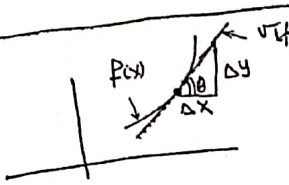
Deritvative الإشتقاق or Differentiation

* تعريفة الرياضي :

$$\boxed{\frac{d\,f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

* تعريفة المهندس: هو ميل المماس للدالة f(x)



وهو

$$\boxed{m = \frac{\Delta y}{\Delta x} = \tan(\theta)}$$

* رمز المشتقة الأولى :

y' or $\frac{dy}{dx}$ or $y^{(1)}$

* رمز المشتقة الثانية :

y'' or $\frac{d^2y}{dx^2}$ or $y^{(2)}$

وهكذا ...

* بعض خصائص الإشتقاق

1. تفاضل، ثابتة: $\frac{d}{dx}(\text{Const.}) = 0$

2. دالة، ثابتة *: $\frac{d}{dx}(k \cdot f(x)) = k * f'(x)$

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3. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

4. $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

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5. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$

$= \frac{\text{المقام * تفاضل البسط} - \text{البسط * تفاضل المقام}}{(\text{المقام})^2}$ ← عربي

$\boxed{6}$ $\frac{d}{dx}\left[f(x)\right]^{n} = n\left[f(x)\right]^{n-1} \cdot f'(x)$ عدد ← n

$\boxed{7}$ $\frac{d}{dx} X^{n} \longrightarrow n X^{n-1}$ ثابت

$\boxed{8}$ $\frac{d}{dx} Ln(x) \longrightarrow \frac{1}{x}$

$\frac{d}{dx} Ln(u) \longrightarrow \frac{1}{u} \cdot u'$

$\boxed{9}$ $\frac{d}{dx} Log_{a}(u) = \frac{1}{u} \cdot u' \cdot Log_{a}(e)$

$\boxed{10}$ $\frac{d}{dx} e^{x} = e^{x}$

$\frac{d}{dx} e^{u} = e^{u} \cdot u'$

$\boxed{11}$ $\frac{d}{dx} a^{u} = a^{u} \cdot u' \cdot Ln(a)$

نفسها * تفاضل أسها

* Ln(العدد).

تفاضل (دالة)$^{\text{دالة}}$

$\boxed{\text{الشيات مهم}}$

$\boxed{12}$ $y = \left(f(x)\right)^{g(x)}$

← بأخذ Ln للطرفين

$\therefore Ln(y) = Ln\left[f(x)\right]^{g(x)}$ ← من خصائص Ln انه بتنزل الأس

بتفاضل الطرفين

$\therefore Ln(y) = \underline{g(x)} \cdot \underline{Ln(f(x))}$

بالضرب $*y$:

$\therefore \frac{1}{y} \cdot y' = g(x) \cdot \frac{1}{f(x)} \cdot f'(x) + Ln(f(x)) \cdot g'(x)$

$$\boxed{\therefore y' = y\left[\frac{g(x)}{f(x)} \cdot f'(x) + Ln(f(x)) \cdot g'(x)\right]} = \boxed{(f)^{g}\left[\frac{g}{f} f' + Ln(f) \cdot g'\right]}$$

Simple Examples

* Find $\frac{dy}{dX}$ for

[1] $y = \underbrace{e^{x^2}}_{u} + \underbrace{Ln(x^2-1)}_{v} + \underbrace{\frac{Log(x)}{Ln(\sqrt{x})}}_{w}$

الحل

$\boxed{y' = u' + v' + w'} \Rightarrow (*)$ $\quad \because u = e^{x^2}$ $\quad \boxed{\therefore u' = e^{x^2} \cdot 2X}$

$\because v = Ln(x^2-1)$ $\quad \boxed{\therefore v' = \frac{1}{x^2-1} \cdot 2X}$

$\because w = \frac{Log(x)}{Ln(\sqrt{x})}$ قسمة دالتين

$\therefore w' = \frac{Ln(\sqrt{x}) \cdot \frac{1}{x} Log(e) - Log(x) \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}{[Ln(\sqrt{x})]^2}$

بالتعويض في (*) #

[2] $y = \underbrace{7^{\sqrt{x\sqrt{x\sqrt{x}}}}}_{u} + \underbrace{Log_3(x^2+Ln(x))}_{v} - \underbrace{\frac{20}{X} \cdot e^{-3\sqrt{x}}}_{w}$

الحل

$\therefore y' = u' + v' - w' \longrightarrow (*)$

$\because u = 7^{\sqrt{x\sqrt{x\sqrt{x}}}} = 7^{\sqrt{x\sqrt{x \cdot x^{1/2}}}} = 7^{\sqrt{x\sqrt{x^{3/2}}}} = 7^{\sqrt{x \cdot x^{3/4}}} = 7^{\sqrt{x^{7/4}}} = 7^{x^{7/8}}$

$\boxed{\therefore u' = 7^{x^{7/8}} \cdot \frac{7}{8} x^{-\frac{1}{8}} \cdot Ln(7)}$ #

$\because v = Log_3[x^2 + Ln(x)] \Longrightarrow$ $\quad \therefore v' = \frac{1}{x^2+Ln(x)} \cdot \left(2x + \frac{1}{x}\right) \cdot Log_3(e)$

$\because w = \frac{20 e^{-3\sqrt{x}}}{X} \Longrightarrow$ $\quad \therefore w' = \frac{x \cdot 20 \cdot e^{-3\sqrt{x}} \cdot (-3)\frac{1}{\sqrt{x}} - 20 e^{-3\sqrt{x}} \cdot 1}{(x)^2}$

حاصل قسمة دالتين

بالتعويض في (*) #

3) $y = \underbrace{[x^2+3x]^9 \cdot Log(Log(x))}_{u} + \underbrace{2\sqrt{x} \cdot Ln[Ln(\log_3 x)]}_{V}$

الحل

$\acute{y} = \acute{u} + \acute{V} \rightarrow \boxed{*}$

$u =$ حاصل ضرب دالتين

$\therefore u' = [x^2+3x]^9 \cdot \frac{Log(e)}{Log(x)} \cdot \frac{1}{x} Log(e) + Log(Log(x)) \cdot 9[x^2+3x]^8 \cdot (2x+3)$

و $V =$ حاصل ضرب دالتين

$\therefore V' = 2\sqrt{x} \cdot \frac{1}{Ln(\log_3(x))} \cdot \frac{1}{\log_3(x)} \cdot \frac{1}{x}\log_3(e) + Ln[Ln(\log_3(x))] \cdot \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x}}$

عوض في $\boxed{*}$ #

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4) $y = \underbrace{e^{\sqrt{\frac{x^2-2}{Ln(x)}}}}_{u} - \underbrace{e^{Ln[Log\sqrt{x-1}]}}_{V} + \underbrace{x^{(x-2)}}_{\omega}$

الحل

$\acute{y} = u' - v' + \omega' \rightarrow \boxed{*}$ $\because u = e^{\sqrt{\frac{x^2-2}{Ln(x)}}}$

$\therefore u' = e^{\sqrt{\frac{x^2-2}{Ln x}}} \cdot \frac{1}{2\sqrt{\frac{x^2-2}{Ln x}}} \cdot \frac{Ln(x)(2x) - (x^2-2)\cdot\frac{1}{x}}{(Ln x)^2}$

$\because V = \cancel{e}^{\cancel{Ln}[Log\sqrt{x-1}]} = Log(\sqrt{x-1})$

$\therefore v' = \frac{1}{\sqrt{x-1}} \cdot \frac{1}{2\sqrt{x-1}} \cdot Log(e)$

$\because \omega = x^{(x-2)}$ دالة (دالة)

بأخذ Ln للطرفين

$\therefore Ln(\omega) = Ln\, x^{(x-2)} = (x-2) \cdot Ln(x)$

بالتفاضل:

$\therefore \frac{1}{\omega} \cdot \omega' = [(x-2) \cdot \frac{1}{x} + Ln(x) \cdot 1]$

بالضرب × ω

$\therefore \omega' = \omega \left[\frac{x-2}{x} + Ln(x)\right]$

$\boxed{\therefore \omega' = x^{x-2}\left[\frac{x-2}{x} + Ln(x)\right]}$

عوض في * #

$\boxed{5}$ $y = \underbrace{Ln\left[Ln\left[Ln\left[\sqrt{x^2+\log(x)}\right]\right]\right]}_{u} - \underbrace{x^{x^{x}}}_{v}$

$y' = u' + v' \rightarrow \boxed{*}$

$$u' = \frac{1}{Ln\left[Ln\left[\sqrt{x^2+\log(x)}\right]\right]} \cdot \frac{1}{Ln\left[\sqrt{x^2+\log(x)}\right]} \cdot \frac{1}{\sqrt{x^2+\log(x)}} \cdot \frac{1}{2\sqrt{x^2+\log(x)}} \cdot \left(2x + \frac{1}{x}\log(e)\right)$$

$\therefore V = x^{x^{x}} \xrightarrow{} \omega \quad = x^{\omega}$

بأخذ Ln للطرفين

$\therefore Ln(V) = Ln\left(x^{\omega}\right)$

$Ln(V) = \omega \cdot Ln(x)$ بتفاضل الطرفين

$\therefore \frac{1}{V} \cdot V' = \omega \cdot \frac{1}{x} + Ln(x) \cdot \omega'$

بالضرب V *

$\therefore V' = V\left[\frac{\omega}{x} + Ln(x) \cdot \omega'\right]$

$\therefore V' = x^{x^{x}}\left[\frac{x^{x}}{x} + Ln(x) \cdot x^{x}\left(1 + Ln(x)\right)\right]$

$\omega = x^{x}$ بأخذ Ln للطرفين

$\therefore Ln\,\omega = x \cdot Ln(x)$ بالتفاضل

$\frac{1}{\omega} \cdot \omega' = x \cdot \frac{1}{x} + Ln(x) \cdot 1$

$\omega' = \omega\left[1 + Ln(x)\right]$

$\boxed{\therefore \omega' = x^{x}\left[1 + Ln(x)\right]}$

عوضنا في $\boxed{*}$ #

$\boxed{6}$ find $\frac{dy}{dx}$ from

حل بإيدك :

$\log\left(x - \sqrt{x}\right)$

$y = x^2 \cdot \log\left[\log\left[Ln\left(\sqrt{x^2+\sqrt{x}}\right)\right]\right] + \left[Ln(x-1)\right]$ #

$\boxed{7}$ $y = \left[\frac{3x-2}{x^2-4}\right]^7 + Ln\left[\log\left[\frac{x^2-1}{\sqrt{x+3}}\right]\right]$ #

(5)

{ التفاضل (الاشتقاقة) الضمني } → (Implicit Derivatives)

* عندما تكون x, y عبارة عن حدود مشتركة ولا يمكن جعل y لوحدها في طرف .

