \#$$

$$\therefore y = x^{-e} \cdot \tan(x^2)$$

$$\therefore \frac{dy}{dx} = x^{-e} \cdot 2 \tan(x^2) \cdot \sec^2(x^2) \cdot 2x$$

$$+ (-e) x^{-e-1} \cdot \tan^2(x^2)$$

$$\frac{dy}{dx} = \frac{4 \tan x^2 \sec^2(x^2)}{x^e} - \frac{\tan^2(x^2) \cdot e}{x^{e+1}} \quad \#$$

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$$y = \underbrace{e^{-x}}_{\text{Solution}} \cdot \underbrace{\cos(2x)}_{\text{Solution}}$$

Find $y'' + 2y' + 5y$

$$y' = e^{-x} (-2 \sin 2x) + \cos(2x) \cdot (-e^{-x})$$

$$y' = -2 \cdot e^{-x} \cdot \sin 2x - e^{-x} \cdot \cos(2x)$$

$$y'' = -2 \cdot e^{-x} \cdot 2 \cos 2x + 2 \cdot e^{-x} \cdot \sin 2x + 2 \cdot e^{-x} \sin 2x$$

$$+ e^{-x} \cos(2x)$$

$$\therefore y'' + 2y' + 5y = -4 \cdot e^{-x} \cdot \cos 2x + 2 \cdot e^{-x} \cdot \sin 2x$$

$$+ 2 \cdot e^{-x} \sin 2x + e^{-x} \cos 2x - 4 \cdot e^{-x} \sin 2x$$

$$- 2 \cdot e^{-x} \cos 2x + 5 \cdot e^{-x} \cos 2x$$

$$= \underline{\underline{\text{Zero}}}$$

* Report: Find $\frac{dy}{dx}$ for

$$y = \tan^2 \left[\sec \left(\operatorname{sech}^{-1} \sqrt{1+x^2} \right) \right]$$

$$\begin{matrix} \cancel{x}(\sin y) \\ u \end{matrix} + \begin{matrix} \cos(x) \\ v \end{matrix} = 0$$

مقدمة في الميكانيكا
الديناميكا

$$U = x \sin(y) \Rightarrow U' = x \left[\frac{\sin(y)}{x} [\sin(y) + \ln(x) \cdot \cos(y)] \right]$$

$$V = y \cos(x) \Rightarrow V' = y \left[\frac{\cos(x)}{y} [\cos(x) y' + \ln(y) \cdot (-\sin(x))] \right]$$

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$$U' + V' = 0$$

$$2 x \left[\frac{\sin(y)}{x} \left[\frac{\sin y}{x} + \ln(x) \cos(y) y' \right] \right] + y \cos(x) \left[\frac{\cos(x)}{y} \left[\ln(y) \cdot \sin(x) \right] \right] = 0$$

cosine of y is zero

$$y' = -x \frac{\sin y}{x} + y \frac{\cos(x)}{x} \frac{\ln(y) \cdot \sin(x)}{\ln(x) \cos(y) + y \frac{\cos x}{y}}$$

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جدول التفاضلات {

الدالة	المشتقة	الدالة	المشتقة
$\sin(u)$	$\cos(u) \cdot u'$	$\sin^{-1}(u)$	$\frac{u'}{\sqrt{1-u^2}}$
$\cos(u)$	$-\sin(u) \cdot u'$	$\cos^{-1}(u)$	$\frac{-u'}{\sqrt{1-u^2}}$
$\sec(u)$	$\sec(u) \cdot \tan(u) \cdot u'$	$\sec^{-1}(u)$	$\frac{u'}{u\sqrt{u^2-1}}$
$\operatorname{cosec}(u)$	$-\operatorname{cosec}(u) \cdot \cot(u) \cdot u'$	$\operatorname{cosec}^{-1}(u)$	$\frac{-u'}{u\sqrt{u^2-1}}$
$\tan(u)$	$\sec^2(u) \cdot u'$	$\tan^{-1}(u)$	$\frac{u'}{1+u^2}$
$\cot(u)$	$-\operatorname{cosec}^2(u) \cdot u'$	$\cot^{-1}(u)$	$\frac{-u'}{1+u^2}$
$\sinh(u)$	$\cosh(u) \cdot u'$	$\sinh^{-1}(u)$	$\frac{u'}{\sqrt{u^2+1}}$
$\cosh(u)$	$\sinh(u) \cdot u'$	$\cosh^{-1}(u)$	$\frac{u'}{\sqrt{u^2-1}}$
$\operatorname{sech}(u)$	$-\operatorname{sech}(u) \cdot \tanh(u) \cdot u'$	$\operatorname{Sech}^{-1}(u)$	$\frac{-u'}{u\sqrt{1-u^2}}$
$\operatorname{cosech}(u)$	$-\operatorname{cosech}(u) \cdot \coth(u) \cdot u'$	$\operatorname{cosech}^{-1}(u)$	$\frac{-u'}{u\sqrt{1+u^2}}$
$\tanh(u)$	$\operatorname{sech}^2(u) \cdot u'$	$\tanh^{-1}(u)$	$\frac{u'}{1-u^2}$
$\coth(u)$	$-\operatorname{cosech}^2(u) \cdot u'$	$\coth^{-1}(u)$	$\frac{-u'}{1-u^2}$
a^u	$a^u \cdot \ln(a) \cdot u'$	e^u	$e^u \cdot u'$
$\log_p(u)$	$\frac{1}{u \cdot \ln p} \cdot u' = \frac{1}{u} \cdot \frac{\ln q}{\ln p} \cdot u'$	$\ln(u)$	$\frac{1}{u} \cdot u'$
$[f]^{-n}$	$n \cdot [f]^{n-1} \cdot f$	$[f]^g$	$[f] \cdot \left[\frac{g}{\ln p} + \frac{1}{\ln p} \cdot \ln f \right]$