سنتر فيوتشر

Subject: إعدادي رياضة

Chapter: تفاضل الدوال المثلثية العكسية

Mob: 0112 3333 122

0109 3508 204

1st dervitves by using defention

المشتقة الأولى : باستخدام التعريف

$$\boxed{y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Ex

by using defention prove that $\frac{d}{dx}(2x) = 2$

بالتعريف

$$y' = \lim_{h \to 0} \frac{2(x+h) - (2x)}{h} = \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{2x}}{h}$$

$$= 2$$

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by defention [by princple] find first der

$y = x^2$

الأولى

$$y' = \lim_{h \to 0} \frac{[x+h]^2 - x^2}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

1

Find $\frac{dy}{dx}$ if $y = 2x^3$

$$y' = \lim_{h \to 0} \frac{2(x+h)^3 - 2x^3}{h}$$

$$y' = \lim_{h \to 0} \frac{2[x^3 + 3x^2h + 3h^2x + h^3] - 2x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^3} + 6x^2h + 6h^2x + 3h^3 - \cancel{2x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[6x^2 + 6hx + 3h]}{\cancel{h}} = 6x$$

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if $y = \sqrt{3x+1}$ Find $\frac{dy}{dx}$

$$y' = \lim_{h \to 0} \frac{\sqrt{1 + 3(x+h)} - \sqrt{1+3x}}{h}$$

بالضرب × المرافق

$$y' = \lim_{h \to 0} \frac{(\sqrt{1+3x+3h} - \sqrt{1+3x})(\sqrt{1+3x+3h} + \sqrt{1+3x})}{h[\sqrt{1+3x+3h} + \sqrt{1+3x}]}$$

$$y' = \lim_{h \to 0} \frac{\cancel{1+3x}+3h - \cancel{(1+3x)}}{h\left[\sqrt{1+3x+3h} + \sqrt{1+3x}\right]}$$

$$\lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}\left[\sqrt{1+3x+3h} + \sqrt{1+3x}\right]} = \frac{3}{2\sqrt{1+3x}}$$

Prove ~~that~~ $\frac{d}{dx} \sin x = \cos x$

$$y' = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \sin h + \cos h \sin h - \sin x}{h}$$

$$= \cos x \lim_{h \to 0} \cancel{\frac{\sin h}{h}}_{1} - (\sin x) \lim_{h \to 0} \frac{1 - \cos(h)}{h}$$

$$= \cos x - \sin x \cdot 2 \lim_{h \to 0} \frac{\sin \frac{h}{2}}{h} \cdot \sin h/2$$

$$= \cos x - (\sin x)(2)\left(\frac{1}{2}\right)(0)$$

تعويض مباشر

س

$$= \cos x$$

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Prove that $\frac{d}{dx} \cos x = -\sin x$

الحل

$y = \cos x \qquad y = \sin\left(\frac{\pi}{2} - x\right)$

$\therefore y' = -\cos\left(\frac{\pi}{2} - x\right) = -\sin x$

OR

$y = \cos x$

$$y' = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= -\sin x \underbrace{\lim_{h \to 0} \frac{\sin h}{h}}_{1} - \cos x \left[\underbrace{\lim_{h \to 0} \frac{(1 - \cos x)}{h}}_{0}\right]$$

زي المثال السابق

$$= -\sin x - 0 = -\sin x$$

Prove that $\frac{d}{dx} e^x = e^x$

$y = e^x$ $\qquad y' = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$

$y' = \lim_{h \to 0} e^x \left[\frac{e^h - 1}{h} \right]$

مفكوك ماكلورين

$\because e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \cdots$

$\therefore y' = \lim_{h \to 0} e^x \left[\frac{\cancel{1} + h + \frac{h^2}{2!} + \frac{h^3}{3!} \cdots \cancel{-1}}{h} \right]$

$= e^x \lim_{h \to 0} \frac{\cancel{h} \left(1 + \frac{h}{2!} + \frac{h^2}{3!} \cdots \right)}{\cancel{h}}$

