

Quine-McCluskey Logic Minimization Project

10 Test Cases

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Example 1: Normal Expression

Please enter your expression (SoP form): $a'bc + ab' + bc'$

Truth Table

a	b	c	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

The Canonical Sum of Products (SoP) is: $(a'bc') + (a'bc) + (ab'c') + (ab'c) + (abc')$

The Canonical Product of Sums (PoS) is: $(a+b+c) * (a+b+c') * (a'+b'+c')$

Coverage Table

Z	2	3	4	5	6
01-	x	x			
-10	x				x
10-			x	x	
1-0			x		x

All Prime Implicants

$a'b : 01- : (2,3)$
 $bc' : -10 : (2,6)$
 $ab' : 10- : (4,5)$
 $ac' : 1-0 : (4,6)$

All Essential Prime Implicants:

$a'b : 01-$
 $ab' : 10-$

The minterms that are not covered by EPIs: (6)

Minimized Expression: $a'b + ab' + ac'$

Example 2: Minterms only (taken from slides)

Minimize: $F(A, B, C, D) = \sum m(0, 2, 5, 6, 7, 8, 10, 12, 13, 14, 15)$

Please enter the minterms: 0 2 5 6 7 8 10 12 13 14 15

Truth Table

a	b	c	d	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

The Canonical Product of Sums (PoS) is: $(a+b+c+d') * (a+b+c'+d') * (a+b'+c+d) * (a'+b+c+d') * (a'+b+c'+d')$

The Canonical Sum of Products (SoP) is: $(a'b'c'd') + (a'b'cd') + (a'bc'd) + (a'bcd') + (a'bcd) + (ab'c'd') + (ab'cd') + (abc'd') + (abc'd) + (abcd') + (abcd)$

Coverage Table

z	0	2	5	6	7	8	10	12	13	14	15
-0-0	x	x				x	x				
--10		x		x			x			x	
1--0						x	x	x		x	
-1-1			x		x				x		x
-11-				x	x					x	x
11--								x	x	x	x

All Prime Implicants

$b'd'$: -0-0 : (0,2,8,10)
 cd' : --10 : (2,6,10,14)
 ad' : 1--0 : (8,10,12,14)
 bd : -1-1 : (5,7,13,15)
 bc : -11- : (6,7,14,15)
 ab : 11-- : (12,13,14,15)

All Essential Prime Implicants:

$b'd'$: -0-0
 bd : -1-1

The minterms that are not covered by EPIs : (6, 12, 14)

Minimized Expression: $b'd' + bd + ab + bc$

Example 3: Minterms and don't cares (taken from slides)

$$f(A, B, C, D) = \sum m(4, 5, 6, 8, 9, 10, 13) + \sum d(0, 7, 15)$$

Please enter the minterms: 4 5 6 8 9 10 13

Please enter the dont care terms: 0 7 15

Truth Table

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

The Canonical Sum of Products (SoP) is: $(a'bc'd') + (a'bc'd) + (a'bcd') + (ab'c'd') + (ab'c'd) + (ab'cd') + (abc'd)$

The Canonical Product of Sums (PoS) is: $(a+b+c+d) * (a+b+c+d') * (a+b+c'+d) * (a+b+c'+d') * (a+b'+c'+d') * (a'+b+c'+d') * (a'+b'+c'+d')$

Coverage Table

Z	4	5	6	8	9	10	13
0-00	x						
-000				x			
100-				x	x		
10-0				x		x	
1-01					x		x
01--	x	x	x				
-1-1		x					x

All Prime Implicants

$a'c'd'$: 0-00 : (0,4)
 $b'c'd'$: -000 : (0,8)
 $ab'c'$: 100- : (8,9)
 $ab'd'$: 10-0 : (8,10)
 $ac'd$: 1-01 : (9,13)
 $a'b$: 01-- : (4,5,6,7)
 bd : -1-1 : (5,13,7,15)

All Essential Prime Implicants:

$a'b$: 01--
 $ab'd'$: 10-0

The minterms that are not covered by EPIs : (9, 13)

