

سنتر فيو تشر

Subject: ..... اسلام

Chapter: ..... تفاصيل الرجال والنساء

Mob: 0112 3333 122

0109 3508 204

1st derivatives by using definition  
تعريف المُسْتَعْدِفِ :

$$\boxed{y' = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}}$$

Ex by using definition prove that  $\frac{d}{dx}(2x) = 2$

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{2(x+h) - (2x)}{h} = \lim_{h \rightarrow 0} \frac{2x+2h-2x}{h} \\ &= 2 \end{aligned}$$

by definition [by principle] find first der

$$y = x^2$$

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{[x+h]^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} = 2x \end{aligned}$$

①

Find  $\frac{dy}{dx}$  if  $y = 2x^3$

$$y' = \lim_{h \rightarrow 0} 2 \frac{(x+h)^3 - 2x^3}{h}$$

$$y' = \lim_{h \rightarrow 0} 2 \frac{[x^3 + 3x^2h + 3h^2x + h^3] - 2x^3}{h} - 2x^3$$

$$= \lim_{h \rightarrow 0} \frac{2x^3 + 6x^2h + 6h^2x + 3h^3 - 2x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h [6x^2 + 6hx + 3h]}{h} = 6x$$

if  $y = \sqrt{3x+1}$  Find  $\frac{dy}{dx}$

$$y' = \lim_{h \rightarrow 0} \frac{\sqrt{1+3(x+h)} - \sqrt{1+3x}}{h}$$

$$y' = \lim_{h \rightarrow 0} \frac{[\sqrt{1+3x+3h} - \sqrt{1+3x}]}{h} \left[ \frac{1}{\sqrt{1+3x+3h} + \sqrt{1+3x}} \right]$$

مما زاد

$$h \rightarrow 0$$

(C)

$$y' = \lim_{h \rightarrow 0} \frac{1+3x+3h - (1+3x)}{h \left[ \sqrt{1+3x+3h} + \sqrt{1+3x} \right]}$$

$$\lim_{h \rightarrow 0} \frac{3h}{h \left[ \sqrt{1+3x+3h} + \sqrt{1+3x} \right]} = \frac{3}{2\sqrt{1+3x}}$$

Prove that  $\frac{d}{dx} \sin x = \cos x$

$$y' = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos x \sin h + \cos h \sin h - \sin x}{h}$$

$$= \cos x \lim_{h \rightarrow 0} \frac{\sin h}{h} - (\sin x) \lim_{h \rightarrow 0} \frac{1 - \cos(h)}{h}$$

$$= \cos x - \sin x \cdot 2 \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \cdot \sin \frac{h}{2}$$

$$= \cos x - (\sin x)(2)(\frac{1}{2})(0)$$

$$= \cos x$$

Prove that  $\frac{d}{dx} \cos x = -\sin x$

$$y = \cos x$$

$$y = \sin(\frac{\pi}{2} - x)$$

$$\therefore y' = -\cos(\frac{\pi}{2} - x) = -\sin x$$

OR

$$y = \cos x$$

$$y' = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= -\sin x \cancel{\lim_{h \rightarrow 0} \frac{\sin h}{h}} - \cos x \cancel{\left[ \lim_{h \rightarrow 0} \frac{(1 - \cos x)}{h} \right]}$$

$$= -\sin x - 0 = -\sin x$$

(2)

Prove that  $\underbrace{\frac{d}{dx} e^x}_{\downarrow} = e^x$

$$y = e^x \quad y' = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}$$

$$y' = \lim_{h \rightarrow 0} e^x \left[ \frac{e^h - 1}{h} \right]$$

$$\therefore e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \dots \quad \text{معنده مجموعه}$$

$$\therefore y' = \lim_{h \rightarrow 0} e^x \left[ \frac{x + h + \frac{h^2}{2!} + \frac{h^3}{3!} \dots}{h} \right]$$

$$= e^x \lim_{h \rightarrow 0} \cancel{h} \left( 1 + \frac{h}{2!} + \frac{h^2}{3!} \dots \right)$$

