

$$E_1 = \vec{OA} + \vec{Ap}$$

$$E_2 = \vec{OB} + \vec{Bc} + \vec{Cp}$$

$$E_{x_1} = l_a \cos \theta_1 + l_b \cos \theta_2$$

$$E_{y_1} = l_a \sin \theta_1 + l_b \sin \theta_2$$

$$E_{x_2} = l_c + l_a \cos \theta_4 + l_b \cos \theta_3$$

$$E_{y_2} = l_a \sin \theta_4 + l_b \sin \theta_3$$

$$E_{x_1} = E_{x_2} \longrightarrow l_a \cos \theta_1 + l_b \cos \theta_2 = l_c + l_a \cos \theta_4 + l_b \cos \theta_3$$

$$l_b \cos \theta_2 = l_c + l_a \cos \theta_4 + l_b \cos \theta_3 - l_a \cos \theta_1 \longrightarrow \boxed{1}$$

$$E_{y_1} = E_{y_2} \longrightarrow l_a \sin \theta_1 + l_b \sin \theta_2 = l_a \sin \theta_4 + l_b \sin \theta_3$$

$$l_b \sin \theta_2 = l_a \sin \theta_4 + l_b \sin \theta_3 - l_a \sin \theta_1 \longrightarrow \boxed{2}$$

For forward kinematics

$$E_x = l_c + l_a \cos \theta_4 + l_b \cos \left[ 2 \tan^{-1} \left[ \frac{-F \pm \sqrt{E^2 + F^2 - G^2}}{G - E} \right] \right]$$

$$E_y = l_a \sin \theta_4 + l_b \sin \left[ 2 \tan^{-1} \left[ \frac{-F \pm \sqrt{E^2 + F^2 - G^2}}{G - E} \right] \right]$$

$$\text{where: } E = 2l_b (l_c + l_a (\cos \theta_4 - \cos \theta_1))$$

$$F = 2l_a l_b (\sin \theta_4 - \sin \theta_1)$$

$$G = l_c^2 + 2l_a^2 + 2l_c l_a \cos \theta_4 - 2l_c l_a \cos \theta_1 - 2l_a^2 \cos (\theta_4 - \theta_1)$$

For Inverse kinematics

$$\theta_1 = 2 \tan^{-1} \left[ \frac{-F_1 \pm \sqrt{E_1^2 + F_1^2 - G_1^2}}{G_1 - E_1} \right]$$

$$\theta_2 = 2 \tan^{-1} \left[ \frac{-F_4 \pm \sqrt{E_4^2 + F_4^2 - G_4^2}}{G_4 - E_4} \right]$$

$$\text{where: } E_1 = -2l_a E_x \quad E_4 = 2l_a (-E_x + l_c)$$

$$F_1 = -2l_a E_y$$

$$F_4 = -2l_a E_y$$

$$G_1 = l_a^2 + E_x^2 + E_y^2 - l_b^2$$

$$G_4 = l_c^2 + l_a^2 - l_b^2 + E_x^2 + E_y^2 - 2l_c E$$