Minimized Expression: $a'b + ab'd' + ac'd$

Example 4: Minterms and don't cares (taken from H.W 2)

$$f(A, B, C, D) = \sum m(2, 3, 7, 9, 11, 13) + \sum d(1, 10, 15)$$

Please enter the minterms: 2 3 7 9 11 13

Please enter the dont care terms: 1 10 15

Truth Table

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

The Canonical Sum of Products (SoP) is: $(a'b'cd') + (a'b'cd) + (a'bcd) + (ab'c'd) + (ab'cd) + (abc'd)$

The Canonical Product of Sums (PoS) is: $(a+b+c+d) * (a+b+c+d') * (a+b'+c+d) * (a+b'+c+d') * (a+b'+c'+d) * (a'+b+c+d) * (a'+b+c'+d) * (a'+b'+c+d) * (a'+b'+c'+d')$

Coverage Table

Z	2	3	7	9	11	13
-01-	x	x			x	
-0-1		x		x	x	
--11		x	x		x	
1--1				x	x	x

All Prime Implicants

$b'c : -01- : (2, 3, 10, 11)$
 $b'd : -0-1 : (1, 3, 9, 11)$
 $cd : --11 : (3, 7, 11, 15)$
 $ad : 1--1 : (9, 11, 13, 15)$

All Essential Prime Implicants:

$cd : --11$
 $b'c : -01-$
 $ad : 1--1$

The minterms that are not covered by EPIs :

Minimized Expression: $cd + b'c + ad$

Example 5: Minterms only (taken from H.W 2)

$$f(A, B, C, D, E) = \sum m(0, 2, 3, 5, 7, 9, 11, 13, 14, 16, 18, 24, 26, 28, 30)$$

Please enter the minterms: 0 2 3 5 7 9 11 13 14 16 18 24 26 28 30

Truth Table

a	b	c	d	e	F
0	0	0	0	0	1
0	0	0	0	1	0
0	0	0	1	0	1
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	1	1
0	0	1	1	0	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	0	1	1
0	1	0	1	0	0
0	1	0	1	1	1
0	1	1	0	0	0
0	1	1	0	1	1
0	1	1	1	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	0	0	1	0
1	0	0	1	0	1
1	0	0	1	1	0
1	0	1	0	0	0
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	0	1	0
1	1	0	1	0	1
1	1	0	1	1	0
1	1	1	0	0	1
1	1	1	0	1	0
1	1	1	1	0	1
1	1	1	1	1	0

The Canonical Sum of Products (SoP) is: $(a'b'c'd'e') + (a'b'c'de) + (a'b'c'd'e) + (a'b'cd'e) + (a'b'cd'e) + (a'bc'd'e) + (a'bcd'e) + (a'bcde') + (ab'c'd'e') + (ab'c'de') + (abc'd'e') + (abc'de') + (abcde')$

The Canonical Product of Sums (PoS) is: $(a+b+c+d+e') * (a+b+c'+d+e) * (a+b+c'+d'+e) * (a+b'+c+d+e) * (a+b'+c'+d+e) * (a+b'+c'+d'+e) * (a'+b+c+d+e) * (a'+b+c'+d+e) * (a'+b+c'+d'+e) * (a'+b'+c+d+e) * (a'+b'+c'+d+e) * (a'+b'+c'+d'+e)$

All Prime Implicants

$a'b'c'd : 0001- : (2,3)$
 $a'b'de : 00-11 : (3,7)$
 $a'c'de : 0-011 : (3,11)$
 $a'b'ce : 001-1 : (5,7)$
 $a'cd'e : 0-101 : (5,13)$
 $a'bc'e : 010-1 : (9,11)$
 $a'bd'e : 01-01 : (9,13)$
 $bcde' : -1110 : (14,30)$
 $b'c'e' : -00-0 : (0,2,16,18)$
 $ac'e' : 1-0-0 : (16,18,24,26)$
 $abe' : 11--0 : (24,26,28,30)$

All Essential Prime Implicants:

$b'c'e' : -00-0$
 $bcde' : -1110$
 $abe' : 11--0$

Coverage Table															
Z	0	2	3	5	7	9	11	13	14	16	18	24	26	28	30
0001-		x	x												
00-11			x		x										
0-011			x				x								
001-1				x	x										
0-101				x				x							
010-1						x	x								
01-01						x		x							
-1110									x						x
-00-0	x	x								x	x				
1-0-0										x	x	x	x		
11--0												x	x	x	x

The minterms that are not covered by EPIs : (3, 5, 7, 9, 11, 13)