الحل
نقوم بتفاضل الطرفين بالنسبة لـ X :

$\boxed{Ex. (8)}$: $y^3 + 3x.y + x^3 - 5 = 0$

لا يمكن جعل y لوحدها في طرف ∴ نفاضل الطرفين بالنسبة لـ X

$$\therefore\ 3y^2 . \frac{dy}{dx} + 3\underbrace{\left[1 . \cancel{\frac{dx}{dx}} . y + \frac{dy}{dx} . x\right]}_{} + 3x^2 . \frac{dx}{dx} = 0$$

حيث أن $X.Y$ عبارة عن حاصل ضرب دالتين

$$3y^2 \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} + 3x^2 = 0$$

خذ $\frac{dy}{dx}$ عامل مشترك من الحدود اللي موجود فيها $\frac{dy}{dx}$

$$\therefore\ \frac{dy}{dx}(3y^2 + 3x) + 3y + 3x^2 = 0$$

$$\therefore\ \frac{dy}{dx} = \frac{-\cancel{3}(y + x^2)}{\cancel{3}(y^2 + x)} = \frac{-(y + x^2)}{(y^2 + x)}$$

#

$$\boxed{9} \quad y^5 + 3x^2y^3 - 7x^6 - 8 = 0$$

بتفاضل الطرفين بالنسبة لـ x :

$$\therefore 5y^4 \cdot \frac{dy}{dx} + 3\left(2x \cdot y^3 + 3y^2 \frac{dy}{dx} \cdot x^2\right) - 42x^5 + 0 = 0$$

$$\therefore 5y^4 \cdot \frac{dy}{dx} + 6xy^3 + 9y^2 \frac{dy}{dx} \cdot x^2 - 42x^5 = 0$$

$$\therefore \frac{dy}{dx}\left[5y^4 + 9y^2x^2\right] + 6xy^3 - 42x^5 = 0$$

$$\therefore \frac{dy}{dx} = \frac{-6xy^3 + 42x^5}{5y^4 + 9x^2y^2} = \tan(\theta)$$

ممكن يعطيني (x,y) أعوضها في معادلة $\frac{dy}{dx}$ وأحسب $\tan\theta$ ثم $\theta = \tan^{-1}(\ )$

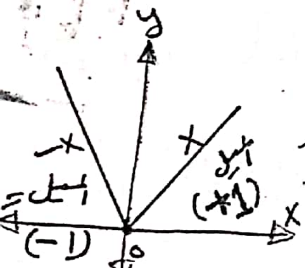
(ملاحظة هامة) : إذا كانت الدالة لها مشتقة عند نقطة

فإنها تكون متصلة عند هذه النقطة .

والعكس ليس بالضرورة أن يكون صحيح (أي إذا كانت

الدالة متصلة عند نقطة فليس بالضرورة أن يكون لها مشتقة عند

هذه النقطة) .



مثال ذلك : الدالة $\boxed{f(x) = |x|}$ متصلة عند $x = 0$

ولكن ليس لها مشتقة عند $x = 0$

مهم

لأن ميل المماس عند الصفر $= +1$ و -1

وليس قيمة واحدة فقط

EX 10 find $\frac{dy}{dx}$ from:

$$e^{x\sqrt{y^2-1}} - 5\ln(x^2-4y) + 7^{x\cdot y} = e^{x^2}$$

الحل : تفاضل ضمني

نتفاضل الطرفين بالنسبة لـ x :

$$\therefore e^{x\sqrt{y^2-1}}\cdot\left[x\cdot\frac{1}{2\sqrt{y^2-1}}\cdot 2y\cdot y' + \sqrt{y^2-1}\cdot 1\right] - 5\cdot\frac{1}{x^2-4y}\cdot(2x-4y')$$

$$+ 7^{x\cdot y}\cdot(xy'+y\cdot 1) = e^{x^2}\cdot 2x$$

$$\therefore \frac{xy\cdot e^{x\sqrt{y^2-1}}}{\sqrt{y^2-1}}\, y' + e^{x\sqrt{y^2-1}}\cdot\sqrt{y^2-1} - \frac{5(2x)}{x^2-4} + \frac{5(4)}{x^2-4y}\, y' + 7^{xy}\cdot x\, y'$$

$$+ 7^{xy}\cdot y = 2x\cdot e^{x^2}$$

$$y'\underbrace{\left[\frac{xy\,e^{x\sqrt{y^2-1}}}{\sqrt{y^2-1}} + \frac{20}{x^2-4y} + 7^{xy}\cdot x\right]}_{B} = \underbrace{-e^{x\sqrt{y^2-1}}\cdot\sqrt{y^2-1} + \frac{10x}{x^2-4} - 7^{xy}\cdot y + 2xe^{x^2}}_{A}$$

$$\therefore y' = \frac{A}{B} \quad \#$$

EX 11 find $\frac{dy}{dx}$ from:

$$\ln[\log(x\cdot y)] - \frac{4x^2}{x^3-y^3} + 5\,7^{x} = e^{y}$$

الحل

نتفاضل الطرفين بالنسبة لـ x

$$\therefore \frac{1}{\log(xy)}\cdot\frac{\log(e)}{xy}\cdot(xy'+y\cdot 1) - \frac{[x^3-y^3]\cdot 8x - 4x^2(3x^2-3y^2\cdot y')}{[x^3-y^3]^2} + 5\cdot 7^{x}\ln 7 = e^{y}y'$$

$$\therefore \frac{x\log(e)\cdot y'}{xy\log(xy)} + \frac{y\log(e)}{xy\log(xy)} - \frac{8x(x^3-y^3)}{(x^3-y^3)^2} + \frac{12x^4}{(x^3-y^3)^2} - \frac{12x^2y^2\, y'}{(x^3-y^3)^2} + 5\cdot 7^{x}\ln 7 = e^{y}\, y'$$

عامل مشترك كما سبق ... كمل .

Chain Rule قاعدة السلسلة

معطى Given $y = f(x)$

و $x = g(t)$

مطلوب $\boxed{\dfrac{dy}{dt}} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt}$

$y \rightarrow x \rightarrow t$

Example 12: if $y = e^{\sin(x^2)} + \ln\left[\log\sqrt{x^2-1}\right]$

و $x = \underbrace{3t^2}\cdot\underbrace{\sin(t)} + \underbrace{2\cos(t)}\cdot\underbrace{\ln(\sqrt{t})}$

Find $\dfrac{dy}{dt}$ (solution)

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

نجيب: $\dfrac{dy}{dx} = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x + \dfrac{1}{\log(\sqrt{x^2-1})} \cdot \dfrac{\log(e)}{\sqrt{x^2-1}} \cdot \dfrac{1}{2\sqrt{x^2-1}} \cdot 2x$

حاصل ضرب دالتين

و $\dfrac{dx}{dt} = 3t^2 \cdot \cos(t) + \sin(t) \cdot (6t) + 2\cos(t) \cdot \dfrac{1}{\sqrt{t}} \cdot \dfrac{1}{2\sqrt{t}}$

$+ \ln(\sqrt{t})\,(-2\sin(t))$

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EX: 13: if $y = (3x^2 + 5x - 1) \cdot 7^{x^2-1}$ حل بإيدك

و $x = 2t^2 \cdot \ln(3t)$

Find $\dfrac{dy}{dt}$ #

الدوال المثلثية

$\sin\theta$ ، $\cos\theta$ ، $\tan\theta$

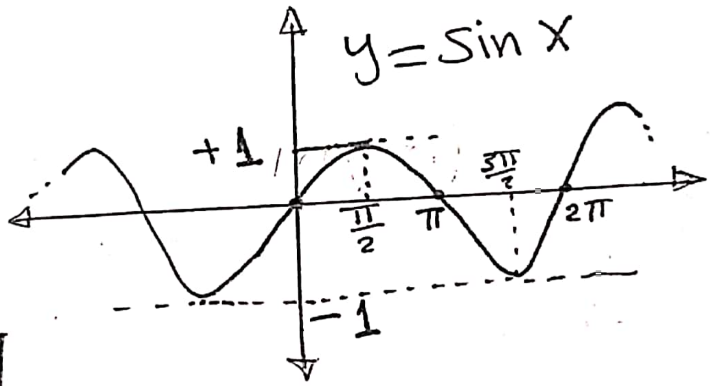
، $\sec\theta = \dfrac{1}{\cos\theta}$

، $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

، $\cot\theta = \dfrac{1}{\tan\theta}$

بعض أشكال الدوال المثلثية وخصائصها:

$\boxed{\text{I} \quad y = \sin(x)}$



مدى محور X — Domain (المجال) $= R$

المدى محور y — $R_f = [-1, 1]$

- الدالة فردية (متماثلة حول نقطة الأصل) odd F^n

$\boxed{\therefore \sin(-x) = -\sin x}$

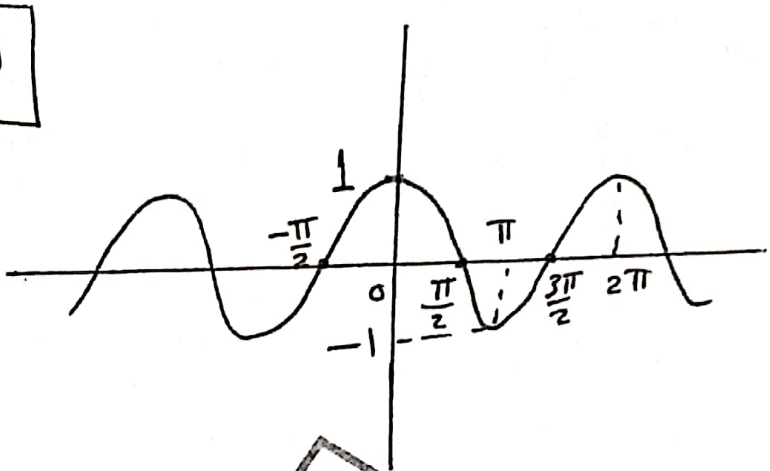
$(f(-x) = -f(x))$

الدالة دورية Periodic : أى أنها تكرر نفسها كل فترة (دورة)