$\therefore y' = e^x \cdot 1 = e^x$

drive dervitve $\log_a x$ and find dervitv $\ln x$

$$y = \log_a x$$

$$y' = \lim_{h \to 0} \frac{\log_a(x+h) - \log x}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \log_a\left(\frac{x+h}{x}\right) = \lim_{h \to 0} \frac{\frac{x}{h} \log\left(1+\frac{h}{x}\right)}{x}$$

$$= \lim_{h \to 0} \frac{\log_a\left(1+h/x\right)^{h/x}}{x} = \frac{\log_a e^1}{x}$$

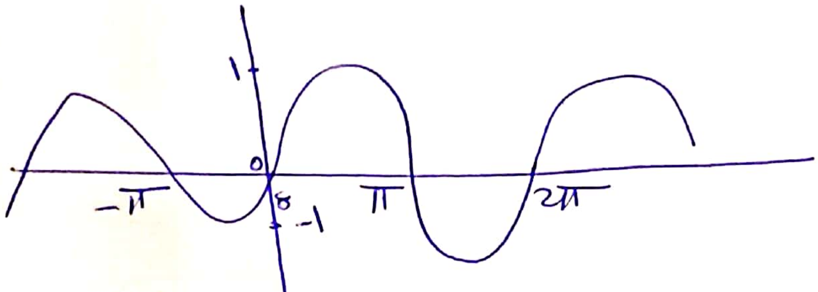
$$= \frac{\ln e}{x \ln a}$$

$$\therefore \frac{d}{dx} \log_a x = \frac{\ln e}{x \ln a} = \frac{1}{x \ln a}$$

Put $a = e$

$$\therefore \frac{d}{dx} \log_e x = \ln x = \frac{1}{x \ln e} = \frac{1}{x}$$

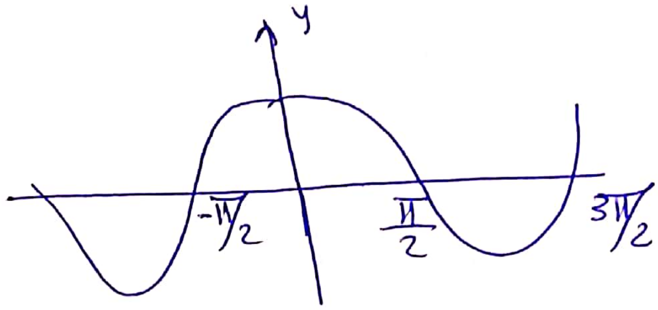
الدوال المثلثية



$y = \sin x$

$D_f = R$

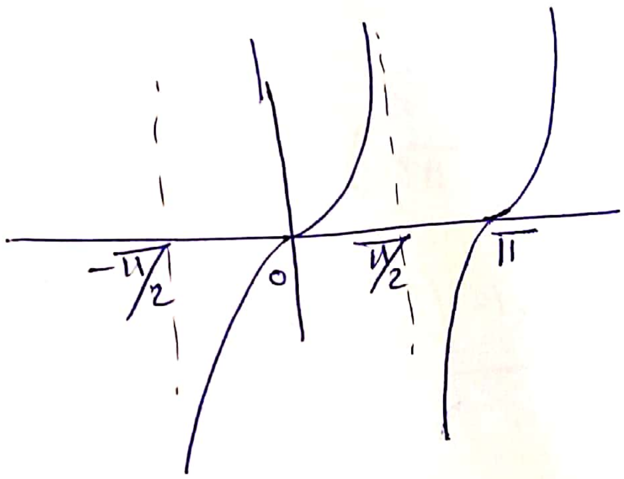