$$\therefore y' = e^x \cdot 1 = e^x$$

⑥

drive deriv. for  $\log_a x$  and find deriv. for  $\ln x$

$$y = \log_a x$$

$$y' = \lim_{h \rightarrow 0} \frac{\log_a(x+h) - \log_a x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \log_a \left( \frac{x+h}{x} \right) = \lim_{h \rightarrow 0} \frac{1}{h} \log_a \left( 1 + \frac{h}{x} \right)$$

$$= \lim_{h \rightarrow 0} \frac{\log_a \left( 1 + \frac{h}{x} \right)^{h/x}}{x} = \frac{\log_a e^1}{x}$$

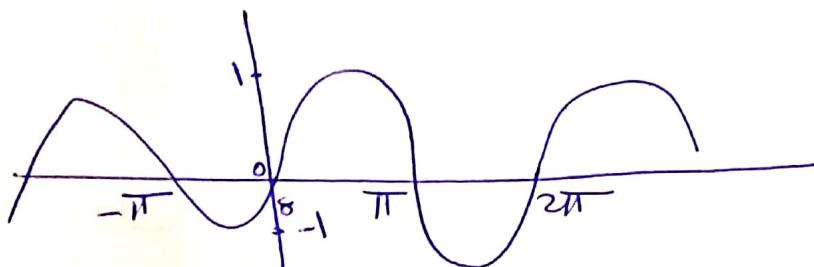
$$= \frac{\ln e}{x \ln a}$$

$$\therefore \frac{d}{dx} \log_a x = \frac{\ln e}{x \ln a} = \frac{1}{x \ln a}$$