ودورتها هي 2π

$\boxed{2}$ $y = \cos(x)$

- المجال $= R$
- المدى $= [-1 , 1]$
- الدالة دورية
- الدالة زوجية ← أي متماثلة حول محور y



$$\therefore f(-x) = f(x)$$

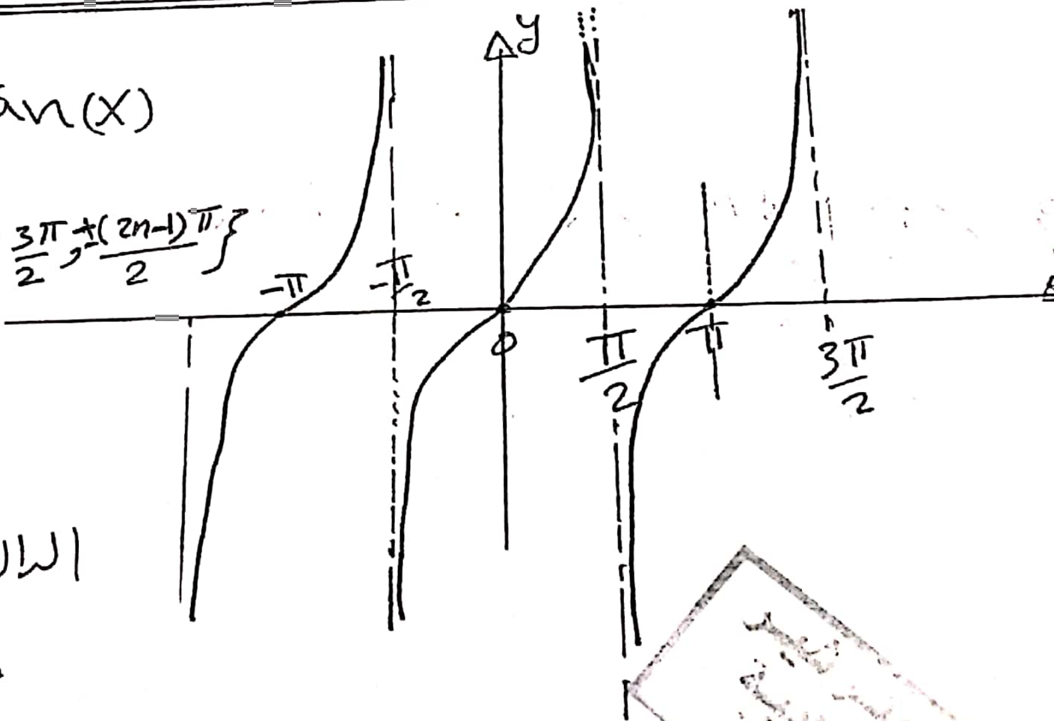
$$\boxed{\therefore \cos(-x) = \cos x}$$

$\boxed{3}$ $y = \tan(x)$

المجال $= R - \left\{ \pm\frac{\pi}{2} , \pm\frac{3\pi}{2} , \pm\frac{(2n-1)\pi}{2} \right\}$

المدى $R_f = R$

الدالة فردية أي متماثلة حول نقطة الأصل



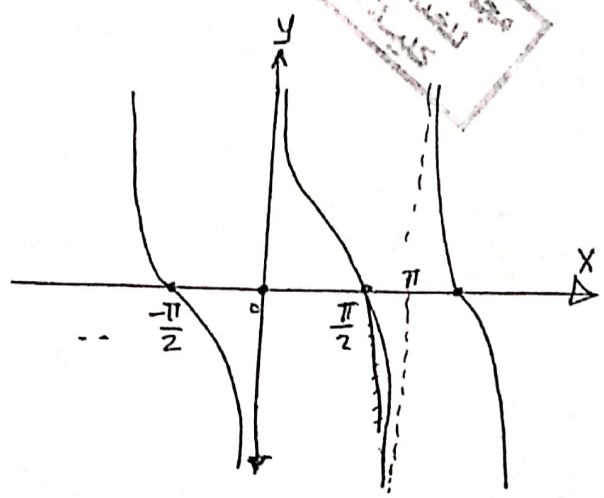
$$\therefore \boxed{\tan(-x) = -\tan(x)}$$

الدالة دورية أيضاً أي تتكرر

$\boxed{4}$ $y = \cot(x) = \dfrac{1}{\tan(x)}$

مجال $D_f = R - \{\pm n\pi\}$

مدى $R_f = R$

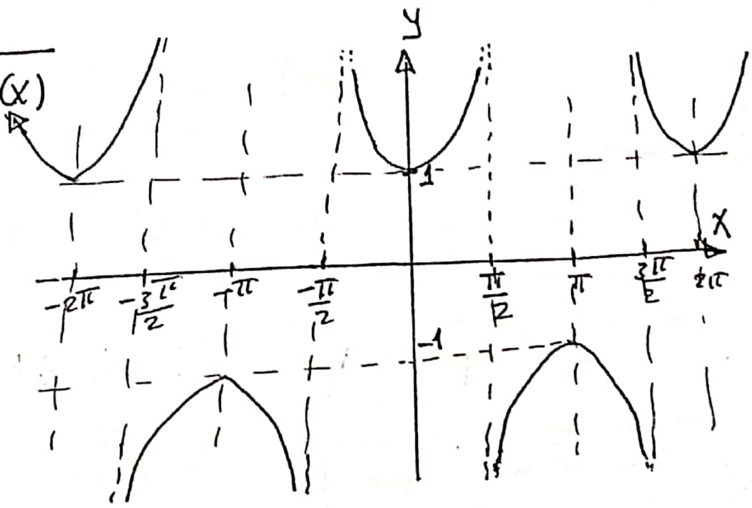


$\boxed{5}$ $y = \sec(x) = \dfrac{1}{\cos(x)}$

$D_f = R - \{\pm\underbrace{(2n+1)}_{}\frac{\pi}{2}\}$

أي عدد فردي
مضروب $* 90^\circ$

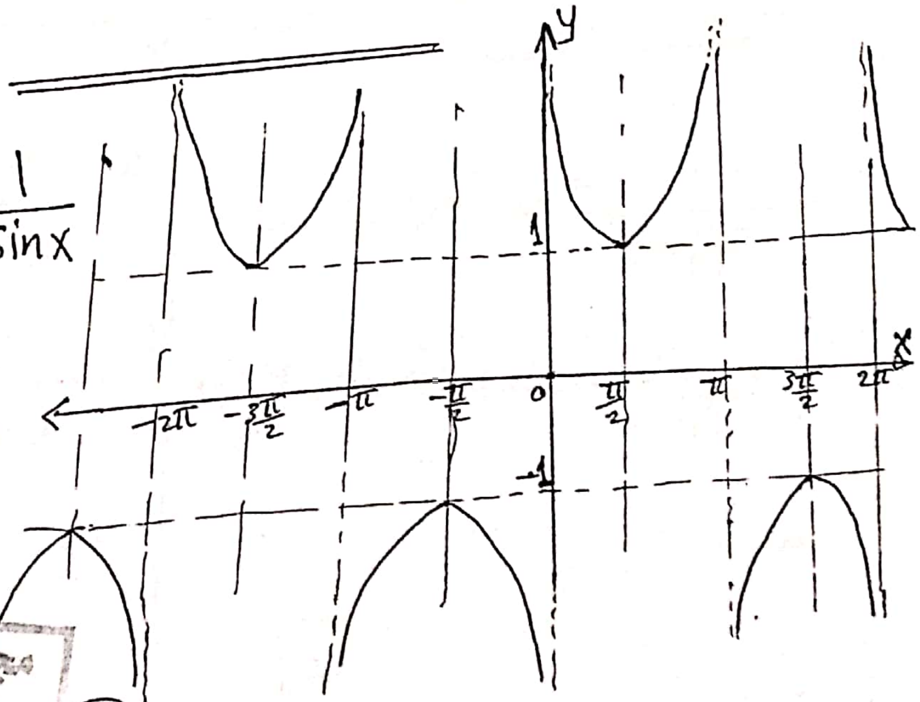
$R_f = R -]-1,1[$



$\boxed{6}$ $y = \operatorname{Cosec}(x) = \dfrac{1}{\sin x}$

$D_f = R - \{\pm n\pi\}$

$R_f = R -]-1,1[$



(حفظ!)

* مشتقات الدوال المثلثية :-

$\sin(x) \xrightarrow{\text{تفاضل}} \cos(x)$

$\sin(u) \longrightarrow \cos(u) \cdot u' \longleftarrow u' = \frac{du}{dx}$ و $u = u(x)$

$\cos(u) \longrightarrow -\sin(u) \cdot u'$

$\tan(u) \longrightarrow \sec^2(u) \cdot u'$

$\cot(u) \longrightarrow -\operatorname{cosec}^2(u) \cdot u'$

$\sec(u) \Longrightarrow \sec(u) \cdot \tan(u) \cdot u'$

$\operatorname{cosec}(u) \longrightarrow -\operatorname{cosec}(u) \cdot \cot(u) \cdot u'$

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* الدوال المثلثية العكسية :-

1. $y = \sin^{-1}(x) \xrightarrow{\text{تفاضل}} \frac{1}{\sqrt{1-x^2}}$

   $\sin^{-1}(u) \longrightarrow \frac{1}{\sqrt{1-u^2}} \cdot u'$

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2. $y = \cos^{-1}(x) \xrightarrow{\text{تفاضل}} \frac{-1}{\sqrt{1-u^2}} \cdot u'$

3. $y = \tan^{-1}(u) \xrightarrow{\text{"}} \frac{1}{1+u^2} \cdot u'$

4. $y = \cot^{-1}(u) \xrightarrow{\text{"}} \frac{-1}{1+u^2} \cdot u'$

5. $y = \sec^{-1}(x) \xrightarrow{\text{تفاضل}} \frac{1}{|u|\sqrt{u^2-1}} \cdot u'$

6. $y = \operatorname{cosec}^{-1}(u) \xrightarrow{\text{"}} \frac{-1}{|u|\sqrt{u^2-1}} \cdot u'$

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إثباتات مشتقات
الدوال المثلثية
العكسية

$y = \sin^{-1}(x) \iff x = \sin y$

Find $\frac{dy}{dx}$

Solution نفرض $\frac{dx}{dy} = \cos y$

$\therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{1}{\cos y}$

$\because \cos^2 y + \sin^2 y = 1$

$\therefore \cos^2 y = 1 - \sin^2 y$

$\therefore \cos y = \sqrt{1 - \sin^2 y}$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 - \sin^2 y}}$

$\because x = \sin y \Rightarrow \therefore \sin^2 y = x^2$

$$\boxed{\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}}$$ ✓ //

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كلية الهندسة

$\therefore\ y = \cos^{-1}(x) \iff x = \cos y$

Find $\frac{dy}{dx}$

$$\sin^2 y + \cos^2 y = 1$$

نوجد $\frac{dx}{dy} = -\sin y$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{-\sin y} = \frac{-1}{\sqrt{1-\cos^2 y}}$$