$R_f = [-1, 1]$



$y = \cos x$

$D_f = R$

$R_f = [-1, 1]$

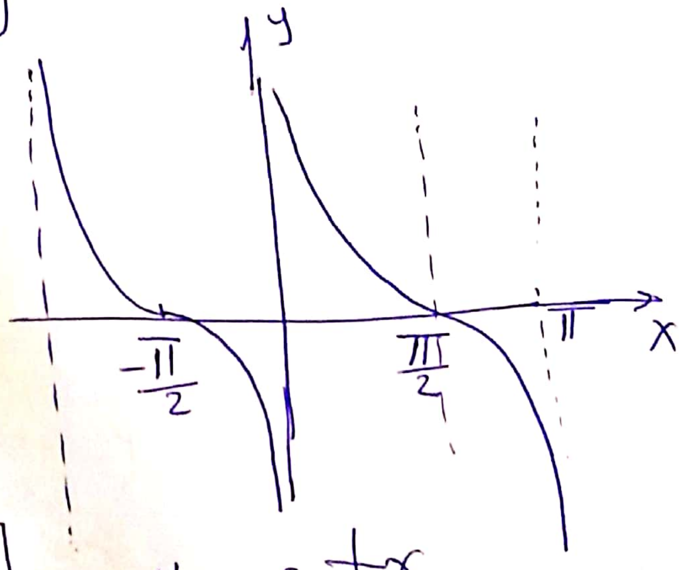


$y = \tan x$

$D_f = R - [\pi/2, 3\pi/2 \ldots]$

$R - [(2n \pm 1)\pi/2]$

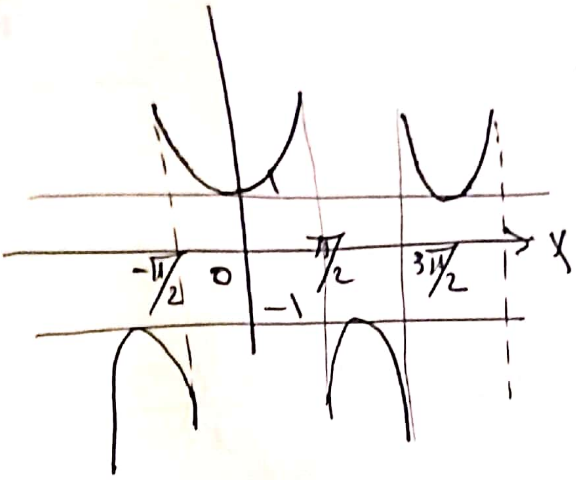
$R_f = R$



$D_f = R - [n\pi]$ $\quad y = \cot x$

$R_f = R$

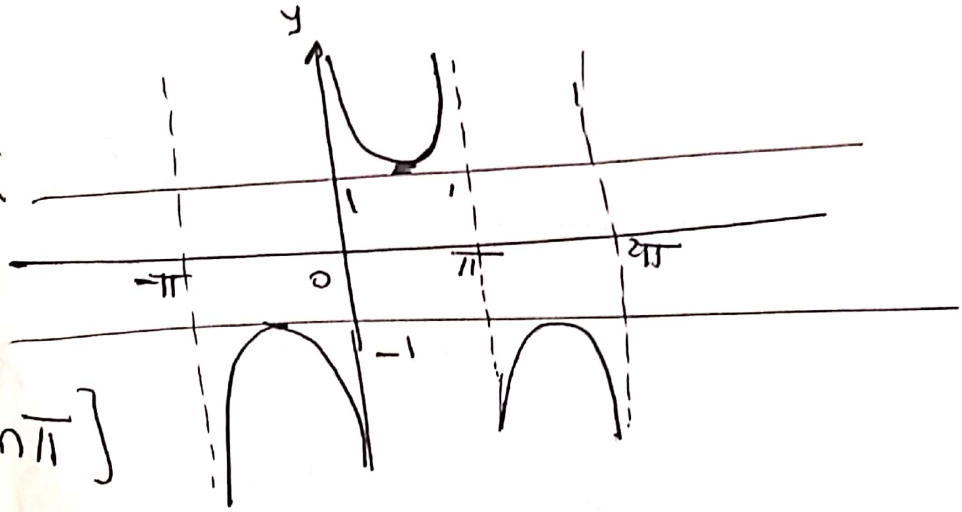
$y = \sec x$



$D_f \quad R - (2n+1)\frac{\pi}{2}$

$R_f = R -]-1,1[$

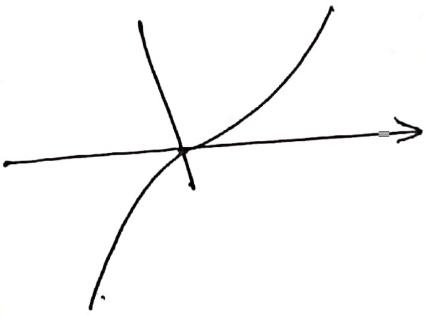
$y = \operatorname{cosec} x$



$D_f = R - (n\pi)$

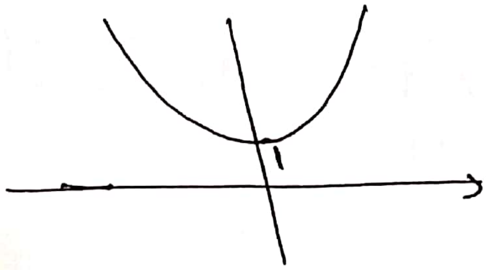
$R_f = R -]-1,1[$

$y = \sinh x$



$D_f = R$

$R_f = R$

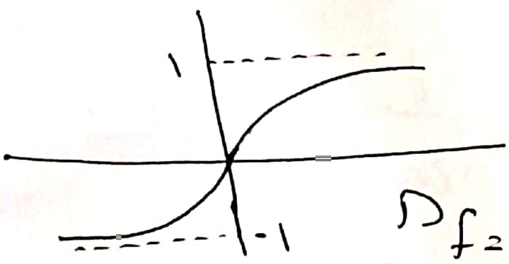


$y = \cosh x$

$D_f = R$

$R_f = [1, \infty[$

$y = \tanh x$



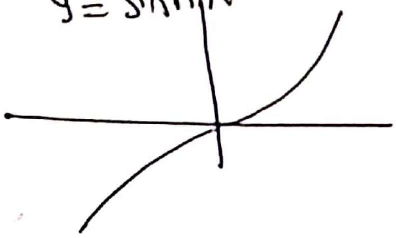
$D_f = R$

$R_f =]-1,1[$

(1)