$$\text{Put } a = e$$

$$\therefore \frac{d}{dx} \log_e x = \ln x = \frac{1}{x \ln e} = \frac{1}{x}$$

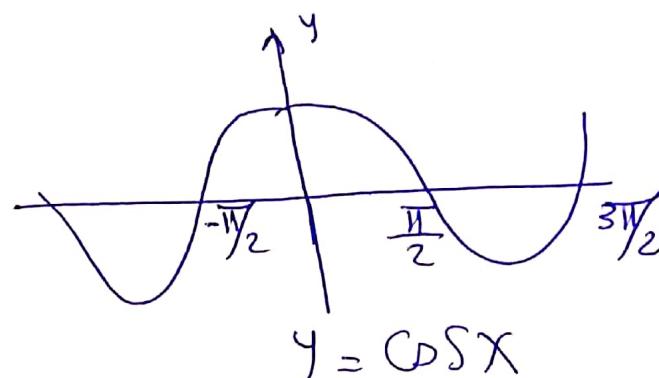
الدوال المثلثية



$$y = \sin x$$

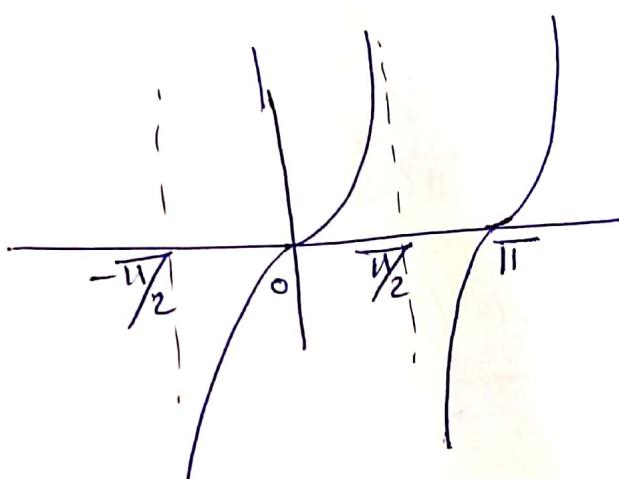
$$D_f = \mathbb{R}$$

$$R_f = [-1, 1]$$



$$D_f = \mathbb{R}$$

$$R_f = [-1, 1]$$

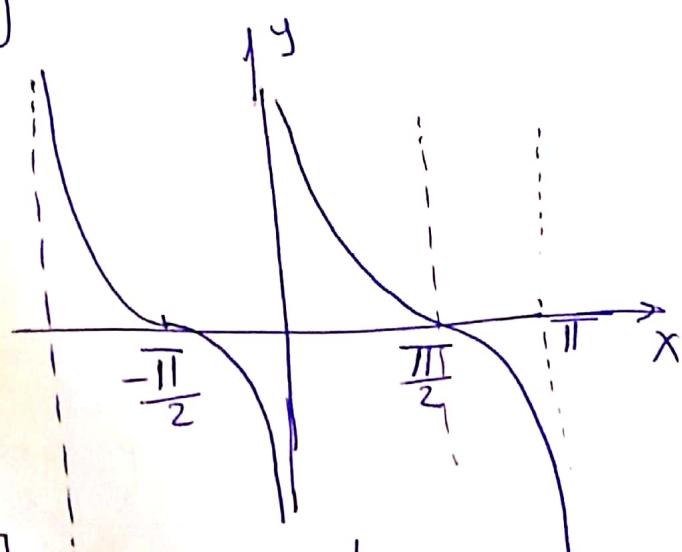


$$y = \tan x$$

$$D_f = \mathbb{R} - [\frac{\pi}{2}, \frac{3\pi}{2}, \dots]$$

$$\mathbb{R} - [(2n+1)\frac{\pi}{2}]$$

$$R_f = \mathbb{R}$$



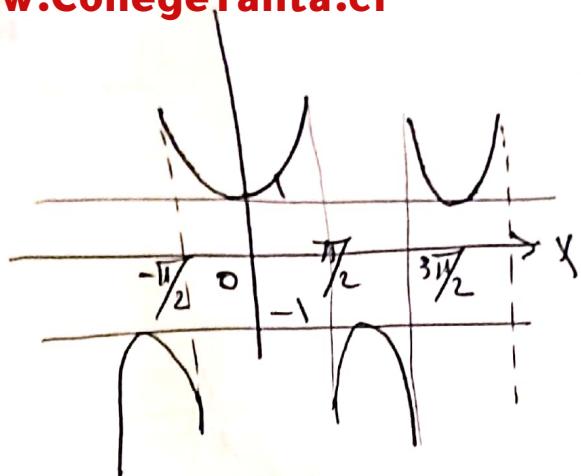
$$D_f = \mathbb{R} - [n\pi]$$

$$R_f = \mathbb{R}$$

$$y = \cot x$$

(\*)