$\because \cos y = x$

$\therefore \cos^2 y = x^2$

$$\therefore \frac{dy}{dx} = \frac{\boxed{-}1}{\sqrt{1-x^2}}$$

الفرق بينه وبين $\sin^{-1}(x)$ هو إشارة السالب

$y = \tan^{-1} x \iff x = \tan y$

$$\frac{dx}{dy} = \sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \boxed{\frac{1}{1+x^2}} \checkmark$$

EX

$\cot^{-1}(x) = y \Leftrightarrow x = \cot y$

$$\frac{dx}{dy} = -\operatorname{cosec}^2 y = -(1+\cot^2 y)$$

$$= -(1+x^2)$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{-1}{1+x^2}} = \frac{1}{\frac{dx}{dy}}$$

ملحوظة :

$$\boxed{\begin{aligned} 1+\tan^2 x &= \sec^2 x \\ 1+\cot^2 x &= \operatorname{cosec}^2 x \end{aligned}}$$

$$y = \sec^{-1}(x) \Longleftrightarrow x = \sec y$$

$$\frac{dx}{dy} = \sec y \cdot \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\because \sec y = x$$

$$, 1 + \tan^2 y = \sec^2 y$$

$$\therefore \tan^2 y = \sec^2 y - 1$$

$$\therefore \tan y = \sqrt{\sec^2 y - 1} = \sqrt{x^2 - 1}$$

$$\boxed{\therefore \frac{dy}{dx} = \frac{\pm 1}{x\sqrt{x^2-1}}}$$

$$\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$$

$$y = \csc^{-1} x \longleftarrow \boxed{x = \csc y}$$

$$\therefore \frac{dx}{dy} = -\csc y \cdot \cot y$$

$$\because 1 + \cot^2 y = \csc^2 y$$

$$\therefore \cot^2 y = \csc^2 y - 1$$

$$\therefore \cot y = \sqrt{\csc^2 y - 1} = \sqrt{x^2 - 1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{\mp 1}{x\sqrt{x^2-1}}$$

2

$$= \frac{-1}{|u|\sqrt{u^2-1}}\, u'$$

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Hyperbolic f^{ns} الدوال الزائدية

← هي دوال تختص بدراسة نقاط ع القطع الزائد . HyPerbola

← تعريفها الرياضي :

$$\boxed{1}\ y = \sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

حفظ

$$\boxed{2}\ y = \cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

$$\boxed{3}\ y = \tanh(x) = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\boxed{4}\ y = \coth(x) = \frac{1}{\tanh x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$$\boxed{5}\ y = \operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^{x} + e^{-x}}$$

$$\boxed{6}\ y = \operatorname{cosech}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^{x} - e^{-x}}$$

مقارنة

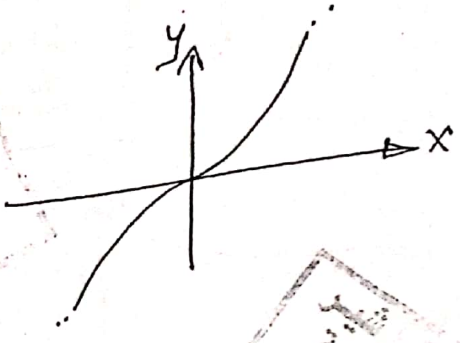
الدوال المثلثية	الدوال الزائدية
	$\cosh^2(x) - \sinh^2(x) = 1$
1. $\cos^2(x) + \sin^2(x) = 1$	$1 - \tanh^2(x) = \operatorname{sech}^2(x)$
2. $1 + \tan^2(x) = \sec^2(x)$	$\coth^2(x) - 1 = \operatorname{cosech}^2(x)$
3. $1 + \cot^2(x) = \operatorname{cosec}^2(x)$	
4. $\cos(0) = 1$, $\sin(0) = 0$	$\cosh(0) = 1$, $\sinh(0) = 0$

* HYPerbolic functions

تسمى الدوال الزائدية

← نسبةً للقطع الزائد .

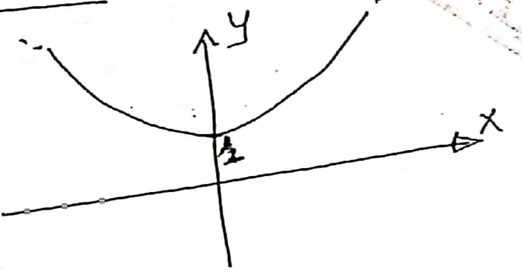
$\boxed{1}$ $y = \sinh(x) = \dfrac{e^{x} - e^{-x}}{2}$



$D_f = \mathbb{R}$

$R_f = \mathbb{R}$

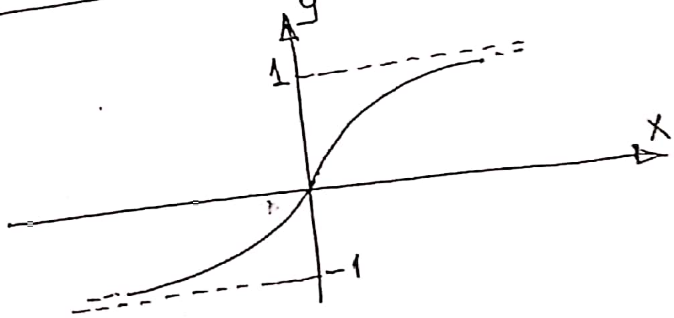
$\boxed{2}$ $y = \cosh(x) = \dfrac{e^{x} + e^{-x}}{2}$



$D_f = \mathbb{R}$

$R_f = [1, \infty[$

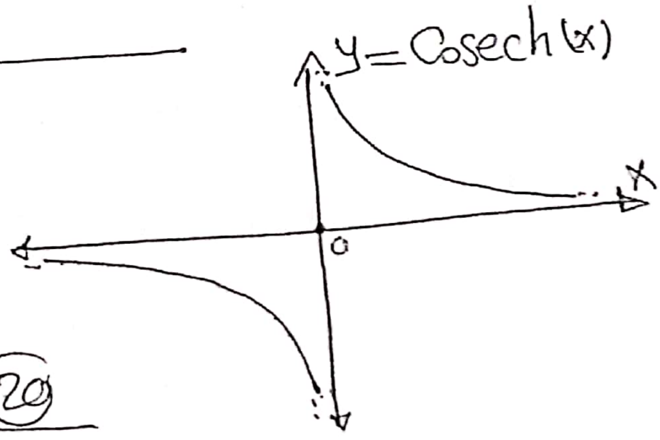
$\boxed{3}$ $y = \tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$



$D_f = \mathbb{R}$

$R_f =]-1, 1[$

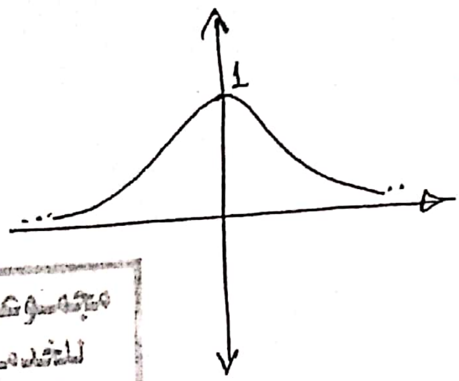
$\boxed{4}$ $y = \operatorname{Cosech}(x) = \dfrac{1}{\sinh(x)}$



$D_f = \mathbb{R} - \{0\}$

$R_f = \mathbb{R} - \{0\}$

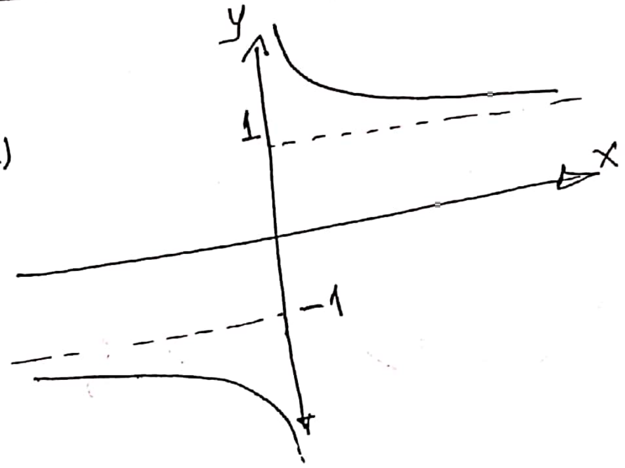
$\boxed{5}$ $y = \mathrm{Sech}(x) = \dfrac{1}{\cosh(x)}$



$D_f = \mathbb{R}$

$R_f =]0, 1]$

$\boxed{6}$ $y = \mathrm{Coth}(x) = \dfrac{1}{\tanh(x)}$



$D_f = \mathbb{R} - \{0\}$

$R_f = \mathbb{R} - [-1, 1]$

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* بعض خصائص الدوال الزائدية ومقارنتها بالدوال المثلثية :

زائدية

مثلثية

$\boxed{1}$ $\sin^2(x) + \cos^2(x) = 1$ $\longleftrightarrow$ $\boxed{1}$ $\cosh^2(x) - \sinh^2(x) = 1$

الإثبات $\Rightarrow \left(\dfrac{e^{x} + e^{-x}}{2}\right)^2 - \left(\dfrac{e^{x} - e^{-x}}{2}\right)^2$

$= \frac{1}{4}\left[\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2 - \cancel{e^{-2x}}\right]$

$= 1$

$\boxed{2}$ $1 + \tan^2(x) = \sec^2(x)$ $\longleftrightarrow$ $\boxed{2}$ $1 - \tanh^2(x) = \mathrm{Sech}^2(x)$

$\boxed{3}$ $1 + \cot^2(x) = \mathrm{Cosec}^2(x)$ $\longleftrightarrow$ $\boxed{3}$ $\coth^2(x) - 1 = \mathrm{Cosech}^2(x)$

ثباتات

كتابة الدوال الزائدية العكسية بدلالة الدالة اللوغاريتمية

1 Prove that $\sinh^{-1}(x) = \ln\left(x+\sqrt{x^2+1}\right)$

Proof: let $y = \sinh^{-1}(x)$

$$\therefore x = \sinh(y) = \frac{e^{y} - e^{-y}}{2}$$

$\therefore e^{y} - e^{-y} = 2x$ بالضرب $* e^{y}$:

$\therefore e^{2y} - 1 = 2xe^{y}$ $\therefore e^{2y} - 2xe^{y} - 1 = 0$

دالة تربيعية في المتغير e^{y} $(e^{y})^2 - 2x(e^{y})^1 - 1 = 0$

Put $e^{y} = z \Rightarrow \therefore \underbrace{z^2}_{A=1} \underbrace{- 2x.z}_{B=-2x} \underbrace{-1}_{C=-1} = 0$

