



إعدادي 2020

التفاضل

المحاضرة الثالثة - نهايات الدوال

م. أمل ياسر



(نهايات الدوال) The limits of functions

8:00

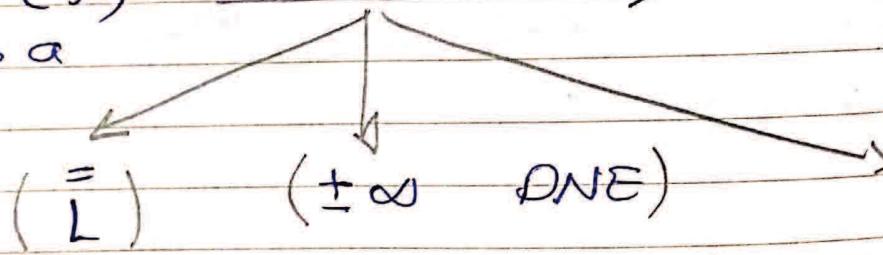
9:00

$$\lim_{\substack{x \rightarrow a}} f(x) = L$$

 $\forall \epsilon > 0 \exists \delta > 0$ such that

$$|f(x) - L| < \epsilon \quad \text{and} \quad |x - a| < \delta$$

$$\lim_{\substack{x \rightarrow a}} f(x) \xrightarrow{\text{direct substitution}}$$



$$\left(\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, \infty^0 \right)$$

حالات غير معروفة

$$\lim_{\substack{x \rightarrow a}} f(x) = L^- \rightarrow \mid \quad \leftarrow \lim_{\substack{x \rightarrow a}} f(x) = L^+$$

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$$L^- = L^+$$

January

الأربعاء

٣٠ ربيع آخر ١٤٢٩ هـ

٩ طوبية ١٧٣٤ ق

١) $\lim_{x \rightarrow 2} \frac{x^2 - 1}{x + 2} = \frac{3}{4}$

٢) $\lim_{x \rightarrow \infty} \frac{x+7}{\frac{1}{x}+2} = \frac{0+7}{\frac{1}{\infty}+2} = \frac{7}{0+2} = \frac{7}{2}$

notes $\rightarrow \frac{1}{\infty} = 0 \quad / \quad \frac{\infty}{1} = \infty \quad /$

$$\frac{1}{\infty} = 0$$

٣) $\lim_{x \rightarrow \infty} \frac{x-2}{5} = \frac{\infty-2}{5} = \frac{\infty}{5} = \infty$

٤) $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3} = \frac{0}{0}$

$$\lim_{x \rightarrow -3} \frac{(x+3)(x-2)}{(x+3)} = (x-2) = -5$$

* Rules of limits :-

8.00

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$$\textcircled{1} \lim_{x \rightarrow a} f(x) = L \quad \text{Lim } \cancel{f(x)}$$

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$$\lim_{x \rightarrow a} g(x) = M$$

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$$\textcircled{2} \lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm$$

 $x \rightarrow a$

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Friday

19

January



الجمعة

١١ طوبية ١٧٣٤ ق

٢ جماد أول ١٤٣٩ هـ

$$\boxed{5} \lim_{x \rightarrow a} c = c \quad \text{C} \rightarrow \text{Constant}$$

$$\boxed{6} \lim_{x \rightarrow a} c f(x) = c \lim_{x \rightarrow a} f(x)$$

$$= c \cdot L$$

$$\boxed{7} \lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$$

$$\boxed{8} \lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$$



٣ جماد أول ١٤٢٩ هـ

السبت

20

January

١٥ طوبه ١٧٣٤ ق

Example ①

$$f(x) = \begin{cases} x^2 & x > 2 \\ \frac{1}{x} & x = 2 \\ x-2 & x < 2 \end{cases}$$

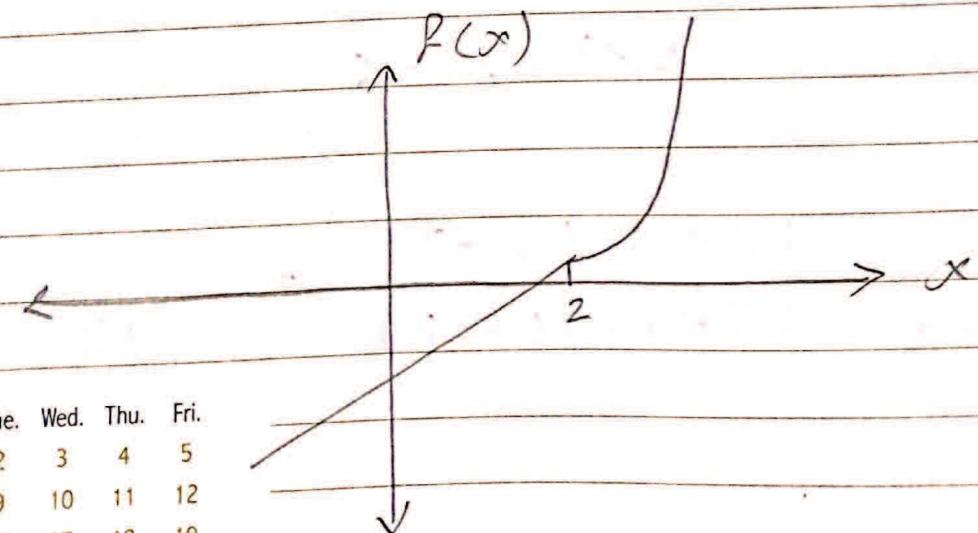
at $x=2$

$$\lim_{\substack{x \rightarrow 2^-}} f(x) = \lim_{x \rightarrow 2^+} x-2 = 0$$

$$\lim_{\substack{x \rightarrow 2^-}} f(x) = \lim_{x \rightarrow 2^-} x^2 = 4$$

 $x \rightarrow 2^+$

$$\lim_{\substack{x \rightarrow 2^+}} f(x) \neq \lim_{x \rightarrow 2^-} f(x) \rightarrow \text{DNE}$$