Drive $\frac{d}{dx}\sinh x = \cosh x$

$y = \sinh x$

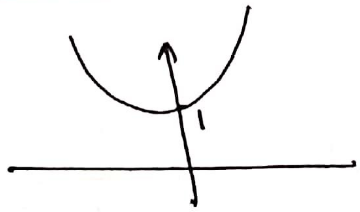


$\because y = \sinh x = \frac{e^{x} - e^{-x}}{2}$

بالتعاوض

$\therefore \frac{dy}{dx} = \frac{1}{2}\left[e^{x} + e^{-x}\right] = \cosh x$

$\frac{d}{dx}\cosh x = \sinh x$

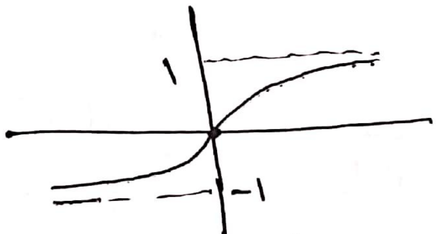


$y = \cosh x$

$y = \cosh x = \frac{e^{x} + e^{-x}}{2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}\left[e^{x} - e^{-x}\right] = \sinh x$

$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$



$y = \tanh x$

و

$\because y = \frac{\sinh x}{\cosh x}$

$\therefore y' = \frac{\cosh x \cdot \cosh x - \sinh x \cdot \sinh x}{\cosh^2 x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$

$\because \cosh^2 x - \sinh^2 x = 1$ ، $\frac{1}{\cosh x} = \operatorname{sech} x$