نحلها بالقانون العام
$Az^2 + Bz + C = 0$
$\therefore z = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$\therefore z = \frac{2(x) \pm \sqrt{4x^2 - 4(1)(-1)}}{2(1)}$$

$$z = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = \frac{\cancel{2}x \pm \cancel{2}\sqrt{x^2+1}}{\cancel{2}} = x \pm \sqrt{x^2+1}$$

$e^{y} = x \pm \sqrt{x^2+1}$ بأخذ Ln للطرفين

$$\therefore \ln e^{y} = \ln\left(x \pm \sqrt{x^2+1}\right)$$

$$\boxed{\therefore y = \ln\left(x + \sqrt{x^2+1}\right) = \sinh^{-1}(x)}$$

السالب مرفوض لأن ما بداخل Ln لازم يكون عدد موجب

(Ln مدخلات دالة)

[2] $\cosh^{-1}(x) = \ln\left(x+\sqrt{x^2-1}\right)$ الحل بأثبت أن

---

[3] Prove that: $\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

الحل

let $y = \tanh^{-1}(x) \Longrightarrow \therefore \frac{x}{1} = \tanh(y) = \frac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$

طرفين * وسطين

$\therefore x e^{y} + x e^{-y} = e^{y} - e^{-y}$ (بالضرب * e^{y})

$\therefore x e^{2y} + x = e^{2y} - 1$ $\quad \therefore x e^{2y} - e^{2y} + x + 1 = 0$

$e^{2y}(x-1) = -(x+1)$ $\quad \therefore e^{2y} = \frac{-(x+1)}{x-1} = \frac{1+x}{1-x}$ بأخذ Ln للطرفين

$\therefore \ln e^{2y} = \ln\left(\frac{1+x}{1-x}\right)$ $\quad \therefore 2y = \ln\left(\frac{1+x}{1-x}\right)$

$$\boxed{\therefore y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = \tanh^{-1}(x)}$$ #

---

[4] Prove that $\coth^{-1}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ Proof

let $y = \coth^{-1}(x)$

$\therefore x = \coth(y) = \frac{\cosh(y)}{\sinh(y)}$

$\therefore x = \frac{e^{y}+e^{-y}}{e^{y}-e^{-y}}$ طرفين * وسطين

$\therefore x e^{y} - x e^{-y} = e^{y} + e^{-y}$ (بالضرب * e^{y})

$\therefore x e^{2y} - x = e^{2y} + 1$

$\therefore x e^{2y} - e^{2y} = x + 1$

$\therefore e^{2y}(x-1) = x + 1$

$\therefore e^{2y} = \frac{x+1}{x-1}$ (بأخذ Ln للطرفين)

$\therefore 2y = \ln\left(\frac{x+1}{x-1}\right)$

$$\boxed{\therefore y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) = \coth^{-1}(x)}$$

#

5 Prove that $\operatorname{sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ (Proof)

let $y = \operatorname{sech}^{-1}(x)$

$\therefore x = \operatorname{sech}(y) = \frac{1}{\cosh(y)}$

$\therefore x = \frac{2}{e^{y}+e^{-y}}$ طرفين * وسطين

$\therefore x e^{y} + x e^{-y} = 2$ (بالضرب * e^y)

$\therefore x e^{2y} + x = 2e^{y}$

$\boxed{\therefore \underbrace{x}_{A} e^{2y} \underbrace{-2}_{B} e^{y} + \underbrace{x}_{C} = 0}$

معادلة تربيعية نحلها بالقانون العام

$\therefore e^{y} = \frac{-(-2) \pm \sqrt{4-4(x)(x)}}{2x}$

$e^{y} = \frac{\cancel{2} \pm \cancel{2}\sqrt{1-x^2}}{\cancel{2}x}$

$e^{y} = \frac{1 \pm \sqrt{1-x^2}}{x}$ بأخذ ln للطرفين

$\therefore y = \ln\left(\frac{1 \pm \sqrt{1-x^2}}{x}\right)$

$\downarrow$

$\therefore \operatorname{sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

والسالب مرفوض لعدم وجود ln لعدد سالب كما سبق في $\cosh^{-1}$, $\sinh^{-1}$

6 Prove that $\operatorname{cosech}^{-1}(x) =$ (Proof)

الحل

let $y = \operatorname{cosech}^{-1}(x)$

$\therefore x = \operatorname{csch}(y) = \frac{1}{\sinh(y)}$

$\therefore x = \frac{2}{e^{y}-e^{-y}}$

$\therefore x e^{y} - x e^{-y} = 2$ ($* e^{y}$)

$\therefore x e^{2y} - x = 2e^{y}$

$\boxed{\therefore \underbrace{x}_{A} e^{2y} \underbrace{-2}_{B} e^{y} \underbrace{-x}_{C} = 0}$

$\therefore e^{y} = \frac{2 \pm \sqrt{4+4(x)(+x)}}{2x}$

$\therefore e^{y} = \frac{\cancel{2} \pm \cancel{2}\sqrt{1+x^2}}{\cancel{2}x}$

بأخذ ln للطرفين

$\therefore y = \ln\left(\frac{1 \pm \sqrt{1+x^2}}{x}\right)$

$\downarrow$

$\therefore \operatorname{csch}^{-1}(x) = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)$

والسالب مرفوض لعدم وجود ln لعدد سالب

#

Dfferentiation

إثباتات

* أوجد مشتقة $y = \sin(x)$

أو

Prove that $\frac{d}{dx}\sin(x) = \cos x$

الحل

$$\therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2\cos\left(x+\frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

معامل عام

$$= \lim_{\Delta x \to 0} \cos\left(x+\frac{\Delta x}{2}\right) \cdot \underbrace{\lim_{\Delta x \to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\left(\frac{\Delta x}{2}\right)}}_{=1}$$

$$= \cos(x+0) =$$

$$= \cos(x)$$

لاحظ

$$\sin(x+\Delta x) - \sin(x) = 2\cos\left(x+\frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)$$

الإثبات

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$$\sin(x+\Delta x) - \sin x =$$

$$= \sin x \cos\Delta x + \sin\Delta x \cos x - \sin x$$

$$= \sin x\,[\cos\Delta x - 1] + \cos x \sin\Delta x$$

$$= \cos x \sin\Delta x - 2 * \tfrac{1}{2}(1-\cos\Delta x)\cdot \sin x$$

$$= \cos x \left(2\sin\tfrac{\Delta x}{2}\cdot\cos\tfrac{\Delta x}{2}\right) - 2\sin^2\!\left(\tfrac{\Delta x}{2}\right)\cdot\sin(x)$$

عامل مشترك

$$= 2\sin\left(\tfrac{\Delta x}{2}\right)\left[\cos x\cdot\cos\tfrac{\Delta x}{2} - \sin x\cdot\sin\left(\tfrac{\Delta x}{2}\right)\right]$$

$$= 2\sin\left(\tfrac{\Delta x}{2}\right)\cdot\cos\left(x+\tfrac{\Delta x}{2}\right) \quad \#$$

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* Prove that $\frac{d}{dx}\cos x = -\sin(x)$

(الحل)

لاحظ أن

$$\cos(x) = \sin\left(\tfrac{\pi}{2} - x\right)$$

$$\therefore \frac{d}{dx}\cos(x) = \frac{d}{dx}\left[\sin\left(\tfrac{\pi}{2}-x\right)\right] = \cos\left(\tfrac{\pi}{2}-x\right)(-1)$$

$$= -\sin(x)$$

---

* $y = \tan(x) = \dfrac{\sin(x)}{\cos x} = \dfrac{\text{بسط}}{\text{مقام}}$

$$\therefore \frac{dy}{dx} = \frac{\cos x\cdot\cos x - \sin x\cdot(-\sin x)}{\cos^2 x}$$

$$\therefore \frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = \boxed{-\operatorname{cosec}^2 x}$$

$$y = \sec x = \frac{1}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cancel{(\cos x) \cdot 0} - (1) \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x \cdot \cos x} = \tan x \cdot \frac{1}{\cos x}$$

$$= \sec x \cdot \tan x$$

$$y = \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\frac{dy}{dx} = \frac{(\sin x \cdot)(0) - (1)\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x \cdot \sin x}$$

$$\because \frac{\cos x}{\sin x} = \cot x \quad , \quad \frac{1}{\sin x} = \operatorname{cosec} x$$

$$\boxed{\therefore \frac{dy}{dx} = -\operatorname{cosec} x \cdot \cot x}$$

▣ إثباتات تفاضلات الدوال الزائدية ▣

$\boxed{1}$ Prove that $\frac{d}{dx}\sinh(x) = \cosh x$

الحل

$\because \sinh(x) = \frac{e^{x} - e^{-x}}{2}$

بتفاضل الطرفين

$\therefore \frac{d}{dx}\sinh(x) = \frac{d}{dx}\left(\frac{1}{2}\left(e^{x} - e^{-x}\right)\right)$

$= \frac{1}{2}\left(e^{x} + e^{-x}\right) = \cosh x$

$\boxed{2}$ Prove that $\frac{d}{dx}\cosh x = \sinh x$

الحل

$\because \cosh(x) = \frac{e^{x} + e^{-x}}{2}$

بتفاضل الطرفين

$\therefore \frac{d}{dx}\cosh(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right) = \sinh(x)$

$\boxed{3}$ Prove that $\frac{d}{dx}\tanh(x) = \operatorname{sech}^2(x)$

الحل

$\because \tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

بتفاضل الطرفين

$\therefore \frac{d}{dx}\tanh(x) = \frac{[e^{x}+e^{-x}][e^{x}+e^{-x}] - [e^{x}-e^{-x}][e^{x}-e^{-x}]}{[e^{x}+e^{-x}]^2}$

$= \frac{[e^{x}+e^{-x}]^2 - [e^{x}-e^{-x}]^2}{[e^{x}+e^{-x}]^2}$

$= \frac{\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2 - \cancel{e^{-2x}}}{[e^{x}+e^{-x}]^2}$

$= \frac{4}{[e^{x}+e^{-x}]^2} = \left[\frac{2}{e^{x}+e^{-x}}\right]^2$

$= \left(\frac{1}{\cosh x}\right)^2 = \operatorname{sech}^2(x)$

$\boxed{4}$ Prove that $\frac{d}{dx}\coth(x) = -\operatorname{cosech}^2 x$

الحل

$\because \coth(x) = \frac{1}{\tanh(x)} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$

بتفاضل الطرفين

$\therefore \frac{d}{dx}(\coth(x)) = \frac{[e^{x}-e^{-x}][e^{x}-e^{-x}] - [e^{x}+e^{-x}][e^{x}+e^{-x}]}{[e^{x}-e^{-x}]^2}$

$= \frac{\cancel{e^{2x}} - 2 + \cancel{e^{-2x}} - \cancel{e^{2x}} - 2 - \cancel{e^{-2x}}}{[e^{x}-e^{-x}]^2}$

$= \frac{-4}{[e^{x}-e^{-x}]^2} = -\left[\frac{2}{e^{x}-e^{-x}}\right]^2$

$= -\left[\frac{1}{\sinh x}\right]^2 = -\operatorname{cosech}^2(x)$

بالمثل نستنتج أن تفاضل

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \cdot \tanh x$

و $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \cdot \coth x$.