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* Find the limits of the function

$$\textcircled{1} \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} \quad \frac{0}{0} \xrightarrow{\text{الخواص}} \text{المcis up} \rightarrow 4$$

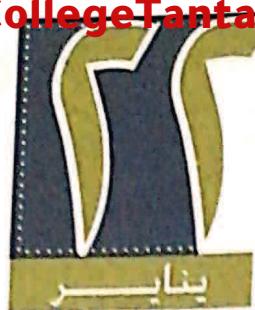
$$\lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x-2)} = \lim_{x \rightarrow 2} (x+2) = 4$$

$$\textcircled{2} \lim_{x \rightarrow 2} \frac{2x^2 + 5x + 2}{5x^2 + 7x - 6} \xrightarrow{\text{المcis up}} \text{الخواص}$$

$$\lim_{x \rightarrow 2} \frac{8 + 10 + 2}{20 + 14 - 6} = \frac{20}{28}$$

$$\textcircled{3} \lim_{x \rightarrow 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6} \xrightarrow{\text{المcis up}} \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{(2x-1)(x-2)}{(5x+3)(x-2)} = \frac{3}{13}$$



٥ جماد أول ١٤٢٩ هـ

Monday

22

January

الإثنين

١٤ طوبية ١٧٣٤ ق

8:00 * notes :-

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$$\textcircled{1} \quad (x^2 - y^2) = (x-y)(x+y)$$

$$\textcircled{2} \quad (x^3 - y^3) = (x-y)(x^2 + xy + y^2)$$

$$\textcircled{3} \quad (x^3 + y^3) = (x+y)(x^2 - xy + y^2)$$

$$\textcircled{4} \quad (x+y)^n = x^n + n(x^{n-1})y$$

$$+ \frac{n(n-1)}{2} (x^{n-2})y^2 +$$

$$- \frac{n(n-1)(n-2)}{3} (x^{n-3})y^3 + \dots$$

$$+ y^n$$

④ $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \frac{0}{0}$

$$\lim_{h \rightarrow 0} \frac{(x+h-x)((x+h)^2 + x(x+h) + x^2)}{h}$$

$$= 3x^2$$

٥ $\lim_{x \rightarrow 1} \frac{\sqrt{x^2+1} - \sqrt{2}}{(x-1)}$ امتحان خارجى

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$$\lim_{x \rightarrow 1} \frac{(\sqrt{x^2+1} - \sqrt{2})(\sqrt{x^2+1} + \sqrt{2})}{(x-1)(\sqrt{x^2+1} + \sqrt{2})}$$

$$\lim_{x \rightarrow 1} \frac{x^2+1-2}{(x-1)(\sqrt{x^2+1} + \sqrt{2})}$$

٦ $\lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)(\sqrt{x^2+1} + \sqrt{2})} = \frac{2}{2\sqrt{2}}$

$$= \frac{1}{\sqrt{2}}$$



limits of trigometric function :-
نهايات الوظائف المثلثية

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin mx}{mx} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x}{x} \neq 1$$

Example ①

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{\tan 5x}$$

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{7x} \cdot \frac{5x}{\tan 5x} \cdot \frac{7x}{5x}$$

$$\frac{7}{5} \lim_{x \rightarrow 0} \frac{\sin 7x}{7x} \cdot \frac{5x}{\tan 5x}$$

$$= \frac{7}{5}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{m \rightarrow 0} (1+m)^{\frac{1}{m}} = e$$

g.f $\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow x_0} g(x) = \infty$

$$\lim f(x) \cdot g(x) = m$$

then $\lim_{x \rightarrow x_0} (1 + f(x))^{g(x)} = c^m$

Example ①

$$\lim_{x \rightarrow 0} (1 + 4 \sin x)^{\frac{3}{x}}$$

$$\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow 0} 4 \sin x = 0$$

$$\lim_{x \rightarrow x_0} g(x) = \lim_{x \rightarrow 0} \frac{x}{3} = \infty$$

$$\lim_{x \rightarrow x_0} f(x) \cdot g(x) = \lim_{x \rightarrow 0} 4 \sin x \left(\frac{3}{x}\right)$$

$$= 12 \lim_{x \rightarrow 0} \frac{\sin x}{x} = 12$$

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Saturday
27
January



السبت

١٠ جماد أول ١٤٣٩ هـ

طبعة ١٧٣٤ ق

$$\lim_{x \rightarrow \frac{\pi}{2}} (1 + \cos x)^{8 \tan x}$$

$$\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \cos x = \lim_{x \rightarrow x_0} g(x) =$$

$$\lim_{x \rightarrow \frac{\pi}{2}} 8 \tan x = \infty$$

$$\lim_{x \rightarrow x_0} f(x) \cdot g(x) = \lim_{x \rightarrow \frac{\pi}{2}} \cos x \cdot 8 \tan x$$

$$= 8 \lim_{x \rightarrow \frac{\pi}{2}} \cos x \cdot \frac{\sin x}{\cos x} = 8$$