Minimized Expression: $b'c'e' + bcde' + abe' + a'c'de + a'bd'e + a'b'ce$

Example 6: Minterms and don't cares (taken from H.W 2)

$$f(A, B, C, D) = \sum m(1, 3, 4, 5, 6, 7, 10, 12, 13) + d(2, 9, 15)$$

Please enter the minterms: 1 3 4 5 6 7 10 12 13
Please enter the dont care terms: 2 9 15

Truth Table

a	b	c	d	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

The Canonical Sum of Products (SoP) is: $(a'b'c'd) + (a'b'cd) + (a'bc'd') + (a'bc'd) + (a'bcd') + (a'bcd) + (ab'cd') + (ab'cd) + (abc'd') + (abc'd)$

The Canonical Product of Sums (PoS) is: $(a+b+c+d) * (a+b+c'+d) * (a'+b+c+d') * (a'+b+c'+d') * (a'+b'+c'+d')$

Coverage Table

Z	1	3	4	5	6	7	10	12	13
-010							x		
0--1	x	x		x		x			
--01	x			x					x
01--			x	x	x	x			
-10-			x	x				x	x
0-1-		x			x	x			
-1-1				x		x			x

All Prime Implicants

$b'cd'$: -010 : (2,10)
 $a'd$: 0--1 : (1,3,5,7)
 $c'd$: --01 : (1,5,9,13)
 $a'b$: 01-- : (4,5,6,7)
 bc' : -10- : (4,5,12,13)
 $a'c$: 0-1- : (2,3,6,7)
 bd : -1-1 : (5,7,13,15)

All Essential Prime Implicants:

$b'cd'$: -010
 bc' : -10-

The minterms that are not covered by EPIs : (1, 3, 6, 7)

Minimized Expression: $b'cd' + bc' + a'c + a'd$

Example 7: Normal Expression (taken from H.W 2)

$$f(A, B, C) = AB + A'C + AB'C$$

Please enter your expression (SoP form): $ab + a'c + ab'c$

Truth Table

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

The Canonical Sum of Products (SoP) is: $(a'b'c) + (a'bc) + (ab'c) + (abc)$

The Canonical Product of Sums (PoS) is: $(a+b+c) * (a+b'+c) * (a'+b+c)$

Coverage Table

Z	1	3	5	6	7
11-				x	x
--1	x	x	x		x

All Prime Implicants

$ab : 11- : (6,7)$
 $c : --1 : (1,3,5,7)$

All Essential Prime Implicants:

$c : --1$
 $ab : 11-$

The minterms that are not covered by EPIs:

Minimized Expression: $c + ab$

Example 8: Normal Expression (taken from H.W 2)

$$f(A, B, C) = A'B + c$$

Please enter your expression (SoP form): $a'b + c$

Truth Table

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

The Canonical Sum of Products (SoP) is: $(a'b'c) + (a'bc') + (a'bc) + (ab'c) + (abc)$

The Canonical Sum of Products (SoP) is: $(a'b'c) + (a'bc') + (a'bc) + (ab'c) + (abc)$

Coverage Table

Z	1	2	3	5	7
01-		x	x		
--1	x		x	x	x

All Prime Implicants

$a'b : 01- : (2, 3)$
 $c : --1 : (1, 3, 5, 7)$

All Essential Prime Implicants:

$c : --1$
 $a'b : 01-$

The minterms that are not covered by EPIs:

Minimized Expression: $c + a'b$

Example 9: Normal Expression (always evaluates to true)

Please enter your expression (SoP form): $a + a'$

Truth Table

a	F
0	1
1	1

The Canonical Sum of Products (SoP) is: $(a') + (a)$

The Canonical Product of Sums (PoS) is: 1

Coverage Table

Z	0	1
-	x	x

All Prime Implicants

: - : (0,1)

All Essential Prime Implicants:

: -

The minterms that are not covered by EPIs:

Minimized Expression: 1

Example 10: Normal Expression (always evaluates as false)

Please enter your expression (SoP form): aa'

Truth Table

a	F
0	0
1	0

The Canonical Sum of Products (SoP) is: 0

The Canonical Product of Sums (PoS) is: $(a) * (a')$

Coverage Table

Z

All Prime Implicants

All Essential Prime Implicants:

The minterms that are not covered by EPIs:

Minimized Expression: 0