مشتقات الدوال الزائدية العكسية

الإثبات مشابه تماماً إثبات مشتقات الدوال المثلثية العكسية

[1] $\boxed{y = \tanh^{-1} x} \longleftrightarrow \boxed{x = \tanh y}$

$$\frac{dx}{dy} = \operatorname{sech}^2 y$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\operatorname{sech}^2 y} = \frac{1}{1-\tanh^2 y}$$

$$\therefore \frac{d}{dx}(\tanh^{-1}(x)) = \frac{1}{1-x^2}$$

قاعدة عامة

$$\therefore \frac{d}{dx} \tanh^{-1}(u) = \frac{1}{1-u^2} \cdot u'$$

---

[2] $y = \sinh^{-1}(x) \longleftrightarrow x = \sinh y$

$$\therefore \frac{dx}{dy} = \cosh y$$

$$\frac{dy}{dx} = \frac{1}{\cosh y}$$

$$\therefore \cosh^2 y - \sinh^2 y = 1$$

$$\cosh y = \sqrt{1+\sinh^2 y} = \sqrt{1+x^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$$

$y = \cosh^{-1} x \iff x = \cosh y$

$$\frac{dx}{dy} = \sinh y$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh x = \sqrt{\cosh^2 x - 1}$$

$$= \sqrt{x^2 - 1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sinh y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 1}}$$

$\boxed{7}$ $y = \coth^{-1} x \iff x = \coth y$

$$\frac{dx}{dy} = -\operatorname{cosech}^2 y$$

$$\therefore 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\frac{dy}{dx} = \frac{-1}{\operatorname{cosech}^2 y}$$

$$= \frac{-1}{\coth^2 x - 1} = \frac{-1}{x^2 - 1}$$

$\boxed{5}$ $y = \operatorname{sech}^{-1} x \Leftrightarrow x = \operatorname{sech} y$

$$\frac{dx}{dy} = -\operatorname{sech} y \cdot \tanh y$$

$\because 1 - \tanh^2 y = \operatorname{sech}^2 y$

$\therefore \tanh y = \sqrt{1 - \operatorname{sech}^2 y}$

$= \sqrt{1 - x^2}$

$$\frac{dy}{dx} = \frac{-1}{\operatorname{sech} y \cdot \tanh y}$$

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{1 - x^2}}$$

$\boxed{6}$ $y = \operatorname{cosech}^{-1} x \Leftrightarrow x = \operatorname{cosech} y$

$$\frac{dx}{dy} = -\operatorname{cosech} y \cdot \coth y$$

$\because \coth^2 y - 1 = \operatorname{cosech}^2 y$

$\therefore \coth y = \sqrt{\operatorname{cosech}^2 y + 1}$

$= \sqrt{x^2 + 1}$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{-1}{\operatorname{cosech} y \cdot \coth y}$$

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2 + 1}}$$

$$\therefore e^{2y} = \frac{-(x+1)}{(x-1)} = \frac{1+x}{1-x}$$

بأخذ Ln للطرفين

$$\therefore 2y = \ln \frac{1+x}{1-x}$$

$$\boxed{\therefore y = \frac{1}{2} \ln \left(\frac{1+x}{1-x}\right) = \tanh^{-1} x}$$

$\boxed{4}$ $y = \coth^{-1} x \longleftrightarrow x = \coth y = \dfrac{e^{y}+e^{-y}}{e^{y}-e^{-y}}$

$$\therefore x e^{y} - x e^{-y} = e^{y} + e^{-y}$$

$$\therefore e^{y}(x-1) - e^{-y}(x+1) = 0$$

بالضرب x e^{y}

$$\therefore e^{2y}(x-1) - (x+1) = 0$$

$$\therefore e^{2y} = \frac{x+1}{x-1}$$

بأخذ ln للطرفين

$$\therefore 2y = \ln \left(\frac{x+1}{x-1}\right)$$

$$\boxed{\therefore y = \coth^{-1}(x) = \frac{1}{2} \cdot \ln \left(\frac{x+1}{x-1}\right)}$$

اكتب $\operatorname{sech}^{-1} x$ و $\operatorname{cosech}^{-1} x$ بدلالة الدالة (ln)

واجب بنفسك

<< مسائل على الإشتقاق (التفاضل) >>

* Find $\frac{dy}{dx}$ for

(1) $y = \underbrace{(x^3-5)}\,.\,\underbrace{\tan^3(x^2+\cos^3(2x))}$

حاصل ضرب دالتين

Solution:

$$\frac{dy}{dx} = (x^3-5)\cdot(3)\cdot\tan^2(x^2+\cos^3(2x))\cdot\sec^2(x^2+\cos^3(2x))$$
$$\cdot(2x+3\cos^2(2x)\cdot(-\sin(2x))\cdot 2)$$
$$+\tan^3(x^2+\cos^3(2x))\cdot(3x^2)$$

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(2) $x^2 - \sin(xy) - \tan^2 y^2 = 0$ دالة ضمنية : نفاضل الطرفين بالنسبة لـ x

(W.r.t. x)

$$\therefore 2x - \cos(x\cdot y)\cdot\left(x\frac{dy}{dx}+y\right) - 2\tan y^2\cdot\sec^2 y^2\cdot 2y\frac{dy}{dx} = 0$$

$$\therefore 2x - y\cos(xy) = x\cos(xy)\frac{dy}{dx} + 4y\tan(y^2)\sec^2 y^2\cdot\frac{dy}{dx}$$

$$2x - y\cos(xy) = \frac{dy}{dx}\left[x\cos xy + 4y\tan y^2\sec^2 y^2\right]$$

$$\therefore \frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos xy + 4y\tan y^2\sec^2 y^2} \quad \#$$

$\boxed{3}$ $y = \underbrace{e^{-x}} \cdot \underbrace{\cos 2x}$ find $y'' + 2y' + 5y$

Solution : $y' = -e^{-x} \cdot 2 \sin 2x - e^{-x} \cos 2x$

$$y' = -2e^{-x} \cdot \sin 2x - e^{-x} \cdot \cos 2x$$

$$y'' = -2e^{-x} \cdot 2\cos 2x + 2e^{-x} \cdot \sin 2x + 2e^{-x} \cos 2x + e^{-x} \cdot \cos 2x$$

$$y'' = -4e^{-x} \cos 2x + 2 \cdot e^{-x} \cdot \sin 2x + 3e^{-x} \cos 2x$$

$$\therefore y'' = 2e^{-x} \cdot \sin 2x - e^{-x} \cos 2x$$

$$\therefore y'' + 2y' + 5y = 2e^{-x} \sin 2x - e^{-x} \cos 2x - 4e^{-x} \sin 2x - 2e^{-x} \cos 2x + 5 \cdot e^{-x} \cdot \cos 2x$$

$$= -2e^{-x} \sin 2x + 2e^{-x} \cos 2x$$

#

$$* \; y = \frac{x^P}{x^m - a^m} \qquad a, P, m = \text{Consts.}$$

حاصل قسمة دالتين

$$\therefore \frac{dy}{dx} = \frac{(x^m - a^m) \cdot P x^{P-1} - x^P (m x^{m-1})}{(x^m - a^m)^2}$$

$$* \; y = \tan(ax+b)$$

$$\frac{dy}{dx} = \sec^2(ax+b) \cdot a$$

$$* \; y = \underbrace{\sin^3 t}\cdot \underbrace{\cos t}$$

$$\frac{dy}{dt} = \sin^3 t \cdot (-\sin t) + 3\sin^2 t \cdot (\cos t) \cdot (\cos t)$$

$$= -\sin^4 t + 3\sin^2 t \cdot \cos^2 t$$

$$* \; y = \sqrt[3]{x^2+x+1} = (x^2+x+1)^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3}(x^2+x+1)^{-\frac{2}{3}}(2x+1) = \frac{2x+1}{3\sqrt[3]{(x^2+x+1)^2}}$$

* $y = \cos(x+y)$ دالة ضمنية

بتفاضل الطرفين بالنسبة لـ (X)

$$\therefore \frac{dy}{dx} = -\sin(x+y)\cdot\left(1+\frac{dy}{dx}\right) = -\sin(x+y) - \frac{dy}{dx}(\sin(x+y))$$

$$\therefore \frac{dy}{dx} + \frac{dy}{dx}\sin(x+y) = -\sin(x+y)$$

$$\therefore \frac{dy}{dx}(1+\sin(x+y)) = -\sin(x+y)$$

$$\therefore \frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)} \quad \#$$

---

* $\cos(xy) = x$; بتفاضل الطرفية بالنسبة لـ(X)

$$\therefore -\sin(xy)\cdot\left[x\frac{dy}{dx}+y\right] = 1$$

$$\therefore -x\sin(xy)\cdot\frac{dy}{dx} - y\sin(xy) = 1$$

$$\therefore \frac{dy}{dx} = \frac{1+\cancel{y}\sin(xy)}{-x\sin(xy)} \quad \#$$

---

* $x^{1/2} + y^{1/2} = a^{1/2}$ بتفاضل الطرفين بالنسبة لـ(X) :

, $a = const$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{-\frac{y}{2}\, x^{-1/2}}{\frac{1}{2}\, y^{-1/2}} = \frac{y^{1/2}}{x^{1/2}} = \sqrt{\frac{y}{x}}$$

---

* if $x = a\cos t$, $y = b\sin t$

find $\frac{dy}{dx}$ Solution

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

Exp where $\frac{dy}{dt} = b\cos t$

$$\frac{dt}{dx} = \frac{1}{\left(\frac{dx}{dt}\right)} \rightarrow = \frac{1}{-a\sin t}$$

$$\therefore \frac{dy}{dt} = b\cos t \cdot \frac{-1}{a\sin t} = \frac{-b}{a}\,\frac{\cos t}{\sin t} = \frac{-b}{a}\cot(t)$$

---

* $x = a\cos^3 t$, $y = b\sin^3 t$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\cancel{3}b\sin^2 t \cdot \cos t}{-\cancel{3}a\cos^2 t \cdot \sin t}$$

$$= \frac{-b}{a}\left(\frac{\sin t}{\cos t}\right)^2 \cdot \frac{\cos t}{\sin t} = \frac{-b}{a}\,\frac{\sin t}{\cos t}$$

$$= \frac{-b}{a}\tan t$$

#

احسب ... ميل ... المماس ... المنحنى ... عند النقطة

* Calculate the Slop of the Tangent for the Curve : $X \cdot \cos y + y \cdot \sin x = \frac{\pi}{2}$ at Point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