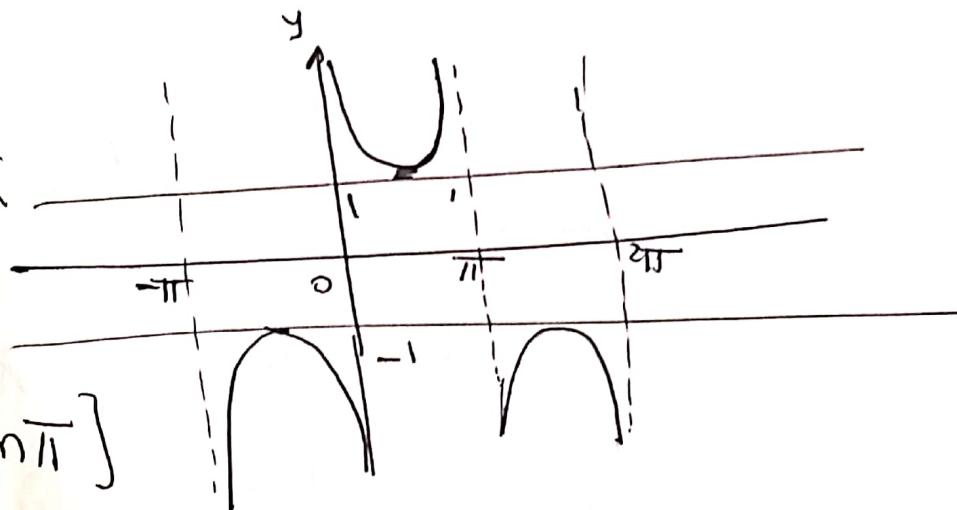
$$y = \sec x$$



$$\mathcal{D}_f = \mathbb{R} - \left( (2n+1)\frac{\pi}{2} \right)$$

$$R_f = \mathbb{R} - \left[ -1, 1 \right]$$

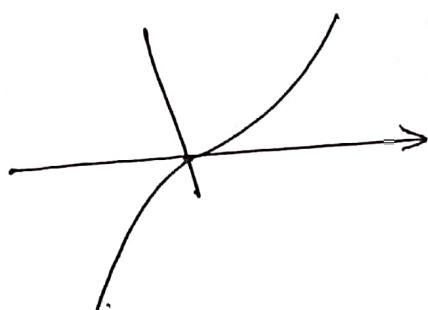
$$y = \csc x$$



$$\mathcal{D}_f = \mathbb{R} - \left[ n\pi \right]$$

$$R_f = \mathbb{R} - \left[ -1, 1 \right]$$

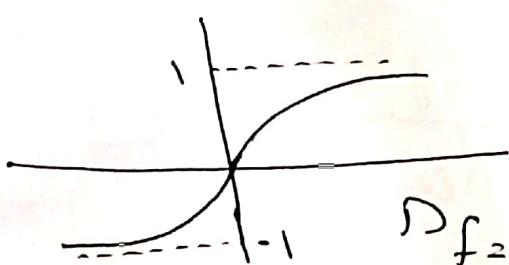
$$y = \sinh x$$



$$\mathcal{D}_f = \mathbb{R}$$

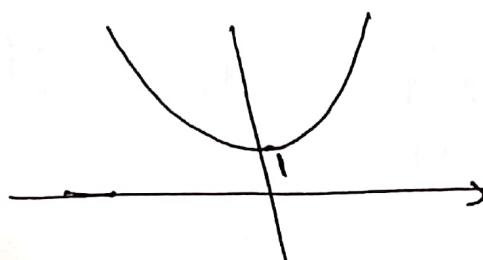
$$R_f = \mathbb{R}$$

$$y = \tanh x$$



$$\mathcal{D}_f = \mathbb{R}$$

$$R_f = \left[ -1, 1 \right]$$



$$y = \cosh x$$

$$\mathcal{D}_f = \mathbb{R}$$

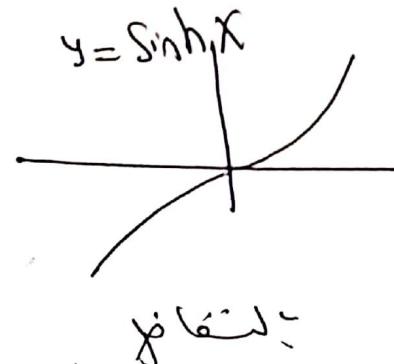
$$R_f = \left[ 1, \infty \right]$$

(1)

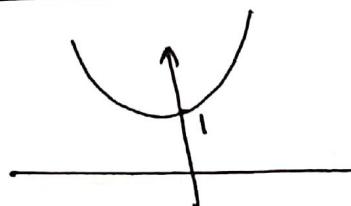
$$\text{Drive } \frac{d}{dx} \sinh x = \cosh x$$

$$\therefore y = \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} [e^x + e^{-x}] = \cosh x$$



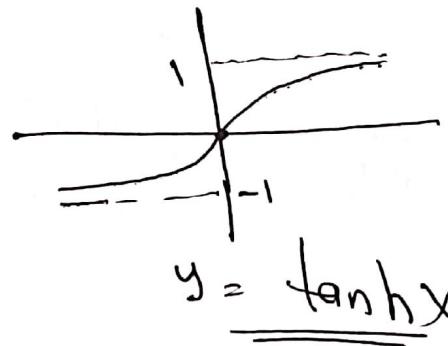
$$\frac{d}{dx} \cosh x = \sinh x$$



$$y = \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} [e^x - e^{-x}] = \sinh x$$

$$\frac{d}{dx} \tanh x = \operatorname{Sech}^2 x$$



$$\therefore y = \frac{\sinh x}{\cosh x}$$

$$\therefore y' = \frac{\cosh x \cdot \cosh x - \sinh x \cdot \sinh x}{\cosh^2 x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$$

$$\therefore \cosh^2 x - \sinh^2 x = 1 \quad , \quad \frac{1}{\cosh x} = \operatorname{Sech} x$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cosh^2 x} = \operatorname{Sech}^2 x$$