$\therefore \frac{dy}{dx} = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$

Prove that

$$\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$$

$$y = \coth x = \frac{\cosh x}{\sinh x}$$

$$y' = \frac{\sinh x \cdot \sinh x - \cosh x \cdot \cosh x}{\sinh^2 x}$$

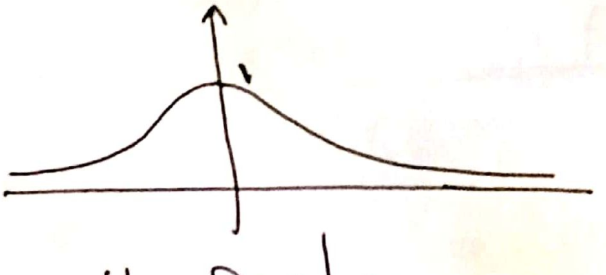
$$= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} = \frac{-1}{\sinh^2 x} = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$y = \operatorname{sech} x \qquad y = \frac{1}{\cosh x}$$

$$y' = \frac{0 - \sinh x}{\cosh^2 x} = -\frac{\sinh x}{\cosh x} \cdot \frac{1}{\cosh x}$$

$$y' = -\tanh x \cdot \operatorname{sech} x$$



$y = \operatorname{sech} x$

$D_f = R$

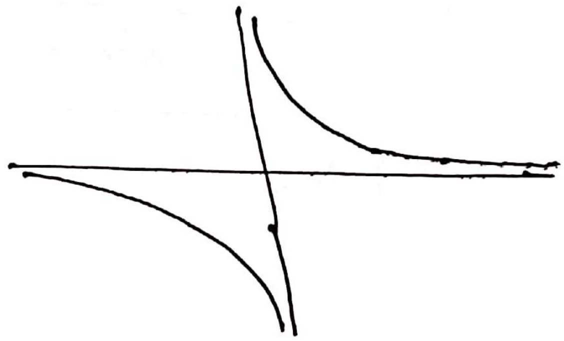
$R_f =]0, 1]$

Prove that $\frac{d}{dx}\operatorname{Csech}x = -\operatorname{Csech}x \coth x$

$y = \operatorname{Csech}x = \frac{1}{\sinh x}$

$y' = \frac{0 - \cosh x}{\sinh^2 x} = -\frac{\cosh x}{\sinh x} \cdot \frac{1}{\sinh x}$

$y' = -\coth x \operatorname{Csech}x$



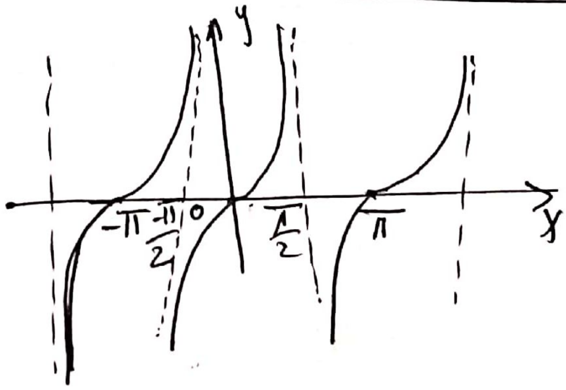
المجال $D_f = R - \{0\}$

المدى $R_f = R - \{0\}$

$y = \operatorname{Csech}x$

Prove that

$\frac{d}{dx}\tan x = \sec^2 x$



$y = \frac{\sin x}{\cos x}$

$y' = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$y' = \frac{1}{\cos^2 x} = \sec^2 x$

Prove that $\frac{d}{dx} \cot x = -\operatorname{Cosec}^2 x$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$y' = \frac{(\sin x)(-\sin x) - \cancel{\sin x}\cos x\,(\cos x)}{(\sin x)^2}$$

$$= -\frac{\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\operatorname{Cosec}^2 x$$