أوجد ميل المماس للمنحنى $x\cos y + y\sin x = \frac{\pi}{2}$ عند النقطة $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

ميل المماس هو المشتقة الأولى
- نحسب أولاً $\frac{dy}{dx}$ له

<< Solution >>

$$x\cos y + y \sin x = \frac{\pi}{2}$$

نتفاضل الطرفين بالنسبة لـ x :

$$\therefore -x \cdot \sin y \cdot \frac{dy}{dx} + \cos y + y\cos x + \frac{dy}{dx}\sin x = 0$$

$$\frac{dy}{dx}\left[\sin x - x\sin y\right] = -\cos y - y\cos x$$

$$\therefore \frac{dy}{dx} = \frac{-\cos y - y\cos x}{\sin x - x\sin y} = \frac{\cos y + y\cos x}{x\sin y - \sin x}$$

$\therefore$ ميل المماس عند النقطة $\left(\overset{x}{\frac{\pi}{2}}, \overset{y}{\frac{\pi}{2}}\right)$

نعوض :

$$\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{2}\right) + \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)\left(\sin\left(\frac{\pi}{2}\right)\right) - \sin\left(\frac{\pi}{2}\right)}$$

$$\tan\theta = \frac{\cos(90^\circ) + \frac{3.14}{2}\cos(90^\circ)}{\frac{3.14}{2}(\sin 90^\circ) - \sin(90^\circ)} = \text{Zero}$$

إذن $\theta = 0 = \tan^{-1}(0)$

>> أى أن المنحنى يوازى محور X <<

$$y^2 \cdot \ln(\cosh^{-1} x^2) + e^{\frac{1}{2}\ln y} \cdot \tan\left[\sqrt{x \cdot \sqrt{x \cdot \sqrt{x}}}\right] = 0$$

find $\frac{dy}{dx}$: <Solution>

$$e^{\frac{1}{2}\ln y} = e^{\ln y^{1/2}} = y^{1/2} = \sqrt{y}$$

$$\sqrt{x \cdot \sqrt{x \cdot \sqrt{x}}} = \sqrt{x\sqrt{x \cdot x^{1/2}}} = \sqrt{x \cdot \sqrt{x^{3/2}}}$$

$$= \sqrt{x \cdot x^{3/4}} = \sqrt{x^{7/4}} = x^{7/8}$$

$$\Rightarrow y^2 \cdot \ln(\cosh^{-1} x^2) + \sqrt{y} \cdot \tan[x^{7/8}] = 0$$

بتفاضل الطرفين بالنسبة لـ x

$$\therefore 2y\frac{dy}{dx} \cdot \ln(\cosh^{-1} x^2) + y^2 \cdot \frac{1}{\cosh^{-1} x^2} \cdot \frac{-1}{\sqrt{1+x^4}} \cdot 2x$$

$$+ \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} \cdot \tan[x^{7/8}] + \sec^2[x^{7/8}] \cdot \frac{7}{8} x^{-1/8} \cdot \sqrt{y} = 0$$

$$\therefore \frac{dy}{dx}\left[2y\ln(\cosh^{-1} x^2) + \frac{\tan(x^{7/8})}{2\sqrt{y}}\right]$$

$$= \left[\frac{2xy}{\cosh^{-1} x^2 \cdot \sqrt{1+x^4}} - \frac{\frac{7}{8}\sqrt{y}\sec^2 x^{7/8}}{x^{1/8}}\right]$$

$$\therefore \frac{dy}{dx} = \left[\frac{2xy}{\cosh^{-1} x^2 \sqrt{1+x^4}} - \frac{\frac{7}{8}\sqrt{y}\sec^2 x^{7/8}}{x^{1/8}}\right] \Big/ \left[2y\ln(\cosh^{-1} x^2) + \frac{\tan x^{7/8}}{2\sqrt{y}}\right]$$

$$y = \sqrt{e^{\sinh^{-1}\sqrt{x}}} + 7^{x^2 \cdot \sin^2 x}$$ find y'

<Solution>

$$y' = \frac{1}{2 \cdot \sqrt{e^{\sinh^{-1}(\sqrt{x})}}} \cdot e^{\sinh^{-1}\sqrt{x}} \cdot \frac{1}{\sqrt{1+x}} \cdot \frac{1}{2\sqrt{x}}$$

$$+ 7^{x^2 \cdot \sin^2 x} \cdot \ln(7) \cdot [x^2 \cdot 2\sin x \cdot \cos x + 2x \sin^2 x]$$

$u = 2\ln[\cot(s)]$, $v = \tan(s) + \cot(s)$.

find $\frac{du}{dv}$. <Solution>

$$\frac{du}{dv} = \frac{du}{ds} \cdot \frac{ds}{dv}$$

where $$\frac{du}{ds} = 2 \cdot \frac{1}{\cot(s)} \cdot (-\csc^2 s)$$

$$\frac{dv}{ds} = \sec^2(s) \cancel{+} \csc^2(s)$$

$$\therefore \frac{ds}{dv} = \frac{1}{\sec^2(s) - \csc^2(s)}$$

$$\therefore \frac{du}{dv} = \left[\frac{-2\csc^2 s}{\cot(s)}\right] \cdot \left[\frac{1}{\sec^2(s) - \csc^2 s}\right]$$ #

$$* \quad y = \operatorname{Log}\big(\underbrace{10^{\sin x}}_{u} \cdot \underbrace{\tan(3x^2)}_{V}\big) + \sin\big(\cos(\underbrace{e^{\sqrt{x}}}_{W})\big)$$

Solution:-

let $y = \operatorname{Log}(U \cdot V) + \sin(\cos(W))$

Where $U = 10^{\sin x}$, $V = \tan(3x^2)$

$W = e^{\sqrt{x}}$

W', V', U' سنحتاج ؟!؟

$$U' = 10^{\sin x} \cdot \ln(10) \cdot \cos x$$

$$V' = \sec^2(3x^2) \cdot 6x = 6x \cdot \sec^2(3x^2)$$

$$W' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{U \cdot V} \cdot \operatorname{Log}(e) \cdot (U \cdot V' + V \cdot U')$$

$$+ \cos(\cos(e^{\sqrt{x}})) \cdot [-\sin(e^{\sqrt{x}}) \cdot W']$$

$$\therefore \frac{dy}{dx} = \frac{1 * \operatorname{Log}(e) \cdot [10^{\sin x} \cdot (6x \cdot \sec^2(3x^2)) + \tan(3x^2)}{(10^{\sin x} \cdot \tan(3x^2))}$$

$$\cdot (10^{\sin x} \cdot \ln(10) \cdot \cos x)]$$

$$+ \cos(\cos(e^{\sqrt{x}})) \cdot [-\sin(e^{\sqrt{x}}) \cdot (e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}})] \ \#$$

* $(\tan x)^{y} = (\tan y)^{x}$ find $\frac{dy}{dx} = y'$

الحل: بأخذ Ln للطرفين

$$\therefore\ y \cdot \ln(\tan x) = x \cdot \ln(\tan y)$$

بتفاضل الطرفين بالنسبة لـ (x):

$$\therefore\ y \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot y' = x \cdot \frac{1}{\tan y} \cdot \sec^2 y \cdot y' + \ln(\tan y) \cdot (1)$$

$$y' \ln(\tan x) - \frac{x \sec^2 y}{\tan y} \cdot y' = \ln(\tan y) - \frac{y \sec^2 x}{\tan x}$$

$$\therefore\ y'\left[\ln(\tan x) - \frac{x \sec^2(y)}{\tan y}\right] = \ln(\tan y) - \frac{y \sec^2 x}{\tan x}$$

$$\therefore\ y' = \frac{\ln(\tan y) - \frac{y \sec^2 x}{\tan x}}{\ln(\tan x) - \frac{x \sec^2(y)}{\tan y}}$$

#

$y = x^{(x^x)}$ find $\frac{dy}{dx}$

Solution:-

ناخذ Ln الطرفين :-

$\therefore Ln\, y = x^x \cdot Ln\, x$

Put $u = x^x$

$\therefore Ln\, y = u \cdot Ln\, x \Rightarrow$ بتفاضل الطرفين بالنسبة لـ (x)

$\therefore \frac{1}{y} \cdot y' = u \cdot \frac{1}{x} + Ln(x) \cdot u'$

$\therefore y' = y\left[\frac{u}{x} + Ln(x) \cdot u'\right]$

باقي لدينا ايجاد u' :

$\therefore u = x^x$ ناخذ ln الطرفين

$\therefore Ln\, u = x \cdot ln\, x \Rightarrow$ بتفاضل الطرفين بالنسبة لـ x

$\therefore \frac{1}{u} \cdot u' = x \cdot \frac{1}{x} + ln\, x$

$\therefore u' = u[1 + ln\, x] = x^x[1 + ln\, x]$

$\therefore \frac{dy}{dx} = y' = \underbrace{x^{x^x}}_{y}\left[\frac{x^x}{x} + ln\, x\left[x^x(1 + ln\, x)\right]\right]$

* Find $\frac{dy}{dx}$ for :-

[1] $$y = 7^{x^2 \cdot \operatorname{Cosec}^2 x} + x^2 \cdot \tan^3(3x^2 + \sin 2x)$$

<Solution>

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$$\frac{dy}{dx} = 7^{x^2 \cdot \operatorname{Cosec}^2 x} \cdot \ln(7) \cdot \left(x^2 \cdot 2\operatorname{Cosec} x \cdot (-\operatorname{Cosec} x \cdot \cot x) + 2x \operatorname{Cosec}^2 x\right) +$$

$$x^2 \cdot \left[3\tan^2(3x^2 + \sin 2x) \cdot \sec^2(3x^2 + \sin 2x) \cdot (6x + 2\cos 2x)\right] + 2x \cdot \tan^3(3x^2 + \sin 2x) .$$

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حل آخر للمثال السابق للتبسيط:

$$y = 7^{(u)} + \underbrace{x^2 \cdot \tan^3(V)}_{\text{حاصل ضرب دالتين}} = 7^u + x^2 \cdot (\tan(V))^3$$

where $u = x^2 \cdot \operatorname{Cosec}^2 x$ و $V = 3x^2 + \sin 2x$

$$\frac{dy}{dx} = 7^u \cdot \ln(7) \cdot u' + \underbrace{x^2}_{\text{الأولى}} \cdot \underbrace{3[\tan(V)]^2 \cdot [\sec^2(V) \cdot V']}_{\text{مشتقة الثانية}}$$

$$+ \underbrace{2x}_{\text{مشتقة الأولى}} \cdot \underbrace{[\tan(V)]^3}_{\text{والثانية}}$$