(q)

Prove that

$$\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$$

$$y = \coth x = \frac{\cosh x}{\sinh x}$$

$$y' = \frac{\sinh x \cdot \sinh x - \cosh x \cdot \cosh x}{\sinh^2 x}$$

$$= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} = \frac{-1}{\sinh^2 x} = -\operatorname{csch}^2 x$$

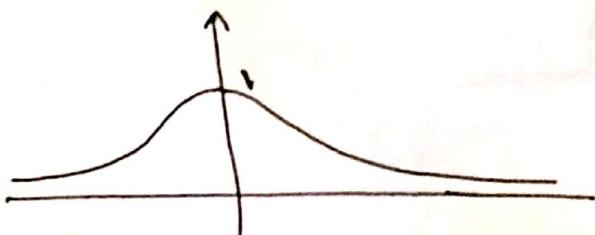
$$\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$y = \operatorname{sech} x$$

$$y = \frac{1}{\cosh x}$$

$$y' = \frac{0 - \sinh x}{\cosh^2 x} = -\frac{\sinh x}{\cosh x} \cdot \frac{1}{\cosh x}$$

$$y' = -\tanh x \cdot \operatorname{sech} x$$



$$y = \operatorname{sech} x$$

$$D_f = R$$

$$R_f = [0, 1]$$

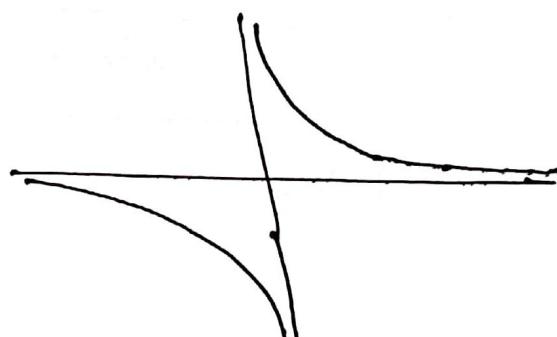
(1)

Prove That  $\frac{d}{dx} \operatorname{Cosech} x = -\operatorname{Cosech} x \coth x$

$$y = \operatorname{Cosech} x = \frac{1}{\sinh x}$$

$$y' = \frac{0 - \cosh x}{\sinh^2 x} = -\frac{\cosh x}{\sinh x} \cdot \frac{1}{\sinh x}$$

$$y' = -\coth x \operatorname{Cosech} x$$

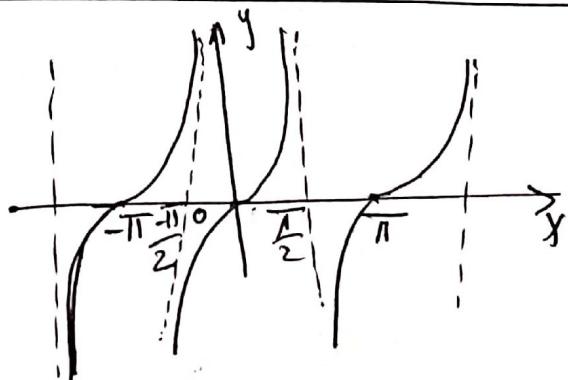


$$\text{J.M} \quad D_f = R - \{0\}$$

$$\rightarrow R_f = R - \{0\} \quad y = \operatorname{Cosech} x$$

Prove That

$$\frac{d}{dx} \tan x = \sec^2 x$$



$$y = \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos x \cos x - \sin x (-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$y' = \frac{1}{\cos^2 x} = \sec^2 x$$

(11)