$\frac{d}{dx} \sec x = \sec x \tan x$ أثبت

$$y = \sec x = \frac{1}{\cos x}$$

$$y' = \frac{0 - (-\sin x)}{\cos x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$y' = \tan x \sec x$$

Prove that $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{Cosec} x \cot x$

$$y = \operatorname{Cosec} x = \frac{1}{\sin x}$$

$$y' = \frac{0 - \cos x}{(\sin^2 x)} = -\frac{\cos x}{\sin x \cdot \sin x}$$

$$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \operatorname{Cosec} x$$

تفاضل الدوال المثلثية العكسية

$y = \sin^{-1} u \qquad y' = \frac{1}{\sqrt{1-u^2}}\, u'$

$y = \cos^{-1} u \longrightarrow y' = \frac{-1}{\sqrt{1-u^2}}\, u'$

$y = \tan^{-1} u \qquad y' = \frac{1}{1+u^2}\, u'$

$y = \cot^{-1} u \longrightarrow y' = \frac{-1}{1+u^2}\, u'$

$y = \sec^{-1} u \qquad y' = \frac{1}{u\sqrt{u^2-1}}\, u'$

$y = \operatorname{cosec}^{-1} u \longrightarrow y' = \frac{-1}{u\sqrt{u^2-1}}\, u'$

تفاضل الدوال الزائدية العكسية

$y = \sinh^{-1} u \qquad y' = \frac{1}{\sqrt{1+u^2}}\, u'$

$y = \cosh^{-1} u \qquad y' = \frac{+1}{\sqrt{u^2-1}}\, u'$

$y = \tanh^{-1} u \qquad y' = \frac{+1}{1-u^2}\, u'$

$y = \coth^{-1} u \qquad y' = \frac{-1}{u^2-1}\, u'$

$y = \operatorname{sech}^{-1} u \qquad y' = \frac{-1}{u\sqrt{1-u^2}}\, u'$

$y = \operatorname{cosech}^{-1} u \qquad y' = \frac{-1}{u\sqrt{u^2+1}}\, u'$

Find $\dfrac{dy}{dx}$ if

$$y = \sin^{-1}(2x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-4x^2}} \cdot (2)$$

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if $y = (x^2+1) \cdot e^{\tan^{-1} x}$

$$\frac{dy}{dx} = 2x\, e^{\tan^{-1} x} + \cancel{(x^2+1)} \cdot e^{\tan^{-1} x} \frac{1}{\cancel{1+x^2}}$$

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if $y = \cos^{-1}\left(\dfrac{1}{\sqrt{x^2-1}}\right)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \frac{1}{x^2-1}}} \cdot \left[\left(-\frac{1}{\cancel{2}}\right)(x^2-1)^{-3/2} \cdot \cancel{2}x\right]$$

$$= \frac{-1}{\sqrt{\frac{x^2-1-1}{x^2-1}}} \left(\frac{-x}{(x^2-1)^{3/2}}\right) = \frac{x\sqrt{x^2-1}}{\sqrt{x^2-2}\ (x^2-1)^{3/2}}$$

$$= \frac{x}{\sqrt{x^2-2} \cdot (x^2-1)}$$

if $y = \operatorname{Cosec}^{-1}(e^{x}) + \tan^{-1}(\sinh x)$

$$\frac{dy}{dx} = \frac{-e^{x}}{e^{x}\sqrt{e^{2x}+1}} + \frac{1}{1+\sinh^{2}x} \cdot \cosh x$$

if $y = \sinh^{-1}(\tan\sqrt{x})$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+\tan^{2}\sqrt{x}}} \cdot \sec^{2}\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

if $\cosh(x+y) + e^{\tanh^{-1}x} + 4^{y} = 0$

$$(1+y')\sinh(x+y) + e^{\tanh^{-1}x}\frac{1}{1-x^{2}} + 4^{y}\cdot y' \ln 4 = 0$$

if $y = \tan^{-1}\tan(\sec\sqrt{x})$

$y = \sec\sqrt{x}$

$$y' = \sec\sqrt{x} \cdot \tan\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

