COLLEGE
Tanta Engineering

إعدادي 2020

التفاضل

المحاضرة الثالثة - نهايات الدوال

م. أمل ياسر

CollegeTanta
015 553 25450

The limits of functions (نهايات الدوال)

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$$\lim_{x \to a} f(x) = L$$

$\forall \epsilon > 0 \ \exists \delta > 0$ such that

$$|f(x) - L| < \epsilon \quad \forall \quad |x - a| < \delta$$

$\lim_{x \to a} f(x)$ direct substitution

- $(\overset{=}{L})$
- $(\pm\infty \quad DNE)$
- $\left(\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, \infty^0\right)$ كميات غير معرفة

$$\lim_{x \to a} f(x) = L^- \rightarrow | \leftarrow \lim_{x \to a} f(x) = L^+$$

$$L^- = L^+$$

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(1) $\lim_{x \to 2} \frac{x^2-1}{x+2} = \frac{3}{4}$

(2) $\lim_{x \to 0} \frac{x+7}{\frac{1}{x}+2} = \frac{0+7}{\frac{1}{0}+2} = \frac{7}{\infty+2} = \frac{7}{\infty} = 0$

notes → $\frac{1}{0} = \infty$ / $\frac{\infty}{1} = \infty$ /

$\frac{1}{\infty} = 0$

(3) $\lim_{x \to \infty} \frac{x-2}{5} = \frac{\infty-2}{5} = \frac{\infty}{5} = \infty$

(4) $\lim_{x \to -3} \frac{x^2+x-6}{x+3} = \frac{0}{0}$

$\lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)} = (x-2) = -5$

* Rules of limits :-

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(1) $\lim_{x \to a} f(x) = L$ ~~$\lim f(x)$~~

$\lim_{x \to a} g(x) = M$

(2) $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm$

$\lim_{x \to a} g(x) = L \pm M$

(3) $\lim_{x \to a} [f(x) \cdot g(x)] = [\lim_{x \to a} f(x)][\lim_{x \to a} g(x)]$

(4) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$

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Friday

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January

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الجمعة

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(5) $\lim_{x \to a} C = C$ ← C → constant

(6) $\lim_{x \to a} C f(x) = C \lim_{x \to a} f(x)$

$= C \cdot L$

(7) $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$

(8) $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$

Example (1)

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$$f(x) = \begin{cases} x^2 & x > 2 \\ \frac{1}{x} & x = 2 \\ x-2 & x < 2 \end{cases}$$ at $x = 2$

1:00 $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} x-2 = 0$

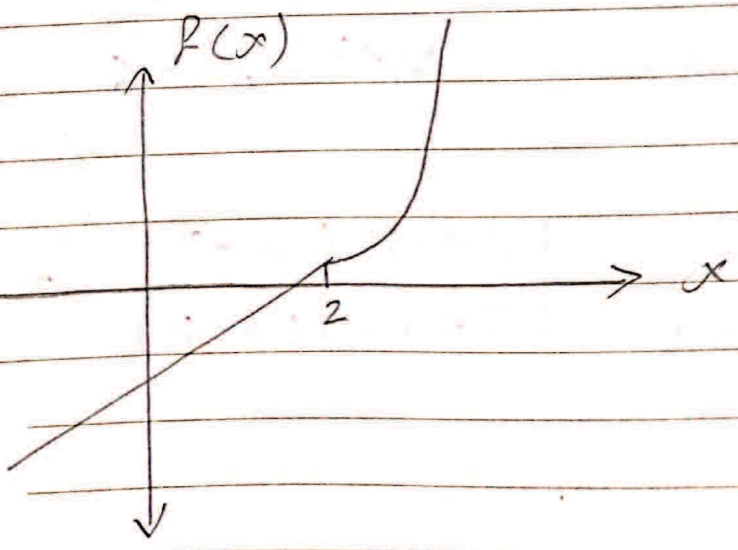
2:00
3:00
4:00

5:00 $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} x^2 = 4$

6:00
7:00

8:00 $\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x) \rightarrow$ DNE

9:00
10:00



* Find the limits of the function

① $\lim_{x \to 2} \frac{x^2-4}{x-2}$ بالتعويض المباشر $\frac{0}{0}$

$\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)} = \lim_{x \to 2} (x+2) = 4$