Prove That  $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$y' = \frac{(\sin x)(-\sin x) - \sin x \cos x (\cos x)}{(\sin x)^2}$$

$$= -\frac{\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$$


---

$$\frac{d}{dx} \sec x = \sec x \tan x \quad \rightarrow$$

$$y = \sec x = \frac{1}{\cos x}$$

$$y' = \frac{0 - (-\sin x)}{\cos x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$y' = \tan x \sec x$$


---

Prove That  $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$$y = \operatorname{cosec} x = \frac{1}{\sin x}$$

$$y' = \frac{0 - \cos x}{(\sin^2 x)} = -\frac{\cos x}{\sin x \cdot \sin x}$$

$$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \operatorname{cosec} x$$



تفاضل الدوال المثلثية المثلثية

$$y = \sin^{-1} u \quad y' = \frac{1}{\sqrt{1-u^2}} u'$$

$$y = \cos^{-1} u \longrightarrow y' = \frac{-1}{\sqrt{1-u^2}} u'$$

$$y = \tan^{-1} u \quad y' = \frac{1}{1+u^2} u'$$

$$y = \cot^{-1} u \longrightarrow y' = \frac{-1}{1+u^2} u'$$

$$y = \sec^{-1} u \quad y' = \frac{1}{u\sqrt{u^2-1}} u'$$

$$y = \cosec^{-1} u \longrightarrow y' = \frac{-1}{u\sqrt{u^2-1}} u'$$

تفاضل الدوال المثلثية المثلثية

$$y = \sinh^{-1} u \quad y' = \frac{1}{\sqrt{1+u^2}} u'$$

$$y = \cosh^{-1} u \quad y' = \frac{+1}{\sqrt{u^2-1}} u'$$

$$y = \tanh^{-1} u \quad y' = \frac{+1}{1-u^2} u'$$

$$y = \coth^{-1} u \quad y' = \frac{-1}{u^2-1} u'$$

$$y = \operatorname{sech}^{-1} u \quad y' = \frac{-1}{u\sqrt{1-u^2}} u'$$

$$y = \operatorname{cosech}^{-1} u \quad y' = \frac{-1}{u\sqrt{u^2+1}} u'$$

Find  $\frac{dy}{dx}$  if

$$y = \sin^{-1}(2x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-4x^2}} \cdot (2)$$

$$\text{if } y = (x^2+1) \cdot e^{\tan^{-1} x}$$

$$\frac{dy}{dx} = 2x e^{\tan^{-1} x} + (x^2+1) \cdot e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$\text{if } y = \cos^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{1}{x^2-1}}} \cdot \left[ \left( -\frac{1}{2} \right) (x^2-1)^{-\frac{3}{2}} \cdot -2x \right]$$

$$= \frac{-1}{\sqrt{\frac{x^2-1-1}{x^2-1}}} \left[ \frac{-x}{(x^2-1)^{\frac{3}{2}}} \right] = \sqrt{x^2-2} \sqrt{\frac{x \sqrt{x^2-1}}{(x^2-1)^{\frac{3}{2}}}}$$

$$= \frac{x}{\sqrt{x^2-2} \cdot (x^2-1)}$$

(48)

$$\text{if } y = \operatorname{Cosec}^{-1}(e^x) + \tan^{-1}(\sinh x)$$

$$\frac{dy}{dx} = \frac{-e^x}{e^x \sqrt{e^{2x} + 1}} + \frac{1}{1 + \sinh^2 x} \cdot \cosh x$$

$$\text{if } y = \sinh^{-1}(\tan \sqrt{x})$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + \tan^2 \sqrt{x}}} \cdot \sec^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$\text{if } \cosh(x+y) + e^{\tanh^{-1} x} + 4^y = 0$$

$$\rightarrow (1+y) \cdot \sinh(x+y) + e^{\tanh^{-1} x} \frac{1}{1-x^2} + 4^y \cdot y' = 0$$

$$\text{if } y = \tan^{-1} \tan(\sec \sqrt{x})$$

$$y = \sec \sqrt{x}$$

$$y' = \sec \sqrt{x} \cdot \tan \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