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if $\tan^{-1} y + \cos^{-1} x = 1$ Find $\frac{dy}{dx}$

$$\frac{1}{1+y^2} y' + \frac{-1}{\sqrt{1-x^2}} = 0$$

if $f(x) = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$, $g(x) = \tan^{-1} x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{x^2}{1+x^2}}} \cdot \left[\frac{\sqrt{1+x^2} - x \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}}{1+x^2} \right.$$

$$= \frac{1}{\sqrt{\frac{1+\cancel{x^2}-\cancel{x^2}}{1+x^2}}} \cdot \left[\frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{1+x^2} \right]$$

$$= \frac{\sqrt{1+x^2}\left(\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}\right)}{1+x^2}$$

$$= \frac{1+\cancel{x^2}-\cancel{x^2}}{1+x^2} = \frac{1}{1+x^2} \quad \#$$

Show that $\frac{df}{dx} = \frac{dg}{dx}$

$$\frac{dg}{dx} = \frac{1}{1+x^2} \quad \#$$

Find $\frac{dy}{dx}$ if

$$y = e^{\cosh x} + \cosh\sqrt{x} + \tanh^3(x)$$

$$\frac{dy}{dx} = e^{\cosh x} \cdot \sinh x + \frac{\sinh\sqrt{x}}{2\sqrt{x}} + 3\tanh^2(x) \cdot \operatorname{sech}^2 x$$

if $x^{\sinh y} + y^{\cosh x} = 5$

$$u + v = 5$$

$$u' + v' = 0$$

$$u = x^{\sinh y}$$

$$\ln u = \sinh y \ln x$$

$$\frac{u'}{u} = y' \cosh y \cancel{\ln x} + \frac{1}{x} \sinh y$$

$$u' = u\left[y' \cosh y \ln x + \frac{\sinh y}{x}\right]$$

$$u' = x^{\sinh y}\left[y' \cosh y \ln x + \frac{\sinh y}{x} \right]$$

$$v = y^{\cosh x}$$

$$\ln v = \cosh x \ln y$$

$$\frac{v'}{v} = \sinh x \ln y + \frac{y'}{y} \cosh x$$

$$v' = v\left[\sinh x \ln y + \frac{y'}{y} \cosh x\right]$$

$$v' = y^{\cosh x}\left[\sinh x \ln y + \frac{y'}{y} \cosh x\right]$$

$$\therefore \quad u' + v' = 0$$

$$y = \operatorname{csch}\sqrt{x} + \operatorname{sech}(x^3)$$

$$\frac{dy}{dx} = -\frac{\operatorname{csch}\sqrt{x} \coth\sqrt{x}}{2\sqrt{x}} - 3x^2 \operatorname{sech} x^3 \tanh x^3$$

$$y = \ln(\cosh x + \sinh x) \qquad \text{Find } \frac{dy}{dx}$$

$$\because \cosh x + \sinh x = e^x \qquad y = \cancel{\ln e^x} = x$$

$$y' = 1$$

if $\tanh^{-1}(xy) + e^{\cosh\sqrt{x}} = 4$

Find $\dfrac{dy}{dx}$

$$\frac{1}{1-(xy)^2}\left[xy' + y\right] + \frac{\sinh\sqrt{x}}{2\sqrt{x}}\, e^{\cosh\sqrt{x}} = 0$$