② $\lim_{x \to 2} \frac{2x^2+5x+2}{5x^2+7x-6}$ بالتعويض المباشر

$\lim_{x \to 2} \frac{8+10+2}{20+14-6} = \frac{20}{28}$

③ $\lim_{x \to 2} \frac{2x^2-5x+2}{5x^2-7x-6}$ بالتعويض المباشر $\frac{0}{0}$

$\lim_{x \to 2} \frac{(2x-1)(x-2)}{(5x+3)(x-2)} = \frac{3}{13}$

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8:00 * notes :-

9:00

10:00

11:00 ① $(x^2-y^2) = (x-y)(x+y)$

12:00

☼ ② $(x^3-y^3) = (x-y)(x^2+xy+y^2)$

1:00

2:00 ③ $(x^3+y^3) = (x+y)(x^2-xy+y^2)$

3:00

4:00 ④ $(x+y)^n = x^n + n(x^{n-1})y$

5:00

☾ $+ \frac{n(n-1)}{2}(x^{n-2})y^2 +$

6:00

7:00 $\frac{n(n-1)(n-2)}{3}(x^{n-3})y^3 + \cdots$

8:00

9:00 $+ y^n$

10:00

(4) $\lim_{h \to 0} \dfrac{(x+h)^3 - x^3}{h} = \dfrac{0}{0}$

$\lim_{h \to 0} \dfrac{(x+h-x)\left((x+h)^2 + x(x+h) + x^2\right)}{h}$

$= 3x^2$

⑤ $\lim_{x \to 1} \frac{\sqrt{x^2+1} - \sqrt{2}}{(x-1)}$ بالضرب في المرافق

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$$\lim_{x \to 1} \frac{(\sqrt{x^2+1} - \sqrt{2})(\sqrt{x^2+1} + \sqrt{2})}{(x-1)(\sqrt{x^2+1} + \sqrt{2})}$$

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$$\lim_{x \to 1} \frac{x^2+1-2}{(x-1)(\sqrt{x^2+1} + \sqrt{2})}$$

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$$\lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)(\sqrt{x^2+1} + \sqrt{2})} = \frac{2}{2\sqrt{2}}$$

7:00
8:00

$$= \frac{1}{\sqrt{2}}$$

9:00
10:00

*limits of trigomertic function :- نهايات الدوال المثلثية

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x}{\sin x} = 1$$

Sat. Sun. Mon. Tue. Wed. Thu. Fri.

$$\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{x}{\tan x} = 1$$

$$\lim_{x \to 0} \frac{\sin mx}{mx} = 1$$

$$\lim_{x \to 0} \frac{\cos x}{x} \neq 1$$

Example (1)

$$\lim_{x \to 0} \frac{\sin 7x}{\tan 5x}$$

$$\lim_{x \to 0} \frac{\sin 7x}{7x} \cdot \frac{5x}{\tan 5x} \cdot \frac{7x}{5x}$$

$$\frac{7}{5} \lim_{x \to 0} \frac{\sin 7x}{7x} \cdot \frac{5x}{\tan 5x}$$

$$= \frac{7}{5}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = \lim_{m\to 0}(1+m)^{\frac{1}{m}} = e$$

$$\text{if } \lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = \infty$$

$$\lim f(x)\cdot g(x) = m$$

$$\text{then } \lim_{x\to x_0}\left(1+f(x)\right)^{g(x)} = e^{m}$$

Example (1)

$$\lim_{x\to 0}(1+4\sin x)^{\frac{3}{x}}$$

$$\lim_{x\to x_0} f(x) = \lim_{x\to 0} 4\sin x = 0$$

$$\lim_{x\to x_0} g(x) = \lim_{x\to 0}\frac{x}{3} = \infty$$

$$\lim_{x\to x_0} f(x)\cdot g(x) = \lim_{x\to 0} 4\sin x\left(\frac{3}{x}\right)$$

$$= 12\lim_{x\to 0}\frac{\sin x}{x} = 12$$

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Saturday

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$$\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{8 \tan x}$$

$$\lim_{x \to x_0} f(x) = \lim_{x \to \frac{\pi}{2}} \cos x = \lim_{x \to x_0} g(x) =$$

$$\lim_{x \to \frac{\pi}{2}} 8 \tan x = \infty$$

$$\lim_{x \to x_0} f(x) \cdot g(x) = \lim_{x \to \frac{\pi}{2}} \cos x \cdot 8 \tan x$$

$$= 8 \lim_{x \to \frac{\pi}{2}} \cos x \cdot \frac{\sin x}{\cos x} = 8$$