⑩

if

$$\tan^{-1} y + \cos^{-1} x = 1$$

Find  $\frac{dy}{dx}$

$$\frac{1}{1+y^2} y' + \frac{-1}{\sqrt{1-x^2}} = 0$$

$$\text{if } f(x) = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right), \quad g(f) = \tan^{-1} x$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}} \cdot \left[ \frac{\sqrt{1+x^2} - x \cdot \frac{2x}{x\sqrt{1+x^2}}}{1+x^2} \right] \\ &= \frac{1}{\sqrt{\frac{1+x^2-x^2}{1+x^2}}} \cdot \left[ \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{1+x^2} \right] \\ &= \frac{\sqrt{1+x^2}}{1+x^2} \left[ \sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}} \right] \\ &= \frac{1+x^2-x^2}{1+x^2} = \frac{1}{1+x^2} \quad \# \end{aligned}$$

Show that  $\frac{df}{dx} = \frac{dy}{dx}$

$$\frac{dg}{dx} = \frac{1}{1+x^2} \quad \#$$



Find  $\frac{dy}{dx}$  if

$$y = e^{\operatorname{cosh} x} + \operatorname{cosh} \sqrt{x} + \tanh^3(x)$$

$$\frac{dy}{dx} = e^{\operatorname{cosh} x} \cdot \operatorname{sinh} x + \frac{\operatorname{sinh} \sqrt{x}}{2\sqrt{x}} +$$

$$3 \tanh^2(x) \cdot \operatorname{sech}^2 x$$

$$\text{if } x^{\operatorname{sinh} y} + y^{\operatorname{cosh} x} = 5$$

$$u + v = 5$$

$$u' + v' = 0$$

$$u = x^{\operatorname{sinh} y}$$

$$\ln u = \operatorname{sinh} y \ln x$$

$$\frac{u'}{u} = y \operatorname{cosh} y \ln x + \frac{1}{x} \operatorname{sinh} y$$

$$u' = u \left[ y \operatorname{cosh} y \ln x + \frac{\operatorname{sinh} y}{x} \right]$$

$$U' = x^{\sinh y} \left[ y' \cosh y \ln x + \frac{\sinh y}{x} \right]$$

$$v = y^{\cosh x}$$

$$\ln v = \cosh x \ln y$$

$$\frac{v'}{v} = \sinh x \ln y + \frac{y'}{y} \cosh x$$

$$v' = v \left[ \sinh x \ln y + \frac{y'}{y} \cosh x \right]$$

$$v' = y^{\cosh x} \left[ \sinh x \ln y + \frac{y'}{y} \cosh x \right]$$

$$\therefore U' + v' =$$

$$y = \operatorname{Cosech} \sqrt{x} + \operatorname{Sech} (x^3)$$

$$\frac{dy}{dx} = -\frac{\operatorname{Cosech} \sqrt{x} \operatorname{Ath} \sqrt{x}}{2\sqrt{x}} - 3x^2 \operatorname{Sech} x^3 \operatorname{tanh} x^3$$

$$\underline{y = \ln(\cosh x + \sinh x)}$$

Find  $\frac{dy}{dx}$

$$\therefore \cosh x + \sinh x = e^x$$

(\*)

$$y = \ln e^x = x$$

$$y' = 1$$

$$\text{if } \tanh^{-1}(xy) + e^{\cosh \sqrt{x}} = 4$$

Find  $\frac{dy}{dx}$

$$\frac{1}{1-(xy)^2} [xy' + y] + \frac{\sinh \sqrt{x}}{2\sqrt{x}} e^{\cosh \sqrt{x}} = 0$$

$$\text{if } y = \underbrace{\sin^3(\cosh x) + 2}_{b} + e^{\tanh^{-1} \sqrt{x}} + x^{\cosh x}$$

$$\frac{dy}{dx} = 3[\sin(\cosh x)]^2 \cdot \cosh(\cosh x) \cdot \sinh x$$

$$+ 2x \tanh x^4 \cdot \ln 2 [\tanh x^4 + x(4x^3) \operatorname{sech}^2 x^4]$$

$$+ e^{\tanh^{-1} \sqrt{x}} \cdot \frac{1}{1+x} \frac{1}{2\sqrt{x}} + \frac{u'}{A}$$

$$\therefore u = x^{\cosh x}$$

$$\ln u = \cosh x \ln x$$

$$\frac{1}{u} u' = \cosh x \frac{1}{x} + \ln x \sinh x$$

$$u' = u \left[ \frac{\cosh x}{x} + \sinh x \ln x \right]$$

مكتبة جامعة

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1 \end{aligned} \quad \left. \right\}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