---

if

$$y = \sin^3(\cosh x) + 2^{x \cdot \tanh x^4} + e^{\tan^{-1}\sqrt{x}} + x^{\cosh x}$$

$$\frac{dy}{dx} = 3\left[\sin(\cosh x)\right]^2 \cdot \cos(\cosh x) \cdot \sinh x$$

$$+ 2^{x\tanh x^4} \cdot \ln 2 \left[\tanh x^4 + x(4x^3)\operatorname{sech}^2 x^4\right]$$

$$+ e^{\tan^{-1}\sqrt{x}} \cdot \frac{1}{1+x}\,\frac{1}{2\sqrt{x}} + \underset{A}{u'}$$

$$\because u = x^{\cosh x}$$

$$\ln u = \cosh x \ln x$$

$$\frac{1}{u}u' = \cosh x \frac{1}{x} + \ln x \sinh x$$

$$u' = u\left[\frac{\cosh x}{x} + \sinh x \ln x\right]$$

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التفاضل لاستقام

$$\sin x = \frac{1}{\operatorname{cosec} x} \qquad \cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\left.\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}\right\}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

خواص اللوغارتم

$$\ln(xy) = \ln x + \ln y$$

$$\ln(x/y) = \ln x - \ln y$$

$$\ln x^n = n \ln x$$

$$\ln x = \log_e x$$

$$\ln e^x = x \qquad e^{\ln x} = x$$

$$\log_y x = \frac{\ln x}{\ln y}$$

$$\log_b x = y \longrightarrow x = b^y$$

$y =$ Constant $\qquad \frac{dy}{dx} = y' = 0$

$$y = x^m \qquad y' = m x^{m-1}$$

$$y = f \cdot g \qquad y' = f \cdot g' + g f'$$

$$y = f/g \qquad y' = \frac{g f' - g' f}{g^2}$$

$$y = (f)^m \qquad y' = m (f)^{m-1} f'$$

$$y = \sqrt{f} \qquad y' = \frac{1}{2\sqrt{f}} f'$$

تفاضل الدوال المثلثية

$y = \sin u \qquad y' = u' \cos u$

$y = \cos u \qquad y' = -u' \sin u$

$y = \tan u \qquad y' = u' \sec^2 u$

$y = \cot u \qquad y' = -u' \csc^2 u$

$y = \sec u \qquad y' = u' \sec u \cdot \tan u$

$y = \csc u \qquad y' = -u' \csc u \cdot \cot u$

$y = \sinh u \qquad y' = u' \cosh u$

$y = \cosh u \qquad y' = u' \sinh u$

$y = \tanh u \qquad y' = u' \operatorname{sech}^2 u$

$y = \coth u \qquad y' = -u' \operatorname{csch}^2 u$

$y = \operatorname{sech} u \qquad y' = -u' \operatorname{sech} u \cdot \tanh u$

$y = \operatorname{csch} u \qquad y' = -u' \operatorname{csch} u \cdot \coth u$

تفاضل الدوال المثلثية العكسية

$$y = \sin^{-1} u \qquad y' = \frac{1}{\sqrt{1-u^2}}\, u'$$

$$y = \cos^{-1} u \longrightarrow y' = \frac{-1}{\sqrt{1-u^2}}\, u'$$

$$y = \tan^{-1} u \qquad y' = \frac{1}{1+u^2}\, u'$$

$$y = \cot^{-1} u \longrightarrow y' = \frac{-1}{1+u^2}\, u'$$

$$y = \sec^{-1} u \qquad y' = \frac{1}{u\sqrt{u^2-1}}\, u'$$

$$y = \csc^{-1} u \longrightarrow y' = \frac{-1}{u\sqrt{u^2-1}}\, u'$$

تفاضل الدوال الزائدية العكسية

$$y = \sinh^{-1} u \qquad y' = \frac{1}{\sqrt{1+u^2}}\, u'$$

$$y = \cosh^{-1} u \qquad y' = \frac{+1}{\sqrt{u^2-1}}\, u'$$

$$y = \tanh^{-1} u \qquad y' = \frac{+1}{1-u^2}\, u'$$

$$y = \coth^{-1} u \qquad y' = \frac{-1}{u^2-1}\, u'$$

$$y = \operatorname{sech}^{-1} u \qquad y' = \frac{-1}{u\sqrt{1-u^2}}\, u'$$

$$y = \operatorname{csch}^{-1} u \qquad y' = \frac{-1}{u\sqrt{u^2+1}}\, u'$$