باقي لدينا إيجاد V' و U' ونعوضهم في المعادلة السابقة ⇐

$$u = \underbrace{x^2}\cdot\underbrace{\operatorname{Cosec}^2 x} = x^2\cdot(\operatorname{Cosec} x)^2$$

$$\therefore \frac{du}{dx} = u' = 2x\cdot\operatorname{cosec}^2 x + x^2\cdot\left[(2)\cdot(\operatorname{Cosec}x)^1\cdot(-\operatorname{Cosec}x\cdot\cot x)\right] \Longrightarrow (1)$$

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$$\therefore V = 3x^2 + \sin 2x$$

$$\therefore V' = 6x + 2\cos(2x) \Longrightarrow (2)$$

بالتعويض بقيم V', u' , u, V في $\frac{dy}{dx}$

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$$\therefore \frac{dy}{dx} = 7^{x^2\cdot\operatorname{Cosec}^2 x}\cdot\ln(7)\cdot\left[2x\operatorname{Cosec}^2 x + x^2\left(2\operatorname{Cosec}x\cdot(-\operatorname{Cosec}x\cdot\cot x)\right)\right]$$

$$+ x^2\cdot 3\left[\tan(3x^2+\sin 2x)\right]^2\cdot\left[\sec^2(3x^2+\sin 2x)\cdot(6x+2\cos(2x))\right] + 2x\tan^3(3x^2+\sin 2x).$$

$y = (\sin x)^x$ find the first derivative w.r.t. x $\equiv \frac{dy}{dx}$

(Solution)

بأخذ ln الطرفين

$\ln y = x \cdot \ln(\sin x) \Rightarrow$ بتفاضل الطرفين بالنسبة لـ x

$\therefore \frac{1}{y} \cdot y' = x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot (1)$

$\therefore y' = y\left[x \cdot \left(\frac{\cos x}{\sin x}\right) + \ln(\sin x)\right]$

$\therefore y' = (\sin x)^x \left[x \cdot \cot x + \ln(\sin x)\right]$

---

* $y = \cos x + \ln(x^2-1) + \underbrace{(\cos x)^x}_{u}$, then $\frac{dy}{dx}$ is ??

Solution

$\frac{dy}{dx} = -\sin x + \frac{1}{x^2-1} \cdot (2x) + u'$

لإيجاد u' :-

where $u = (\cos x)^x \Rightarrow$ بأخذ ln الطرفين

$\ln u = x \cdot \ln(\cos x) \Rightarrow$ «بتفاضل الطرفين بالنسبة لـ x»

$\therefore \frac{1}{u} \cdot u' = x \cdot \frac{1}{\cos x} \cdot (-\sin x) + \ln(\cos x) \cdot (1)$

$\therefore u' = [-x \cdot \tan x + \ln(\cos x)] \cdot (\cos x)^x$

$\therefore \frac{dy}{dx} = -\sin x + \frac{2x}{x^2-1} + (\cos x)^x [\ln(\cos x) - \cancel{x \tan} \; x \tan x]$

#

$$y = Ln\left[\sqrt{\frac{1+\sin x}{1-\sin x}}\right] \qquad \text{find } \frac{dy}{dx}$$

Solution

$$y = Ln\left[\frac{1+\sin x}{1-\sin x}\right]^{1/2} = \frac{1}{2} Ln\left[\frac{1+\sin x}{1-\sin x}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\left[\frac{1+\sin x}{1-\sin x}\right]} \cdot \left[\frac{(1-\sin x)\cdot(\cos x) - (1+\sin x)(-\cos x)}{(1-\sin x)^2}\right]$$

$$= \frac{\cancel{(1-\sin x)}}{2(1+\sin x)} \cdot \frac{\cos x - \cancel{\sin x \cos x} + \cos x + \cancel{\sin x \cos x}}{(1-\sin x)^{\cancel{2}}}$$

$$= \frac{\cancel{2}\cos x}{\cancel{2}(1+\sin x)(1-\sin x)} = \frac{\cos x}{1-\sin^2 x}$$

$$= \frac{\cancel{\cos x}}{\cos^2 x} = \frac{1}{\cos x} = \boxed{\sec x}$$

$$* \quad y = e^{2\cdot Ln x} + 3^x + (\cos x)^{\sin x}$$

$$y = \cancel{e}^{Ln x^2} + 3^x + u$$

$$y = x^2 + 3^x + u$$

$$\therefore \frac{dy}{dx} = 2x + 3^x \ln(3) + u'$$

نوجد الـ u' كالتالي

where $u = (\cos x)^{\sin x}$

$\therefore \ln(u) = \sin x \cdot \ln(\cos x)$ → باخذ ما لطرفين

$\therefore \dfrac{1}{u} \cdot u' = \left(\sin x \cdot \dfrac{1}{\cos x} \cdot (-\sin x) + \ln(\cos x) \cdot \cos x\right)$ → بتفاضل الطرفين w.r.t. (x)

$\therefore u' = u\left[-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)\right]$

$u' = (\cos x)^{\sin x} \left[-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)\right]$

$\therefore \dfrac{dy}{dx} = 2x + 3^x \ln(3) + (\cos x)^{\sin x} \left[-\sin x \cdot \tan x + \cos x \cdot \ln(\cos x)\right]$ #

$* \; y = \underbrace{x^{-e}}_{} \cdot \underbrace{\tan^2(x^2)}_{}$ (رمز ثابت)

$\therefore \dfrac{dy}{dx} = x^{-e} \cdot 2\tan(x^2) \cdot \sec^2(x^2) \cdot 2x + (-e)\, x^{-e-1} \cdot \tan^2(x^2)$

$\dfrac{dy}{dx} = \dfrac{4 \tan x^2 \sec^2(x^2)}{x^e} - \dfrac{\tan^2(x^2) \cdot e}{x^{e+1}}$ #

$$y = e^{-x} \cdot \cos(2x) \qquad \text{Find } y'' + 2y' + 5y$$

Solution

$$y' = e^{-x}(-2\sin 2x) + \cos(2x) \cdot (-e^{-x})$$

$$y' = -2 \cdot e^{-x} \cdot \sin 2x - e^{-x} \cdot \cos(2x).$$

$$y'' = -2e^{-x} \cdot 2\cos 2x + 2e^{-x} \cdot \sin 2x + 2e^{-x} \sin 2x + e^{-x} \cos(2x)$$

$$\therefore y'' + 2y' + 5y = -4 e^{-x} \cdot \cos 2x + 2e^{-x} \sin 2x + 2e^{-x} \sin 2x + e^{-x}\cos 2x - 4e^{-x} \sin 2x - 2 \cdot e^{-x} \cdot \cos 2x + 5e^{-x} \cos 2x$$

$= \text{Zero}$

---

* **Report**: Find $\frac{dy}{dx}$ for

$$y = \tan^2\left[\sec\left(\operatorname{sech}^{-1}\sqrt{1+x^2}\right)\right]$$

$$\underbrace{x^{(\sin y)}}_{u} + \underbrace{y^{\cos(x)}}_{v} = 0$$

$$u = x^{\sin(y)} \Rightarrow u' = x^{\sin(y)}\left[\frac{\sin y}{x} + \ln(x)\cdot\cos(y)\, y'\right]$$

$$v = y^{\cos(x)} \Rightarrow v' = y^{\cos(x)}\left[\frac{\cos x}{y}\, y' + \ln(y)\cdot(-\sin(x))\right]$$

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$$u' + v' = 0$$

$$x^{\sin(y)}\left[\frac{\sin y}{x} + \ln(x)\cos(y)\, y'\right] + y^{\cos(x)}\left[\frac{\cos x}{y}\, y' - \ln(y)\cdot\sin(x)\right] = 0$$

عشان نوحد y' ع طرف لوحده

$$y' = \frac{-x^{\sin y}\,\frac{\sin y}{x} + y^{\cos(x)}\ln(y)\sin(x)}{x^{\sin y}\ln(x)\cos(y) + y^{\cos x}\,\frac{\cos x}{y}} \quad \#$$

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{جدول التفاضلات}

الدالة ←	المشتقة	الدالة ←	المشتقة
$\sin(u)$	$\cos(u) . u'$	$\sin^{-1}(u)$	$\frac{u'}{\sqrt{1-u^2}}$
$\cos(u)$	$-\sin(u) . u'$	$\cos^{-1}(u)$	$\frac{-u'}{\sqrt{1-u^2}}$
$\sec(u)$	$\sec(u). \tan(u) . u'$	$\sec^{-1}(u)$	$\frac{u'}{u\sqrt{u^2-1}}$
$\mathrm{cosec}(u)$	$-\mathrm{cosec}(u). \cot(u) . u'$	$\mathrm{cosec}^{-1}(u)$	$\frac{-u'}{u\sqrt{u^2-1}}$
$\tan(u)$	$\sec^2(u) . u'$	$\tan^{-1}(u)$	$\frac{u'}{1+u^2}$
$\cot(u)$	$-\mathrm{cosec}^2(u) . u'$	$\cot^{-1}(u)$	$\frac{-u'}{1+u^2}$
$\sinh(u)$	$\cosh(u) . u'$	$\sinh^{-1}(u)$	$\frac{u'}{\sqrt{u^2+1}}$
$\cosh(u)$	$\sinh(u) . u'$	$\cosh^{-1}(u)$	$\frac{u'}{\sqrt{u^2-1}}$
$\mathrm{sech}(u)$	$-\mathrm{sech}(u). \tanh(u) . u'$	$\mathrm{Sech}^{-1}(u)$	$\frac{-u'}{u\sqrt{1-u^2}}$
$\mathrm{cosech}(u)$	$-\mathrm{cosech}(u). \coth(u) . u'$	$\mathrm{cosech}^{-1}(u)$	$\frac{-u'}{u\sqrt{1+u^2}}$
$\tanh(u)$	$\mathrm{sech}^2(u) . u'$	$\tanh^{-1}(u)$	$\frac{u'}{1-u^2}$
$\coth(u)$	$-\mathrm{cosech}^2(u) . u'$	$\coth^{-1}(u)$	$\frac{-u'}{1-u^2}$
a^u	$a^u . \ln(a) . u'$	e^u	$e^u . u'$
$\log_p(u)$	$\frac{1}{u \cdot \ln p} . u' = \frac{1}{u} \cdot \log_p e \cdot u'$	$\ln(u)$	$\frac{1}{u} . u'$
$[f]^n$	$n . [f]^{n-1} . f'$	$[f]^g$	$[f]^g \cdot [\frac{g}{f} \cdot f' + g' \cdot \ln f]$