①

القواعد المهمة

$$\ln(xy) = \ln x + \ln y$$

$$\ln(x/y) = \ln x - \ln y$$

$$\ln x^n = n \ln x$$

$$\ln x = \log_e x$$

$$\ln e^x = x$$

$$e^{\ln x} = x$$

$$\log_y x = \frac{\ln x}{\ln y}$$

$$\log_b x = y \rightarrow x = b^y$$


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$$y = \text{constant}$$

$$\frac{dy}{dx} = y' = 0$$

$$y = x^m$$

$$y' = m x^{m-1}$$

$$y = f \cdot g$$

$$y' = f \cdot g' + g \cdot f'$$

$$y = f/g$$

$$y' = \frac{g \cdot f' - g' \cdot f}{g^2}$$

$$y = (f)^m$$

$$y' = m (f)^{m-1} f'$$

$$y = \sqrt[m]{P}$$

(c)

$$y' = \frac{1}{2\sqrt{P}} f'$$

$$y = \sin u$$

$$y' = u' \cos u$$

$$y = \cos u$$

$$y' = -u' \sin u$$

$$y = \tan u$$

$$y' = u' \sec^2 u$$

$$y = \cot u$$

$$y' = -u' \operatorname{cosec}^2 u$$

$$y = \operatorname{sech} u$$

$$y' = u' \operatorname{sech} u \cdot \tan u$$

$$y = \operatorname{cosech} u$$

$$y' = -u' \operatorname{cosech} u \cdot \cot u$$


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$$y = \sinh u$$

$$y' = u' \cosh u$$

$$y = \cosh u$$

$$y' = u' \sinh u$$

$$y = \tanh u$$

$$y' = u' \operatorname{sech}^2 u$$

$$y = \coth u$$

$$y' = -u' \operatorname{sech}^2 u$$

$$y = \operatorname{sech} u$$

$$y' = -u' \operatorname{sech} u \cdot \tanh u$$

$$y = \operatorname{cosech} u$$

$$y' = -u' \operatorname{cosech} u \cdot \coth u$$

تفاضل الدوال المثلثية (الجزء الثاني)

$$y = \sin^{-1} u$$

$$y' = \frac{1}{\sqrt{1-u^2}} u'$$

$$y = \cos^{-1} u \longrightarrow$$

$$y' = \frac{-1}{\sqrt{1-u^2}} u'$$

$$y = \tan^{-1} u$$

$$y' = \frac{1}{1+u^2} u'$$

$$y = \cot^{-1} u \longrightarrow$$

$$y' = \frac{-1}{1+u^2} u'$$

$$y = \sec^{-1} u$$

$$y' = \frac{1}{u\sqrt{u^2-1}} u'$$

$$y = \csc^{-1} u \longrightarrow$$

$$y' = \frac{-1}{u\sqrt{u^2-1}} u'$$

تفاضل الدوال المثلثية (الجزء الثاني)

$$l = \sinh^{-1} u$$

$$y' = \frac{1}{\sqrt{1+u^2}} u'$$

$$l = \cosh^{-1} u$$

$$y' = \frac{+1}{\sqrt{u^2-1}} u'$$

$$l = \tanh^{-1} u$$

$$y' = \frac{+1}{1-u^2} u'$$

$$l = \coth^{-1} u$$

$$y' = \frac{-1}{u^2-1} u'$$

$$l = \operatorname{sech}^{-1} u$$

$$y' = \frac{-1}{u\sqrt{1-u^2}} u'$$

$$l = \operatorname{cosech}^{-1} u$$

$$y' = \frac{-1}{u\sqrt{u^2+1}} u'$$

